Full complex Fresnel holograms displayed on liquid crystal devices

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Abstract

We propose a method to display full complex Fresnel holograms by adding the information displayed on two analogue ferroelectric liquid crystal spatial light modulators. One of them works in real-only configuration and the other in imaginary-only mode. The Fresnel holograms are computed by backpropagating an object at a selected distance with the Fresnel transform. Then, displaying the real and imaginary parts on each panel, the object is reconstructed at that distance from the modulators by simple propagation of light. We present simulation results taking into account the specifications of the modulators as well as optical results. We have also studied the quality of reconstructions using only real, imaginary, amplitude or phase information. Although the real and imaginary reconstructions look acceptable for certain distances, full complex reconstruction is always better and is required when arbitrary distances are used.

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1. Introduction

Q.1

Many optical applications require an accurate display of full complex information, which in an ideal case would involve the use of devices able to simultaneously modulate amplitude and phase over a wide range of values. Last years, liquid crystal devices (LCD) have become the most frequently used spatial light modulators (SLM) in the field of optical image processing. Even though their modulation capabilities are limited, different strategies have been developed in order to modulate the full complex plane by using these devices.

A first option is to use codifying methods with a single panel, which implies a loss of resolution. There are several methods for codifying complex information into only positive values to be represented in a transmissive support, as reported in [1]. More recent studies present methods to codify the complex function into a single modulator; Serati and Bauchert [2] propose a codifying method using Fourier transform properties with a single panel, Birch *et al* [3] use a method that consists of a macropixel where the complex information is codified and Stolz *et al* [4] have studied the properties of the coupled phase and amplitude domains in the

implementation of diffractive optical elements (DOEs) onto a single twisted-nematic liquid crystal (TNLC) display.

A second approach is to use at least two panels, which entails the design of a more complex optical system. These systems are based on coupling two devices; Juday and Florence [5] and Gregory *et al* [6] proposed the two common architectures to do this, one to combine amplitude and phase as a product and another as an addition of the two parts. Amako *et al* [7] and Gonçalves *et al* [8] used a 4f configuration to couple two devices working in amplitude-mostly and phase-mostly regimes to obtain the product architecture. Tudela *et al* [9] used a simplified coupling optical architecture. Nowadays there are devices able to work in real-only and imaginary-only configurations; therefore it is possible to represent the complex unity plane by using two of these panels in an optical system in which the real and imaginary parts are added.

Q.2

In this work we present an implementation of this additive method to represent Fresnel holograms by using two reflective analogue ferroelectric liquid crystal (AFLC) SLMs from Boulder Nonlinear Systems (BNS) [10, 11]. The idea is to compute a digital hologram in order to recover an object at a certain distance from the plane where the hologram

will be displayed. In section 2 we explain the modulation capabilities of the AFLC SLMs used in this work and in section 3 we detail the method of displaying Fresnel holograms separating the backpropagated complex information into its real and imaginary parts. This procedure is illustrated in section 4 where some simulated reconstructions are presented and a study of the information distribution of the propagated complex information is performed. In section 5 we present the experimental optical set-up and some optical results.

2. Modulation capabilities of analogue ferroelectric liquid crystal spatial light modulators

In this work we use two reflective 8-bit AFLC SLMs with 128×128 pixels [11]. The grey-level (gl) value displayed on each pixel determines the applied electric field across it. On each pixel, the modulator acts as a pure rotative device with a switchable orientation of its optic axis depending of the applied electric field, between $\pm\pi/8$ (for gl between 0 and 255). The Jones matrix for the modulator having its optic axis at an angle θ to the y direction can be expressed (in the x - y coordinates, see figure 1) as:

$$\mathbf{W}(\theta) = \mathbf{R}(-\theta)\mathbf{L}_2(0)\mathbf{R}(\theta) = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \quad (1)$$

where constant phases have been omitted. $\mathbf{L}_2(0)$ is the Jones matrix for a half-wave retarder with its slow axis in the y direction (which is taken as the bisection of the SLM's possible optic axes), and $\mathbf{R}(\theta)$ is the matrix for a coordinate rotation by an angle θ :

$$\mathbf{L}_2(0) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \mathbf{R}(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (2)$$

If one modulator (SLM1) is placed after a polarizing beam-splitter cube (PBSC), as indicated in figure 1(a), the incident light is linearly polarized along the x -axis as only the p -component is transmitted. Then, light is modulated by the AFLC panel and is reflected back, and only the s -component exits the beam-splitter. This is equivalent to placing a polarizer in the y direction. The Jones matrix for the whole optical system is

$$\mathbf{J}_1(\theta_1) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{W}(\theta_1) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \sin 2\theta_1 \end{pmatrix}. \quad (3)$$

This gives real-only modulation when the applied electric field rotates the optic axis between $-\pi/8 \leq \theta_1 \leq \pi/8$, depending on the gl value of each particular pixel. This is shown in figure 1(b). The cross ($\times$) indicates the SLM modulation when no electric field is applied ($\theta_1 = 0$ or $\text{gl} = 128$). The lower and upper limits are indicated with a circle ($\circ$), for $\theta_1 = -\pi/8$ ($\text{gl} = 0$), and with a square ($\square$), for $\theta_1 = \pi/8$ ($\text{gl} = 255$).

If a second modulator (SLM2) is placed on the other arm of the PBSC, the incident light is now linearly polarized along the y -axis and only the p -component (x direction) exits the beam-splitter, as indicated in figure 2(a). It can be easily shown that the Jones matrix in this case becomes

$$\mathbf{J}_2(\theta_2) = \begin{pmatrix} \sin 2\theta_2 \\ 0 \end{pmatrix} \quad (4)$$

Figure 1. (a) Spatial light modulator 1 (SLM1) configuration and (b) operating curve.

Figure 2. (a) Spatial light modulator 2 (SLM2) configuration and (b) operating curves for full complex modulation.

which again gives real-only modulation. If the information from both panels is considered at the exit of the PBSC, the resulting Jones matrix has two real-only components ($\sin 2\theta_1$ and $\sin 2\theta_2$), where in the general case different gl values are displayed on each modulator, giving different orientation angles θ_1 and θ_2 . In order to have both real-only and imaginary-only configurations, a quarter-wave plate (L_4) with its slow axis parallel to the y direction is placed at the exit of the beam-splitter, as it introduces a phase delay of $\pi/2$ between the two perpendicular light polarization components:

$$\mathbf{L}_4(0)[\mathbf{J}_1(\theta_1) + \mathbf{J}_2(\theta_2)] = \begin{pmatrix} \sin 2\theta_2 \\ -i \sin 2\theta_1 \end{pmatrix} \quad \text{with} \quad \mathbf{L}_4(0) = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}. \quad (5)$$

Finally, both components are added by means of a linear polarizer with its transmission axis at $\pi/4$ to the x -axis, and full complex modulation is achieved:

$$\begin{aligned} \mathbf{P}(\pi/4)\mathbf{L}_4(0)[\mathbf{J}_1(\theta_1) + \mathbf{J}_2(\theta_2)] \\ = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \sin 2\theta_2 \\ -i \sin 2\theta_1 \end{pmatrix} = \frac{1}{2}(\sin 2\theta_2 - i \sin 2\theta_1) \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \end{aligned} \quad (6)$$

This corresponds (except for the constant factor $\frac{1}{2}$) to adding the two distributions shown in figure 2(b). When displaying all the available gls on both panels, this set-up ensures that the whole complex plane is covered. However, conventional nematic liquid crystals are not designed for real-only modulation, and thus cannot be used in this adding method.

3. Fresnel holograms

The system is used to display Fresnel holograms. The idea is to compute a digital hologram in order to recover an object at a certain distance from the plane where it will be displayed. The hologram is obtained by backpropagating the information from the object, $f(x, y)$, using the inverse Fresnel transform to a distance $-d$:

$$\begin{aligned} U(x, y, -d) &= \text{Fresnel}[f(x, y), -d] \\ &= \frac{e^{-i\frac{2\pi d}{\lambda}}}{i\lambda d} \int_{-\infty}^{\infty} f(x', y') \exp\left[-\frac{i\pi}{\lambda d}((x - x')^2 \right. \\ &\quad \left. + (y - y')^2)\right] dx' dy' \end{aligned} \quad (7)$$

where (x, y) and (x', y') are the transverse coordinates in the planes $z = -d$ and 0 respectively.

The integral part of the Fresnel transform in equation (7) can be regarded as a convolution of two functions, the function of the object to be backpropagated, $f(x, y)$, and the exponential part:

$$g(x, y) = \frac{e^{-i\frac{2\pi d}{\lambda}}}{i\lambda d} \exp\left[-\frac{i\pi}{\lambda d}((x - x')^2 + (y - y')^2)\right]. \quad (8)$$

By applying the Fourier transform ($\mathcal{FT}$) to equation (7) we can change the convolution of the two functions into the product of their Fourier transforms:

$$\begin{aligned} \mathcal{FT}[f(x, y) * g(x, y)] &= \mathcal{FT}[f(x, y)]\mathcal{FT}[g(x, y)] \\ &= F(u, v)G(u, v). \end{aligned} \quad (9)$$

The Fourier transform of the exponential part, $G(u, v)$, has a mathematical known solution, that for a distance $-d$ is

$$G(u, v) = \exp\left[\frac{-i2\pi d}{\lambda}\right] \exp[-i\pi\lambda d(u^2 + v^2)]. \quad (10)$$

The Fourier transform of the object, $F(u, v)$, can be computed using the fast Fourier transform (FFT) algorithm. Then the backpropagated distribution $U(x, y, -d)$ can be obtained as:

$$U(x, y, -d) = \frac{e^{-i\frac{2\pi d}{\lambda}}}{i\lambda d} \mathcal{FT}^{-1}[F(u, v) \exp[-i\pi\lambda d(u^2 + v^2)]] \quad (11)$$

where the inverse Fourier transform is again calculated using the FFT algorithm. Computing the Fresnel transform using this method avoids possible sampling problems in the discretization of the exponential function $g(x, y)$ (see equation (8)) in the integral of equation (7). This function $g(x, y)$ oscillates very rapidly as its frequency is inversely proportional to the wavelength λ , which is much smaller than the distance d and the object size ($f(x, y)$). However, the Fourier transformed function $G(u, v)$ (see equation (10)) varies much more slowly as the frequency is directly proportional to λ , and thus its sampling is less critical. More information about computing Fresnel diffraction can be found in [12] and [13].

Finally the complex information to be displayed, $U(x, y, -d)$, can be represented as the sum of its real and imaginary parts:

$$U(x, y, -d) = \text{Re}[U(x, y, -d)] + i \text{Im}[U(x, y, -d)]. \quad (12)$$

Figure 3. (a) Original object and (b) simulation of the full complex reconstruction at 240.88 mm.

These two parts are separately displayed on two different panels, one in the real-only configuration and the other in the imaginary-only configuration. By adding these two wavefronts and by simple propagation of light, the original object is recovered at a distance d from the modulators.

4. Simulated results

We have performed some simulations using the operating modes and characteristics of our real panels (see figure 2(b)). Each of them is an 8-bit AFLC SLM with 128×128 pixels and a pixel pitch of $40 \mu\text{m}$ [11] and the light wavelength used is 658 nm. These characteristics determine the resolution and total size of the holograms.

The simulation process starts by fitting the real and imaginary values obtained from the backpropagated information about the original object (figure 3(a)) into the real-only and imaginary-only operating curves respectively. Then, the adjusted values are propagated to the desired plane (240.88 mm in this case), where the two parts are added and the reconstruction of the image is achieved (figure 3(b)). The same process is carried out using the information from just one of the panels to see the differences between reconstructions obtained only from the real or the imaginary part (figures 4(a) and (b) respectively). The simulated full complex reconstruction recovers the original object, although a little noise appears inside the shadow of the building and the letters. This is due to the limited number of gls that can be addressed in the panels, which results in a loss of precision when adjusting the complex distribution to the operating curves. In the case when only the real or the imaginary part is used, we can see that the reconstruction at this distance is much better with the real part. This gives an acceptable reconstruction with noise around the objects, while with the imaginary part the reconstruction is very noisy and it is more difficult to identify the object.

We have calculated the RMS (root mean squared) error (see equation (13)) of reconstructions at several distances when using the different parts of the backpropagated complex distribution (real, imaginary, amplitude or phase), as well as the full complex one, to study how the information from the wavefront is distributed when it is propagated.

$$\text{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^N (A_{\text{reconst}}[i] - A_{\text{orig}}[i])^2}. \quad (13)$$

This RMS error is computed by comparing the amplitude values of the original object (A_{orig}) and the reconstruction

Figure 4. Simulations of (a) real-only and (b) imaginary-only reconstructions at 240.88 mm.

($A_{reconst}$) at each distance for the N pixels of the images. The results presented in figure 5 show the curves for the full complex, the amplitude-only, the phase-only reconstructions and the envelopes of the extreme values of the RMS curves for the real-only and imaginary-only reconstructions. We use the adjustment to the panels' operating curves for the full complex, real-only and imaginary-only cases, whereas the amplitude-only and the phase-only reconstructions were computed without taking into account the effects of the modulator, as the AFLC SLMs used in this work are not very suitable for these configurations.

The real-only and imaginary-only reconstructions have a complementary behaviour: their RMS errors oscillate between a top and a lower level—when the RMS error of the real part (white dot) is in the lower level, the RMS error of the imaginary part (black dot) is in the top one, and vice versa. Other distances present an intermediate performance, where the results of both individual reconstructions are poor. The distance of 240.88 mm used in this work corresponds to a minimum in the RMS error of the real-only reconstruction (see figure 4(a)), which is better than when considering only the imaginary part (figure 4(b)). We have chosen it on purpose to show the possibility of using a single modulator to obtain acceptable results. Nevertheless, this option only works with certain fixed distances. To explain this behaviour for the real and imaginary parts we have to consider the backpropagated information from the object to be reconstructed. If we use only the real part we have

$$\begin{aligned} \text{Re}[U(x, y, -d)] &= \frac{1}{2}(U(x, y, -d) + U(x, y, -d)^*) \\ &= \frac{1}{2}(U(x, y, -d) + U(x, y, d)). \end{aligned} \quad (14)$$

When this part is propagated from $z = -d$ to 0, where we want to reconstruct the object, we have:

$$\begin{aligned} \text{Fresnel}[\text{Re}[U(x, y, -d)], d] &= \frac{1}{2}(\text{Fresnel}[U(x, y, -d), d] + \text{Fresnel}[U(x, y, d), d]) \\ &= \frac{1}{2}(U(x, y, 0) + U(x, y, 2d)). \end{aligned} \quad (15)$$

Similarly, when using only the imaginary part,

$$\text{Im}[U(x, y, -d)] = \frac{1}{2i}(U(x, y, -d) - U(x, y, d)) \quad (16)$$

which gives after propagation to $z = 0$:

$$\begin{aligned} \text{Fresnel}[\text{Im}[U(x, y, -d)], d] &= \frac{1}{2i}(U(x, y, 0) - U(x, y, 2d)). \end{aligned} \quad (17)$$

Figure 5. RMS error of the reconstructions at different distances. For the real and imaginary parts only the envelopes of the maxima and minima are plotted.

Figure 6. RMS error of the reconstructions for 1000 different distances in a small zone. For the real-only and imaginary-only parts the oscillating behaviour is shown.

Not much information is lost when the field is reconstructed using real-only information, because $U(x, y, 2d)$ is superimposed on $U(x, y, 0)$. However, $U(x, y, 2d)$ contributes to background noise. When the imaginary part is used, the reconstruction is very noisy because it presents the difference between $U(x, y, 0)$ and $U(x, y, 2d)$.

At other distances d we can achieve a better reconstruction using the imaginary part than using the real part. In fact, their corresponding RMS values oscillate with the distance, with a period equal to $\lambda/2$ as shown in figure 6. Considering the Fresnel diffraction equation (7) at $z = 2d$ the phase term $\exp(i4\pi d/\lambda)$ has the value 1 for distances that obey $d = m\lambda/2$, m being a natural number. This gives a constructive interference for the real part (minimum RMS) and destructive for the imaginary part (maximum RMS). For $d = \frac{2m+1}{4}\lambda/2$ the value of the phase is -1 and thus we have the inverse case, a constructive interference for the imaginary part (minimum RMS) and a destructive interference for the real part (maximum RMS).

In figure 5 we also plot the RMS errors when reconstructing using a single LCD in amplitude-only or phase-only modes, for completeness. In this case the backpropagated values are not adjusted to the modulation curves of the panels that we use in this work because these configurations are not the most suitable for these distributions. The quality of the reconstructions using amplitude-only or phase-only at any distance is never better than when using either the real-

Figure 7. Simulations of full complex reconstructions (a) at a distance of 240.88 mm (focusing the letters) and (b) at a distance of 324.84 mm (focusing the building's shadow).

Figure 8. Detail of the optical set-up.

only or the imaginary-only configurations. The error in the reconstructions using only the phase of the wavefront (which contains high-frequency information) is the highest one and it is practically constant. This is because the phase contains high-frequency information so that the bulk of the object is lost. The error in the amplitude reconstruction (which contains low-frequency information) increases with the distance and it is small for short distances as the behaviour predicted by the geometrical optics prevails.

In any case, the best reconstruction is always obtained when full complex modulation is attained, and the small RMS error shown in figure 5 is due to the limited number of gls that can be addressed in the panels, as mentioned above.

Finally, we have also simulated the simultaneous reconstruction of two objects at two different distances in order to illustrate the possibility of reconstructing volume information from a single complex distribution. This is obtained by adding objects backpropagated at different distances. Full complex reconstruction is required in this case to ensure that the RMS error is always low for all distances. We have used the building's shadow and the letters in figure 3(a) as two separate objects to be recovered at distances of 324.84 and 240.88 mm respectively. Each object is backpropagated at the selected reconstruction distance. Then the two distributions obtained are added to form a single complex distribution, which is adjusted to the panels' modulation curves (the real part to the real-only curve and the imaginary part to the imaginary-only curve). When propagating this resulting distribution we recover each object at the corresponding reconstruction distance. This is shown in figure 7(a) where the letters are

Figure 9. Experimental full complex reconstruction at 240.88 mm.

recovered at 240.88 mm, and in figure 7(b) where the building's shadow is recovered at 324.84 mm while the other object is defocused. These results show the possibility of using a single complex distribution to display volume information, and it is a first step to develop a technique to compute Fresnel holograms of 3D objects, where it will be necessary to take into account occultations and overlaps of the real objects.

5. Experimental set-up and results

We have designed an optical set-up using two AFLC SLMs. It consists of the two modulators arranged in an interferometric configuration, which uses a PBSC to achieve the addition of the real and the imaginary information displayed on each of the panels, as well as a quarter-wave plate and a polarizer at the exit (figure 8). We use a collimated diode laser source of 658 nm, with a beam diameter of about 8 mm. The camera can be displaced to the position where the object is reconstructed, at a distance from both SLMs equal to the backpropagated distance of the computed Fresnel hologram. A piezoelectric micrometer is used to move one of the SLMs to have $\lambda/2$ accuracy in the relative distance of the panels. In order to guarantee that light intensity is equal for both arms, the incident light is linearly polarized at 45° from the x -axis. The different orientation of the SLMs has also been corrected to ensure a proper superimposition of images displayed on both panels. This also requires an accurate alignment of the optical system so that pixel-to-pixel correspondence is fulfilled.

We have reconstructed the object in figure 3(a) at the same distance that was performed in the simulation. Figure 9 shows the experimental reconstruction using the full complex information, while figures 10(a) and (b) correspond to the reconstructions using only the real information and only the imaginary information respectively. Figures 11(a) and (b) show the reconstructions focusing the two objects at different distances. In the reconstructions we can appreciate some noise due to a Fresnel square diffraction pattern. This corresponds to the aperture of the SLM because, in practice, the reflectivity of the modulator is never zero even if in the theoretical operating curves (figure 2(b)) the grey level $gl = 128$ (indicated with a cross) should give a zero amplitude modulation. In the experimental full complex reconstructions we can also appreciate a non-constant background. This is due to the fact that the interference pattern of the panels (when both of them display a constant image), being at exactly the same distance, is not constant within the dimensions of the displays, as can be seen close to the corners in figures 9 and 11. This is

Figure 10. Experimental reconstructions at 240.88 mm (a) using only the real part and (b) using only the imaginary part.

Figure 11. Experimental full complex reconstructions (a) at 240.88 mm (focusing the letters) and (b) at 324.84 mm (focusing the building's shadow).

consequence of the non flatness of the reflective modulators (better than $\lambda/4$ and typically $\lambda/8$, from the manufacturer). If these effects are not considered, we can see that the experimental results are in good agreement with the simulated ones.

6. Conclusions

We have implemented a method to display full complex Fresnel holograms by adding the information displayed on two AFLC SLMs, one of them working in a real-only configuration and the other one in an imaginary-only mode. The Fresnel hologram to be displayed is computed by backpropagating an object to be reconstructed by using a Fresnel transform, which is calculated avoiding sampling problems. Then, displaying the real and imaginary parts on each panel, the object is reconstructed at that distance from the modulators by simple propagation of light, without the need to the use a lens. Another advantage of this system is that no codification is required to display full complex information, therefore the resolution is

only restricted to that of the panels. The simulation shows good reconstructions, although a little noise appears due to the limited number of gls (256) of the modulators. We have obtained optical results by using a compact experimental set-up that adds the information displayed on both panels working in their suitable configurations. These optical results show a good agreement with the simulations taking into account the limitations of the optical elements.

A study of the RMS error of the reconstructions using only real, imaginary, amplitude or phase parts shows the behaviour of the complex distribution at different distances: using only the real part or the imaginary part we can obtain visually acceptable reconstructions but only for concrete distances. The best reconstruction is always the full complex one and it gives a very little RMS error caused by the discrete values of the modulation curves. Finally, results of simultaneous reconstruction of objects at different distances are presented. This shows the possibility of using a single distribution in order to reconstruct volume information.

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