

# Mathematical Thinking and Learning

## 5th through 8th Grade Students' Difficulties in Constructing Bar Graphs: Data Organization, Data Aggregation and Integration of a Second Variable --Manuscript Draft--

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<b>Abstract:</b>	<p>Studies that consider the displays that students create to organize data are not common in the literature. This paper compares 5th through 8th graders' difficulties with the creation of bar graphs constructed using either raw data (Study 1, n=155) or a provided table (Study 2, n=152). Data in Study 1 showed statistical differences for the type of data organization but not for grade level. Students' primary problem was choosing a format that integrated a second variable and aggregating data. In contrast, when the table was provided (Study 2), we observed that 7th-8th graders outperformed 5th-6th graders. We interpret these results in terms of older students' better data interpretation competence. We conclude that students' difficulties in bar graphing can be traced to their tabulation processes. Data organization is an essential bridging tool for understanding the essence of the data and representing them graphically, and educators should devote to it the attention it deserves.</p>



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Editor *Mathematical Thinking and Learning*  
Dr. Lyn D. English  
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Dear Lyn,

Enclosed please find a copy of the final revision of the manuscript entitled, *5<sup>th</sup> through 8<sup>th</sup> Grade Students' Difficulties in Constructing Bar Graphs: Data Organization, Data Aggregation and Integration of a Second* (MTL-D-13-00029). All of your comments and all of Reviewer 5's have been addressed in the revision. Reviewer 1's comment 1 and 2 are commented in the response table but did not generate any change in the manuscript. We interpreted them as additional comments rather than "must" suggestions. Please tell us if we need to make any additional changes. All changes are highlighted in the manuscript.

The Figures, Tables and Appendices are all the same excepte for Table 3 and Table 7 (there were small errors in the title). The paper contains 3 Appendices, 11 figures and 8 tables and its length is 35 pages long excluding cover page, letter, abstract, references, tables and figures. Aso, we have removed anonimity by adding two of our previous papers that are referred to in the manuscrit, and also, we have added the acknowledgments and proof read the text. We hope that the paper is close to publication.

We again, are very grateful to the reviewers', as well as to your thorough revision of the manuscript. We hope that you will find that it is suitable for publication.

With kind regards,

Merce

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All changes in the new manuscript are marked with the control of changes

Reviewer 1 comments	Authors' response
<p>1. Given that the students come from multiple classrooms in multiple schools and each class is split among the two studies, it may help the readers if the authors create a graphic showing how the participant pool was created. This is just an idea but perhaps the authors could show 5 circles representing the 5 schools. Inside each school circle, a smaller circle or circles could be drawn to indicate the class or classes that came from that school. Each class circle could be labeled as a primary or secondary classroom. Each class circle could be further split with a line and the number of students that were assigned to study 1 and to study two could be labeled. I'm not sure if this is helpful but I thought I would suggest it as a way to help readers.</p>	<p>We considered that adding a graphic would not help understand the sample, it may even add more confusion.</p> <p>We compared data between grades and between schools and we did not find significant differences. That's why we decided not to differentiate schools and we pooled the groups into primary and secondary.</p>
<p>2. Although I like the new headers in the discussion, the second paragraph in the section titled Differences in Graphing from a Provided Table or from a Self-made Table seems to be more about the lack of progression from primary to secondary which seems like it would be a more appropriate paragraph for the previous section.</p>	<p>We agree that in the Section titled Differences in Graphing from a Provided Table or from a Self-made Table we talk about graphing performance differences between educational levels. However, we want to emphasize the idea that the explanation for the differences can be found in the condition (creating or interpreting a table). So we think it is better not to move the text.</p>
3. Minor issues	
<p>3.1. Page 3 and page 8 - consider iterative instead of "reiterative." I think of reiterate as about saying something again versus iterate as about doing something repeatedly.</p>	Replaced
<p>3.2. The last sentence on page 8 starts with "This leads to think that providing..." How about "this leads us to think"?</p>	Replaced
<p>3.3. Also in that last sentence on page 8, "transform between different representational displays" seems awkward. How about "iteratively transform particular representational displays into the other types of displays (lists, tables, graphs, etc.)"?</p>	Replaced
<p>3.4. On page 11, do you mean that teachers worked on tables for 3 to 6 lesson and graphs for 3 to 6 lessons or 3 to 6 lessons in total?</p>	<p>Clarified. The new text now reads: The time devoted to activities related to both graphs and tables ranged in total, from three to six class sessions.</p>
<p>3.5. On the last two sentences on the last</p>	<p>No, the term "non canonical" is correct but we</p>

paragraph on page 17, don't you mean deviated from a canonical table?	have rewritten it to make it clearer
3.6. Extra period at end of first full sentence at top of page 22 ("?constructing bar graphs. .The nature?").	Removed
3.7. On page 25, which Appendix is being referred to	Reference added
3.8. On page 25, it says "He drew seven bars, including one for each gender at each of the four height intervals." This would imply eight bars rather than seven.	Explained in the new text
3.9. You might consider giving the Spanish word and English translation for words contained in samples of student work when described.	Translation from Catalan to English added for all Figures
3.10. Near the top of page 35, in the sentence that says "but also, the must "sketch" something "inside" the frame," change the to they	Changed
3.11. On pages 27, 42, 43 and 45, change the appendix seriation to A, B and C rather than 1, 2 and 3.	Changed

Reviewer 5 comments	Authors' response
<p>1. My primary concern still is that whatever is published, it should not convey the conclusion that by-hand graphing always must precede graphing by tool. I think that such a conclusion is an overgeneralization that is not a finding of this study. Perhaps there's a way that the editor can work with the authors' on the language pertaining to this point to convey the authors' point(s) without creating the impression in the reader that by-hand always needs to occur first.</p>	<p>To clarify this point, we have added a new sentence. The new text reads:  “ With our results we do not aim to conclude that by-hand graphing tasks should always precede the construction of graphs using computer spreadsheets. However, we think that by-hand activities should not be neglected. We observed how they help novice students explicit all the steps involved in the process, and also, they can help teachers identify their students’ main difficulties. “</p>
<p>2. Some clarification of what those essential decisions are would be helpful. Also, what is the referent for they in "they are automatically 'drawn'?" Are the decisions "drawn," or are conclusions drawn, or are graphs drawn?</p>	<p>A new text has been added to clarify this point and illustrate it with an example  As we saw in the Results Section (Figure 7), from Pat’s graph we can tell that she is able to identify the frequencies for each gender in each interval. She chooses to represent height in the y-axis and frequencies in the x-axis. She draws vertical bars rather than follow the convention of horizontal bars for frequencies being represented in the x-axis. Pat then has to choose a single value within the given interval to mark it in the y-axis. Also along the process, Pat probably realizes that she needs to represent frequencies clearer, and she decides to add them inside the bars. Also, she has to take the decision of representing gender, and for that, she splits the x-axis frame into two halves, one for each gender value. She does not use the convention of the legend to integrate a second variable (gender) in the graph, either. In contrast, when students like Pat are asked to do the same task using a spreadsheet, they choose the axes, label them, set the scaling, and the “sketch” is drawn automatically from the table provided. The steps illustrated above in Pat’s example are essential decisions that are not made explicit when students use a spreadsheet; they are automatically “drawn”.</p> <p>The text above refers to the text in the Results Section, in page 26, that adds more detail to understand Pat’s example</p> <p>“The values of the y-axis match the size of the bars, which represent approximate values for height intervals; for instance, to represent the</p>

	<p>interval “more than 169 cm”, Pat marked 170 in the y-axis. Regarding the x-axis, she split it into two halves, the left one for boys and the right one for girls, and scaled each half from 0 to 50 (the maximum frequency in the provided Table is 51). Also, she placed the bars in the corresponding x-value (except for the last column, that should be in <math>x=6</math>), and she decided to write the frequencies with the specific gender inside the bars (the first bar reads “23 boys of more than 169” at the y-value of 170; the second, “40 boys between 130 and 149” at the y-value of 135; the third, it repeats “23 boys of more than 169” at the y-value of 150; the fourth, “16 girls” at the y-value of 30; the fifth, “20” at the y-value of 5; the sixth includes an error in the interval limits, “51 girls from 159 to 159” at the y-value of 157; and the eighth bar, at the y-value of 170, it reads 6 girls of more than 169”). We observe several inconsistencies in the representation that show Pat’s struggle with data representation in spite of her signs of good table interpretation. She missed to represent frequencies in the height interval of 150-169 cm, for boys and girls respectively. She also missed to represent the value of frequency “0” for boys smaller than 130 cm. In contrast, Pat repeated the data in bar 1 and bar 3. She misplaced the value of 6 for the frequency of girls “taller than 169 cm”. She indicated a frequency of 16 at the y-value of 30 instead of 130”. And finally, there is a value of 20 in the fifth bar difficult to interpret. These results reveal Pat’s struggle with the representation in a bar graph of the data provided in the Table.</p>
<p>3. The real issue, I think, is that there is a potential difference between asking, "how many boys and how many girls are shorter than 130 cm" and "how many boys are shorter than 130 cm and how many girls are shorter than 130 cm." The latter suggests separate attention to the boys and the girls; the former (used in the study) might suggest lumping together both genders into a total number that were shorter than 130 cm.</p> <p>So, I believe that some acknowledgement of the role of the task wording would be informative to the reader.</p>	<p>We have added a new sentence that acknowledges the ambiguity in the task prompt in page 33:</p> <p>“The first three seemed to be influenced by the understanding of the data prior organization in a provided table, whereas the legend was difficult even with data already organized in a table. Nonetheless we are aware of the fact that differences for gender differentiation could be explained by slight ambiguity in the wording of the task prompt in the make-table condition. If it had explicitly repeated: “how many boys are shorter than 130 cm and how many girls are shorter than 130 cm” instead of “how many boys</p>

	and how many girls are shorter than 130 cm", frequencies for gender differentiation in the make-table condition might have been higher."
4. I have a fourth point, but it is not an outgrowth of the previous round of reviews, it is something that I noticed in reviewing the article this time. There are a large number of chi-square tests conducted (over 60, I believe). When one conducts 100 tests using the .05 level of significance, one can expect, on average, 5 to be significant by chance. Conducting 60 tests at the .05 level can, on average, lead to 3 significant results being significant by chance alone. I do not think that it would be appropriate to have the authors revise all the tables and text to use a .001 significance level, but it might be appropriate to note that, given the number of tests, one might expect that several are significant by chance and not due to inherent differences in the study's conditions that are being compared	<p>We have added a note (iii) in page 18, that acknowledges it:</p> <p>Given the total number of tests, one might expect that several are significant by chance. However, most of the results are significant at .001 level.</p>

## **5<sup>th</sup> through 8<sup>th</sup> Grade Students' Difficulties in Constructing Bar Graphs: Data Organization, Data Aggregation and Integration of a Second Variable**

### **Abstract**

Studies that consider the displays that students create to organize data are not common in the literature. This paper compares 5<sup>th</sup> through 8<sup>th</sup> graders' difficulties with the creation of bar graphs constructed using either raw data (Study 1, n=155) or a provided table (Study 2, n=152). Data in Study 1 showed statistical differences for the type of data organization but not for grade level. Students' primary problem was choosing a format that integrated a second variable and aggregating data. In contrast, when the table was provided (Study 2), we observed that 7<sup>th</sup>-8<sup>th</sup> graders outperformed 5<sup>th</sup>-6<sup>th</sup> graders. We interpret these results in terms of older students' better data interpretation competence. We conclude that students' difficulties in bar graphing can be traced to their tabulation processes. Data organization is an essential bridging tool for understanding the essence of the data and representing them graphically, and educators should devote to it the attention it deserves

*Keywords:* bar graphs, table construction, table interpretation, data aggregation, representation of frequencies



The ability to identify and use information that is presented in various formats, such as graphs and tables, has become a crucial skill in not only science and mathematics learning (diSessa, 2004; Latour, 1990; Lemke, 2002; Petrosino, 2003) but also understanding the statistical data encountered in everyday life (Franklin, et al., 2007). Consequently, one of the outstanding challenges of compulsory education throughout the world is to foster the skills that are necessary for the use of information that is presented graphically.

Our perspective on representational phenomena follows Kaput's general correspondence model (1998): any representation supposes a correspondence (a referential relation) between two entities that an actor or observer elaborates. This perspective differs from other authors, such as Nemirovsky, Tierney, and Wright (1998), who maintain that symbols are not frequently experientially distinguished from referents. We follow Kaput's correspondence model because it allows analyzing the way subjects construct the relation between the referent and the referee. We think that this relation is fundamental in any representational behavior (symbolic play, linguistic, graphical, notational, etc.). Kaput's model fits with a central aim of our study: to consider how actors (students, in our case) elaborate or "construct" this referential relationship. In the case of graphs, some relevant aspects are stated as follows.

First, graphs are external representations, that is, material objects with a set of specific graphic features. External representation cannot to be confused with internal representation, the corresponding hypothesized mental construct (Kaput, 1998; Martí & Pozo, 2000; Tolchinsky, 2003). Our view is that in the process of interpreting and constructing a graph, students construct dynamic and reciprocal relationships between internal and external representations. This dialogue between internal and external

representations is visualized in the main difficulties students experience in the process of constructing a graph. For this reason, we address this process in the present study.

Second, experiences with external representations, especially when students have to construct them, suppose an iterative ~~reiterative~~ process of transformation between different external representations (Duval, 1995) (for example, between written lists of information, ordered lists, tables and graphs). For this reason, in this study, we emphasize the study of the relationships between lists, tables and graphs.

Finally, graphs, like most external representations, are social objects that follow conventional rules of meaning (see below for a specific analysis of the rules that apply to graphs). In this sense, the appropriation of a graph implies a mediated educational activity. Accordingly, the present study addresses grade level to control for the role of general education in graph competence.

### **Research on Graphing Difficulties**

Analyses of students' graphing techniques have been common in the science and mathematics education literature of the last two decades (Friel, Curcio, & Bright, 2001; Janvier, 1987; Leinhardt, Zaslavsky, & Stein, 1990; Postigo & Pozo, 2004; Shah & Hoeffner, 2002; Watson & Fitzallen, 2010). However, research has investigated children's construction of graphs less frequently (diSessa, 2004; Nemirovsky, Tierney, & Wright, 1998; Watson & Fitzallen, 2010), and research devoted to bar graphs is even less common (Äberg-Bengtsson, 2006; Lehrer & Schauble, 2000, 2007; Lehrer & Romberg, 1996; Watson & Moritz, 2000, 2001; Wu & Krajcik, 2006).

Kosslyn (1989, 2006) addressed the analysis of graphs from a cognitive perspective. He conducted a perceptual analysis of the issues that must be taken into account in graph construction to communicate the information effectively. As Kosslyn put it (2006, p.23): "All graphs, no matter how different individual examples may

look, are created from the same components. Typically, they have three primary elements: the framework, the specifier, and the labels.” The framework sets the stage, indicating the measures of the independent and the dependent variables. The specifiers are the lines, bars, point symbols, or other marks that specify particular relationships among the things represented by the framework. They map parts of the framework and are often based on a pair of values (i.e., the bar on a bar graph). On the other hand, labels in the form of letters, words, or pictures convey the framework or specifier information. For instance, each leg of the framework bears a label naming the variables (both dependent and independent). Thus, graphing requires not only an understanding of each of the components but also a style of organization that facilitates our seeing the relationships among them (Kosslyn, 2006).

Accordingly, research in the field of mathematics education has determined that graphing includes the processes of axes selection and labeling, scaling, unit identification, and plotting in Cartesian space (Leinhardt, et al., 1990; Parmar & Signer, 2005; Rangecroft, 1991). As Leinhardt, et al. (1990) reported, the most difficult step in graph construction is establishing the axes and scales with the appropriate labels (i.e., in Kosslyn’s terms, establishing the relationship between the graph’s components). We agree that the creation and labeling of appropriate axes and scales are important parts of the process of bar graph construction, but Kosslyn (2006) states that it is essential to understand the interrelated connections between single constituents (framework, specifier, and labels) before putting them together to choose the most suitable one for each particular set of data. We need to understand “how well each mark specifies particular relations among the things being represented by the framework” (Kosslyn, 2006, p.24). For this purpose, preliminary forms of data organization are essential in graphing because they allow students to organize raw data

before creating plots, facilitating their understanding of the types of variables with which they are working and their selection of the type of plot that is most appropriate for the data (Friel, et al., 2001). Considering Kaput's (1998) correspondence model, this process of organization of data show the progressive construction of the referential relationship between data and graphing representation.

More specifically, regarding bar graphs, these are considered a particular way to represent information. They are defined as data displays that illustrate data using blocks (bars) and involve the use of equal-width class intervals (Friel, et al., 2001). Bars indicate frequencies and are read on the vertical scale. Äberg-Bengtsson (2006) performed qualitative analyses of the bar and pie graphs constructed by 7- to 12-year-old students using Microsoft Excel with guidance. This author reported that even the youngest students demonstrated excellent performance in representing frequencies graphically. Äberg-Bengtsson (2006) did not report difficulties with data organization on the two axes or with crossing the variables when two variables were to be represented in the graph. To explain the high performance of her students, Äberg-Bengtsson (2006) acknowledged the influence of the students' prior work with computer spreadsheets on their manual design of bar graphs, particularly when the students were guided appropriately. Furthermore, Ainley, Nardy, and Pratt (2000), using an instructional approach that includes computerized activities, showed that children as young as 8 and 9 years old were able to use and interpret graphs, including extrapolating and interpolating data. However, as they highlight, the skills related to the construction of graphs, such as scaling axes and plotting points accurately, were not observed as students' strengths. We hypothesize that the early grade-school students in Äberg-Bengtsson's study (2006) and the study by Ainley, et al. (2000), performed impressively because the data were already displayed and organized in a

table, and thus, some important graphic decisions had already been made. Because these decisions had already been made, graphing became an easier task for the students. Ainley, et al. (2000) highlight the fact that “in using spreadsheets, organizing the layout of data on the spreadsheet so that it can be graphed efficiently emerges as an important analytic skill” (Ainley, Nardy, & Pratt, 2000, p.6). Our claim is that, regardless of whether the task is completed with a computer or by hand, data organization skills and an understanding of structure are essential for graphing.

Aligning with this claim, Lehrer and Schauble (2000, 2007) analyzed students’ progress in creating graphs from a modeling perspective by integrating science learning and the development of inscriptional competence. Lehrer and Schauble (2000, 2007) explored ways in which children learn to use new mathematical resources to understand science, especially through “the development, evaluation and revision of models” (p.151). They showed the close relationship between structuring and displaying data and argue that structure is constructed, not inherent. In a long-term classroom study of elementary school students, the authors demonstrated that students’ difficulties in constructing bar graphs were associated with their resistance to data abstraction, the avoidance of literality, and the removal of redundant information. Grouping data appropriately to create a graph was a source of difficulty prior to creating the bar graphs. In their analysis, the authors explained that children must do the following:

...shift their attention beyond particular cases to consider aggregates (i.e., distributions) of objects, while continuing to maintain a sense of the relation between individual cases and the aggregate. A coordination that can be a quite conceptual challenge that ends up by developing

attributes and conventions of data displays (Lehrer & Schauble, 2000, p. 113).

Similarly, Wu and Krajcik (2006) examined the progression of graphing competence when students were asked to create a graph using raw, self-collected data during a case study conducted in a science class that allowed students to utilize their own inscriptional practices. These authors also demonstrated that the greatest difficulty in constructing a bar graph lies in the organization of the data into a form that can be represented in the coordinate system and properly aggregated and organized. This organization requires (1) crossing of the variables and (2) aggregation of the resulting values after the variables have been crossed; these steps also correspond to the two basic steps of constructing a frequency table. The authors mention the crucial importance of supporting these particular steps in the graphing process. The studies of Lehrer and Schauble (2000) and Wu and Krajcik (2006) are thus relevant to ours because they describe the difficulties that students experience when organizing data for proper graph creation (in Kaput's model terminology, the construction of the referential relationship between data and its graphic representation). However, the reported studies did not test the hypothesized role of expertise in organizing data into tables when constructing bar graphs by means of an experimental design and the current study aims to do precisely this. In the next section, we present theoretical work on the role of tables in graphing.

### **Tables and Their Role in Graphing**

Bertin's matrix theory (2000/2001) presents a semiotic analysis of the properties of data tabulation and graphing and their role in describing relationships among data. He clearly established that tables are prerequisites for graphing. A table allows certain reordering and classification operations to be performed and a double-

entry structure to be applied to the data; essentially, tables cross-classify items (Marti, 2008). In a double-entry structure, the values of one variable are shared with the values of another variable, and these values are expressed in the cells that are generated during the cross-tabulation (Duval, 2003; Novick & Hurley, 2001; Piaget & Inhelder, 1967; Tversky, Kugelmass, & Winter, 1991). These features are essential to the process of building a table and, consequently, the graphing process. Consistent with Bertin (2000/2001), Friel, et al. (2001) made the following claim:

Tables appear to be used in two ways. One way is as a type of data display. Tables may also be used for organizing information as an intermediate step to creating graphical representations. The graph maker may need to organize data in tables (e.g., frequency tables) before graphs can be made (p. 127).

Bertin's theoretical analysis (2000/2001) demonstrated that tables are essential for bar graphing. However, very few researchers have addressed the difficulties that emerge when students must organize data in tabular forms (Brizuela & Lara-Roth, 2002; Marti, 2008; Novick & Hurley, 2001; [Marti, Garcia-Mila, Gabucio, & Konstantinidou, 2011](#)). These authors have obtained results that indicate that table construction is a cognitively demanding process, especially for novices. On the other hand, there is some research in the field of mathematics education that shows that cognitive ability such as abstract-reasoning shows a significant relationship with graphing ability among 7th, 9th, and 11th graders (Berg & Phillips, 1994; Dillashaw & Okey, 1980; Padilla, McKenzie, & Shaw, 1986). In contrast, Roth and McGinn (1997), studying graphing as practice, suggested the lack of competence being explained in terms of "experience and degree of participation rather than exclusively in terms of cognitive ability" (p. 92). This leads [us](#) to think

that providing opportunities to iteratively transform particular representational displays into the other types of displays (lists, tables, graphs, etc.)~~reiteratively transform between different representational displays (lists, tables, graphs, etc.)~~

(Duval, 1995) and progressively organize raw data graphically (English, 2012) may elicit students' competence.

The aims of the present research ~~will be~~are pursued by addressing three research goals:

(1) Trace the main difficulties faced by middle school students in creating bar graphs to the students' level of expertise in formatting data into tables (Study 1).

(2) Analyze how these difficulties change across grade levels (Study 1). We chose to compare upper primary school students (fifth and sixth grades) with lower secondary school students (seventh and eighth grades) with the assumption that the change in the difficulties in graphing performance might be explained by general expertise in data manipulation and data formatting acquired through general instruction.

(3) Compare the prior graphing difficulties with those encountered by the students when they are provided with data in table form (Study 2).

### **Study 1. The Construction of a Graph Based on the Construction of a Table**

As cognitive tools, tables assist us in organizing and grouping data effectively to ensure that the data are better understood and represented. By asking students to construct a table using a list of data that they will subsequently use to create a bar graph, we were able to analyze the relationship between the types of tables constructed and the resulting graphs.

### **Method**



**Design and participants.** A cross-sectional design including one between-subject factor was used in the analysis. The dependent variable was the students' performance in constructing a graph. The factor was the student's educational level, which was one of two levels: 1) primary education (fifth- and sixth-grade students) and 2) secondary education (seventh- and eighth-grade students). We chose to compare upper primary school students with lower secondary school students because according to general patterns in educational systems (Watson & Fitzallen, 2010), the transfer from primary to secondary education implies the implementation of curricular changes in the manner in which tables and graphing are addressed. At both levels, a basic competence is the management of various forms of data organization, although the perspective differs. In the Spanish educational system, in primary education, this general aim appears to be broadly explicated in the mathematics curriculum, specifically in the area of "managing information: probability and randomness." In contrast, in secondary education, two blocks of mathematics content ("functions and graphs" and "statistics and probability") clearly refer to activities concerning the interpretation (but not the construction) of tables and bar graphs.<sup>1</sup>

Beyond general education regulation, to know more about whether and how the participants' teachers in the schools taught tables and graphs content, we gave them a written questionnaire with the following questions: (1) Have you explicitly taught your students how to construct a double entry table/ a bar graph? (2) Before we handed out the task to your students did you work on any activity that specifically referred to learning how to construct bar graphs / tables? (3) How much time did you approximately spend teaching these activities? (4) Did you hand out activities that involved the use of tables or graphs already constructed? (5) Do you use the textbook

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<sup>1</sup> Spanish National Curriculum for School Mathematics.

as a reference for teaching content? What textbook do you use? Do you use complementary material? The questionnaire was answered by 11 teachers (5 primary and 6 secondary). We conclude that most of the teachers addressed the teaching of tables through examples and exercises for which the table was a means to reach another goal. None of them explicitly explained how to construct tables. Answers for graphs were similar to tables. The time devoted to activities related to both graphs and tables ranged in total from three to six class sessions. Finally, all teachers relied on their textbooks to draw examples and exercises from, and two of them mentioned the use of complementary material mostly taken from newspapers (one primary school and one secondary school). We reviewed the textbooks used and they all included tables and graphs but we did not observe any explicit teaching on how to construct or interpret them. They were treated as complementary data or as exercises whose solution required the construction or the interpretation of a table or a graph.

The study sample included 155 students drawn from the five public schools located in the same middle class neighborhood in Barcelona (Spain). Students were located at one of two educational levels: there were 71 primary school students (mean age = 10.8, range = 10.0-11.2) and 84 secondary school students (mean age = 13.7, range = 12.3-14.4). Since the task was presented as a regular activity related to the curriculum, all students in the mathematics class participated in the study.

**Procedure and materials.** The data were gathered in a classroom setting. Students worked individually. The time devoted to the task was one class session (55 minutes) during the second semester of the school year. They were provided with an answer booklet that outlined the specific instructions (see below), along with a general explanation to contextualize the task. The answer booklet consisted of three pages containing descriptive headings: a page for the table, a page for the graph, and a third

page to be used as scratch paper. The data sheet with a list of 25 names, the ordinal number of students in the class, and the ages and heights of these students was added as a separate sheet of paper (see the list in Appendix A). The content of the data used in the assignment was sufficiently simple to limit the confounding effect of students' lack of understanding of the data on their ability to plot them (Gerber, Boulton-Lewis, & Bruce, 1995). Students were told that they could ask the experimenter any question they might have on the material. Their own teachers were in the class with the experimenter. The specific instructions in the material provided were as follows:

*Students from another school have gathered the following data (see the list). With these data, you should (1) create a TABLE and (2) create a GRAPH to show how many boys and how many girls are shorter than 130 cm, how many are between 130 and 149 cm, how many are between 150 and 169 cm, and how many are taller than 169 cm.*

**Scoring.** According to several authors (Friel, et al., 2001; Klass, 2012), the most appropriate format for the type of data (a list of individuals who indicated gender and height) and the type of demand (how many boys and girls are taller than...) was the bar graph. According to Klass (2012), pie charts are used to represent the distribution of the categorical components of a single variable. As a general rule, pie charts should be used rarely, if at all, for comparing two or more variables. Given that each variable is represented by a pie, the reader is forced to draw comparisons across the two pie charts, and it is more difficult for the eye to discern the relative size of pie slices than it is to assess relative bar length.

Graphing performance was coded according to the criteria that were determined to be relevant in the literature review. We integrated Kosslyn's (2006) proposal that there are three main components of graphing (framework, specifier and

labels) with the definition of bar graphs by Friel, et al. (2001) and the analysis of Cartesian graphs by Leinhardt, et al. (1990). The first criterion, format, corresponds to Kosslyn's framework concept (whether students represented height and frequencies on the y- and x-axes, respectively). The second and third criteria (gender differentiation and gender integration, respectively) are based on Kosslyn's concept of the specifier. In addition, the analysis of bar graphs by Friel, et al. (2001) suggests using bar graphs to represent the relationship between two categorical variables. Again, the analyses of Cartesian graphs by Kosslyn (2006) and Leinhardt, et al. (1990) inspired the fourth criterion (labeling the second variable with the legend) and the seventh and eighth criteria for axes (x and y) labeling. Finally, the fifth (coherence between what the participants represented and the labels that they used) and the sixth criteria (the scaling of the measures on the axes) were based on the need to measure the relationship between the components described above (see the detailed coding rubrics for the bar graph in Appendix B).

## **Results and Discussion**

Because participants were not specifically asked to construct a bar graph (see the task above), the students could choose the format that they believed was the best way to represent the data. This situation allows us to analyze the degree to which the students understood the correspondence between the nature of the data and the method of representing these data (Klass, 2012).

Thus, prior to analyzing the students' specific difficulties with bar graphs, we examined the formats that the students selected to represent data involving frequencies that cross two variables. As indicated in Table 1, half of the students chose to create a bar graph (50.7% of primary school students and 51.2% of secondary school students). The distribution of the other half of the students was as follows. Among the primary

school students, 25.3% (18 out of 71) submitted blank answer sheets and 23.9% (17 out of 71) chose a format other than a bar graph. Among the secondary school students, 40.4% (34 out of 71) submitted blank answer sheets, and 8.3% (7 out of 84) chose a format other than the bar graph. We will return to the students who left the answer sheet blank in the next section. The formats chosen were pie charts (6 primary students and 4 secondary students), tables (4 primary students and 2 secondary students) and lists, text, or drawings (7 primary students and 1 secondary student).

Insert Table 1 about here

To examine the first research objective, which is the analysis of the students' difficulties in constructing bar graphs, we included in the analysis only the students who constructed a Cartesian graph in applying the scoring rubrics. Thus, the final sample consisted of 79 students (36 primary school students and 43 secondary school students; see Table 1). All of the student productions were double coded, and the inter-coder reliability was 90%. Discrepancies were resolved through discussion.

For the present task, the specific goals were to *create (1) a TABLE and (2) a GRAPH to show how many boys and how many girls are shorter than....* Thus, an understanding of the goal implies a solid understanding of the data according to the two variables gender and height intervals and the ability to choose the framework and the specifier for easily answering the question.

By examining the frequencies with which the students' graphs satisfied each of the listed criteria we can identify the most common difficulties that the students encountered in creating bar graphs. The results in Table 2 indicate the frequency distribution and the statistical comparisons of the two educational levels. We found that none of the frequency comparisons yielded significant differences between the two groups (primary and secondary students). We also observed that gender

integration and legend were the least satisfied criteria (less than 40% in both groups). These criteria were followed by the format, gender differentiation and scaling criteria, which were satisfied in approximately 70% of the graphs. Finally, the criterion of coherence between the representation and the labeling of the two axes was satisfied by approximately 90% of the students (see Table 2).

Insert Table 2 about here

These results provide an answer to the differences by grade level. Nearly all students at both educational levels correctly labeled the axes on their graphs, and more than three-quarters of the students designed graphs with the correct scaling and represented the data coherently in terms of labeling. Unexpectedly, we did not observe significant differences between educational levels for any criteria, regardless of the difficulty associated. We found that general education did not appear to affect the students' competency in graphing.

**Analysis of graphs created according to the tables constructed.** To explain this lack of a difference between educational levels, we hypothesize that the ability to format the data properly prior to the creation of a bar graph might be a stronger factor that interfered with the potential effect of general education. Thus we argue that the students who understood and formatted the data correctly in table form exhibited no difficulties with the representation of data in bar graphs.

The design of the study allowed us to trace the students' graphing competence to the nature of the tables that they created prior to making their graphs. To this end, this section defines the types of tables used. Based on prior theoretical analyses (Duval, 2003; Novick & Hurley, 2001; Tversky, et al., 1991), we categorized the tables produced by the students according to whether they showed a double-entry structure with frequencies (for a thorough analysis of students' tables, see [Marti, et al., 2011](#)).

Tables were coded as (1) the double-entry structure with frequencies when they showed one of the variables displayed along the horizontal dimension and the other in the vertical dimension, and (2) with frequencies if the cells were filled with the frequencies corresponding to the values resulting from the cross tabulation (see Figure 1). For the sake of simplicity from now on we will refer to these tables with the label of canonical and those that do not satisfy either one or both conditions, will be labeled as non-canonical. Examples of non-canonical tables that do not satisfy the first condition include single or double lists that are either unorganized or organized according to height, gender (“nens” for boys and “nenes” for girls), or both and single lists organized according to both height and gender but lacking frequencies. Other examples of non-canonical tables that do not satisfy the second condition include double-entry cell structures that contain the names or heights of students or that simply display crossed cells rather than displaying frequencies [see Figure 2(a)]<sup>1</sup>.

Insert Figure 1 about here

Insert Figures 2(a) and 2(b) about here

It was hypothesized that canonical tables would enable students to organize data effectively and allow for the easy transfer of the data into a graph. Of the 36 primary school students, 18 created non-canonical tables and 18 created canonical tables. Among the 43 secondary school students, 29 created non-canonical tables and 14 created canonical tables. Here, we want to add a note regarding those students who might partially understand the data but do not succeed in organizing them according to conventional practices. In order to consider the students with partial understanding, we include a qualitative analysis that looks at two subcategories within the non-canonical group: those whose tables only satisfied the first condition (double entry structure but no frequencies), and those whose tables only satisfied the second condition (indicated

frequencies but did not organize the data in a double entry display) and we further look at the type of graph they created. The graphs of the first subgroup were: 19 cases (see Figure 2a and 4a for examples. The heading of the table in Figure 4a shows Catalan labels: “nom” for name, “cognom” for last name, “sexe” for sex, and “mida” for height measure) in which students wrote all the names of the cases for each height interval in the cell that resulted from the cross tabulation of data; 8 cases who left the answer sheet blank; 6 cases had a format that did not respond to the task, and 5 had a format with frequencies and the variable gender integrated in the bar graph, for which we assume they had to count the cases in their table cells. The graphs of the second subgroup, those who made a table indicating frequencies without a double entry structure, were less frequent. There were six participants, two of them did not make any graph, two of them made a graph without indicating only heights but no gender, and finally, two of them made two separate bar graphs one for each gender. From these results we can conclude that the understanding of the double entry structure is essential for bar graphing far more than data aggregation into frequencies. In the former case, students can count the cases in the cells and represent them accordingly, while the reverse does not apply: displays with frequencies without the double entry structure shows a much poorer understanding.

Finally, we want to add a comment on the blank sheet answers. The analysis of the tables that the students constructed for later data plotting allowed us to trace the types of tables that the students who left their answer sheets blank constructed. With the exception of 2 students, all of the primary school students who left the answer sheet blank ~~deviated-departed~~ from a non-canonical table. Similarly, of the 34 secondary students who left the answer sheet blank, 29 ~~deviated-departed~~ from a non-canonical table and only 5 constructed a canonical one.



In the next analysis, we examine the percentages of student graphs that satisfied the criteria according to the type of table constructed. The frequencies and percentages are presented in Table 3 and Figure 3. Table 4 shows the statistical analyses comparing the frequencies (and percentages) between educational levels for each table-group (canonical and non-canonical) and Table 5 shows the statistical analysis comparing the frequencies (and percentages) according to the type of table for each educational level. Given that the data were not normally distributed we performed nonparametric tests<sup>iii</sup>.

Insert Table 3 about here

Insert Table 4 about here

Insert Table 5 about here

Insert Figure 3 about here

We observe in Figure 3 that the students who experienced the greatest difficulties were those who created a non-canonical table prior to creating their graphs. Although, as shown in Table 4 and Figure 3, none of the criteria comparisons between educational levels for each table-group (non-canonical and canonical) was statistically significant, when we compared frequencies according to the type of table constructed (Table 5), we observed that in contrast to those who constructed a non-canonical table, the corresponding frequencies of those who created canonical tables increased significantly for both educational levels: format [primary education:  $\chi^2(1)=18, p=.001$  and secondary education:  $\chi^2(1)=11.1, p=.001$ ]; gender differentiation [primary education:  $\chi^2(1)=12, p=.001$  and secondary education:  $\chi^2(1)=12.9, p=.001$ ]; gender integration [primary education:  $\chi^2(1)=7.5, p=.006$  and secondary education:  $\chi^2(1)=6.5, p=.011$ ]; legend [primary education:  $\chi^2(1)=16.8, p=.001$  and secondary education:  $\chi^2(1)=6.0, p=.014$ ]; and scaling [primary education:  $\chi^2(1)=6.4, p=.01$  and

secondary education:  $\chi^2(1)=4.7, p=.031$ ]. In contrast, the coherence criterion showed significant differences only for primary education due to the fact that the canonical group showed a percentage of 100% (18/18),  $\chi^2(1)=4.8, p=.030$ . Finally, the type of table constructed did not show any effect on axes labeling, percentages were over 90% in both groups.

The frequencies of students' graphs in the non-canonical group that satisfied each particular criterion were low. The lowest frequencies were for gender integration and legend. For gender integration, the frequency was 3 out of 18 (17%) for primary school students and 4 out of 29 (14%) for secondary school students and, for legend, the frequency was 1 out of 18 (5%) for primary school students and 3 out of 29 (10%) for secondary school students. In contrast, the frequencies for format, gender differentiation and scaling were slightly higher; they were 6 out of 18 (33%) for format, 7 out of 18 (39%) for gender differentiation, and 9/18 (50%) in primary education students. Similarly, for secondary education, the frequency for format was 14 out of 29 (48%), for gender differentiation was 10 out of 29 (34%) and that for scaling was 15/29 (52%) (See Table 3 and Figure 3).

We find a great contrast when we compare these frequencies with the frequencies of those who created canonical tables. The frequencies for format reached 100% at both educational levels (18/18 for primary school students and 14/14 for secondary school students), and they were higher than 90% for gender differentiation (17/18 for primary school students and 13/14 for secondary school students). Additionally, following the same difficulty pattern as in the non-canonical group, frequencies for gender integration and legend were slightly lower. For the former, the frequency was 11 out of 18 (61%) for primary school students and 7 out of 14 (50%) for secondary school students, the frequency for legend was 13 out of 18 (71%) for

primary school students and 6 out of 14 (43%) for secondary school students. Finally, the frequency for scaling was a little higher, 16/18 (89%) for primary students and 12/14 (86%) for secondary students.

Therefore, we again confirm that difficulty is associated with the integration of the second variable in the graph (see Table 3), illustrated by the difficulty with the criterion gender integration and, consequently, the legend criterion. Next, we illustrate these figures using some of the students' graphs and tables.

**Qualitative analysis of the students' graphs and tables.** Figures 2(a) and 2(b) show the table and graph, respectively, constructed by Marta (a secondary school student). Marta's table contains the double-entry structure, but it does not display frequencies and was thus coded as non-canonical. The table has one row for each name on the data list, with the names being structured by gender, and the columns include the four height intervals. This organization is a common practice that demonstrates students' resistance to the avoidance of literality and progression toward abstraction (Lehrer & Schauble, 2007): the graph demonstrates a lack of abstraction that prevents the use of frequencies; it only superficially resembles a bar graph and shows no data aggregation. Instead, the x-axis contains all of the names of the students on the data list structured into two gender categories, and the y-axis contains the four height intervals. Gender is differentiated but not integrated; thus, there is no need for a legend.

Similarly, Xenia's table and graph are shown in Figures 4(a) and 4(b), respectively. Xenia's table is a double list of names structured by height, and gender is marked as an added label in the third column. The higher literality of this table deviates from the double-entry structure. The students know how tables should appear and create formats that resemble them, but they fail to use the double entry and

frequencies. Xenia's and Marta's tables demonstrate the same problem (the lack of data aggregation into frequencies) in different manners. Xenia's graph is structured according to the names of the students on the data list (x-axis); thus, her graph includes no data aggregation and, subsequently, includes no frequencies. Consistent with their respective tables, Xenia's graph is slightly more literal than Marta's because Xenia's graph indicates specific heights for each name and resembles a scatter plot.

Insert Figures 4(a) and 4(b) about here

As explained in the discussion of coding criteria, the graph format must include frequencies on either the y-axis or the x-axis and height intervals on the x-axis or the y-axis. As indicated in Figures 2(b) and 4(b), a common error among students who constructed graphs using non-canonical tables was the representation of the heights of the students on the y-axis by drawing one bar for each student and placing the names of the students on the x-axis, defined as the case-value plots, precursors of bar-graphs that require neither data aggregation nor abstraction into frequencies (Watson & Fitzallen, 2010). Eight primary and 11 secondary school students drew graphs using this structure; all of these graphs belonged to the non-canonical group. Thus, students demonstrated problems with the representation of frequencies on the y-axis and exhibited a bias for concreteness by associating the variable of height on the y-axis with the name of a student on the x-axis.

Difficulties differentiating gender were observed in the graphs created by students who began with a non-canonical table. We observed different degrees of gender differentiation. For instance, the graphs in Figure 2(b) and Figure 5 clearly differentiate gender; however, they do not represent the integration of gender and height. In both figures, the x-axis provides the names of the students, and the y-axis shows the height intervals. Figure 5 presents the heights for each student and labels the

y-axis using height intervals, whereas Figure 2(b) presents the students at each height interval, which is closer to satisfying the format criterion.

Insert Figure 5 about here

These results provide answers to the two research questions that were posed in this paper. Concerning the first question, our data confirm the hypothesis that the primary difficulties encountered by the students in creating bar graphs were related to difficulties in organizing the data in double-entry tables including frequencies. With regard to the second research question, we have observed the lack of differences among middle school educational levels in constructing bar graphs. The nature of the tables that were constructed (canonical or non-canonical) was a much stronger factor than educational level and led to graphs that differed clearly in quality. However, although the students' graphs may have satisfied the format criterion, even when they chose not to create a canonical table, gender integration and legend appeared to be difficult for many students.

Do these results confirm that students' difficulties lie in the process of constructing a table? Would the students' difficulties in creating bar graphs change if they were provided with data that were already in table form? In the next section, we present a study in which we analyze whether the provision of data that are already in table form affects students' graphing performance.

### **Study 2. The Construction of a Graph Based on Data Provided in a Table**

The third research question addresses the issue of whether the difficulties faced by middle school students in creating bar graphs change when these students are provided with data that are already in table form. That is, what (if any) of the difficulties that were encountered in Study 1 persist when the data are already

organized in table form, and what difficulties are reduced? The underlying hypothesis is that table interpretation may be easier than table production.

Consequently, if certain graph criteria are satisfied differently when the students are provided with a table, we can claim that tables provide information that would otherwise be excessively difficult to process when students must construct such a table themselves. This hypothesis is based on studies conducted in the fields of language development (Clark, 1993) or literacy (Ferreiro & Teberosky, 1983), which indicate that production is more difficult than interpretation

## **Method**

**Design and participants.** The design for this study was identical to that of Study 1. The dependent variable was the performance demonstrated by students in constructing bar graphs, but in this study, the data were already organized in a table that was provided to the students. A total of 152 students were selected from the same schools used in Study 1. Students were also drawn from two educational levels: 68 primary school students (fifth and sixth grades; mean age = 10.9, range = 10.1-11.4) and 84 secondary school students (seventh and eighth grades; mean age = 13.6, range = 12.5-14.3). Similarly to Study 1, of these 152 students, the graphs produced by 23 students (14 primary school students and 9 secondary school students) were not included in the analysis because they were in the form of a pie chart, a table, or text. Additionally, 5 students in the primary education group and 2 in the secondary education left the answer sheet blank. Therefore, data obtained from a total of 122 students were included in the analysis. The final sample consisted of 49 primary school students and 73 secondary school students.

**Procedure and materials.** The data were gathered in the classroom. We provided the students with an answer booklet outlining the specific instructions (see below):<sup>2</sup>

*Look at the following table carefully. We want you to create a GRAPH using this table so that it is easier to compare boys' and girls' heights:*

Insert Table 6 about here

## **Results and Discussion**

As in Study 1, we compared the frequencies with which the criteria were satisfied by the graphs of the primary and secondary school students. In contrast to the results of Study 1, we observed that the graphs of the secondary school students outperformed those of the primary school students in terms of format [ $\chi^2(1) = 6.2, p = .014$ ], gender integration [ $\chi^2(1) = 4.9, p = .027$ ], legend [ $\chi^2(1) = 4.7, p = .029$ ] and coherence [ $\chi^2(1) = 9.2, p = .002$ ]. Almost all of the secondary school students made a bar graph correctly in terms of format (97%), gender differentiation (92%), and coherence (94%). Only half of them (53%) integrated gender and only one third (33%) provided a legend. In contrast, 88% of primary school students created a graph that satisfied the format criterion, only 33% integrated gender, very few (16%) used a legend, and 73% represented and labeled coherently. It is interesting to note that, almost all differentiated gender (94%) (See frequencies and percentages in Table 7).

Insert Table 7 about here

This lack of differences between educational levels regarding the gender differentiation criterion is easily explained by the provided table's explicit indication of gender. Consequently, both primary and secondary students were able to use this information to represent the data in the bar graph. However in contrast with secondary students, primary students did not appear to be able to use other information implicit

in the table, such as cross-tabulated frequencies, resulting in significant differences in the chi-square analysis of format, gender integration and legend criteria. Thus, we could hypothesize that the secondary school students' expertise in interpreting tables was higher.

Because the present study is part of a larger project that focuses on both graph and table construction and interpretation, this hypothesis can be confirmed by means of another set of data from the wider project. These data on graph interpretation were gathered simultaneously with the data on graph construction by randomly splitting the classes into three subgroups: the first group of participants was assigned to Study 1, the second to Study 2 and finally, the third to a table interpretation task. Thus, with this extra set of data ([Gabucio, Marti, Enfedaque, Gilabert, & Konstantinidou, 2010](#)), we were able to respond to the issue of whether secondary school students had greater expertise in interpreting tables.

Two hundred students were given a test in which they were asked to interpret a table that was very similar to the table used in Study 2, where heights were replaced by weight and the totals were reduced to 50, keeping the rest identical. We found significant differences between the test means of students at the two educational levels; younger students demonstrated clear difficulties with the items in the test that pertained to the double-entry structure. For instance, younger compared to older students had difficulties with questions such as: *The numbers that appear inside the cells correspond to: a) weights; b) number of people; c) age; d) height* (see Appendix in [Gabucio, et al., 2010, ~~name deleted to maintain the integrity of the review process~~](#) for the interpretation test).

**Qualitative analysis on the students' graphs from a provided table.** In the following paragraphs, we qualitatively illustrate these results using examples. Jon



(Figure 6) drew eight bars, one for each interval value and gender. He marked the values of the interval limits on the y-axis (0, 130, 149-150, 169). He did not label the x-axis, but he indicated gender (“nens” for boys or “ nenes” for girls) inside the bar at each height interval. The sizes of the bars appear to correspond to the height interval limits rather than to the frequencies in the table or to the students’ heights (as in Figure 4b o Figure 5), although there are three bars for girls in height 149, but none in height 130.

Insert Figure 6 about here

Pat’s graph (Figure 7) is interesting because it illustrates a secondary school student’s understanding of the frequencies in the table that resulted in an unsuccessful attempt to represent those frequencies in the graph. The graph shows heights on the y-axis and frequencies on the x-axis. The values of the y-axis match the size of the bars, which represent approximate values for height intervals; for instance, to represent the interval “more than 169 cm”, Pat marked 170 in the y-axis. Regarding the x-axis, she split it into two halves, the left one for boys and the right one for girls, and scaled each half from 0 to 50 (the maximum frequency in the provided Table is 51). Also, she placed the bars in the corresponding x-value (except for the last column, that should be in  $x=6$ ), and she decided to write the frequencies with the specific gender inside the bars (the first bar reads “23 boys of more than 169” at the y-value of 170; the second, “40 boys between 130 and 149” at the y-value of 135; the third bar repeats “23 boys of more than 169” at the y-value of 150; the fourth, “16 girls” at the y-value of 30; the fifth, “20” at the y-value of 5; the sixth includes an error in the interval limits, “51 girls from 159 to 159” at the y-value of 157; and the eighth bar reads 6 girls of more than 169” at the y-value of 170). We observe several inconsistencies in Pat’s bar graph. She missed to represent frequencies in the height interval of 150-169 cm, for

boys and girls respectively. She also missed to represent the value of frequency “0” for boys smaller than 130 cm. In contrast, Pat repeated the data in bar 1 and bar 3. She misplaced the value of 6 for the frequency of girls “taller than 169 cm”. She indicated a frequency of 16 at the y-value of 30 instead of 130”. And finally, there is a value of 20 in the fifth bar difficult to interpret. These results reveal Pat’s struggle with the graphical representation of the data provided in the Table in spite of her signs of proper table interpretation.

Finally Ferran’s graph (Figure 8) indicates clear gender integration, with a bar graph that appears correct. However, when we examine the axes, although the columns represent the values of the frequencies of the table, the y-axis is labeled “height in cm” and the x-axis is labeled “students”, segmenting the axis with the correct height intervals. Thus, the table exhibits a clear incoherence between what is represented and what is labeled. This graph is in contrast to Figures 2(b) and 5 in Study 1 in terms of the attempts to integrate gender into the graphs. Figures 7 and 8 indicate that although secondary school students have a better understanding of tables than primary school students, even the graphs of secondary students are not straightforward. The following examples show that although the students may understand tables, their graphs still indicate difficulties regarding the representation of gender integration and the legend.

Insert Figure 7 about here

Insert Figure 8 about here

**Study 1 and Study 2 comparison.** In addition to the differences between the primary and secondary school students in graph performance based on the provided table, we believe that it is worthwhile to compare the results in Study 1 with the results in Study 2. Our final analysis compares the percentages of students’ graphs that

satisfied the criteria when the graphs were created using their own tables or using a table provided. Although the two task prompts were not identical, given that students in each classroom were randomly split into two groups and assigned to simultaneously perform the task for each study, the comparison is reasonable. We did not analyze the data with a between-group design with one condition (to make a table or read a table) because as previously mentioned, the task prompts were necessarily different to address the task's goal.

In primary education, the comparison of graphing performance (percentage of criteria satisfied) yielded significant differences between the graphs that were created based on a constructed table and the graphs that were created based on a provided table for the criteria gender differentiation and legend. Gender differentiation was better satisfied by graphs from a provided table and legend was better satisfied by graphs from one's own table (see the respective data for graphs that were based on a constructed table in Table 2 and based on a provided table in Table 7 and see statistical results in Table 8; for comparison, Appendix [C](#) presents data from Table 2 and Table 7 in a single Figure).

Insert Table 8 approx. here

In secondary education, all frequencies were higher for the graphs made from a provided table for format, gender differentiation, gender integration and scaling. As if having the table and being able to interpret it thoroughly had helped the students construct a better graph (statistical analysis are provided in Table 8). Format, gender differentiation, gender integration and legend are criteria closely related to the understanding of the structure of the data, and its consequent representational management. The first three seemed to be influenced by the understanding of the data prior organization in a provided table, whereas the legend was difficult even with data

already organized in a table. Nonetheless we are aware of the fact that differences for gender differentiation could be explained by a slight ambiguity in the wording of the task prompt in the make-table condition. If it had explicitly repeated: “how many boys are shorter than 130 cm and how many girls are shorter than 130 cm” instead of “how many boys and how many girls are shorter than 130 cm”, frequencies for gender differentiation in the make-table condition might have been higher.

Contrary to the belief that tables that are similar to the table presented in our study are straightforward, students’ lack of understanding of such tables may have implications for their ability to graph depicted data. In contrast to the expertise of secondary school students, the lower expertise of primary school students is hypothesized to be the cause of their low graphing performance when they were asked to create a graph using a provided table.

### **General Discussion**

Although the construction and comprehension of tables have been neglected topics in the literature, the results of this research suggest that these topics deserve further study in the field of learning and instruction. As Ainley (2000) states, the widespread use of graphs in advertising and in the news assumes that graphs clearly communicate their meaning, but this assumption conflicts with the results of the research on pupils' difficulties with graphing in mathematics and science. In response to the first goal of the study, our data provide evidence that students have significant difficulties with bar graphs. In analyzing how students construct a table before constructing a bar graph, we have shown that their main difficulties are related to structuring the data from two variables and understanding that a set of items can satisfy two criteria after the variables have been integrated and the numbers have been aggregated accordingly. We think that these difficulties indicate that students, when

constructing a graph, have to engage in a complex process that relates data with graphic aspects. As Kaput's model points out (1998), this process involves the construction of a referential relationship between two different entities, the data and the graph that represents them. Thus, we do not believe that tables are transparent cognitive tools that can be used properly by all primary and secondary students during the problem-solving process; instead, an accurate understanding of certain implicit information in tables is required for graphing. Lehrer and Schauble (2007) sought to determine what is transparent and what is obscure in the modeling practices of primary school students. Our results show that what makes graphs obscure and their appropriation difficult and neglected may be the lack of understanding of the double-entry structure of the data that underlie tables and bar graphs. This lack of understanding was apparent at both educational levels.

### **The Relationship between Graphing and Tabulating: Cross-classification and Data Aggregation**

To interpret the relationship between the students' difficulties with graphing and their difficulties with tabulating, we discuss the students' difficulties in constructing a canonical table. Based on the analysis of the students' constructions, we identified two cognitive requirements of table construction: a) comprehension of the cross-classification of variables and b) comprehension that frequencies are abstract numbers that result from the aggregation of individual cases.

The cross-classification of variables is the essence of tables: any data located in the cells (in our case, frequencies) are related both to one condition of one variable and to another condition of the other variable. From a logical point of view, cross-classification requires the coordination of two dimensions (which occurs, for example, when one must classify objects that are red and square and distinguish them from

objects that are red and round, blue and square or blue and round). Unlike simple or additive classification, multiplicative classification requires the coordination of two (or more) dimensions. According to Piaget, these multiplicative classifications are possible during the concrete operational stage (approximately 7-8 years) (Piaget & Inhelder, 1967). Why, then, did some of the participants in our Study 1 have difficulties in the cross-classification of variables considering that, according their ages, they most likely do not have any difficulty with multiplicative classifications?

We believe that the difficulties in table construction are related to graphic constraints rather than to the students' general competence with logic. First, to construct a table, the students must anticipate and draw the number of rows and columns according to the values of the two dimensions (gender and height). Thus, both variables must be explicitly considered and represented in the table margins, and the students must understand the difference between margins and cells. Second, the students must also understand that cells in the same row (or column) are related to the value of the variable indicated in the margin. Thus, they must understand that the meaning of the data included in the cells depends on spatial conventions (Brizuela & Lara-Roth, 2002; Marti, 2008; Novick & Hurley, 2001; [Marti, et al., 2011](#)).

The second cognitive requirement (the computation of frequencies) is related to the specificity of the table that the students must construct (i.e., a table of frequencies). Although logically correct, it is not sufficient for students to place all of the cases that fulfill the conditions indicated in the margins in each cell. To fulfill the goal of the task, the total number of cases that fulfill both conditions must be computed. With respect to the need to provide frequencies, the students appeared very reluctant to remove information from the display, even at the cost of redundancy within their tables or graphs. Lehrer and Schauble (2000, 2007) and Wu and Krajcik

(2006) also reported these difficulties in observational studies in which students had to manipulate row data. These studies specifically reported the students' resistance to avoiding literality. Lehrer and Schauble's analysis of the modeling performed by middle school students revealed that the heuristic employed by the students was "more stuff is better" (2000, p. 60). This tendency to include all information in a display reveals a resistance to removing redundant information, which was also a typical characteristic of the tables and graphs constructed by the students in the current study, which controlled for data manipulation.

To explain the students' resistance to removing redundant information, Lehrer and Schauble (2007, p.158) consider "objectifying data as a mental step away from the cases that the data represent to treat the data as objects in their own right." Rather than taking a case view of the data, objectifying the data entails taking an aggregate view in which the data themselves are the object of manipulation, inspection, and conjecture. For instance, in their study about plant growth, the authors observed how contentious 3rd graders became when graph midpoints did not correspond to any particular case-value. Just as children in Lehrer and Schauble's study (2000) found it counterintuitive to accept that a value deemed to be typical did not appear in the original distribution, they also tended to find the elimination of redundant information counterintuitive. Also from data modeling approach, English (2012) shows how first grade children engage in basic activities that are crucial to graphic representation: structuring of data, detection of redundant information, or awareness of the need to eliminate unnecessary features when they asked modify their initial representations.

### **Progression between Primary and Secondary Education**

Based on the data related to our second goal, cross-classification and its representation in a graphic display, as well as the aggregation of data into frequencies,

appeared to be equally demanding for primary and secondary school students. How can we explain these difficulties and the lack of progression between the primary and secondary school levels? We have observed that logic development related to competence in handling multiplicative structures cannot be an explanation. Some students may have been able to overcome these challenges because of their experience with tables during school instruction (Roth & McGinn, 1997). However, unexpectedly, we found no significant improvements in the table construction performance of secondary students compared with primary students. We argue that the reason for this lack of improvement is that according to our review of the students' textbooks and their teachers' practices (see the questionnaire above), table construction is not an explicit object of instruction among these students' school practices. These leads to the importance of explicitly teaching the graphic and cognitive requirements necessary for constructing a table.

These results contrast with those obtained by Äberg-Bengtsson (2006) and Ainley, et al. (2000) in their studies of first- and second-grade primary school students. These authors used Microsoft Excel to explicitly teach how to create a graph. As they acknowledge, using Microsoft Excel to create a graph involves a linear procedure that requires one to follow a particular sequence of steps. Any explicit teaching of the steps required to construct graphs and tables is beneficial regardless of whether they learn by hand or using a computer. Students should have the option to learn about creating graphs and tables, to eventually be able to exhaustively explain what they have created in both instances, with technology or without it.

According to Ainley, et al. (2000), the use of computers in education has the potential to revolutionize the ways in which children learn graphing skills. But, as they claim, the ability to produce graphs with a computer does not simply remove the need



for traditional pencil-and-paper skills. The authors suggest that the mastery of graphing requires, among other things, the practical skills required to produce graphs by hand. For this reason, it is important for students to struggle with the decisions involved in the manual construction of a graph using raw data because they appropriate data and develop a better understanding of them. With our results we do not aim to conclude that by-hand graphing tasks should always precede the construction of graphs using computer spreadsheets. However, we think that by-hand activities should not be neglected. We observed how they help novice students explicit all the steps involved in the process, and also, they can help teachers identify their students' main difficulties. The question is whether our students would have created the graphs and tables that illustrate their struggle if they had been asked to represent the data using an Excel spreadsheet. How would they have represented Figure 6 or Figure 7 using a spreadsheet? The first step in graph construction is not only different but also essential in the process. When students are asked to construct a graph on a paper from a set of raw data, they must not only organize the blank sheet of paper to draw the format with the axis, but also, they must “sketch” something “inside” the frame resulting from setting the axis. As we saw in the Results Section (Figure 7), from Pat's graph we can tell that she is able to identify the frequencies for each gender in each interval. She chooses to represent height in the y-axis and frequencies in the x-axis. She draws vertical bars rather than follow the convention of horizontal bars for frequencies being represented in the x-axis. Pat then has to choose a single value within the given interval to mark it in the y-axis. Also along the process, Pat probably realizes that she needs to represent frequencies clearer, and she decides to add them inside the bars. Also, she has to take the decision of representing gender, and for that, she splits the x-axis frame into two halves, one for each gender value. She does not use

the convention of the legend to integrate a second variable (gender) in the graph, either. In contrast, when students like Pat are asked to do the same task using a spreadsheet, they choose the axes, label them, set the scaling, and the “sketch” is drawn automatically from the table provided. The steps illustrated in Pat’s example are essential decisions that are not made explicit when students use a spreadsheet; they are automatically “drawn”. The fact that novice students are free to “sketch” the data within the frame established by the axis, opens a window for teachers to observe their mental struggle, and consequently, allows a more precise intervention addressing the specific graphing difficulties.

### **Differences in Graphing from a Provided Table or from a Self-made Table**

On the other hand, our results also indicated that creating a graph from an existing table appeared to be easier than creating a graph from a self-made table, but only for secondary school students. Thus, contrary to the belief that tables similar to the table presented in our study are straightforward, the students’ lack of expertise in understanding of such tables may have implications for their ability to graph the depicted data. In contrast to the expertise of secondary school students, the lower expertise of primary school students is hypothesized to be the cause of their low graphing performance when they were asked to create a graph using a provided table.

Another difficulty that appeared across educational levels for those students who departed from the canonical table involved gender integration. Creating a single graph that integrated heights for both genders (boys and girls) appeared to be a highly demanding task. As previously mentioned, the students tended to overcome this obstacle by constructing two separate graphs. Fewer than 50% of the students constructed graphs that integrated gender. Our claim is that this difficulty may stem from the graphical constraint of condensing all of the information into a two-

dimensional structure. That is, having all levels of one of the variables on one axis (i.e., the x-axis) and the frequencies on the other axis (i.e., the y-axis) leaves no dimension for the second variable that must eventually show the values of all subcategories. In contrast, in Study 1, there were no differences in table construction between the primary and secondary school students. The secondary school students outperformed the primary school students in terms of table interpretation but not in terms of the construction of canonical tables because, as mentioned, their school coursework tend to emphasize the use and interpretation of printed tables rather than the manipulation and proper formatting of raw data.


We conclude that students' difficulties in bar graphing can be traced to their tabulation processes. Data organization is an essential bridging tool for understanding the essence of the data and representing them graphically. One-variable bar graphs are introduced very early in the school practices, since they appear easy to start representing data. However the step to a two-variable bar graph is done quite automatically without any explicit consideration to the new processes involved in such step. We show in the present paper that data aggregation and cross-classification to integrate the second variable are two of these difficult processes, and show the essential role of tables in such step. Students are often taught, more or less explicitly, traditional representational systems, as isolated topics (English, 2012), or as means to solve other problems. As English puts it: "The need for classroom experiences that provide opportunities to structure and display data in ways that students choose and to analyze and revise their creations are important in addressing these early difficulties" (2012, page 17). We agree with English (2012) that modeling opportunities are to be provided, but we argue that these opportunities must provide specific scaffolds that address the difficult steps involved in graphing.

### Aknowledgements

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## References

- Äberg-Bengtsson, L. (2006). "Then you can take half . . . almost"—Elementary students learning bar graphs and pie charts in a computer-based context. *Journal of Mathematical Behavior* 25, 116–135, <http://dx.doi.org/10.1016/j.jmathb.2006.02.007>.
- Ainley, J. (2000). Transparency in graphs and graphing tasks. An iterative design process. *Journal of Mathematical Behavior*, 19, 365–384, [http://dx.doi.org/10.1016/S0732-3123\(00\)00052-3](http://dx.doi.org/10.1016/S0732-3123(00)00052-3).
- Ainley, J., Nardi, E., & Pratt, D. (2000). Towards the construction of meaning for trend in active graphing. *International Journal of Computers for Mathematics Learning*, 5(2), 85-114. <http://dx.doi.org/10.1023/A:1009854103737>.
- Berg, C. & Phillips, D. G. (1994). An investigation of the relationship between logical thinking structures and the ability to construct and interpret line graphs. *Journal of Research in Science Teaching*, 31, 323–344, <http://dx.doi.org/10.1002/tea.3660310404>.
- Bertin, J. (2000/2001). Matrix theory of graphics. *Information Design Journal*, 10, 5–19.
- Brizuela, B., & Lara-Roth, S. (2002). Additive relations and function tables. *Journal of Mathematical Behavior*, 20, 309–319, [http://dx.doi.org/10.1016/S0732-3123\(02\)00076-7](http://dx.doi.org/10.1016/S0732-3123(02)00076-7).
- Clark, E. (1993). Acquisitional principles in lexical development. In S. A. Gelman & J. Byrnes (Eds.), *Perspectives on language and thought. Interrelations in development* (pp. 31–72). Cambridge, UK: Press Syndicate of the University of Cambridge.
- Dillashaw, F. G. & Okey, J. R. (1980). Test of the integrated science process skills for

- secondary science students. *Science Education*, 64, 601–608,  
<http://dx.doi.org/10.1002/sce.3730640506>
- diSessa, A. A. (2004). Metarepresentation: Native competence and targets for instruction. *Cognition and Instruction*, 22, 293–331,  
[http://dx.doi.org/10.1207/s1532690xci2203\\_2](http://dx.doi.org/10.1207/s1532690xci2203_2).
- Duval, R. (2003). L'organisation visuelle de l'information en tableaux [The visual organisation of information in tables]. *Spirale. Revue de Recherche en Éducation*, 32, 1–28.
- Duval, R. (1995). *Sémiosis et pensée humaine. Registres sémiotiques et apprentissages intellectuels*. Bern: Peter Lang.
- Ferreiro, E., & Teberosky, A. (1983). *Literacy before schooling*. Portsmouth, NH: Heinemann Educational Books Inc.
- English, L. (2012). Data modeling with first-grade students. *Educational Studies in Mathematics*, 81, 15-30, DOI 10.1007/s10649-011-9377-3.
- anklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for assessment and instruction in statistics education (GAISE) report: A preK-12 curriculum framework*. Alexandria, VA: American Statistical Association. Retrieved from <http://www.amstat.org/education/gaise/>
- Friel, S. N., Curcio, F., & Bright, G. (2001). Making sense of graphs: Critical factors influencing comprehension and instructional implications. *Journal for Research in Mathematics Education*, 32, 124–158,  
<http://dx.doi.org/10.2307/749671>.
- [Gabucio, F., Marti, E., Enfedaque, J., Gilabert, S. & Konstantinidou, K. \(2010\). Niveles de comprensión de las tablas en alumnos de primaria y secundaria \(Levels of graph comprehension in primary and secondary school students\).](#)

*Cultura y Educación*, 22(2), 183-197, DOI:

<http://dx.doi.org/10.1174/113564010791304528>.

- Gerber, R., Boulton-Lewis, G., & Bruce, C. (1995). Children's understanding of graphic representations of quantitative data. *Learning and Instruction*, 5, 77–100, [http://dx.doi.org/10.1016/0959-4752\(95\)00001-J](http://dx.doi.org/10.1016/0959-4752(95)00001-J).
- Janvier, C. (1987). *Problems of representation in mathematics learning and problem solving*. Hillsdale, NJ: Lawrence Erlbaum Associates.
- Kaput, J.J. (1998). Representations, inscriptions, descriptions and learning: A kaleidoscope of windows. *Journal of Mathematical Behavior*, 17(2), 265-281.
- Klass, G. (2012). *Just plain data analysis: Finding, presenting, and interpreting social science data*. New York: Rowman and Littlefield Publishers, Second Edition.
- Kosslyn, S. M. (1989). Understanding charts and graphs. *Applied Cognitive Psychology*, 3, 185–226, <http://dx.doi.org/10.1002/acp.2350030302>.
- Kosslyn, S. M. (2006). *Graph design for the eye and the mind*. New York, NY: Oxford University Press.
- Latour, B. (1990). Drawing things together. In M. Lynch & S. Woolgar (Eds.), *Representation in scientific practice* (pp. 19–68). Cambridge, MA: MIT Press.
- Lehrer, R., & Romberg, T. (1996). Exploring children's data modeling. *Cognition and Instruction*, 14, 69–108, [http://dx.doi.org/10.1207/s1532690xci1401\\_3](http://dx.doi.org/10.1207/s1532690xci1401_3).
- Lehrer, R., & Schauble, L. (2000). Model-based reasoning in mathematics and science. In R. Glaser (Ed.), *Advances in instructional psychology*, (Vol. 5, pp. 101–159). Mahwah, NJ: Lawrence Erlbaum Associates.
- Lehrer, R., & Schauble, L. (2007). Contrasting emerging conceptions of distribution in contexts of error and natural variation. In M. Lovett & P. Shah (Eds.), *Thinking*

*with data* (pp.149–176). Mahwah, NJ: Lawrence Erlbaum Associates.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs and graphing: Tasks, learning and teaching. *Review of Educational Research*, 60, 1–64, <http://dx.doi.org/10.3102/00346543060001001>.

Lemke, J. (2002). Enseñar todos los lenguajes de la ciencia: Palabras, símbolos, imágenes y acciones [Teach all languages of science: Words, symbols, images and actions]. In M. Benlloch (Ed.), *La educación en ciencias. Ideas para mejorar su práctica* [Education in science: Ideas to improve its practice] (pp. 159–186). Barcelona, Spain: Paidós.

Marti, E. (2008). Tables as cognitive tools in primary education. In C. Andersen, N. Scheuer, M. P. P. Echeverría, & E. Teubal (Eds.), *Representational systems and practices as learning tools in different fields of knowledge*. (pp. 133–148). Rotterdam, The Netherlands: Sense Publishers.

Martí, E., Garcia-Mila, M., Gabucio, F., & Konstantinidou, K. (2011). The construction of a double entry table: A study of primary and secondary school students' difficulties. *European Journal of Psychology of Education*, 26 (2), pp. 215-235, [ttp://dx.doi.org/10.1007/s10212-010-0046-1](http://dx.doi.org/10.1007/s10212-010-0046-1)

Martí, E. & Pozo, J. I. (2000). Más allá de las representaciones mentales: La adquisición de los sistemas externos de representación. *Infancia y Aprendizaje*, 90, 11-30, <http://dx.doi.org/10.1174/021037000760087946>.

Nemirovsky, R., Tierney, C., & Wright, T. (1998). Body motion and graphs. *Cognition and Instruction*, 16, 119–172, [http://dx.doi.org/10.1207/s1532690xci1602\\_1](http://dx.doi.org/10.1207/s1532690xci1602_1).



- Novick, L. R., & Hurley, S. M. (2001). To matrix, network, or hierarchy. That is the question. *Cognitive Psychology*, 42, 158–216.  
<http://dx.doi.org/10.1006/cogp.2000.0746>.
- Padilla, M. J., McKenzie, D. L. & Shaw, E. L. (1986). An examination of the line graphing ability of students in grades seven through twelve. *School Science and Mathematics*, 86, 20–26.
- Parmar, R. S., & Signer, B. R. (2005). Sources of error in constructing and interpreting graphs: A study of fourth- and fifth-grade students with LD. *Journal of Learning Disabilities*, 38, 250–261,  
<http://dx.doi.org/10.1177/00222194050380030601>.
- Petrosino, A. (2003). Commentary: A framework for supporting learning and teaching about mathematical and scientific models. *Contemporary Issues in Technology and Teacher Education*, 3, 288–299.
- Piaget, J., & Inhelder, B. (1967). *La genèse des structures logiques élémentaires* [The genesis of elementary logical structures]. Neuchâtel, Switzerland: Desclaux et Niestlé.
- Postigo, Y., & Pozo, J. I. (2004). On the road to graphicacy: The learning of graphical representation systems. *Educational Psychology*, 24, 623–644,  
<http://dx.doi.org/10.1080/0144341042000262944>.
- Rangecroft, M. (1991). Graphwork – Developing a progression. Part 1 – The early stages. *Teaching Statistics*, 13, 44–46, <http://dx.doi.org/10.1111/j.1467-9639.1991.tb00163.x>.
- Roth, W.-M., & Bowen, G. M. (1999). Complexities of graphical representations during ecology lectures: an analysis rooted in semiotics and hermeneutic

- phenomenology. *Learning and Instruction*, 9, 235–255,  
[http://dx.doi.org/10.1016/S0959-4752\(98\)00029-2](http://dx.doi.org/10.1016/S0959-4752(98)00029-2).
- Roth, W.-M. & McGinn, M. (1997). Graphing: Cognitive ability or practice? *Science Education*, 81, 91–106, <http://dx.doi.org/10.1023/A:1017909815902>
- Shah, P., & Hoeffner, J. (2002). Review of graph comprehension research: Implications for instruction. *Educational Psychology Review*, 14, 47–69,  
<http://dx.doi.org/10.1023/A:1013180410169>.
- Tochinsky, L. (2003). *The cradle of culture and what children know about writing and numbers before being taught*. Mahwah, NJ: Lawrence Erlbaum.
- Tversky, B., Kugelmass, S., & Winter, A. (1991). Cross cultural and developmental trends in graphic productions. *Cognitive Psychology*, 23, 515–557,  
[http://dx.doi.org/10.1016/0010-0285\(91\)90005-9](http://dx.doi.org/10.1016/0010-0285(91)90005-9).
- Watson, J. & Fitzallen, N. (2010). *Report for the NSW Department of Education and Training*. Education & Training. NSW Government (NSW, Australia).
- Watson, J. M., & Moritz, J. B. (2000). The longitudinal development of understanding of average. *Mathematical Thinking and Learning*, 2, 11–50,  
[http://dx.doi.org/10.1207/S15327833MTL0202\\_2](http://dx.doi.org/10.1207/S15327833MTL0202_2).
- Watson, J. M., & Moritz, J. B. (2001). Development of reasoning associated with pictographs: Representing, interpreting, and predicting. *Educational Studies in Mathematics*, 48, 47–81, <http://dx.doi.org/10.1023/A:1015594414565>.
- Wu, H.-K., & Krajcik, J. S. (2006). Inscriptional practices in two inquiry-based Classrooms: A case study of seventh graders' use of data tables and graphs. *Journal of Research in Science Teaching*, 43, 63–95,  
<http://dx.doi.org/10.1002/tea.20092>.

## Appendix A1. List provided to make the table

	First name	Last name	Age		Height	
1	ANA	ALIAGA	12	years	126	cm
2	PAU	BLANCO	14	years	174	cm
3	SILVIA	CANO	14	years	165	cm
4	LUCAS	DE LOS SANTOS	13	years	171	cm
5	MARTA	FERNÁNDEZ	13	years	154	cm
6	FATIMA	GARCÍA	13	years	148	cm
7	OLGA	GARCÍA	14	years	172	cm
8	MIQUEL	GISBERT	14	years	173	cm
9	NURIA	GÓMEZ	13	years	156	cm
10	AINHOA	GRI	13	years	147	cm
11	ANDREU	GUILLÉN	13	years	157	cm
12	ISMAEL	JIMÉNEZ	13	years	165	cm
13	MONTSE	MARTÍN	13	years	142	cm
14	JOANA	MARTÍNEZ	12	years	134	cm
15	MARC	MOLERO	13	years	146	cm
16	ENRIC	MOLINA	12	years	141	cm
17	EVA	MOLINA	12	years	128	cm
18	ALICIA	MORENO	13	years	151	cm
19	VICTOR	PAJARES	13	years	167	cm
20	SARA	PEIRO	12	years	144	cm
21	JOAN	PLANAS	12	years	134	cm
22	JUDITH	ROMERO	13	years	139	cm
23	FRANCESC	SÁNCHEZ	13	years	149	cm
24	AITOR	TOLEDO	13	years	160	cm
25	SERGI	VEGA	13	years	155	cm

## Appendix **B2**. Coding criteria

### 1) Format

Yes: Participant represents height on the y-axis and frequency on the x-axis or vice-versa.

No: Participant represents height on the y-axis and writes students' names on the x-axis; participant represents nothing on the y-axis and any of the above on the x-axis, or vice-versa; participant mixes the information between and within axes; participant represents nothing, but labels the information on the axes.

### 2) Gender differentiation

Yes: The gender variable is differentiated in the same graph.

No: Boys and girls are not identified by gender

### 3) Gender integration

Yes: The gender variable is integrated into the same graph.

No: Boys and girls are represented on two attached graphs; on separate graphs; or are not represented differentially.

### 4) Legend

Yes: The legend is present.

No: There is no legend.

### 5) Coherence between representations and labels

Yes: Participant represents what is labeled on both axes.

No: There is a discrepancy between what the participant represents and what is labeled on one of the axes; or there is a discrepancy on both axes.

### 6) Scaling

Yes: The measures are proportional.

No: The measures are not proportional.

## 7) Y-axis label

Yes: The y-axis is labeled, indicating heights, frequencies or names.

No: The y-axis is labeled, mixing the above; or the y-axis is not labeled.

## 8) X-axis label

Yes: The x-axis is labeled, indicating heights, frequencies or names.

No: The x-axis is labeled, mixing the above; or the x-axis is not labeled.

Appendix C.

Insert Figure Appendix 3. Approx. here

---

<sup>1</sup> Figures labeled (a) and (b) correspond to the sequence of the tables and graphs of the same students.

<sup>2</sup> The specific question in the task instructions was slightly altered. If the question in the second task was identical to the question in the first task, the students could have answered the question by simply reading the table.

<sup>iii</sup> Given the total number of tests, one might expect that several are significant by chance. However, most of the results are significant at .001 level.

Figure 1

[Click here to download high resolution image](#)

	♀	♂
~ 130	1	1
130 ~ 149	6	3
150 ~ 169	4	5
+ 169	1	2



Figure 2a  
[Click here to download high resolution image](#)

Fes servir aquest espai com esborrany.

Noies	Noms	+ 169	150-169 cm	149-130 cm	- 130 cm
	Yajze				X
	Alba			X	
	Susay			X	
	Miriam			X	
	Marta			X	
	Clara			X	
	Esmeralda			X	
	Beatriz			X	
	Mónica			X	
	Maria			X	
	Ester			X	
	Esther			X	
	Claudia				X
	Sara			X	
	Mireia				X
	Irene	X			
Nois	Jordi			X	X
	Sergio R			X	
	Sergio F			X	
	Marc C			X	
	Marc L			X	
	Marc V			X	
	Guillem			X	
	Carles	X			
	Javi			X	
	Carles			X	
	Juan			X	
	Sigrid			X	

Figure 2b  
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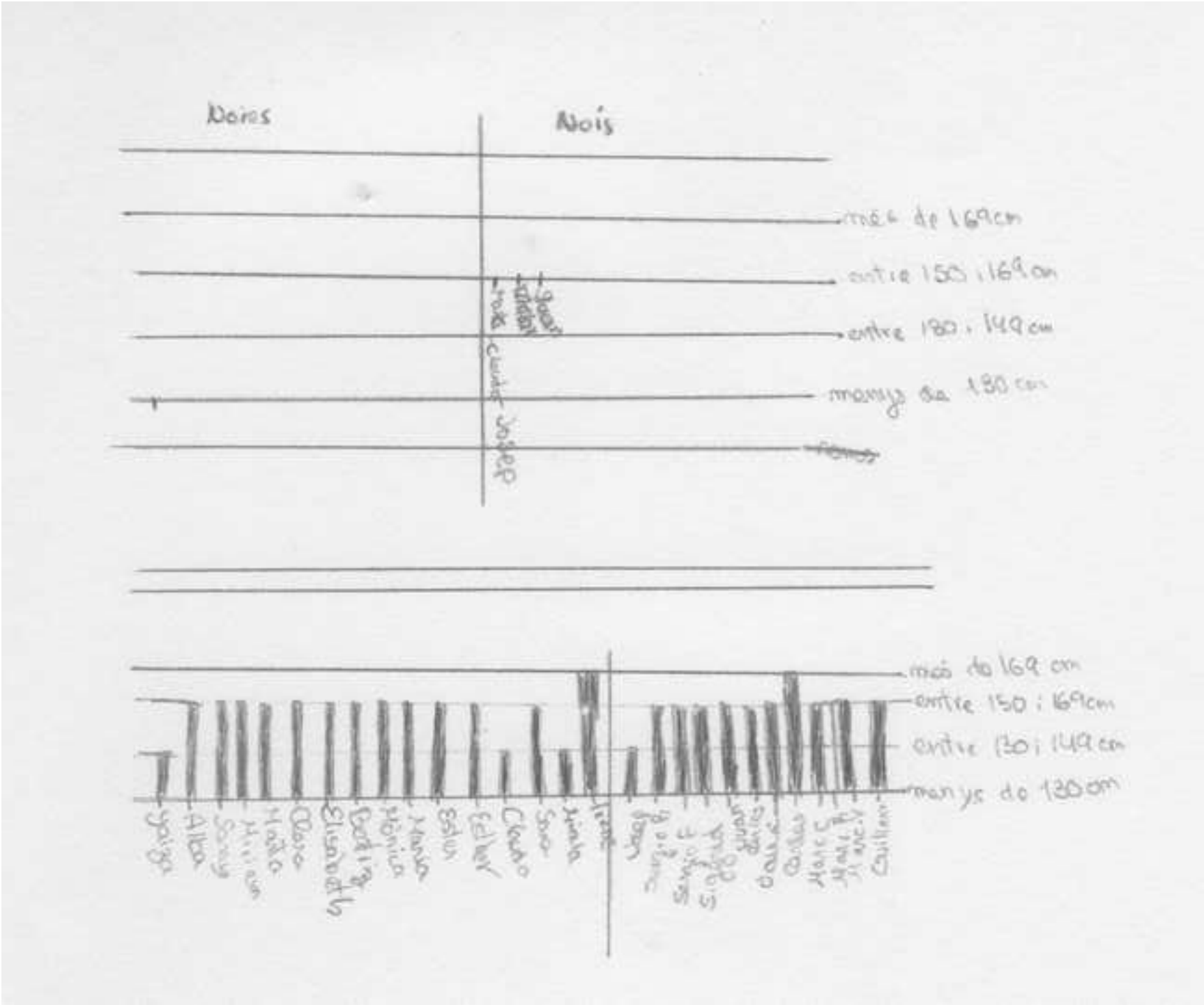


Figure 3

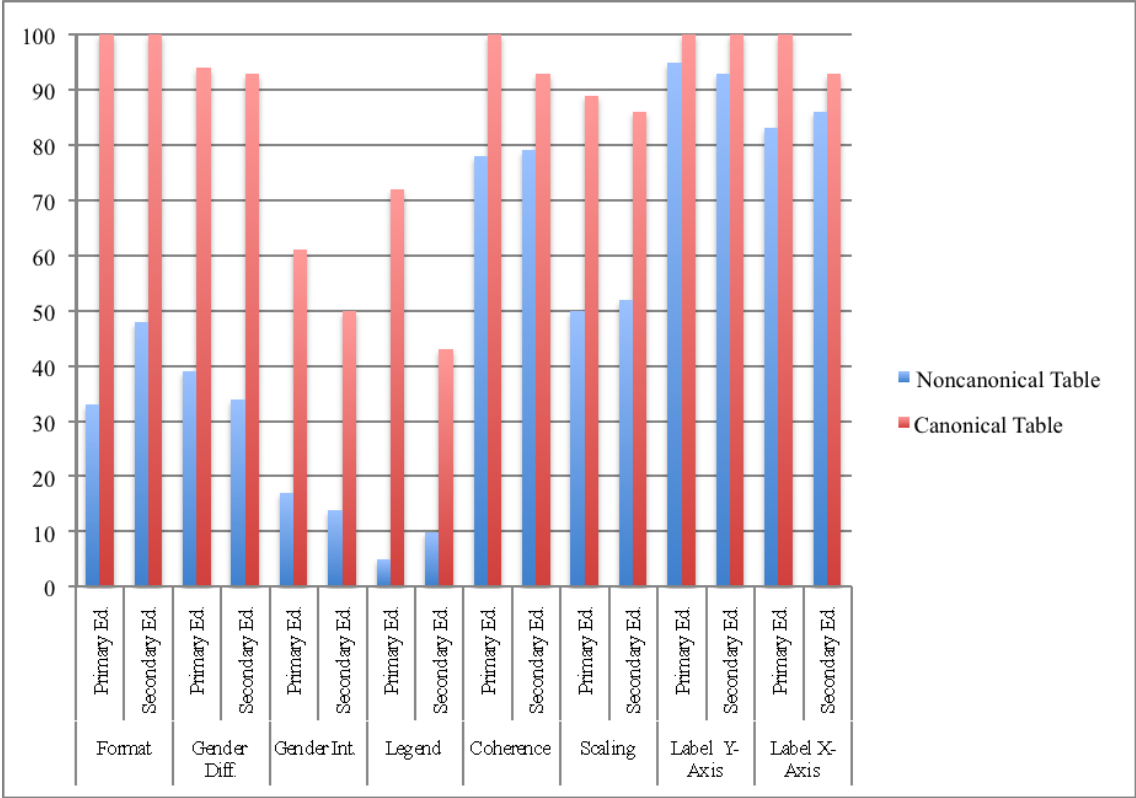


Figure 3. Distribution of Percentage of Graphs that Satisfied the Criteria according to Table Constructed and Educational Level

Figure 4a  
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	Nom	Cognom	Sexe	Mida
Menys de 130 cm	-Ana	-Aliaga	-Femení	-126 cm
	-Eug	-Molina	-Femení	-128 cm
entre 130 i 149 cm	-Fatima	-García	-Femení	-148 cm
	-Ainhac	-Grè	-Femení	-147 cm
	-Kontse	-Martín	-Femení	-142 cm
	-Joana	-Martínez	-Femení	-134 cm
	-Marc	-Molero	-Masculí	-146 cm
	-Enric	-Molina	-Masculí	-141 cm
	-Sara	-Peiro	-Femení	-144 cm
	-Joan	-Planas	-Masculí	-134 cm
	-Judith	-Roquer	-Femení	-132 cm
	-Francesc	-Sánchez	-Masculí	-149
	-Silvia	-Cano	-Femení	-165 cm
	-María	-Fernandez	-Femení	-154 cm
entre 150 i 169 cm	-Núria	-Gómez	-Femení	-156 cm
	-Andreu	-Guillem	-Masculí	-157 cm
	-Ismael	-Jiménez	-Masculí	-165 cm
	-Alícia	-Moreno	-Femení	-151 cm
	-Víctor	-Rojas	-Masculí	-167 cm
	-Àlex	-Toledo	-Masculí	-160 cm
	-Sergi	-Vega	-Masculí	-155 cm
	-Pau	-Blanco	-Masculí	-174 cm
	-Lluís	-De los Santos	-Masculí	-171 cm
més de 169 cm	-Olga	-García	-Femení	-172 cm
	-Miquel	-Gisbert	-Masculí	-173 cm

Figure 4b  
[Click here to download high resolution image](#)

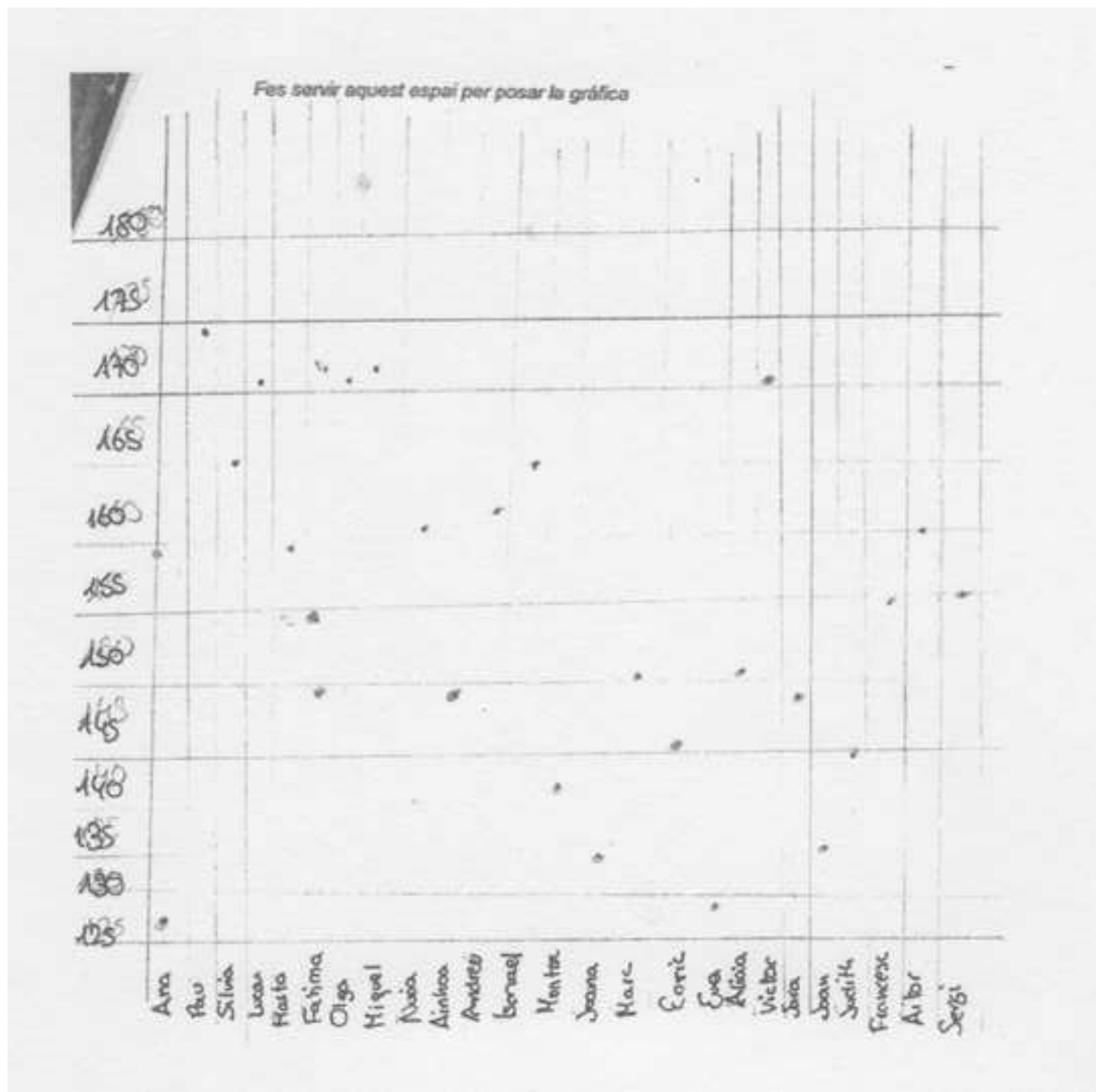


Figure 5  
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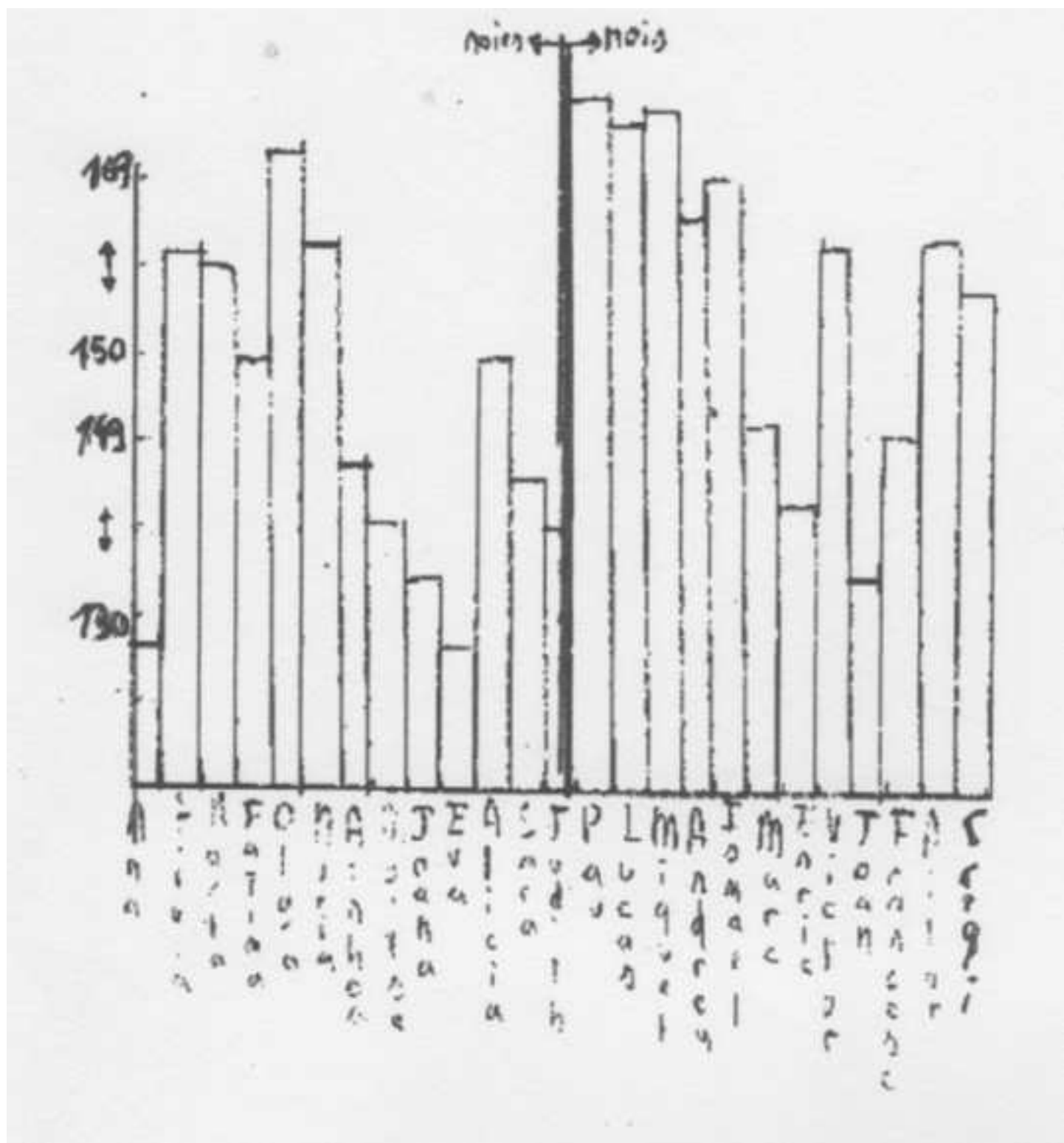


Figure 6  
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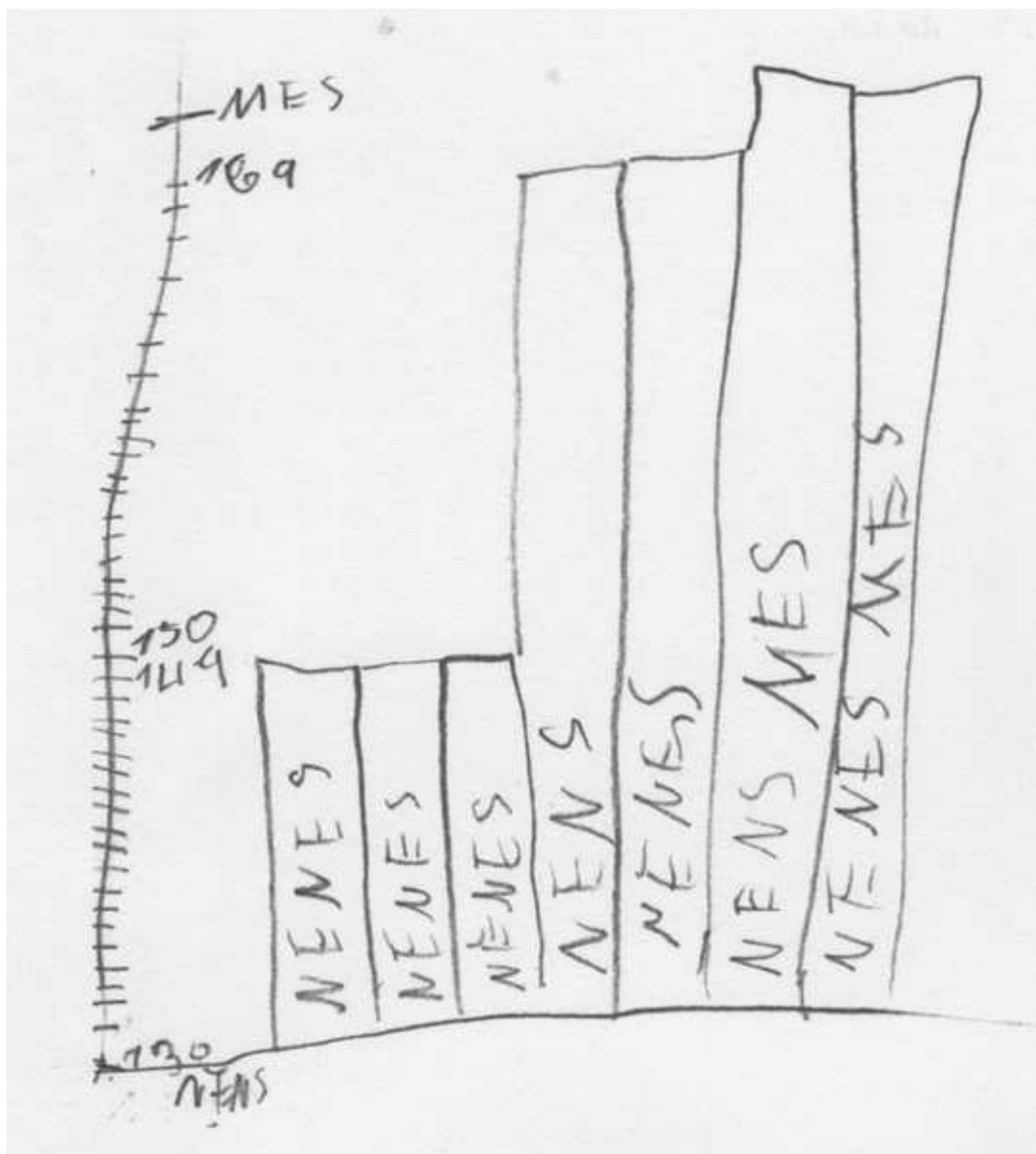




Figure 7  
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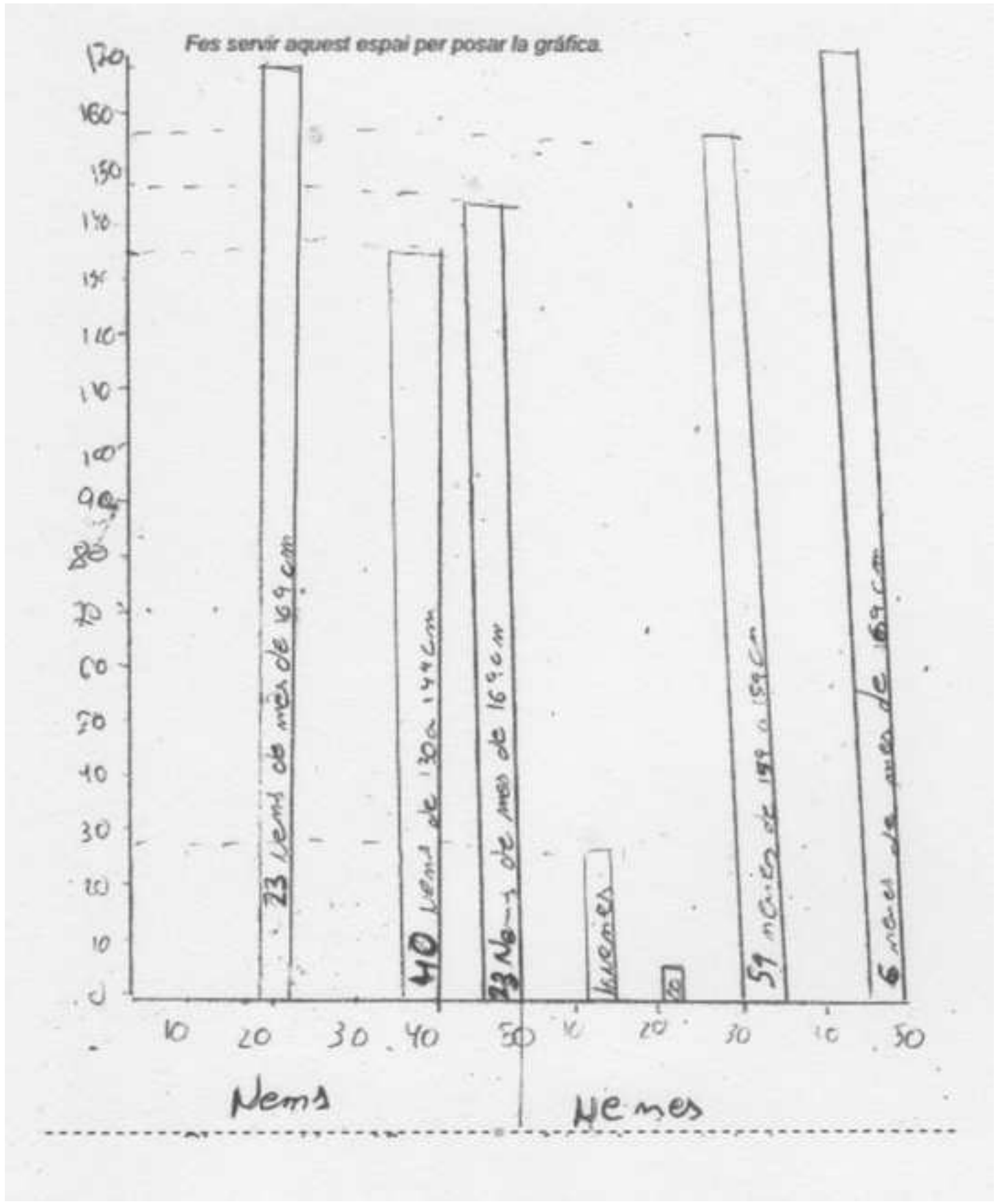




Figure 8  
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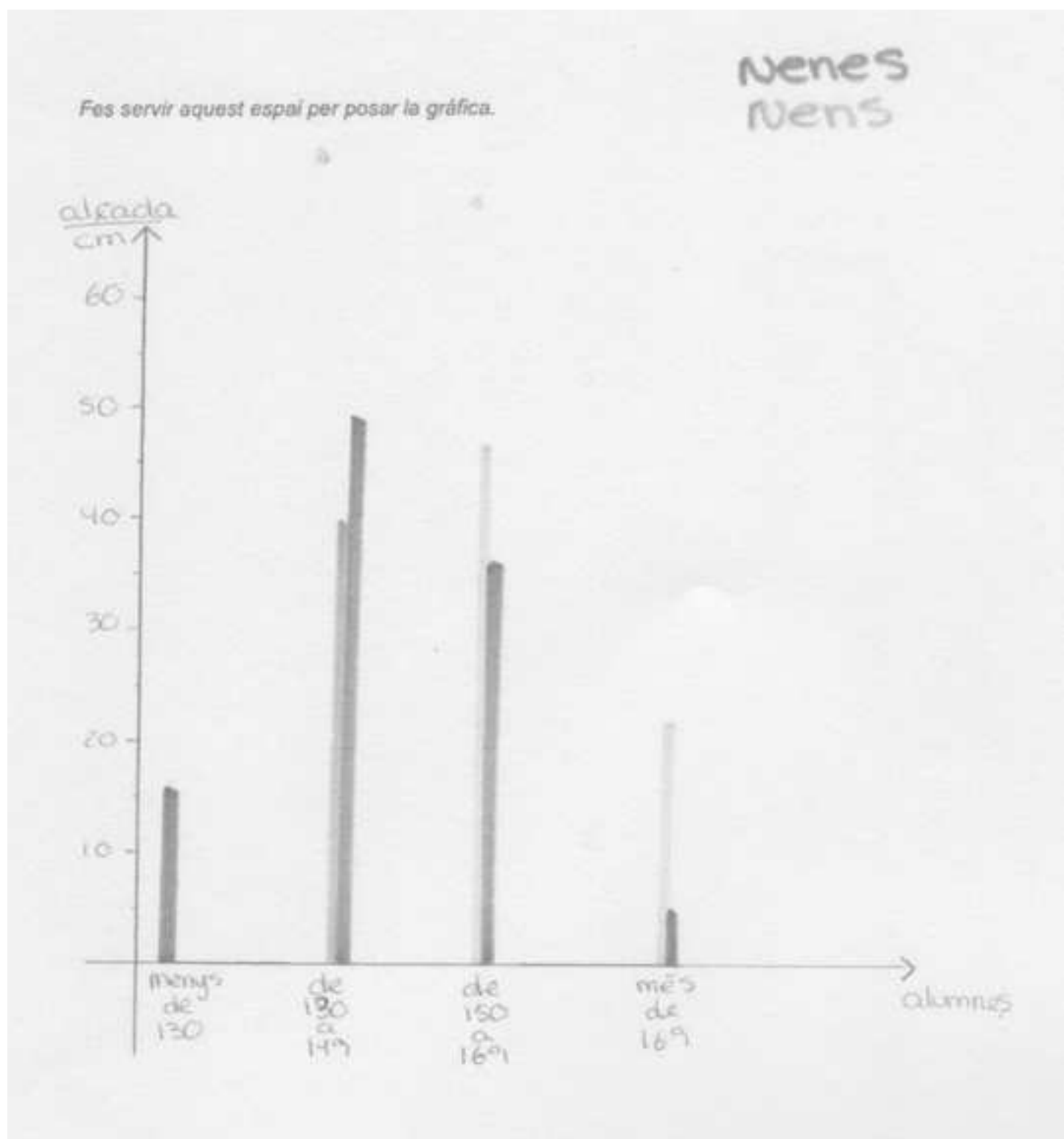


Table 1. Distribution of frequencies (and percentages) of the products of the participants according to educational level

	Blank	No Bar-Graph	Bar-Graph	Total
Primary Education	18 (25.3%)	17 (23.9%)	36 (50.7%)	71 (100%)
Secondary Education	34 (40.4%)	7 (8.3%)	43 (51.2)	84 (100%)
Total	52 (33.5%)	24 (15.4%)	79 (50.9%)	155 (100%)

Table 2. Distribution of frequencies (and percentages) of student graphs that satisfied each criterion following table construction according to educational levels and statistical results

	Primary Education	Secondary Education	
	n=36	n=43	Chi-square analysis
Format	24 (667%)	28 (65%)	ns
Gender Differentiation	24 (67%)	23 (53%)	ns
Gender Integration	14 (39%)	11 (26%)	ns
Legend	14 (39%)	9 (21%)	ns
Coherence	32 (89%)	36 (84%)	ns
Scaling	25 (69%)	27 (63%)	ns
Label y-axis	35 (97%)	41 (95%)	ns
Label x-axis	33 (92%)	38 (88%)	ns

Table 3. Distributions of frequencies (and percentages) of student graphs that satisfied each criterion according to the constructed table for each educational level

	Primary Education		Secondary Education	
	Non-canonical	Canonical	Non-canonical	Canonical
	(n=18)	(n=18)	(n=29)	(n=14)
Format	6 (33%)	18 (100%)	14 (48%)	14 (100%)
Gender diff.	7 (39%)	17 (94%)	10 (34%)	13 (93%)
Gender int.	3 (17%)	11 (61%)	4 (14%)	7 (50%)
Legend	1 (5%)	13 (72%)	3 (10%)	6 (43%)
Coherence	14 (78%)	18 (100%)	23 (79%)	13 (93%)
Scaling	9 (50%)	16 (89%)	15 (52%)	12 (86%)
Label y-axis	17 (95%)	18 (100%)	27 (93%)	14 (100%)
Label x-axis	15 (83%)	18 (100%)	25 (86%)	13 (93%)

Table 4. Chi-squared analysis of the association between the criteria of constructed graphs according to educational level for each type of table constructed

	Non-canonical table	Canonical table
	Primary Ed. vs. Secondary Ed.	Primary Ed. vs. Secondary Ed.
Format	$\chi^2(1)=1.01, p=.314$	constant
Gender differentiation	$\chi^2(1)=0.09, p=.760$	$\chi^2(1)=.034, p=.859$
Gender integration	$\chi^2(1)=0.071, p=.788$	$\chi^2(1)=0.39, p=.530$
Legend	$\chi^2(1)=0.365, p=.545$	$\chi^2(1)= 2.8, p=.09$
Coherence	$\chi^2(1)=0.61, p=.432$	$\chi^2(1)=1.3, p=.249$
Scaling	$\chi^2(1)=0.02, p=.967$	$\chi^2(1)=0.07, p=.788$
Label y-axis	$\chi^2(1)=0.657, p=.418$	constant
Label x-axis	$\chi^2(1)=0.695, p=.406$	$\chi^2(1)=1.28, p=.245$

Table 5. Chi-squared analysis of the association between the criteria of constructed graphs according to the type of tables constructed for each educational level

	Primary education	Secondary education
	Non-canonical vs. Canonical	Non-canonical vs. Canonical
Format	$\chi^2 (1)=18, p=.001$	$\chi^2 (1)=11.1, p=.001$
Gender differentiation	$\chi^2 (1)=12, p=.001$	$\chi^2 (1)=12.9, p=.001$
Gender integration	$\chi^2 (1)=7.5, p=.006$	$\chi^2 (1)= 6.5, p=.011$
Legend	$\chi^2 (1)=16.8, p=.001$	$\chi^2 (1)= 6.0, p=.014$
Coherence	$\chi^2 (1)=4.8, p=.030$	$\chi^2 (1)=1.27, p=.260$
Scaling	$\chi^2 (1)=6.4, p=.010$	$\chi^2 (1)=4.7, p=.031$
Label y-axis	$\chi^2 (1)=1.0, p=.310$	$\chi^2 (1)=3.25, p=.070$
Labels x-axis	$\chi^2 (1)=3.2, p=.070$	$\chi^2 (1)=4.0, p=.524$

Table 6. Table provided for the task: *Boys’ and Girls’ Height Groupings (in cm)*

	<i>Shorter than 130 ( &lt; 130 )</i>	<i>Between 130 and 149</i>	<i>Between 150 and 169</i>	<i>Taller than 169 ( &gt; 169 )</i>
<i>Boys</i>	0	40	48	23
<i>Girls</i>	16	51	36	6
<i>Total</i>	16	91	84	29

Table 7. Distribution of frequencies (and percentages) of student graphs based on a provided table that satisfied the criteria according to educational level

	Primary Education	Secondary Education	
	(n=49)	(n=73)	Statistical test
Format	40 (82%)	71 (97%)	$\chi^2 (1)=6.2, p=.014$
Gender diff.	46 (94%)	67 (92%)	ns
Gender int.	16 (33%)	39 (53%)	$\chi^2 (1)=4.9, p=.027$
Legend	8 (16%)	24 (33%)	$\chi^2 (1)=4.7, p=.029$
Coherence	36 (73%)	69 (94%)	$\chi^2 (1)=9.2, p=.002$
Scaling	35 (71%)	58 (79%)	ns
Label y-axis	46 (94%)	73 (100%)	ns
Label x-axis	44 (90%)	68 (93%)	ns



Table 8. Chi-squared analysis of the association between the criteria of constructed graphs according condition for each educational level

	Chi-square analysis	
	Primary Education	Secondary Education
Format	ns	$\chi^2 (1)=22.3, p=.001$
Gender Differentiation	$\chi^2 (1)=10.1, p=.001$	$\chi^2 (1)=22.8, p=.001$
Gender Integration	ns	$\chi^2 (1)=8.5, p=.03$
Legend	$\chi^2 (1)=5.5, p=.019$	ns
Coherence	ns	ns
Scaling	ns	$\chi^2 (1)=4.5, p=.033$
Label y-axis	ns	ns
Label x-axis	ns	ns

Figure Appendix C

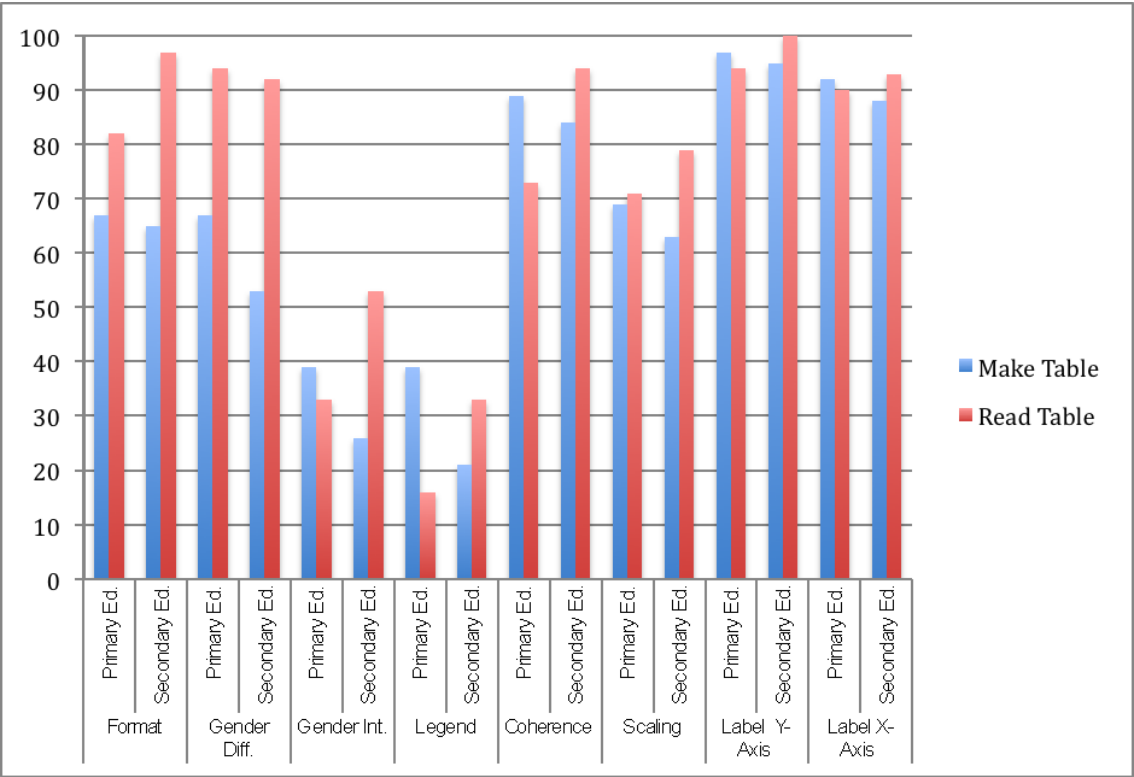


Figure Appendix C. Distribution of Percentage of Graphs that Satisfied the Criteria according to Condition (Make Table - Read Table) and Educational Level

Figure Captions

Figure 1. Marc’s canonical table. Primary student.

Figure 2(a). Marta’s non-canonical table. Secondary student.

Figure 2(b). Marta’s graph. Secondary student.

Figure 3. Distribution of Percentage of Graphs that Satisfied the Criteria according to Table Constructed and Educational Level

Figure 4(a). Xenia’s non-canonical table. Primary student.

Figure 4(b). Xenia’s graph. Primary student.

Figure 5. Melani’s graph. Non-canonical table. Primary student.

Figure 6. Jon’s graph from the provided table. Primary student.

Figure 7. Pat’s graph from the provided table. Secondary student.

Figure 8. Fer’s graph from the provided table. Secondary student.

Figure Appendix C. Distribution of Percentage of Graphs that Satisfied the Criteria according to Table Condition