Intergenerational Mobility under Education-Effort Complementarity

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Abstract: In this paper, we build a model that, according to the empirical evidence, gives raise to oscillations in wealth within a dynasty while keeping intergenerational persistence in education attainment. The mechanism that we propose is based on the interaction between effort and wealth suggested by the Carnegie effect, according to which wealthier individuals make less effort than the poorer. The oscillations in wealth arise from changes in the effort exerted by different generations as a response to both inherited wealth and college premium. Our mechanism generates a rich social stratification with several classes in the long run as a consequence of the combination of different levels of education and effort. Furthermore, we generate a large mobility in wealth among classes even in the long run. Our model highlights the role played by the minimum cost on education investment, the borrowing constraints, and the complementarity between effort and education.

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1. Introduction

In this paper, we present a mechanism generating intergenerational mobility and social stratification based on the interaction between wealth, education, and labor effort. Our analysis is motivated by two empirical facts. First, there exist empirical support for the so called "Carnegie conjecture", according to which those individuals who receive a large inheritance are tempted to put small effort in productive activities so that they may end up enjoying a small amount of wealth.[1] Several empirical papers have documented a negative relationship between labor supply and the amount of inheritance individuals receive. This reduction in the labor supply takes the form of a reduction in the number of hours worked, an early retirement decision, or direct job quitting (see Brown et al., 2010; Cox, 2014; Elinder et al., 2012; Erlend et al., 2012; Holtz-Eakin et al., 1993; Joulfaian and Wilhelm, 1994, 2006). The size of the negative effect of inheritance in the overall labor income found in these papers is very heterogeneous and depends crucially on both the period of the life cycle where the intergenerational transmission of wealth takes place and the expected or unexpected nature of inheritances. We should mention however that there is another channel through which the amount of inheritance could be positively correlated with earnings since it may favor entrepreneurship as it tends to make less binding the liquidity constraints associated with starting a new business and, moreover, the probability of success of that business increases with the amount of initial capital (see Cox, 2014; and Holtz-Eakin, 1994). We will abstract from entrepreneurship decisions in our analysis and restrict our focus on the effort decision in a regular labor market.

The second empirical fact motivating our analysis is the observed high intergenerational persistence of education especially within highly educated families (see Checchi et al., 1999, and Hertz et al., 2008). In particular, Hertz et al. estimated the correlation between years of schooling between fathers and their children for a large sample of 42 countries. One of the most striking results of their analysis is that the strongest correlations (with values of the correlation coefficient above 0.6) appear in South America (Peru, Ecuador, Panama, Chile, Brazil, Colombia and Nicaragua) and other countries like Egypt, Sri Lanka, and Pakistan where the credit constraints to finance education seem to be quite pervasive. The high persistence in education attainment in South America is also found in Behrman et al. (2001). However, Nordic countries display lower estimates of intergenerational education correlations (Chevalier et al, 2009 and Hertz et al., 2008), which is consistent with the idea that intergenerational educational persistence is lower in countries with a strong welfare state devoting a large fraction of public spending to education so that borrowing constraints have weaker effects on human capital investment.

Relying upon the previous empirical evidence, we aim to build a model that gives raise to oscillations in wealth within a dynasty while keeping intergenerational persistence in education. The mechanism that we propose is based on the interaction between effort and wealth suggested by the Carnegie conjecture. In particular, the oscillations in wealth arise from changes in the effort exerted by different generations

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[1] Andrew Carnegie, the famous 19th century steel businessman, stated: "the parent who leaves his son enormous wealth generally deadens the talents and energies of the son, and tempts him to live a less useful and less worthy life than he otherwise would ..." (Carnegie, 1962).

as a response to both inherited wealth and education return (or college premium). Our mechanism generates a rich social stratification with four classes in the long-run: (1) A poor class composed of unskilled individuals who do not make effort; (2) a rich class composed of skilled individuals who make effort; (3) a middle class composed of unskilled individuals who make effort; and (4) another middle class composed of skilled individuals who do not exert effort. Moreover, we generate large mobility among classes even in the long run. In particular, we obtain both upward and downward mobility and long-run cycles between the two classes of unskilled individuals and between the two classes of skilled individuals. These oscillations are in fact a direct consequence of the Carnegie conjecture: when an individual receives a large inheritance he exerts small effort so that the wealth of the family decreases. Since the next generation receives a small inheritance, their members make more effort and the wealth of the family increases again. This strong and deterministic mobility in wealth agrees with the studies reported by Cochell and Zeeb (2005), according to which six out of ten affluent families will lose the family fortune by the end of the second generation and nine out of ten will loose it by the end of the third generation. Our model will achieve however a deterministic reversal of fortune in just one generation. This extreme form of mobility will allow us to highlight the key assumptions underlying the mechanism at work.

Our theory combines several ingredients. First, we assume that investment in human capital is indivisible so that a minimum level of expenditure on education is required to acquire human capital. Second, we assume that individuals face a borrowing constraint so that only those with a sufficiently high level of initial wealth can afford the cost of education. These two assumptions impose a barrier on human capital investment for poor individuals.

Third, we assume that labor supply is endogenous and indivisible in the sense that individuals have to choose an occupation. We assume that an occupation is the set of productive activities that require a similar labor effort and, moreover, individuals will derive disutility from the amount of effort they exert. Furthermore, to generate a trade off in the occupation choice, we assume that labor earnings are an increasing function of both the human capital level and the amount of effort exerted by the worker. We can thus illustrate the difference in the behavior of intergenerational mobility of earnings and of education by taking into account the observed large intergenerational mobility in occupations inside each skill class. Under the assumption that the level of effort (and thus of earnings) associated with different occupations requiring the same level of skill is very heterogeneous, we could attribute the observed large intergenerational mobility in lifetime income to the sizeable differences among the average earnings of different occupations. In this respect, Zylbergerg (2013) documents the persistence in the level of skill between parents and sons together with a large variability in earnings within each level of skill. He reports average annual earnings of occupations with high education requirements of around $63,000 with a standard deviation of $25,000 and, as he says, "fathers in some well-paid occupations (surgeons) are very likely to have sons in average-salary occupations (teachers), without reneging on the long-term perspectives of the dynasty."

Finally, our results rely crucially on the following natural assumption: human capital and labor effort are strong complements when determining labor earnings.

In other words, the return from effort is higher for the more educated individuals than for the unskilled individuals. This assumption is quite standard in the literature dealing with labor and education (see Aguiar and Hurst, 2007; and Karasiotou, 2012). Moreover, there is empirical evidence suggesting the realism of this assumption. For instance, data from the OCDE Labor Force Statistics show that better-educated workers exhibit larger participation rates, retire later, and work more hours.

The main contribution of our model is to show that the existence of a rich social structure with several classes relies both on the differences among wages imposed by technology and on the policies implemented by the government. Thus, non-marginal changes in either wages or fiscal policy may alter the social structure and thus cause dramatic changes in wealth inequality. On the one hand, the changes in the wage distribution occurred in recent years, where we have witnessed an increase in the wage dispersion among different occupations, a rise in the skill premium, and a relative decrease of wages in occupations requiring an intermediate level of skill, have affected indeed the structure of social classes in the economy (see Acemoglu and Autor, 2011; Autor and Dorn, 2013; and Autor et al., 2016). On the other hand, the effect of changes in policy is analyzed in Section 7, where we show that under a strong welfare state that sets a very low cost of education, the class of educated individuals exerting effort disappears as the lifetime income net of education cost for the educated will be so high that effort will be discouraged according to the Carnegie conjecture.

Our paper is mainly related with that of Degan and Thibault (2012) where the Carnegie conjecture is explicitly modelled as the amount of effort (and thus of labor income) depends on the endogenous amount of inheritance individuals receive. The different constellations of parameter values concerning bequest motive and effort cost considered by these authors give raise to a plethora of patterns of dynamic accumulation of wealth. Our model differs from that of Degan and Thibault because we introduce accumulation of human capital. The acquisition of human capital through education faces a borrowing constraint so that only the individuals who have received a sufficiently large amount of inheritance can afford the indivisible cost of education. Therefore, the bequests left by a parent will play a triple role as they condition the initial wealth of their children, the amount of effort they will exert, and the skill level they will acquire through formal education. Both effort and skill will determine in turn the level of lifetime income of the next generation within the dynasty.

Our analysis is also related with the literature on the role of borrowing constraints in order to prevent individuals from acquiring education when there is an indivisible cost associated with schooling. As was pointed out by Galor and Zeira (1993), the access to education by the poorest individuals depends on whether they can borrow or not. When there are capital market imperfections resulting in borrowing constraints, those individuals with a level of wealth lying below some threshold value cannot afford the cost of education.[2] Intergenerational transfers from parents to children could help to ameliorate the negative effects of borrowing constraints on the accumulation of human capital. However, in an environment with credit market imperfections, only those individuals who receive a sufficiently large inheritance can invest in human capital (see Becker and Tomes, 1976; Eckstein and Zilcha, 1994; or Behrman et al., 1995).

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[2]Dynarski (2002) and Keane (2002) analyze empirically the effect of borrowing constraints on decisions concerning human capital acquisition.

Regarding the dynamics of wealth distribution, Galor and Zeira (1993) show that, if one assumes credit market imperfections and an indivisible cost of education, then the distribution of inherited wealth entirely determines the accumulation of human capital and the dynamics of the subsequent distribution of wealth. Note that in our model each individual will decide how much to invest in her own human capital. Other papers in this strand of literature attribute instead this decision to the parents (Galor and Moav, 2004 and 2006; Alonso-Carrera, et al., 2012).

The paper is organized as follows. Section 2 presents the basic model of intergenerational transmission of wealth and of individual decisions concerning education and effort. Section 3 characterizes the dynamics of bequests. Section 4 and 5 characterize the dynamics of bequest, effort, and human capital in the short and in the long run, respectively, for the relevant case where education investment is potentially profitable for all individuals. Section 6 analyzes the inequality of wealth in the long run. Section 7 discusses the relationship between the characteristics of the welfare state and social stratification. Section 8 concludes the paper.

2. The model

Let us consider an overlapping-generations economy (OLG) where individuals live for two periods and have offspring at the end of the first period of life. The exogenous number of children per parent is $n > 0$, i.e., the gross rate of population growth is n. In the first period of his life an individual born in period $t-1$ receives an inheritance b_{t-1} from his parent. This inheritance can be devoted to save the amount s_{t-1} or to pay for education through formal schooling. In the second period of their lives, individuals work, receive a salary w_t, get a gross return R_t per unit of saving, consume the amount c_t, and leave the amount b_t of bequest to each of their offspring. We index a generation by the period at which their members work. Thus, the budget constraints in the two periods of life for an individual belonging to generation t are

$$b_{t-1} = x_{t-1} + s_{t-1} \qquad (2.1)$$

and

$$w_t + R_t s_{t-1} = c_t + n b_t, \qquad (2.2)$$

where x_{t-1} is the amount invested in education.

We assume that education has a fixed indivisible cost μ and impose the typical borrowing constraint on education acquisition so that individuals can only pay for their own education if the amount of inheritance is larger than the fixed cost μ of education. This borrowing constraint implies that $x_{t-1} \leq b_{t-1}$ or, equivalently, $s_{t-1} \geq 0$.[3]

Agents derive utility from the amount consumed in the second period of their lives and from the bequest they leave to each of their descendents. Therefore individuals display a "joy of giving" motivation for bequests (or "warm-glow" altruism) as in Abel (1985) and Yaari (1965). Moreover, we assume that individuals may exert effort when

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[3] The model could be reformulated along the lines of Alonso-Carrera et. al. (2012), Galor and Moav (2004 and 2006), or Zilcha (2003) in order to allow parents to pay directly for the education of their children.

they work and this effort results in a loss of utility. We assume the following logarithmic functional form:

$$U_t = \ln c_t + \beta \ln b_t - \rho e_t, \tag{2.3}$$

where e_t is the level of effort, which is assumed to be a discrete variable taking the values 1 for the workers who exert effort or 0 for the workers who do not exert effort, $e_t \in \{0, 1\}$. As we have also mentioned in the Introduction, we can associate the level of effort with a given occupation so that there are occupations in the economy that require the same level of skill but different amount of effort. We are thus assuming for simplicity that there are only two occupations for each level of skill. Given this assumed discrete nature of effort, the assumption of linear disutility from effort is made without loss of generality.

Individuals live in a small open economy with a constant returns to scale technology. Hence, the gross rate R_t of return on capital is exogenously given as it has to be equal to the international rate of return and, thus, since the capital-efficiency units of labor ratio is fully determined by R_t, the wage rate $\overline{w}_t$ per efficiency unit of labor is also given. We assume that both rates are constant along time, $R_t = R > 0$ and $\overline{w}_t = \overline{w}$ for all t. The number of efficiency units supplied by a worker born in period $t-1$ depends on both his level of human capital h_t and the amount of effort e_t he exerts according to the strictly positive function $\varepsilon(h_t, e_t)$.

We consider a simple form of technological indivisibility in the production of human capital. In particular, the individual level of capital can take two values depending on whether the investment in education is below or above the fixed indivisible cost μ of education. Thus, the level of human capital at period t is given by the following function:

$$h_t = \begin{cases} 1 & \text{if } x_{t-1} \geq \mu, \\ \\ 0 & \text{if } x_{t-1} < \mu. \end{cases} \tag{2.4}$$

A level of human capital equal to 1 corresponds to educated (or skilled) workers, whereas a level equal to zero corresponds to uneducated (or unskilled) workers. Obviously, the optimal investment in education for individuals who cannot afford the minimum cost μ is $x_{t-1} = 0$, whereas those individuals who end up being educated will choose $x_{t-1} = \mu$. Observe that those individuals who receive an inheritance b_{t-1} strictly smaller than μ cannot invest in education even if they wish to do so. Therefore, the salary compensation w_t of a worker with the level h_t of human capital exerting the amount e_t of effort will be equal to $\varepsilon(h_t, e_t)\overline{w}$. Since the wage $\overline{w}$ per efficiency unit is constant, to ease the notation we define the earning function $w(h_t, e_t) \equiv \varepsilon(h_t, e_t)\overline{w}$ so that

$$w_t = w(h_t, e_t). \tag{2.5}$$

We assume that the earning function $w(h_t, e_t)$ satisfies the following assumption:

Assumption A

(a) $w(h_t, 1) > w(h_t, 0)$ for all values of h_t;

(b) $w(1, e_t) > w(0, e_t)$ for all values of e_t; and

(c) $w(1, 1) - w(1, 0) > w(0, 1) - w(0, 0)$

The previous assumption is very plausible. Parts (a) and (b) say that wages are increasing in human capital and effort, while part (c) means that both arguments of the function $w(\cdot, \cdot)$, human capital and effort, are complementary, i.e., the function w is supermodular since to exert effort is more profitable for skilled individuals than for unskilled ones. Note that Assumption A does not allow us to make a comparison between the labor income $w(1,0)$ of educated individuals who do not make effort and the labor income $w(0,1)$ of non-educated individuals who make effort. Note that part (c) can be rewritten as

$$w(1,1) - w(0,1) > w(1,0) - w(0,0), \qquad (2.6)$$

which means that education is more profitable for the individuals that are willing to exert positive effort. The complementarity between effort and education implies that a rich individual who can afford the cost of education but is not willing to exert effort may end up not investing in education when the wage premium of education under no effort is small. Similarly, a poor individual who cannot pay for his education may choose a low level of effort when the wage premium associated with effort is too low for non-educated individuals.

The problem faced by a generic individual of generation t is to find the values of c_t, b_t, e_t, and h_t in order to maximize (2.3) subject to

$$w_t + Rb_{t-1} - Rx_{t-1} = c_t + nb_t, \qquad (2.7)$$

(2.4), $e_t \in \{0,1\}$, and $x_{t-1} \leq b_{t-1}$. Note that the constraint (2.7) follows from combining (2.1) and (2.2) and eliminating the saving s_{t-1}.

We solve this problem in two steps. First, given the values of education investment x_{t-1} and effort e_t, we obtain the following optimal values for consumption and bequest:

$$c_t = \frac{w_t + Rb_{t-1} - Rx_{t-1}}{1 + \beta} \qquad (2.8)$$

and

$$b_t = \frac{\beta (w_t + Rb_{t-1} - Rx_{t-1})}{n(1 + \beta)}. \qquad (2.9)$$

Next, we evaluate the utility function (2.3) at the optimal level of consumption (2.8) and bequests (2.9) and use (2.5) to obtain the indirect utility

$$U(h_t, e_t) = (1 + \beta) \ln [w(h_t, e_t) - R\, X(h_t) + Rb_{t-1}] - \rho e_t + M, \qquad (2.10)$$

where M is a constant and $X(h_t)$ is the function mapping human capital into education investment, which is implicitly defined by (2.4),

$$x_{t-1} = X(h_t) = \begin{cases} \mu & \text{if } h_t = 1, \\ \\ 0 & \text{if } h_t = 0. \end{cases}$$

Then, we solve for the optimal values of effort and human capital (or, equivalently, of investment in education). Note that the optimal decisions will depend on the inheritance received by individuals. The optimal decisions on education investment and effort are obtained from the direct comparison between different utility levels. To simplify this comparison we define $\theta \equiv \exp [\rho/(1 + \beta)] > 1$ so that, using (2.10), we obtain the following implications:

1. The utility of an unskilled agent who makes positive effort is larger than the utility of an agent who acquires education but does not make positive effort if $U_t(0,1) > U_t(1,0)$, that is, if

$$\widetilde{b_1} \equiv \frac{w(0,1) - \theta[w(1,0) - R\mu]}{(\theta - 1)R} > b_{t-1}.$$

2. Non-educated agents decide to make effort if $U(0,1) > U(0,0)$, that is, if

$$\widetilde{b_2} \equiv \frac{w(0,1) - \theta w(0,0)}{(\theta - 1)R} > b_{t-1}. \qquad (2.11)$$

3. Agents not exerting effort decide to invest in education if $U(1,0) > U(0,0)$, that is, if

$$w(1,0) - w(0,0) > R\mu.$$

4. Agents exerting effort decide to invest in education if $U(1,1) > U(0,1)$, that is, if

$$w(1,1) - w(0,1) > R\mu.$$

5. Educated agents decide to make positive effort if $U(1,1) > U(1,0)$, that is, if

$$\widetilde{b_3} \equiv \frac{w(1,1) - \theta w(1,0)}{(\theta - 1)R} + \mu > b_{t-1}. \qquad (2.12)$$

6. The utility of an agent who acquires education and make positive effort is larger than the utility of an agent who does neither educate nor make positive effort if $U_t(1,1) > U_t(0,0)$, that is, if

$$\widetilde{b_4} \equiv \frac{w(1,1) - \theta w(0,0) - R\mu}{(\theta - 1)R} > b_{t-1}.$$

Implications 1, 2, 5 and 6 highlight the role of the amount of bequests in order to induce workers to exert effort. When the amount of inheritance individuals receive is too large, the marginal utility of their consumption and bequest turns out to be small and, thus, they optimally decide not to make effort as the cost of effort is larger than the potential increase in utility arising from the amounts of own consumption and left bequest. We have thus made explicit the mechanism lying behind the Carnegie conjecture discussed in the Introduction.

To close the characterization of each individual's optimal plan, we should compare the threshold levels of bequests $\widetilde{b_1}$, $\widetilde{b_2}$, $\widetilde{b_3}$ and $\widetilde{b_4}$. First, we obtain that $\widetilde{b_4} > \widetilde{b_2}$ and $\widetilde{b_3} > \widetilde{b_1}$ if and only if

$$w(1,1) - w(0,1) > R\mu.$$

Note that the previous condition means that education is profitable for at least those agents who exert effort. Second, we obtain that $\widetilde{b_4} > \widetilde{b_3}$ and $\widetilde{b_2} > \widetilde{b_1}$ if and only if

$$w(1,0) - w(0,0) > R\mu. \qquad (2.13)$$

This condition holds when education is profitable for those agents who do not exert effort, which implies that it is also profitable for the agents who exert effort as follows from part (c) of Assumption A. Finally, we obtain that $\widetilde{b}_3 > \widetilde{b}_2$ if and only if

$$\theta < \widetilde{\theta}_1 \equiv \frac{w(1,1) - w(0,1) - R\mu}{w(1,0) - w(0,0) - R\mu}. \tag{2.14}$$

The numerator of (2.14) is the skill premium net of education cost for those individuals who exert positive effort, whereas the denominator is the net skill premium for those who do not make effort. Therefore, we have that $\widetilde{b}_3 > \widetilde{b}_2$ when the net labor income gain from education for individuals exerting effort is sufficiently large relative to the net labor income gain for the individuals who do not make effort.

3. The dynamics of bequests

The characterization of the equilibrium dynamics in this economy depends crucially on part (c) of Assumption A, according to which the effort premium is higher for the skilled individuals than for the unskilled ones or, equivalently, the education premium of those agents who make positive effort is larger than the education premium of the individuals who do not make positive effort (see (2.6)). This assumption is compatible with the following configurations of the parameter values characterizing the wage premia and education cost:

Configuration 1. $R\mu > w(1,1) - w(0,1)$.

Here the capitalized cost of education is larger than the increase in wage due to education for the individuals exerting effort. Therefore, according to (2.6) education is never profitable and no agent decides to be educated. Since agents never get educated in this scenario, the threshold $\widetilde{b}_2$ is the unique relevant threshold. Therefore, agents make positive effort if $b_{t-1} < \widetilde{b}_2$ and make no effort if $b_{t-1} > \widetilde{b}_2$.

Configuration 2. $w(1,1) - w(0,1) > R\mu > w(1,0) - w(0,0)$.

Here education is profitable only for those agents who exert effort. It is immediate to see that the thresholds $\widetilde{b}_1$ and $\widetilde{b}_3$ are not relevant for the dynamics of bequest and, moreover, $\widetilde{b}_2 < \widetilde{b}_4$ in this case. On the one hand, if $\widetilde{b}_2 < b_{t-1} < \widetilde{b}_4$, then $U(1,1) > U(0,0) > U(1,0)$ and $U(1,1) > U(0,1)$ for any value of $\widetilde{b}_1$ and $\widetilde{b}_3$. The previous first inequality follows from the fact that $b_{t-1} < \widetilde{b}_4$. The second inequality comes from the fact that education is not profitable for those individuals who do not make effort. The third inequality follows from the fact that education is profitable for agents who make positive effort. On the other hand, if $b_{t-1} > \widetilde{b}_4$ then $U(0,0) > U(1,1) > U(0,1)$ and $U(0,0) > U(1,0)$ for any value of $\widetilde{b}_1$ and $\widetilde{b}_3$. The first inequality follows from the fact that $b_{t-1} > \widetilde{b}_4$. The second inequality arises from the fact that education is profitable for those individuals who make effort. The third inequality comes from the fact that education is not profitable for agents who do not make positive effort. Therefore, the only relevant inequality for the dynamics of bequest in this scenario is $\widetilde{b}_2 < \widetilde{b}_4$. Finally, under this configuration, we should distinguish between the following two cases:

(a) If $b_{t-1} < \mu$ then agents cannot afford the cost of education. In this case, agents make positive effort if $b_{t-1} < \widetilde{b}_2$ and make no effort if $b_{t-1} > \widetilde{b}_2$.

(b) If $b_{t-1} > \mu$ then agents can afford the cost of education. However, they will exert positive effort and, thus, they will become educated, if and only if $b_{t-1} < \widetilde{b}_4$. Otherwise, they will never acquire education nor exert effort.

Configuration 3. $w(1,0) - w(0,0) > R\mu$.

Here all agents want to invest in education since it is always profitable to become skilled regardless of the effort level exerted by workers. Under this configuration, the thresholds $\widetilde{b}_1$ and $\widetilde{b}_4$ are not relevant for the dynamics of bequests because they are respectively smaller and larger than $\widetilde{b}_3$. If $b_{t-1} < \widetilde{b}_3$, then $U(1,1) > U(1,0) > U(0,0)$ and $U(1,1) > U(0,1)$ for any value of $\widetilde{b}_1$ and $\widetilde{b}_4$. The first inequality follows from the fact that $b_{t-1} < \widetilde{b}_3$, whereas the second and third inequalities come from the fact that education is always profitable. On the contrary, if $b_{t-1} > \widetilde{b}_3$, then $U(1,0) > U(1,1) > U(0,1)$ and $U(1,0) > U(0,0)$ for any value of $\widetilde{b}_1$ and $\widetilde{b}_4$. The first inequality follows from the fact that $b_{t-1} > \widetilde{b}_3$, whereas the second and third inequalities arise from the fact that education is always profitable. Therefore, the values of μ, $\widetilde{b}_2$ and $\widetilde{b}_3$ fully determine the dynamics of bequests. In this scenario, we should distinguish between the same two cases appearing in the previous parameter Configuration 2:

(a) If $b_{t-1} < \mu$ then agents cannot afford the cost of education. In this case, agents make positive effort if $b_{t-1} < \widetilde{b}_2$ and make no effort if $b_{t-1} > \widetilde{b}_2$.

(b) If $b_{t-1} > \mu$ then agents can afford the cost of education. However, they will exert positive effort depending on the values of $\widetilde{b}_3$.

We will conduct a detailed study of this Configuration 3 in the next section.

For all parameter configurations, we can use (2.9) to write the equilibrium dynamics of bequest as the following difference equation:

$$b_t \equiv B(b_{t-1}, h_t, e_t) = \frac{\beta R}{n(1+\beta)} \left[b_{t-1} + \frac{w(h_t, e_t)}{R} - X(h_t) \right]. \tag{3.1}$$

As it is customary in these models, we need a high rate n of population growth, a low rate R of return on saving and a small intergenerational discount factor β in order to prevent wealth from growing unboundedly across generations within the same dynasty. The following assumption imposes accordingly the boundedness of the sequence of bequests within a dynasty:

Assumption B

$$\lambda \equiv \frac{\beta R}{n(1+\beta)} < 1. \tag{3.2}$$

We can represent the dynamics of bequest in the (b_{t-1}, b_t) space by fixing the values h_t of human capital and e_t of effort. In this way, we obtain that the dynamics of bequest is characterized by the piecewise linear function $B(b_{t-1}, h_t, e_t)$ and the thresholds of

11

inherited bequest μ, $\widetilde{b}_2$, $\widetilde{b}_3$ and $\widetilde{b}_4$, which determine in turn the endogenous values of human capital h_t and effort e_t. We will use the following notation: $B^1(b_{t-1}) \equiv B(b_{t-1}, 0, 0)$, $B^2(b_{t-1}) \equiv B(b_{t-1}, 0, 1)$, $B^3(b_{t-1}) \equiv B(b_{t-1}, 1, 0)$ and $B^4(b_{t-1}) \equiv B(b_{t-1}, 1, 1)$. From Assumption A we directly obtain that $B^3(b_{t-1}) < B^4(b_{t-1})$ and $B^1(b_{t-1}) < B^2(b_{t-1})$. Moreover, we can obtain the following additional orderings:

(i) $B^3(b_{t-1}) > B^1(b_{t-1})$ when $w(1,0) - w(0,0) > R\mu$, that is, when education is profitable for all agents regardless of the effort they exert. This is the aforementioned Configuration 3 described above.

(ii) $B^4(b_{t-1}) > B^2(b_{t-1})$ when $w(1,1) - w(0,1) > R\mu$, that is, when education is profitable for those agents who make positive effort. This situation can appear when the economy is under the Configurations 2 or 3 described above.

The fixed points of the bequest function (3.1) are the potential steady states for the amount of bequest. These four potential steady states values are given by

$$b_1^* = B^1(b_1^*) = \left(\frac{\lambda}{1-\lambda}\right)\left[\frac{w(0,0)}{R}\right], \tag{3.3}$$

$$b_2^* = B^2(b_2^*) = \left(\frac{\lambda}{1-\lambda}\right)\left[\frac{w(0,1)}{R}\right], \tag{3.4}$$

$$b_3^* = B^3(b_3^*) = \left(\frac{\lambda}{1-\lambda}\right)\left[\frac{w(1,0)}{R} - \mu\right], \tag{3.5}$$

and

$$b_4^* = B^4(b_4^*) = \left(\frac{\lambda}{1-\lambda}\right)\left[\frac{w(1,1)}{R} - \mu\right]. \tag{3.6}$$

In the next section we will characterize the transitional dynamics driven by the bequest functions $B^i(\cdot)$, $i = 1, 2, 3, 4$. We will analyze the evolution of bequest, effort and human capital when the investment in education is always profitable, which corresponds to the parametric Configuration 3. In the Appendix A we conduct the analysis for the other two configurations.

4. Transitional dynamics when education is always profitable

In this section, we characterize the one-period transition of the endogenous variables, inheritance, effort, and human capital, across generations when (2.13) holds. Note that, given an initial value of inheritance, individuals choose the optimal values of human capital, effort and bequest left to the descendants and, moreover, the levels of human capital and effort fully determine total individual lifetime income. To conduct a comprehensive analysis, we should consider the three parameter configurations discussed in the previous section. However, since we are interested in an economy where four classes of individuals emerge, namely, educated rich, educated poor, uneducated rich, and educated poor individuals, we will restrict our analysis to Configuration 3 in the previous section, which corresponds to a situation where the investment in acquiring human capital is always profitable regardless of the amount of effort. On

the one hand, under Configuration 1 the cost of education is so high that nobody will acquire education so that no educated individuals will appear in the long-run equilibrium. On the other hand, under the parameter Configuration 2, the class formed by the educated individuals with the smallest earnings (i.e., educated individuals who do not make positive effort) will not appear in equilibrium since education is only profitable for the individuals exerting effort. Therefore, we are going to assume from now on that $w(1,0) - w(0,0) > R\mu$, which from the supermodularity of the earning function (see part (c) in Assumption A) implies that $w(1,1) - w(0,1) > R\mu$ so that the education premium is always larger than the capitalized value of education cost regardless of the effort level. In this case, all the branches of the bequest function (3.1) may be operative. Moreover, as it was established in the previous section, we know that $B^1(b_{t-1}) < B^2(b_{t-1}) < B^4(b_{t-1})$ and $B^1(b_{t-1}) < B^3(b_{t-1}) < B^4(b_{t-1})$ for all b_{t-1}.

Given b_{t-1} individual decisions on bequests, education, and effort will depend on the education cost μ and the values of the thresholds $\widetilde{b}_2$ and $\widetilde{b}_3$. Hence, we should distinguish among several cases depending on the ranking among the values of μ, $\widetilde{b}_2$ and $\widetilde{b}_3$. We know from the previous section that $\widetilde{b}_2 < \widetilde{b}_3$ if and only if $\theta < \widetilde{\theta}^1$ (see (2.14)). We next proceed with the analysis of all these cases:

Case 1. $\mu > \max\left\{\widetilde{b}_2, \widetilde{b}_3\right\}$.

Here the evolution of bequests, education and effort is given by

$$\{b_t, h_t, e_t\} = \begin{cases} \left\{B^2(b_{t-1}), 0, 1\right\} & \text{if } b_{t-1} < \widetilde{b}_2, \\ \\ \left\{B^1(b_{t-1}), 0, 0\right\} & \text{if } b_{t-1} \in \left[\widetilde{b}_2, \mu\right), \\ \\ \left\{B^3(b_{t-1}), 1, 0\right\} & \text{if } b_{t-1} \geq \mu. \end{cases}$$

Observe that the value of the threshold $\widetilde{b}_3$ is irrelevant in this case. The dynamics of the variables b_{t-1}, h_t, and e_t is fully governed by the relationship between μ, $\widetilde{b}_2$ and the potential steady states b_1^*, b_2^* and b_3^*. Since in this case the number of potential steady-states can be at most three, only three types of individuals (or social classes) may appear in the long run: (i) unskilled agents who exert effort, (ii) unskilled who make no effort, and (iii) skilled who do not exert effort. Moreover, several stationary dynamics, which involve different social classes in the long run, are possible: we can have locally stable social classes and cycles involving switches between two social classes.

Case 2. $\widetilde{b}_3 < \mu < \widetilde{b}_2$.

In this case the transition of b_t, h_t, and e_t is given by

$$\{b_t, h_t, e_t\} = \begin{cases} \left\{B^2(b_{t-1}), 0, 1\right\} & \text{if } b_{t-1} < \mu \\ \\ \left\{B^3(b_{t-1}), 1, 0\right\} & \text{if } b_{t-1} \geq \mu. \end{cases}$$

The relevant dynamics is fully determined by the relationship between μ and the potential steady states b_1^* and b_2^*. It is straightforward to see that there will be at most two potential steady states and, hence, only two social classes may appear in the

long run one: the class of unskilled individuals exerting effort and the class of skilled individuals making no effort.

Case 3. $\widetilde{b}_2 < \mu < \widetilde{b}_3$.

Here the transition of the endogenous variables is given by

$$\{b_t, h_t, e_t\} = \begin{cases} \left\{B^2\left(b_{t-1}\right), 0, 1\right\} & \text{if } b_{t-1} < \widetilde{b}_2, \\[2mm] \left\{B^1\left(b_{t-1}\right), 0, 0\right\} & \text{if } b_{t-1} \in \left[\widetilde{b}_2, \mu\right), \\[2mm] \left\{B^4\left(b_{t-1}\right), 1, 1\right\} & \text{if } b_{t-1} \in \left[\mu, \widetilde{b}_3\right), \\[2mm] \left\{B^3\left(b_{t-1}\right), 1, 0\right\} & \text{if } b_{t-1} \geq \widetilde{b}_3. \end{cases} \tag{4.1}$$

In the next section we will show that in this scenario the dynamics of the variables b_t, h_t, and e_t is fully determined by the relationship between μ, $\widetilde{b}_2$, $\widetilde{b}_3$ and the potential steady states. Note that in this case there are at most four potential stationary values of bequests b_1^*, b_2^*, b_3^* and b_4^*, with $b_1^* < b_2^* < b_4^*$ and $b_3^* < b_4^*$. Here many configurations are possible in the long run. For instance, if $\widetilde{b}_2 < b_1^* < \mu < b_2^* < b_4^* < \widetilde{b}_3$ the economy converges towards a two-class society where the two classes correspond to the steady states b_1^* and b_4^*, with unskilled workers not exerting effort and with skilled workers making effort, respectively. Other configurations are possible like, for instance, a two-class society with the classes being locally stable, a three-class society with two classes forming a cycle and the other being locally stable, a single social class constituting a stable stationary equilibrium, a four class-society where two classes form a cycle and the other two form another cycle. The latter case, which could arise under some additional parametric assumptions, will be of special interest for us since it allows the possibility of delivering four social classes in the long run.

Case 4. $\mu < \min\left\{\widetilde{b}_2, \widetilde{b}_3\right\}$.

In this case the evolution of bequests, education and effort is given by

$$\{b_t, h_t, e_t\} = \begin{cases} \left\{B^2\left(b_{t-1}\right), 0, 1\right\} & \text{if } b_{t-1} < \mu, \\[2mm] \left\{B^4\left(b_{t-1}\right), 1, 1\right\} & \text{if } b_{t-1} \in \left[\mu, \widetilde{b}_3\right), \\[2mm] \left\{B^3\left(b_{t-1}\right), 1, 0\right\} & \text{if } b_{t-1} \geq \widetilde{b}_3. \end{cases}$$

Observe that in this case the threshold $\widetilde{b}_2$ is irrelevant. Moreover, the relevant dynamics of the endogenous variables is fully determined by the relationship between μ, $\widetilde{b}_3$, and the potential steady states. Since in this case the number of steady states can be at most three, only three classes may appear in the long run: (i) a class with unskilled agents who exert effort, (ii) a class with skilled agents who make no effort, and (iii) a class with skilled agents who exert effort. Several stationary situations are possible in this case: we can have locally stable classes and cycles involving switches between two classes.

5. The dynamics of dynastic wealth and the existence of cycles

In this section we will analyze the long-run dynamics of lifetime income and bequests within a given dynasty by using the equilibrium transition of lifetime income and bequests characterized in the previous section. The dynamics of lifetime income depends on the return from education (i.e., the education premium), the values of the thresholds of bequest for which individuals switch their decisions concerning effort and education, and the values of potential steady states of bequests. We have seen in the previous section that a large number of cases arises for the dynamics of dynastic wealth in spite of the simplicity of our model. In order to comply with the empirical evidence presented in the Introduction, we focus here in a dynamic equilibrium displaying intergenerational persistence in education levels but high intergenerational mobility in wealth. This implies that we should consider those parametric configurations that allow the economy to generate four wealth classes: (i) non-educated individuals who do not make effort; (ii) non-educated individuals who make effort; (iii) educated individuals who do not make effort; and (iv) educated individuals who make effort. Finally, according to our main objective, we will analyze under which conditions the education status is intergenerational preserved while wealth status is not.

From the Case 3 in the previous section, we observe that the previous four-classes scenario occurs only if the two following conditions simultaneously hold :

(a) The education is always profitable regardless the level of effort, $w(1,0) - w(0,0) > R\mu$.

(b) The thresholds of bequests characterizing the bequest function satisfy $\widetilde{b}_2 < \mu < \widetilde{b}_3$.

We now characterize the conditions on the parameters of the model ensuring that $\widetilde{b}_2 < \mu < \widetilde{b}_3$. First, we know that $\widetilde{b}_2 < \widetilde{b}_3$ if and only if $\theta < \widetilde{\theta}^1$ (see (2.14)). Secondly, we obtain that $\mu > \widetilde{b}_2$ if and only if

$$\theta > \widetilde{\theta}^2 \equiv \frac{w(0,1) + R\mu}{w(0,0) + R\mu}. \tag{5.1}$$

Therefore, the threshold $\widetilde{b}_2$ is smaller than the education cost when the utility gain obtained by non-educated individuals from making effort is sufficiently small. To gain some intuition about the previous condition, consider an individual who has received an amount of inheritance equal to the education cost μ and has decided not to become educated. This marginal individual will prefer not to exert effort if $U(0,0) > U(0,1)$, which using (2.10) becomes

$$(1+\beta)\ln[w(0,0) + R\mu] + M > (1+\beta)\ln[w(0,1) + R\mu] - \rho + M.$$

After simplifying the previous inequality becomes in turn the condition (5.1). This inequality implies that an individual receiving an amount of inheritance slightly smaller than μ obviously becomes uneducated and decides not to exert effort. Therefore, he will leave a small bequest to their direct descendants that will not enable them to acquire education. From inspection, we see that inequality (5.1) means that the effort premium in terms of utility for non-educated individuals is small so that the relatively richest

unskilled individuals will decide optimally not to exert effort so that the accumulation of wealth within the dynasty will never allow their members to pay for the education cost. This explains the intergenerational persistence in the low educational levels. Finally, we get that $\widetilde{b}_3 > \mu$ if and only if

$$\theta < \widetilde{\theta}^3 \equiv \frac{w(1,1)}{w(1,0)}. \tag{5.2}$$

Thus, the threshold $\widetilde{b}_3$ is larger than the education cost when the utility gain obtained by educated individuals from making effort is sufficiently large. Similarly, we can consider an individual who has received an amount of inheritance equal to the education cost μ and has decided to acquire education This marginal individual will prefer to exert effort if $U(1,1) > U(1,0)$, which using (2.10) becomes

$$(1+\beta)\ln[w(1,1)] - \rho + M > (1+\beta)\ln[w(1,0)] + M.$$

After simplifying the previous inequality becomes in turn the condition (5.2). Hence, an individual receiving an amount of inheritance slightly larger than μ obviously becomes educated and decides to exert effort. Therefore, he will leave an amount of bequest to their direct descendants that will enable them to acquire education. Again, from inspection, we see that inequality (5.2) means that the effort premium in terms of utility for educated individuals is large so that the poorest skilled individuals will find profitable to exert effort so that the amount of wealth transmitted intergenerationally by means of bequests will be always sufficiently large so as to cover the education cost. This explains the intergenerational persistence in the high educational levels. Therefore, the previous inequalities (5.1) and (5.2) imply the intergenerational segmentation between educated and non-educated individuals. Observe that under the previous conditions the four branches of the bequest function (3.1) are operative (see (4.1)).

The condition $\theta \in \left(\widetilde{\theta}^2, \widetilde{\theta}^3\right)$ highlights the role of complementarity between education and effort in determining the dynamics of wealth as (5.1) and (5.2) imply together a complementarity in terms of utility between education and effort: the premium in terms of utility from making effort is small for non-educated individuals, whereas this premium is large for the educated ones. An economy with four classes does not arise in the absence of complementarity in terms of utility between education and effort. On the one hand, if $\theta < \widetilde{\theta}^2$, then the income gain obtained by non-educated individuals from making effort is not sufficiently small and, therefore, $\mu < \widetilde{b}_2$. We have shown in the previous section that there are three wealth classes at most in this case as the class of unskilled workers do not making effort does not arise. On the other hand, if $\theta > \widetilde{\theta}^3$, then the income gain obtained by educated individuals from making effort is not sufficiently large and, therefore, $\widetilde{b}_3 < \mu$. We have also shown that there are also three wealth classes at most in this case as there will be no skilled workers exerting effort. Therefore, the existence of four social classes requires two types of complementarity between education and effort: complementarity in terms of labor earnings and complementarity in terns of utility.

As we have said in the previous section, even if the necessary conditions for a four-class society we have just discussed hold, the economy may exhibit different dynamics depending on the relationship between the thresholds and the potential steady states

of bequests. Let us focus our analysis on a particular case where the economy exhibits four classes with very strong persistence of the education status within a dynasty and extreme mobility in wealth within each skill type. In fact, this extreme mobility will take the form of a deterministic cycle driven by the forces lying behind the Carnegie conjecture. To this end we need to assume that the bequest function (3.1) does not exhibit any fixed point so that the potential fixed points b_1^*, b_2^*, b_3^*, and b_4^* satisfy the following conditions: $b_1^* < \widetilde{b}_2 < b_2^*$ and $b_3^* < \widetilde{b}_3 < b_4^*$. In Figure 1 we show the bequest function when these conditions hold together with $\theta \in \left(\widetilde{\theta}^2, \widetilde{\theta}^3\right)$.

[Insert Figure 1]

We know that $B^1\left(b_{t-1}\right) < B^2\left(b_{t-1}\right) < B^4\left(b_{t-1}\right)$ and $B^3\left(b_{t-1}\right) < B^4\left(b_{t-1}\right)$ for all b_{t-1}. Concerning the relationship between $B^2\left(b_{t-1}\right)$ and $B^3\left(b_{t-1}\right)$, we know from (3.1) that $B^2\left(b_{t-1}\right) < B^3\left(b_{t-1}\right)$ if and only if

$$w\left(1,0\right) - R\mu > w\left(0,1\right), \tag{5.3}$$

that is, when the minimum labor income that can get an educated individual net of education cost is larger than the maximum labor income that can obtain a non-educated individual. Note that from part (c) of Assumption A, condition (5.3) implies that education is always profitable, namely, $w\left(1,0\right) - w\left(0,0\right) > R\mu$. Therefore, condition (5.3) imposes a stronger profitability condition on education. This is indeed the case depicted in Figure 1. However, we do not need to impose this condition for obtaining the type of dynamics we are looking for.

Figure 2 displays a possible dynamics of bequests for our benchmark economy, where bequests do not converge to any of the potential steady states and the economy converges to a four-class society. In the long run, the fraction of educated dynasties will be in a cycle where generations that make no effort and leave an amount of bequest equal to $\bar{b}_4$ alternate with generations that exert effort and leave a bequest equal $\bar{b}_3$. The fraction of non-educated dynasties will also be in a cycle where generations that do not exert effort and leave an amount of bequest equal to $\bar{b}_2$ alternate with generations that make effort and leave a bequest equal to $\bar{b}_1$.

[Insert Figure 2]

Note that the previous two cycles can also arise even when $B^2\left(b_{t-1}\right) > B^3\left(b_{t-1}\right)$ as can be seen in the situation depicted in Figure 3.

[Insert Figure 3]

As was pointed out in the previous section, the dynamics that may emerge under the parametric Case 3 depends crucially on the relationship between $\widetilde{b}_2$, $\widetilde{b}_3$, μ and the potential steady states of bequest. In fact, the existence of the two cycles illustrated in Figures 2 and 3 requires the following additional restrictions. On the one hand, the cycle governing unskilled dynasties exists if and only if

$$\bar{b}_1 \in \left(\widetilde{b}_2, \mu\right) \quad \text{and} \quad \bar{b}_2 \in \left(0, \widetilde{b}_2\right). \tag{5.4}$$

17

Clearly, under these conditions, we have $B^2\left(\bar{b}_2\right) = \bar{b}_1$ and $B^1\left(\bar{b}_1\right) = \bar{b}_2$, which guarantees the existence of the cycle followed by non-educated dynasties. Moreover, the cycle followed by educated individuals emerges if and only if

$$\bar{b}_3 \in \left(\widetilde{b}_3, \infty\right) \quad \text{and} \quad \bar{b}_4 \in \left(\mu, \widetilde{b}_3\right). \tag{5.5}$$

We see that, under the previous conditions, $B^3\left(\bar{b}_3\right) = \bar{b}_4$ and $B^1\left(\bar{b}_4\right) = \bar{b}_3$, which proves the existence of the cycle governing educated dynasties. In Appendix B, the previous two conditions are characterized in terms of the parameter values of the economy. Moreover, we show through a numerical example that conditions (5.4) and (5.5) are compatible with the previous conditions (5.1) and (5.2).

Conditions (5.4) and (5.5) have an easy interpretation. On the one hand, the fact that the values of $\bar{b}_1$ and $\bar{b}_2$ are smaller than the education cost μ prevents unskilled dynasties from investing in human capital. Therefore, all the dynasties starting with an amount of inheritance smaller than μ eventually converge to the cycle defined by the pair $\left(\bar{b}_2, \bar{b}_1\right)$ where generations remain unskilled. On the other hand, all the dynasties with an initial inheritance larger than the education cost μ will converge to the cycle characterized by the pair $\left(\bar{b}_4, \bar{b}_3\right)$ where generations remain skilled. These latter dynasties enjoy an initial wealth that allows them to purchase education and, moreover, they find very profitable to maintain their education status across generations.

In the Appendix C we explicitly characterize the previous two long-run cycles under all the aforementioned conditions. On the one hand, the bequests of non-educated individuals oscillate between the following two values:

$$\bar{b}_1 = \frac{\lambda\left[w\left(0,1\right) + \lambda w\left(0,0\right)\right]}{\left(1 - \lambda^2\right) R} \tag{5.6}$$

and

$$\bar{b}_2 = \frac{\lambda\left[w\left(0,0\right) + \lambda w\left(0,1\right)\right]}{\left(1 - \lambda^2\right) R}. \tag{5.7}$$

The members of generations who inherit the amount $\bar{b}_1$ do not make effort and leave amount $\bar{b}_2$ to their descendants, whereas the individuals of the generations inheriting the amount $\bar{b}_2$ make effort and leave the amount $\bar{b}_1$. Observe that $\bar{b}_2 < \bar{b}_1$. On the other hand, the bequests of educated individuals oscillate between the following two values:

$$\bar{b}_3 = \frac{\lambda\left[w\left(1,1\right) + \lambda w\left(1,0\right)\right]}{\left(1 - \lambda^2\right) R} - \frac{\lambda\mu}{1 - \lambda} \tag{5.8}$$

and

$$\bar{b}_4 = \frac{\lambda\left[w\left(1,0\right) + \lambda w\left(1,1\right)\right]}{\left(1 - \lambda^2\right) R} - \frac{\lambda\mu}{1 - \lambda}. \tag{5.9}$$

The members of generations inheriting the amount $\bar{b}_3$ do not make effort and leave the amount $\bar{b}_4$ to their descendants, whereas the individuals of the generations that inherit the amount $\bar{b}_4$ make effort and leave the amount $\bar{b}_3$. Finally, observe that $\bar{b}_4 < \bar{b}_3$.

Note that our benchmark economy with cycles does not exhibit mobility in human capital in the long-run, whereas it exhibits a very strong mobility in effort and, thus, in lifetime income and bequests. The stronger mobility in wealth relative to the mobility in education levels is supported by the empirical evidence as we have argued in the Introduction.

18

6. Inequality in the long run and comparative statics

In this section, we will characterize the long-run inequality emerging in the benchmark economy displaying endogenous cycles. First, we will perform a wealth comparison between individuals with the same human capital and different effort. Concerning non-educated individuals, we have two types of individuals: (i) individuals who inherit $\bar{b}_1$ and do not make effort (i.e., they receive a large inheritance and a small labor income) so that their lifetime income is given by $R\bar{b}_1 + w(0,0)$; and (ii) individuals who inherit $\bar{b}_2$ and make effort (i.e., they receive a small inheritance and a large labor income) so that their lifetime income is given by $R\bar{b}_2 + w(0,1)$. Therefore, the wealth inequality between non-educated individuals is

$$\left[R\bar{b}_2 + w(0,1)\right] - \left[R\bar{b}_1 + w(0,0)\right] = \frac{w(0,1) - w(0,0)}{1+\lambda} > 0. \tag{6.1}$$

With respect to educated individuals, we have two types of individuals: (i) individuals who inherit $\bar{b}_3$ and do not make effort (i.e., they receive a large inheritance and a small labor income) so that their lifetime income is given by $R\bar{b}_3 + w(1,0)$; and (ii) individuals who inherit $\bar{b}_4$ and make effort (i.e., they receive a small inheritance and a large labor income) so that their lifetime income is given by $R\bar{b}_4 + w(1,1)$. Therefore, the wealth inequality between non-educated individuals is

$$\left[R\bar{b}_4 + w(1,1)\right] - \left[R\bar{b}_3 + w(1,0)\right] = \frac{w(1,1) - w(1,0)}{1+\lambda} > 0. \tag{6.2}$$

We observe that a large inheritance discourages the effort of individuals in the spirit of the Carnegie conjecture. Furthermore, the difference in inheritance is more than compensated by the difference in labor income. Therefore, the educated individuals who receive the larger inheritance will be the poorest among the class of educated individuals. The same applies for the class of non-educated individuals.

We can now compare the wealth between individuals with different human capital but exerting the same amount of effort. The difference of wealth between educated and non-educated individuals who do not make effort is

$$R\bar{b}_3 + w(1,0) - \left[R\bar{b}_1 + w(0,0)\right]$$
$$= \frac{\lambda\left[w(1,1) - w(0,1)\right] + w(1,0) - w(0,0)}{1-\lambda^2} - \frac{\lambda R\mu}{1-\lambda} > 0, \tag{6.3}$$

whereas the difference of wealth between educated and non-educated individuals who make effort is

$$R\bar{b}_4 + w(1,1) - \left[R\bar{b}_2 + w(0,1)\right]$$
$$= \frac{\lambda\left[w(1,0) - w(0,0)\right] + w(1,1) - w(0,1)}{1-\lambda^2} - \frac{\lambda R\mu}{1-\lambda} > 0. \tag{6.4}$$

Obviously, educated individuals exhibit a larger wealth than non-educated individuals when they make the same effort. This follows from applying to (6.3) and (6.4) the condition $w(1,0) - w(0,0) > R\mu$ and the existence conditions $\bar{b}_1 \in \left(\tilde{b}_2, \mu\right)$, $\bar{b}_2 \in \left(0, \tilde{b}_2\right)$, $\bar{b}_3 \in \left(\tilde{b}_3, \infty\right)$ and $\bar{b}_4 \in \left(\mu, \tilde{b}_3\right)$.

We can now analyze the effects on the long-run distribution and its associated inequality of marginal variations in the fundamentals of the benchmark economy. Let us start by considering three marginal shocks hitting this economy and their equivalence in terms of fiscal policy reforms, where the corresponding changes in taxes or subsidies will be devoted to government spending and not returned to the individuals.

1. A marginal reduction in the education cost μ. This is equivalent to an increase in the rate of a education subsidy. We see from (6.1), (6.2), (6.3) and (6.4) that this policy change results only in an increase in inequality between educated and non-educated individuals. This is so because the individuals who educate their children will have now more disposable wealth as they have to pay less for education.

2. A reduction in the education and effort premiums. This implies a decrease in labor income gaps $w(1,1) - w(1,0)$, $w(0,1) - w(0,0)$, $w(1,1) - w(0,0)$ and $w(1,0) - w(0,1)$. To this end, we have to change three of the four labor earnings (for instance, $w(1,1)$, $w(1,0)$ and $w(0,1)$) by different amounts. To study this kind of shock is equivalent to study the effects of proportional or progressive taxation on labor income. Obviously, this will result in a reduction of inequality between any pair of two classes in this economy

3. A marginal decrease in the saving return R. This is equivalent to raise the flat tax rate on capital income. We see from (6.1), (6.2) and the definition of λ in (3.2) that, as λ is increasing in R, a decrease in the return R results in larger inequality both within the class of educated people and within the class of uneducated people. To understand this effect note that the poorest individuals both within the class of educated and within the class of uneducated have received an inheritance larger than the respective richest individuals. In spite of this larger inheritance they have become poorer because they have exerted less effort. Therefore, the difference in gross capital income between the richest and the poorest $(R\left(\bar{b}_2 - \bar{b}_1\right)$ for the uneducated and $R\left(\bar{b}_4 - \bar{b}_3\right)$ for the educated) increases as the return R becomes lower since $\bar{b}_2 < \bar{b}_1$ and $\bar{b}_4 < \bar{b}_3$. Finally, the comparison concerning the degree of inequality within a class of individuals exerting the same amount of effort is generally ambiguous.

The previous three types of shocks we have just mentioned could alter the social stratification when its introduction is non-marginal. Obviously, a big shock may alter the long-run number of social classes. In the next section we will analyze the impact of a particular sizeable policy shock affecting the characteristics of the welfare state.

7. Welfare state and social stratification

The dynamics of dynastic wealth changes dramatically when some of the conditions generating the previous benchmark economy do not hold. Let us first see what would happen when the relationship between $\widetilde{b}_2$, $\widetilde{b}_3$, μ and the potential steady states of bequest differs from that of the benchmark economy.

If $\widetilde{b}_2 < b_1^*$, then the cycle of non-educated individuals will not emerge. Hence, the non-educated individuals will not make effort and will leave a level of bequest equal to b_1^* in the long run. If $\widetilde{b}_2 > b_2^*$, then the cycle of non-educated individuals will not arise. Hence, all the non-educated individuals will make effort and will leave a level of bequest equal to b_2^* in the long run. If $\widetilde{b}_3 < b_3^*$, then the cycle of educated individuals will not emerge. Hence, the educated individuals will not make effort and will leave a level of bequest equal to b_3^* in the long run. Finally, if $\widetilde{b}_3 > b_4^*$, then the cycle of educated individuals will not arise. Therefore, all the educated individuals will make effort and will leave a level of bequest equal to b_4^* in the long run.

From the previous argument, we can conclude that big shocks can lead the benchmark economy to potentially loose some of its four classes. We are now going to illustrate the argument with an example where the education costs is subjected to a sizeable shock. In particular, we are going to assume that a reform in the welfare state is introduced so that the cost of education faced by individuals is reduced dramatically. Note that the threshold $\widetilde{b}_2$ is independent of the education cost μ but the threshold $\widetilde{b}_3$ decreases by the same amount as the cost μ does (see (2.11) and (2.12)). Moreover, the value of the fixed point b_3^* rises as μ decreases (see 3.5). In Figure 4 we depict the situation emerging after this non-marginal change: the cycle involving educated individuals disappears as b_3^* has become larger than $\widetilde{b}_3$ and, hence, all the educated individuals end up not exerting effort. This is so because to exert effort is no longer necessary to preserve the skill level across individuals belonging to the same dynasty. Moreover, the size of the population that becomes educated increases due to the reduction in μ. This mechanism driving the change in the level of effort exerted by skilled individuals complements the one suggested by Prescott (2004), where changes in labor supply were motivated by labor taxes, whereas our mechanism relies directly on the generosity of the welfare state.

[Insert Figure 4]

Note also that, if the decrease in the education cost is very large, we could arrive at a situation where $\mu < \widetilde{b}_2$ and then the cycle of uneducated individuals also disappears and there is only one social class in the long run formed by skilled individuals exerting no effort as it can be seen in Figure 5.

[Insert Figure 5]

A similar analysis leading to the elimination of some social classes can be conducted through sizeable changes in the relative distance between the four wages faced by the potential four classes of our economy. These changes in wages could be a consequence of progressive taxation or of skill-specific technological shocks.

8. Conclusions

In this paper we have characterized the conditions under which an economy could display simultaneously stationary cycles in wealth and persistence in the education attainment across generations. The oscillations of wealth arise because individuals who receive a large inheritance optimally decide not to exert effort in their occupations,

21

which agrees with the idea underlying the Carnegie conjecture. The resulting lifetime income of these individuals becomes smaller than that of their parents and then they leave a small amount of bequest, which forces the next generation to exert effort again. The model displays the realistic feature that unskilled individuals get smaller bequests than skilled individuals. This property, together with the existence of a fixed indivisible cost of schooling and a borrowing constraint on education investment, forces the direct descendants of unskilled individuals to remain unskilled. However, the descendants of skilled individuals can afford the cost of education thanks to the larger inheritance they receive. Therefore, we obtain a perfect persistence of the education status even tough this persistence is compatible with fluctuations of wealth both inside the class of educated individuals and inside the class of uneducated ones. Our model generates thus a rich social class structure with rich skilled workers, poor unskilled workers together with relative poor skilled workers and relative rich unskilled workers.

Our model is deterministic and all the fluctuations of wealth are endogenous. It is straightforward to generate transitions from each class to any of the other three classes by introducing an exogenous variable, like a class-idiosyncratic productivity shock affecting the relationship between effort, human capital, and wage compensation. However, our non-stochastic model allows us to highlight the role that the complementarity between effort and education plays in order to generate this rich class structure exhibiting intergenerational persistence in education levels. When such a complementarity is appropriately modified, the number of classes could decrease dramatically and mobility in the levels of human capital could arise.

Our analysis provides thus new insights on the factors and policies that either prevent or promote societies characterized by equal opportunity and efficient use of resources. Our model has also obvious implications for economic development as it may explain quite naturally differences in wealth per capita across countries and the existence of poverty traps as a consequence of different education costs, tax systems, or technologies. Moreover, our model directly links the changes in the wage distribution across occupations and the new complemenatarities among different levels and types of skill that technological change has brought about in recent years with the dramatic modification of the social structure (see Autor and Dorn, 2013).

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Appendix

A. Transitional dynamics when education is not always profitable

In this appendix we characterize the transition of bequest, effort and human capital when the investment in education is either not profitable for all agents or when it is only profitable for the agents who exert effort. These cases correspond to the parametric Configurations 1 and 2, respectively, which were presented in Section 3.

1. Education is not profitable: $R\mu > w(1,1) - w(0,1)$.

Under this configuration, no individual wants to invest in education because the education premium is always smaller than the present value of education cost with independence of the level of effort exerted. Hence, only the branches $B^1(b_{t-1})$ and $B^2(b_{t-1})$ of the bequest function (3.1) are operative, with $B^1(b_{t-1}) < B^2(b_{t-1})$ for all b_{t-1}. Therefore, given b_{t-1} individuals must first decide if they want to make a positive effort. This decision depends on the value of the threshold $\widetilde{b}_2$. Agents make positive effort if $b_{t-1} < \widetilde{b}_2$ and make no effort if $b_{t-1} > \widetilde{b}_2$. The transition of bequests, education and effort is then given by

$$\{b_t, h_t, e_t\} = \begin{cases} \left\{B^2(b_{t-1}), 0, 1\right\} & \text{if } b_{t-1} < \widetilde{b}_2, \\ \\ \left\{B^1(b_{t-1}), 0, 0\right\} & \text{if } b_{t-1} \geq \widetilde{b}_2. \end{cases}$$

The dynamics of the endogenous variables is determined by the relationship between $\widetilde{b}_2$ and the potential steady states b_1^* and b_2^*. It is straightforward to see that the following dynamics may arise:

(a) If $\widetilde{b}_2 > b_2^*$, then the economy converges to a one-class society with $\{b_t, x_{t-1}, e_t\} = \{b_2^*, 0, 1\}$, i.e., with only unskilled individuals who make effort.

(b) If $\widetilde{b}_2 < b_1^*$, then the economy converges to a one-class society with $\{b_t, x_{t-1}, e_t\} = \{b_1^*, 0, 0\}$, i.e., with only unskilled individuals who make no effort.

(c) If $\widetilde{b}_2 \in (b_1^*, b_2^*)$, then the economy does not converge to a steady state. The economy follows instead a cycle. In this case, poor dynasties make positive effort and accumulate wealth. When they reach a sufficient large level of bequests, their descendants do not make positive effort and disaccumulate wealth, which makes them poor again. Dynasties eventually converge to a cycle along which poor generations with positive effort alternate with rich generations wit no effort. Note that in this case the economy thus converges to a two-class society in the long-run.

2. Education is only profitable for workers who make effort: $w(1,1) - w(0,1) > R\mu > w(1,0) - w(0,0)$.

Here the education premium for individuals who make effort is larger than the present value of the education cost. However, this is not the case for the individuals who

do not make effort. This implies that the individuals who acquire education should also make positive effort. Hence, the branch $B^3(b_{t-1})$ of the bequest function (3.1) is not operative in this scenario. Moreover, under this configuration, we have that $B^1(b_{t-1}) < B^2(b_{t-1}) < B^4(b_{t-1})$ for all b_{t-1}.

Given the amount of inheritance b_{t-1}, the individual decision concerning the amount of bequest left, education, and effort depends obviously on the education cost μ and the values of the thresholds $\widetilde{b}_2$ and $\widetilde{b}_4$. Hence, we should distinguish between several cases depending on the raking of μ, $\widetilde{b}_2$ and $\widetilde{b}_4$. However, we already know from Section 3 that $\widetilde{b}_2 < \widetilde{b}_4$. We next analyze all the possible cases that may arise under this configuration:

Case 1. $\widetilde{b}_2 < \widetilde{b}_4 < \mu$.

Here, the transition of bequests, education and effort is given by

$$\{b_t, h_t, e_t\} = \begin{cases} \left\{B^2(b_{t-1}), 0, 1\right\} & \text{if } b_{t-1} < \widetilde{b}_2, \\ \\ \left\{B^1(b_{t-1}), 0, 0\right\} & \text{if } b_{t-1} \geq \widetilde{b}_2. \end{cases}$$

Observe that in this case the threshold $\widetilde{b}_4$ is irrelevant. The dynamics of the endogenous variables is fully driven by the relationship between $\widetilde{b}_2$ and the potential steady states b_1^* and b_2^*. Since in this case the number of steady states can be at most two, the following three configurations may appear in the long run:

(a) If $\widetilde{b}_2 > b_2^*$, then the economy converges to a one-class society with $\{b_t, x_{t-1}, e_t\} = \{b_2^*, 0, 1\}$, i.e., with only unskilled individuals who make effort.

(b) If $\widetilde{b}_2 < b_1^*$, then the economy converges to a one-class society with $\{b_t, x_{t-1}, e_t\} = \{b_1^*, 0, 0\}$, i.e., with only unskilled individuals who make no effort.

(c) If $\widetilde{b}_2 \in (b_1^*, b_2^*)$, then the economy does not converge to a steady state. The economy follows instead a cycle. In this case, the poor dynasties make positive effort and accumulate wealth and, once they reach a sufficient large level of bequests, their descendants do not make positive effort and disaccumulate wealth, which makes them poor again. Dynasties eventually approach a cycle along which poor generations with positive effort alternate with rich generations without effort. The economy thus converges to a two-class society in the long-run.

Case 2. $\widetilde{b}_2 < \mu < \widetilde{b}_4$.

In this case the transition of b_t, h_t, and e_t

$$\{b_t, h_t, e_t\} = \begin{cases} \left\{B^2(b_{t-1}), 0, 1\right\} & \text{if } b_{t-1} < \widetilde{b}_2, \\ \\ \left\{B^1(b_{t-1}), 0, 0\right\} & \text{if either } b_{t-1} \in \left[\widetilde{b}_2, \mu\right) \text{ or } b_{t-1} \geq \widetilde{b}_4, \\ \\ \left\{B^4(b_{t-1}), 1, 1\right\} & \text{if } b_{t-1} \in \left[\mu, \widetilde{b}_4\right). \end{cases}$$

The dynamics of bequest and lifetime income is then fully determined by the relationship between the bequest threshold $\widetilde{b}_2$ and the potential steady states b_1^* and b_2^*,

and the relationship between the bequest threshold $\widetilde{b}_4$ and the potential steady states b_3^* and b_4^*. Since in this case the number of steady states can be at most three, only three classes may appear in the long run: (i) the class of unskilled who exert effort, (ii) the class of unskilled who make no effort, and (iii) the class of skilled who exert effort. Several stationary dynamics are possible: we can have locally stable classes and cycles involving switches between two classes.

Case 3. $\mu < \widetilde{b}_2 < \widetilde{b}_4$.

In this case the transition of bequests, education and effort is given by

$$
\{b_t, h_t, e_t\} = \begin{cases} \left\{ B^2\left(b_{t-1}\right), 0, 1 \right\} & \text{if } b_{t-1} < \mu, \\[2ex] \left\{ B^4\left(b_{t-1}\right), 1, 1 \right\} & \text{if } b_{t-1} \in \left[\mu, \widetilde{b}_4\right), \\[2ex] \left\{ B^1\left(b_{t-1}\right), 0, 0 \right\} & \text{if } b_{t-1} \geq \widetilde{b}_4. \end{cases}
$$

Observe that in this case the value of the threshold $\widetilde{b}_2$ is irrelevant. The dynamics of bequest and lifetime income is then fully determined by the relationship between the education cost μ, the bequest threshold $\widetilde{b}_4$, and the three potential steady states of bequests (b_1^*, b_2^* and b_4^*). As in the previous case, since the number of steady states can be at most three, only three classes may appear in the long run: (i) the class of unskilled who exert effort, (ii) the class of unskilled who make no effort, and (iii) the class of skilled who exert effort. Similarly, several stationary dynamics are possible: we can have locally stable classes and cycles involving switches between two classes.

B. Conditions for the existence of two cycles

Using (2.11), (5.6) and (5.7), conditions (5.4) for the existence of the cycle governing non-educated families can be written as

$$
\left[\lambda\left(\theta-1\right)-\left(1-\lambda^2\right)\right] w\left(0,1\right) + \left(\theta-\lambda^2\right) w\left(0,0\right) > 0, \tag{B.1}
$$

$$
\lambda\left[w\left(0,1\right)+\lambda w\left(0,0\right)\right] < \left(1-\lambda^2\right) R\mu, \tag{B.2}
$$

and

$$
\left(\lambda^2\theta-1\right) w\left(0,1\right) + \left[\lambda\left(\theta-1\right)+\theta\left(1-\lambda^2\right)\right] w\left(0,0\right) < 0. \tag{B.3}
$$

Similarly, using (2.12), (5.8) and (5.9), conditions (5.5) for the emergence of the cycle followed by educated individuals become

$$
\left[\lambda\left(\theta-1\right)-\left(1-\lambda^2\right)\right] w\left(1,1\right) + \left(\theta-\lambda^2\right) w\left(1,0\right) > \left(\theta-1\right)\left(1+\lambda\right) R\mu, \tag{B.4}
$$

$$
\lambda\left[w\left(1,0\right)+\lambda w\left(1,1\right)\right] > \left(1+\lambda\right) R\mu, \tag{B.5}
$$

and

$$
\left(\lambda^2\theta-1\right) w\left(1,1\right) + \left[\lambda\left(\theta-1\right)+\theta\left(1-\lambda^2\right)\right] w\left(1,0\right) < \left(\theta-1\right)\left(1+\lambda\right) R\mu. \tag{B.6}
$$

Solving for θ in inequality (B.1), we see that this inequality holds if and only if $\theta > \underline{\theta}_2$, where

$$\underline{\theta}_2 \equiv \frac{\left(1 + \lambda - \lambda^2\right) w\left(0, 1\right) + \lambda^2 w\left(0, 0\right)}{\lambda w\left(0, 1\right) + w\left(0, 0\right)}.$$

Inequality (B.2) can be expressed as a constraint on the maximum value of λ. To see this, we rewrite the inequality as the following second order polynomial inequality in the slope λ of the bequest function:

$$P\left(\lambda\right) \equiv \lambda^2 \left(w\left(0, 0\right) + R\mu\right) + \lambda w\left(0, 1\right) - R\mu < 0.$$

Note that the unique positive solution for $P\left(\lambda\right) = 0$ is

$$\overline{\lambda} = \frac{-w\left(0, 1\right) + \sqrt{w\left(0, 1\right)^2 + 4\left[w\left(0, 0\right) + R\mu\right]R\mu}}{2\left(w\left(0, 0\right) + R\mu\right)}.$$

so that (B.2) holds if and only if $\lambda < \overline{\lambda}$.

Solving for θ in inequality (B.3), we see that this inequality holds if and only if $\theta < \overline{\theta}_1$, where

$$\overline{\theta}_1 \equiv \frac{w\left(0, 1\right) + \lambda w\left(0, 0\right)}{\lambda^2 w\left(0, 1\right) + \left(1 + \lambda - \lambda^2\right) w\left(0, 0\right)}.$$

If we solve for θ in inequality (B.4), we get that this inequality can be written as $\theta > \underline{\theta}_1$, where

$$\underline{\theta}_1 \equiv \frac{(1 + \lambda - \lambda^2)w\left(1, 1\right) + \lambda^2 w\left(1, 0\right) - (1 + \lambda)R\mu}{\lambda w\left(1, 1\right) + w\left(1, 0\right) - (1 + \lambda)R\mu}.$$

Note that the denominator of the previous expression is positive since $w\left(1, 1\right) > w\left(1, 0\right) > R\mu$ under the Configuration 3 we are considering.

Similarly, inequality (B.5) can be expressed as a constraint on the minimum value of λ. To see this, we rewrite the inequality as the following second order polynomial inequality in the slope λ of the bequest function:

$$Q\left(\lambda\right) \equiv \lambda^2 w\left(1, 1\right) + \lambda\left[w\left(1, 0\right) - R\mu\right] - R\mu > 0.$$

The unique positive solution for $Q\left(\lambda\right) = 0$ is

$$\underline{\lambda} = \frac{-\left[w\left(1, 0\right) - R\mu\right] + \sqrt{\left[w\left(1, 0\right) - R\mu\right]^2 + 4w\left(1, 1\right)R\mu}}{2w\left(1, 1\right)}.$$

so that (B.5) holds whenever $\lambda > \underline{\lambda}$.

Finally, inequality (B.6) holds if and only if

$$\theta < \overline{\theta}_2 \equiv \frac{w\left(1, 1\right) + \lambda w\left(1, 0\right) - \left(1 + \lambda\right)R\mu}{\lambda^2 \left[w\left(1, 1\right) - w\left(1, 0\right)\right] + \left(1 + \lambda\right)\left[w\left(1, 0\right) - R\mu\right]},$$

where the denominator of $\overline{\theta}_2$ is positive.

Therefore, we can summarize our previous analysis by saying that conditions (5.4) and (5.5) for the existence of two cycles are equivalent to the following:

$$\lambda \in \left(\underline{\lambda}, \overline{\lambda}\right) \quad \text{and} \quad \theta \in \left(\max\left\{\underline{\theta}_1, \underline{\theta}_2\right\}, \min\left\{\overline{\theta}_1, \overline{\theta}_2\right\}\right).$$

We next provide an example under which all the conditions that give rise to the existence of two cycles with extreme intergenerational mobility in the amount of inheritance and absolute persistence in education levels are satisfied. Consider thus the following values for the four wages of the economy:

$$w(0,0) = 0.5, \quad w(0,1) = 1.15, \quad w(1,0) = 2.5, \quad w(1,1) = 5,$$

We choose the values of μ, R, n, and β so that $\lambda = 0.17$, $R\mu = 0.3$ and $\theta = 1.91$. Under this parameter configuration, we get that $\overline{\lambda} = 0.225$, $\underline{\lambda} = 0.109$ so that $\lambda \in \left(\underline{\lambda}, \overline{\lambda}\right)$. Moreover, in this case we get $\overline{\theta}_1 = 2.0454$, $\overline{\theta}_2 = 1.9174$, $\underline{\theta}_1 = 1.809$, $\underline{\theta}_2 = 1.9076$ so that $\theta \in \left(\max\left\{\underline{\theta}_1, \underline{\theta}_2\right\}, \min\left\{\overline{\theta}_1, \overline{\theta}_2\right\}\right).$

Note that this example satisfies the condition under which education is always profitable for all individuals since $w(1,0) - w(0,0) = 2 > R\mu = 0.3$. Finally, the conditions (5.1),

$$\frac{w(0,1) + R\mu}{w(0,0) + R\mu} = 1.81 < \theta = 1.91$$

and

$$\frac{w(1,1)}{w(1,0)} = 2 > \theta = 1.91,$$

are also satisfied.

C. Characterization of cycles in the benchmark economy

Next, we explicitly find the two cycles that arise in the Benchmark Economy.

(a) We will first characterize the cycle that emerges for non-educated individuals. To this end we use Figure 6. The cycle implies that those dynasties with a initial bequest below the education cost μ converge to a cycle along which their bequests oscillate between two social classes characterized by the bequest values $B^1(b_{t-1})$ and $B^2(b_{t-1})$. More precisely, they oscillate between point A and C in Figure 6. Observe that the point A corresponds to $\{b_t, h_t, e_t\} = \{\overline{b}_2, 0, 1\}$, whereas point C corresponds to $\{b_t, h_t, e_t\} = \{\overline{b}_1, 0, 0\}$. In order to compute the bequest levels $\overline{b}_1$ and $\overline{b}_2$, we use the fact that the cycle defines the square $\overline{ABCD}$. Hence, the following conditions should hold in a cycle:

$$\overline{b}_1 - \left(\lambda \overline{b}_1 + a\right) = \overline{b}_1 - \overline{b}_2, \tag{C.1}$$

and

$$\overline{b}_2 - \left(\lambda \overline{b}_2 + c\right) = \overline{b}_2 - \overline{b}_1, \tag{C.2}$$

with

$$a = \frac{\lambda w(0,0)}{R},$$

and

$$c = \frac{\lambda w(0,1)}{R}.$$

By solving the system (C.1)-(C.2), we obtain

$$\bar{b}_1 = \frac{\lambda a + c}{1 - \lambda^2},$$

and

$$\bar{b}_2 = \frac{\lambda c + a}{1 - \lambda^2},$$

which become equal to the expressions (5.6) and (5.7), respectively. In order to prove the existence of this cycle, we should ensure that $\bar{b}_1$ and $\bar{b}_2$ are in the operative part of policy functions $B^1(b_{t-1})$ and $B^2(b_{t-1})$, respectively. From Figures 2 or 3 we see that this hold in the benchmark economy when $\bar{b}_2 \in \left(0, \tilde{b}_2\right)$ and $\bar{b}_1 \in \left(\tilde{b}_2, \mu\right).$

[Insert Figure 6]

(b) Using the same procedure as before we characterize the cycle that emerges for the educated individuals. The existence of a cycle implies that those dynasties with a initial bequest above μ converge to a cycle of two social classes along which their bequests oscillate between $B^3(b_{t-1})$ and $B^4(b_{t-1})$. More precisely, they oscillate between two points: $\{b_t, h_t, e_t\} = \{\bar{b}_3, 1, 0\}$ and $\{b_t, h_t, e_t\} = \{\bar{b}_4, 1, 1\}$, with

$$\bar{b}_3 = \frac{\lambda m + n}{1 - \lambda^2}, \tag{C.3}$$

and

$$\bar{b}_4 = \frac{\lambda n + m}{1 - \lambda^2}, \tag{C.4}$$

where

$$m = \lambda \left[\frac{w(1,0)}{R} - \mu \right],$$

and

$$n = \lambda \left[\frac{w(1,1)}{R} - \mu \right].$$

After some algebra, (C.1) and (C.2) become equal to (5.8) and (5.9), respectively. This cycle exist if $\bar{b}_3$ and $\bar{b}_4$ are in the operative part of policy functions $B^3(b_{t-1})$ and $B^4(b_{t-1})$, respectively. By using Figures 2 or 3, we see that this happens in the benchmark economy when $\bar{b}_3 \in \left(\tilde{b}_3, \infty\right)$ and $\bar{b}_4 \in \left(\mu, \tilde{b}_3\right).$
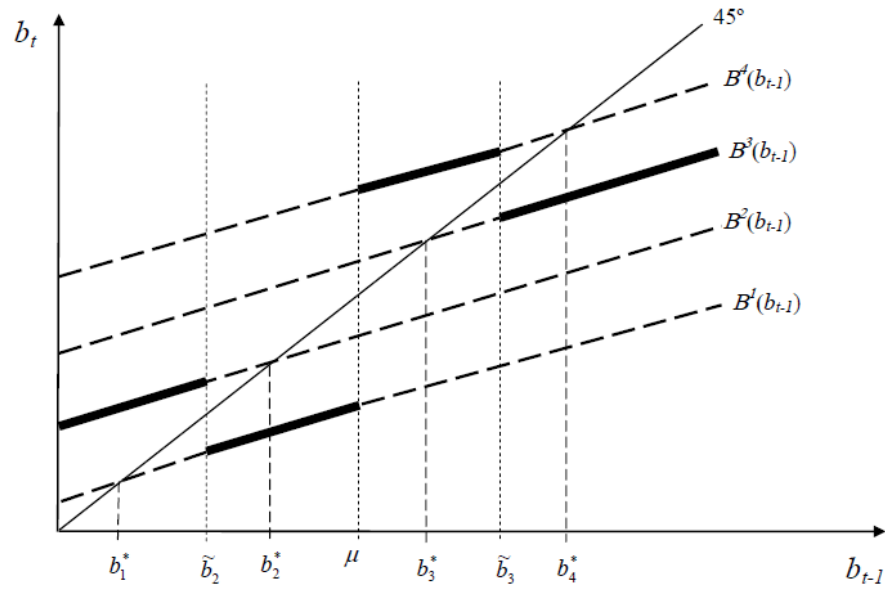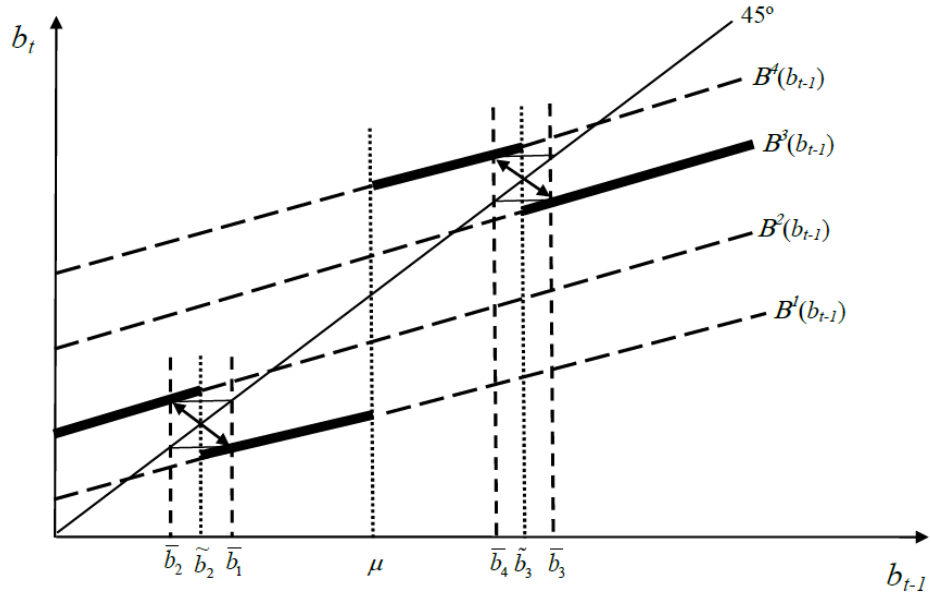
Figure 1. The bequest function.

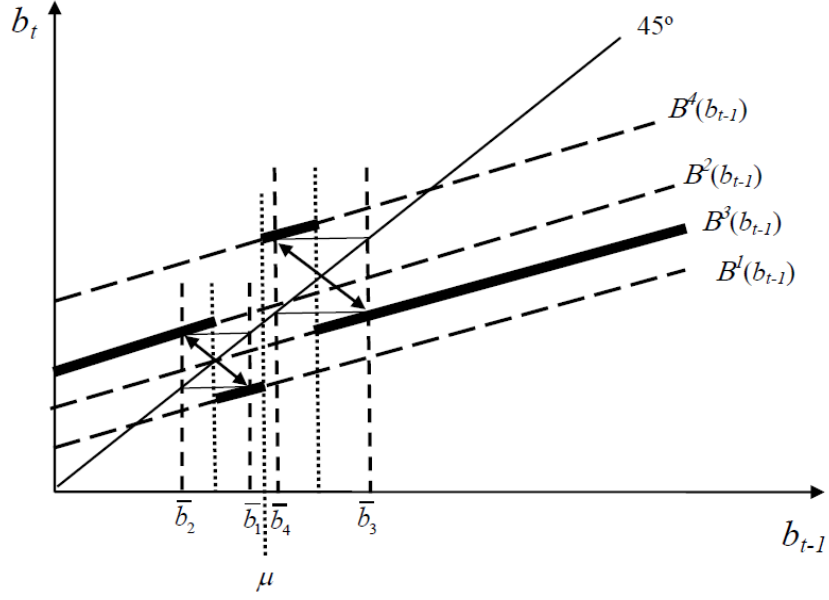Figure 2. Cycles in the benchmark economy when $B^3 > B^2$.

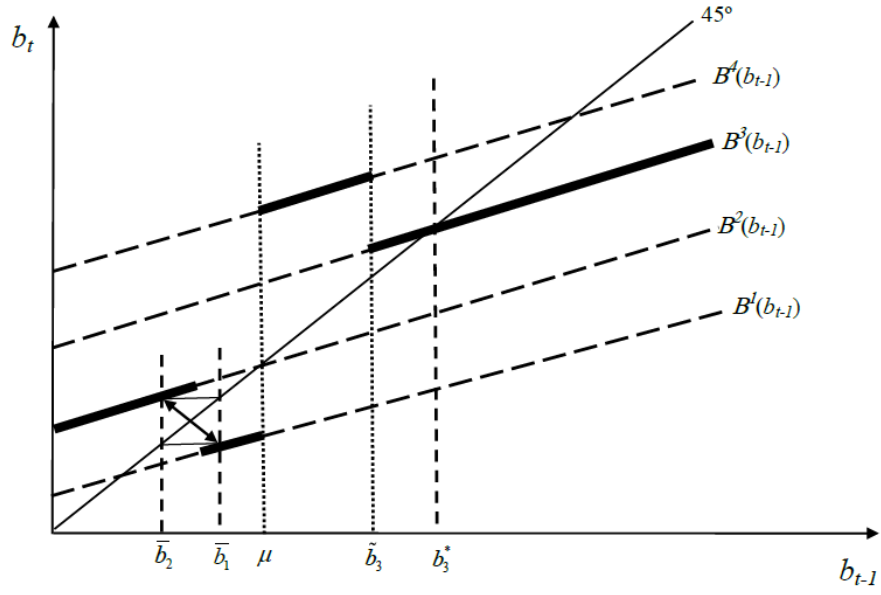Figure 3. Cycles in the benchmark economy when $B^2 > B^3$.

Figure 4. Equilibria after a big reduction in the cost μ of education.
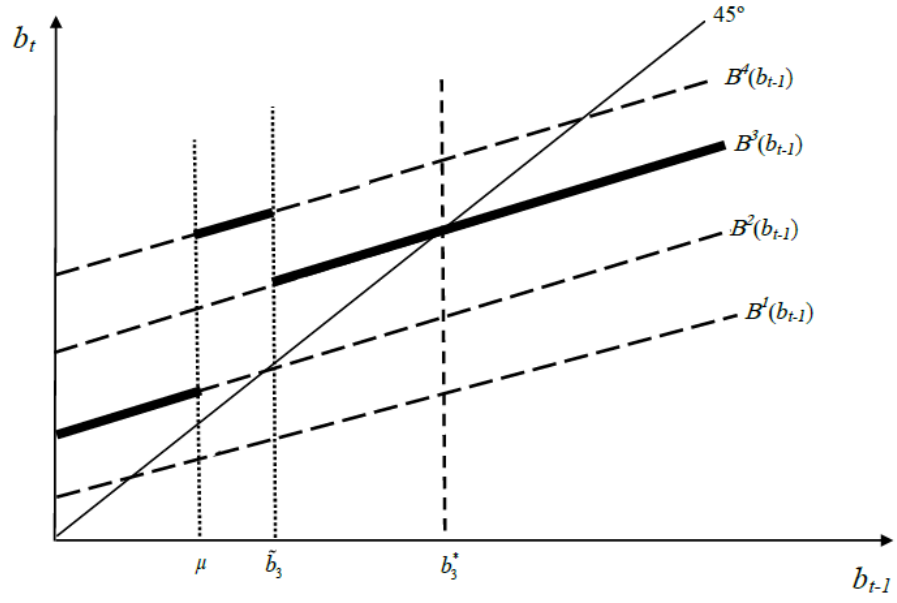
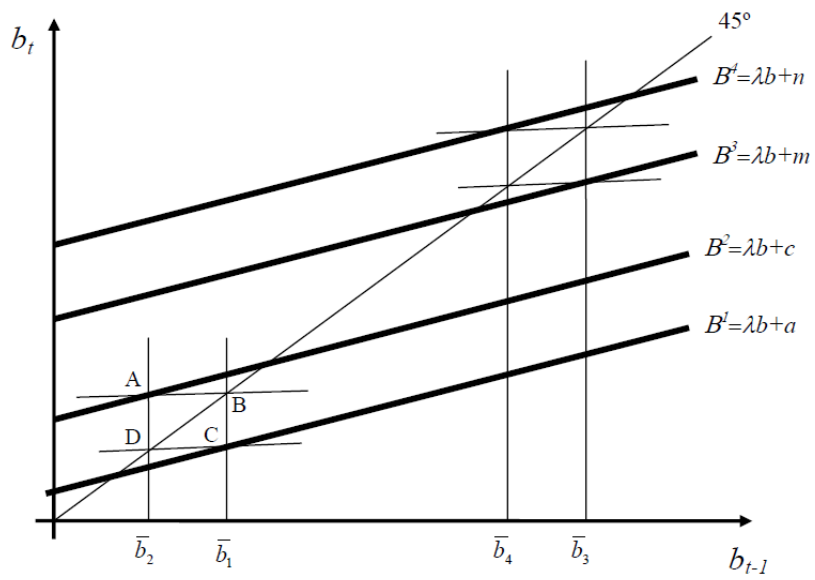Figure 5. Equilibria after a very big reduction in the cost μ of education.

Figure 6. Characterization of cycles.