Quantum cavitation in liquid ³He: Dissipation effects

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We have investigated the effect that dissipation may have on the cavitation process in normal liquid ³He. Our results indicate that a rather small dissipation decreases sizeably the quantum-to-thermal crossover temperature T^* for cavitation in normal liquid ³He. This is a possible explanation of why recent experiments have not yet found clear evidence of quantum cavitation at temperatures below the T^* predicted by calculations which neglect dissipation. [S0163-1829(99)00429-4]

Quantum cavitation in superfluid liquid ⁴He has been unambiguously observed using ultrasound experimental techniques.[1,2] These experiments have shown that quantum cavitation takes over thermal cavitation at a temperature (T) around 200 mK, in good agreement with theoretical calculations,[3,4] so that the problem of cavitation in liquid ⁴He can be considered as satisfactorily settled.

The crossover temperature corresponding to ³He has also been calculated,[3,4] predicting that $T^* \sim 120$ mK. It turns out that preliminary results obtained in a recent experiment[5] have not shown clear evidence of quantum cavitation for temperatures even below that value. However, the phenomenon has been firmly established as a stochastic process. A possible explanation is that thermal cavitation is still the dominant process down to temperatures lower than predicted.

The method of Ref. 4 (see also Ref. 6) is based, on the one hand, in using a density functional that reproduces the thermodynamical properties of liquid ³He at zero temperature (equation of state, effective mass, etc.), as well as the properties of the ³He free surface. A major advantage of using a density functional is that one can handle bubbles in the vicinity of the spinodal region, where they are not empty objects[3,4] and any attempt to describe the critical bubble in terms of a sharp surface radius fails.[7] On the other hand, we have used a functional-integral approach especially well suited to find T^*. This gives us some confidence on the values obtained for the crossover temperature, and inclines us to think that any appreciable discrepancy between theory and experiment has to be attributed not to the method itself, but to some physical ingredient which has been overlooked in the formalism. One such ingredient in the case of liquid ³He is dissipation, which is known[8] to decrease T^*. Since ⁴He is superfluid below the lambda temperature, we are actually treating both quantum fluids within the same framework, the behavior of ⁴He being accounted for by the dissipationless version of the general formalism.

Our starting point is the real time Lagrangian density $\mathcal{L}(\rho,s)$:

$$\mathcal{L}(\rho,s) = m\dot{\rho}s - \mathcal{H}(\rho,s), \qquad (1)$$

where $\rho(\mathbf{r},t)$ denotes the particle density, m the ³He atomic mass, and $s(r,t)$ is the velocity potential, i.e., the collective velocity is $\mathbf{u}(\mathbf{r},t) = \nabla s(\mathbf{r},t)$. The Hamiltonian density $\mathcal{H}(\rho,s)$ reads

$$\mathcal{H}(\rho,s) = \frac{1}{2}m\rho\mathbf{u}^2 + [\omega(\rho) - \omega(\rho_m)], \qquad (2)$$

where $\omega(\rho)$ is the grand potential density of the system and ρ_m is the density of the metastable homogeneous liquid. We refer the reader to Ref. 4 and references therein for details.

To describe the dynamics in the dissipative regime while still being able to deal with inhomogeneous ³He, which is crucial for a proper description of cavitation in liquid helium, we have introduced a phenomenological Rayleigh's dissipation function[9,10] $\mathcal{F}$:

$$\mathcal{F} = \frac{1}{2}\xi\frac{\dot{\rho}^2}{\rho^2}. \qquad (3)$$

From Lagrange's equations,

$$\frac{\partial}{\partial t}\left(\frac{\delta\mathcal{L}}{\delta\dot{x}}\right) - \frac{\delta\mathcal{L}}{\delta x} = -\frac{\partial\mathcal{F}}{\partial\dot{x}}, \qquad (4)$$

with x being either s or ρ, one gets the continuity and motion equation, respectively:

$$\dot{\rho} + \nabla(\rho\mathbf{u}) = 0 \qquad (5)$$

$$m\left\{\frac{\partial u_k}{\partial t} + u_i\nabla_k u_i\right\} = -\nabla_k\left(\frac{\delta\omega}{\delta\rho}\right) + \nabla_k\left[\xi\frac{1}{\rho^2}\nabla(\rho\mathbf{u})\right]. \quad (6)$$

For an homogeneous fluid, the equation of motion ressembles the Navier-Stokes equation[11]

$$m\rho\left\{\frac{\partial u_k}{\partial t} + u_i\nabla_k u_i\right\} = -\nabla_k P + \eta\Delta u_k + \left(\zeta + \frac{1}{3}\eta\right)\nabla_k(\nabla\cdot\mathbf{u}), \qquad (7)$$

where P is the pressure. For liquid ³He at low T, dissipation depends on the mean free path of quasiparticles, and a precise estimation of the magnitude of this effect in the tunneling process is difficult. Since our interest here is to explore

PRB <u>60</u>   3048

TABLE I. Equation of state, sound velocity, energy barrier, and quantum action ($\xi=0$) near the spinodal point.

ρ (Å^{-3})	P (bar)	c_s (m/s)	$\Delta\Omega$ (K)	$\mathcal{S}/\hbar$
.0123	-3.08	42.3	1.3	16.0
.0124	-3.07	47.8	1.8	19.4
.0125	-3.06	52.7	2.5	22.9
.0126	-3.05	57.4	3.2	26.8
.0127	-3.03	61.8	4.0	31.2
.0128	-3.01	66.1	4.9	36.0
.0129	-2.99	70.1	5.8	41.5
.0130	-2.96	74.1	6.9	47.8
.0131	-2.93	77.9	8.0	55.0
.0132	-2.90	81.7	9.3	63.5
.0133	-2.86	85.3	10.6	73.5
.0134	-2.83	88.9	12.1	85.5
.0135	-2.78	92.4	13.7	100.3
.0136	-2.74	95.9	15.5	119.2
.0137	-2.69	99.3	17.4	144.0
.0138	-2.64	102.7	19.6	175.4
.0139	-2.59	106.1	21.9	212.6
.0140	-2.53	109.4	24.6	256.6

the effect of a small viscosity on T^*, we have adopted the pragmatic point of view of identifying ξ with $\zeta+\eta/3$ and presenting results for different ξ's close to the experimental η value (it is known that at low temperatures, the shear viscosity coefficient η is much larger than the bulk viscosity coefficient ζ, see, for example, Ref. 12). Using the macroscopic viscosity coefficient, one should have in mind that we are likely overestimating the dissipation effects.

To obtain T^* we have proceeded as indicated in Ref. 4, writing the above equations in imaginary time $\tau=it$ and linearizing them around the critical bubble density ρ_0, seeking solutions of the kind:

$$\rho(\mathbf{r},\tau)\equiv\rho_0(r)+\rho^1(r)e^{-i\omega_s\tau}. \qquad (8)$$

Upon linearization, we end up with the following equation for ω_s and $\rho^1(r)$:
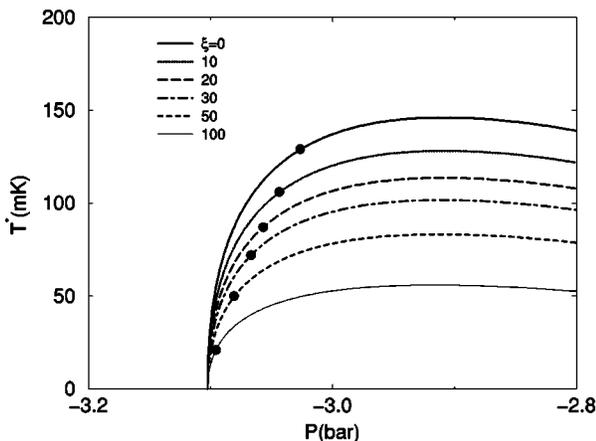
FIG. 1. T^* as a function of pressure for different ξ values (in μP). The homogeneous cavitation pressure $P_h(T^*)$ is shown as circles for $(Vt)_{exp}=10^8$ Å^3 s.
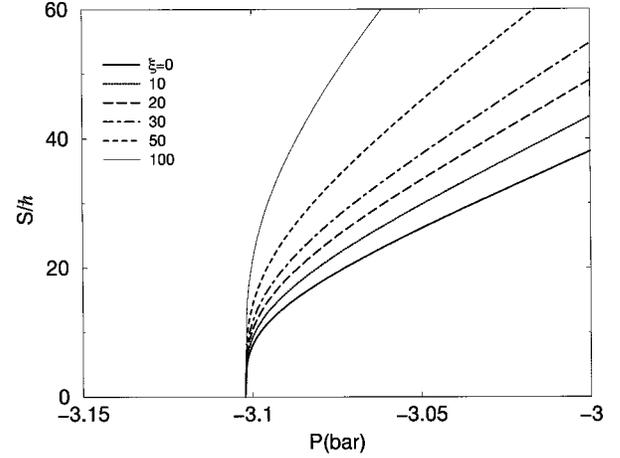
FIG. 2. Effective quantum action in $\hbar$ units as a function of pressure for the values of ξ employed in Fig. 1.

$$\mathcal{M}\rho^1(r)\equiv[m\omega_s^2-\mathcal{M}_1-\xi\omega_s\mathcal{M}_2]\rho^1(r)=0. \qquad (9)$$

The differential operators $\mathcal{M}_1$ and $\mathcal{M}_2$ in Eq. (9) are, respectively, the linearization of

$$\nabla\left\{\rho\nabla\left(\frac{\delta\omega}{\delta\rho}\right)\right\} \quad \text{and} \quad \nabla\left\{\rho\nabla\left(\frac{1}{\rho^2}\right)\right\}, \qquad (10)$$

in which only first-order terms in $\rho^1(r)$ and its derivatives have been kept.[4] Since ξ depends on the density[12] as $\rho^{5/3}$, in actual calculations we have made a local-density approximation, using as form factor in Eq. (3) the expression $1/[\rho_{sat}^{5/3}\rho^{1/3}(r)]$, where ρ_{sat} is the density of the liquid at $T=0$ and $P=0$, and ξ is then density independent.

Equation (9) is a fourth-order linear differential, generalized eigenvalue equation, whose physical solutions have to fulfill $\rho^{1'}(0)=\rho^{1'''}(0)=0$, and fall exponentially to zero at large distances. We have solved it as indicated in Ref. 4. Once the largest dissipation-renormalized frequency ω_s has been determined,[13] the crossover temperature is obtained as $T^*=\hbar\omega_s/(2\pi)$.

Table I collects the equation of state near the spinodal point ($\rho_{sp}=0.011\,91$ Å^{-3}, $P_{sp}=-3.102$ bar), and other quantities which are of interest to analyze the experimental results.[2] Our spinodal point compares very well with recent Monte Carlo calculations[14] ($\rho_{sp}=0.0121$Å^{-3}, $P_{sp}=-3.12\pm0.10$ bar), and also with other phenomenological approaches.[3,15]

We show T^* in Fig. 1 as a function of pressure for different ξ values. In particular, $\xi=100\mu$P roughly corresponds to the experimental value[16] of η at $P=0$ and $T=100$ mK. The associated effective quantum action $\mathcal{S}$ obtained as $\mathcal{S}=\Delta\Omega/T^*$, where $\Delta\Omega$ is the maximum of the energy barrier, is displayed in Fig. 2.

Figure 3 shows $\rho^1(r)$ at $P=-3$ bar for three ξ values, as well as the critical bubble density $\rho_0(r)$. The linearized continuity equation $\rho^1(r)\propto\nabla(\rho_0\mathbf{u})$ implies that $\rho^1(r)$ must have nodes, as it imposes that the integral of $\rho^1(r)$ is zero when taken over the whole space.

When ξ is small enough and the $\mathcal{M}_2$ term in Eq. (9) can be treated perturbatively, a straightforward calculation yields
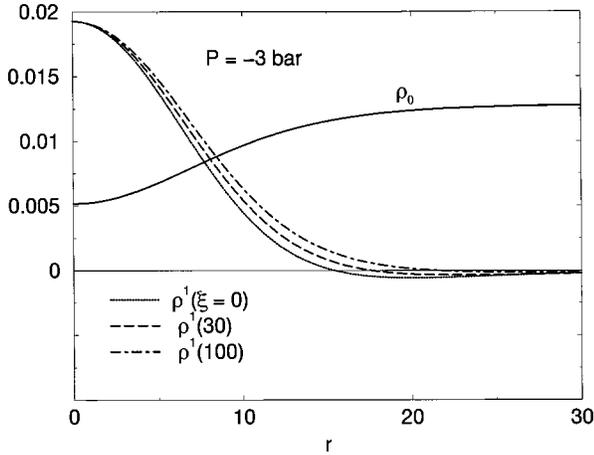
FIG. 3. Particle density profile $\rho_0(r)$ in Å^{-3} for $P = -3$ bar (solid line), as well as $\rho^1(r)$ densities for three ξ values (arbitrary units).

$$\omega_s = \sqrt{\omega_{0,0}^2 + \left(\frac{\xi\mu_2}{2m}\right)^2} - \frac{\xi\mu_2}{2m}, \qquad (11)$$

where we have used a standard matrix notation[9,10] to denote as $\omega_{0,0}$ and $|\rho_0^{1(0)}\rangle$ the higher frequency solution of the non-viscous problem $(m\omega_{0,n}^2 - \mathcal{M}_1)|\rho_n^{1(0)}\rangle = 0$, and have defined $\mu_2 \equiv -\langle\rho_0^{1(0)}|\mathcal{M}_2|\rho_0^{1(0)}\rangle > 0$. Equation (11) is similar to that given in Ref. 8 for the dissipation-renormalized frequency ω_s in the case of frequency-independent damping.

Figures 1 and 2 indicate that for viscosity values of the order of the experimental one, a sizeable decrease of the crossover temperature occurs. However, the present model still predicts that a transition from thermal to quantum cavitation takes place in liquid ^{3}He.

We finally obtain the homogeneous cavitation pressure P_h from the equation[4]

$$1 = (Vt)_{exp} J_0 e^{-S}, \qquad (12)$$

taking for the experimental volume $\times$ time $(Vt)_{exp}$ a typical value of 10^8 Å^3 s, which corresponds to ^{4}He experiments.[2] We have adopted for J_0 the same prescription as in Ref. 4. Figure 4 shows the homogeneous cavitation pressure as a function of T for the ξ values we have been using.
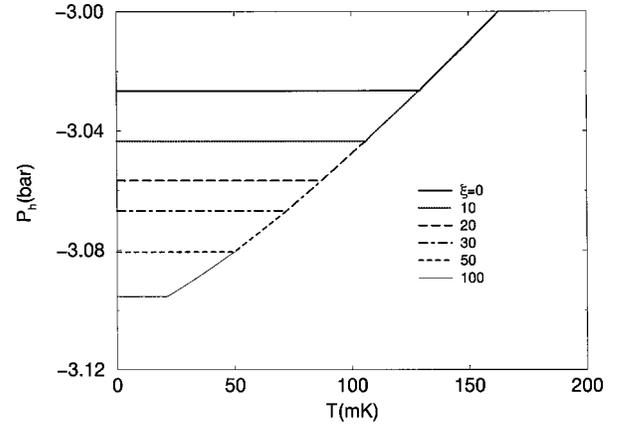
FIG. 4. Homogeneous cavitation pressure P_h as a function of T for $(Vt)_{exp} = 10^8$ Å^3 s, and the ξ values employed in Figs. 1 and 2.

In conclusion, we have developed a phenomenological model to size the effect of dissipation in the cavitation process in liquid ^{3}He that allows one to handle realistic critical cavitation configurations near the spinodal line, and to treat both helium isotopes within the same frame, using the dissipationless limit of the method in the case of ^{4}He. The results we have obtained indicate that for liquid ^{3}He even a moderate dissipation may reduce the crossover temperature in a non-negligible amount, displacing the homogeneous cavitation pressure towards the spinodal value. Viscosity may then be the reason of the inconclusive results for quantum cavitation reported in Ref. 5 which, if confirmed, would indicate that dissipation plays a crucial role in quantum cavitation in liquid helium. The experimental study of cavitation in undersaturated ^{3}He-^{4}He mixtures might then uncover a structure much richer than that theoretically described in Ref. 6, since ^{4}He is still superfluid and ^{3}He is in the normal phase. This would open the possibility of studying the influence of dissipation in the cavitation process varying the ^{3}He concentration.

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