Stability of vortex lines in liquid ^{3}He-^{4}He mixtures at zero temperature

Dora M. Jezek
Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, RA-1428 Buenos Aires, Argentine

Montserrat Guilleumas
Dipartimento di Fisica, Università di Trento, 38050 Povo, Italy

Martí Pi and Manuel Barranco
Departament d'Estructura i Constituents de la Matèria, Facultat de Física, Universitat de Barcelona, E-08028 Barcelona, Spain

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At low temperatures and ^{3}He concentrations below $\sim$6.6%, there is experimental evidence about the existence in liquid helium mixtures of stable vortices with ^{3}He-rich cores. When the system is either supersaturated or submitted to a tensile strength, vortices lose stability becoming metastable and eventually completely unstable, so that their cores freely expand. Within a density functional approach, we have determined the pressure-^{3}He concentration curve along which this instability appears at zero temperature. [S0163-1829(97)14017-6]

The structure, stability, and dynamics of quantized vortices in superfluid ^{4}He have been extensively studied either experimentally or theoretically (for a systematic review see Ref. 1, and for recent work, see for example Refs. 2–7, and references therein). However, less work has been done in the case of ^{3}He-^{4}He solutions.[1] An interesting aspect of the vortex structure in these mixtures was disclosed by Williams and Packard,[8] who provided experimental evidence that in diluted ^{3}He-^{4}He solutions at low temperature (T), stable vortices present ^{3}He condensation onto the core.

In this work we address the problem of the stability of a vortex line in ^{3}He-^{4}He mixtures as a function of pressure (P) and ^{3}He concentration (x). In particular, we will discuss the implications that the existence of ^{3}He-rich vortices may have on the critical supersaturation of isotopic helium mixtures, and will determine the $T=0$ vortex spinodal line as a function of x.

The interest in studying the stability of vortex lines at negative pressures stems from recent attempts to describe the phenomenon of quantum cavitation in liquid ^{4}He.[3] In particular, it has been found[2] that at $T=0$, vortex lines become unstable at $P\sim-8$ bar, whereas the spinodal point is at $P\sim-9.4$ bar,[9] thus quantitatively showing that vorticity raises the spinodal line (see also Ref. 3).

In ^{3}He-^{4}He mixtures the situation is more complex. Indeed, a vortex may become unstable either increasing the ^{3}He concentration or submitting the solution to a tensile strength that may originate a negative pressure. A characteristic of the zero temperature P-x phase diagram that makes more intrincate the study of vortex lines in this system is the existence of a demixing line $P_d(x)$, experimentally determined from saturation up to 20 bar.[10] A recent calculation[11] has found that the line continues down to $x\sim$ 2.4% and $P\sim-3.1$ bar, which is the $T=0$ spinodal point of pure ^{3}He. The existence in the negative pressure, metastable region of this equilibrium line between pure ^{3}He and the mixture, affects the description of cavitation caused either by bubble growing[11,12] or by vortex destabilization.

Instabilities caused by supersaturation were studied in Ref. 13 within the so-called hollow core model (HCM) adapted to helium mixtures by replacing the hollow by a ^{3}He-rich core. This study was carried out at positive pressures, where the model works better. We have predicted that at $P=0$, vortices become completely unstable for $x>8.2$%.

Although HCM might seem too crude a model, at positive pressures and close to the demixing line it bears the basic physical ingredients, so it can be used as a useful guide to understand the appearance of unstable vortices. Let us just recall that within HCM, the total energy per unit vortex length as a function of the core radius R reads[13]

$$\Omega_{\mathrm{HCM}}=SR+VR^2+E_0\ln(R_\infty/R). \tag{1}$$

It is the sum of an S "surface term," plus a V "volume term," plus a kinetic energy term. When the vortex is in the stable phase all three terms are positive and only stable vortices may exist.

On the contrary, in the metastable phase the factor V in the volume term becomes negative. Depending on whether the system is underpressured or supersaturated, this factor is proportional either to the difference ΔP between the pressure in the hollow and in the bulk or to the difference $\Delta\mu$ between the chemical potential of ^{3}He in the mixture and of pure ^{3}He.[12] As a consequence, the stable vortex becomes metastable and there exists a critical vortex configuration for which the potential barrier has a maximum located at a core radius R_c. The potential barrier vanishes at the saddle configuration when the metastable vortex becomes critical, so the vortex core freely expands. For a given x, it happens at a pressure we shall call vortex spinodal pressure.

These arguments can be made quantitative within the density functional approach. To this end, we resort to the density functional of Refs. 11,12 which describes the basic thermodynamical properties of liquid helium mixtures at zero temperature. This approach, phenomenological in its very nature, has proven to be a very robust way to address the description of the vortex structure in pure ^{4}He, comparing well with microscopic approaches.[2] We would like to point out that recent attempts to microscopically calculate the vortex structure[5–7] have just addressed two-dimensional, pure ^{4}He vortices, and that the description of quantized vortices in helium mixtures is not within their reach.

For the sake of simplicity, we address the problem of a vortex line. Using cylindrical coordinates and taking the vortex line as the z axis, the density profiles depend on the r distance to the z axis and are obtained solving the Euler-Lagrange equations

$$\frac{\delta\omega(\rho_3,\rho_4)}{\delta\rho_q}=0, \quad q=3,4, \tag{2}$$

where $\omega(\rho_3,\rho_4)$ is the grand potential density functional[11,12] to which we have added a centrifugal term $\hbar^2\rho_4 n^2/(2m_4 r^2)$ (Ref. 2) associated with the superfluid flow. We choose the quantum circulation number $n=1$ because it corresponds to the most stable vortex,[14] m_4 is the ^{4}He atomic mass, and ρ_q are the particle densities of each helium isotope.

For given P and x, Eqs. (2) are solved imposing that at long distances from the z axis, ρ_q equals that of the metastable, homogeneous liquid ρ_q^h [x is simply $\rho_3^h/(\rho_3^h+\rho_4^h)$], and that ρ_4 and the r derivative of ρ_3 are zero on the z axis. Notice that the metastable and critical configurations are solutions of these equations for the *same* P and x conditions. The barrier height per unit vortex length is

$$\Delta\Omega=2\pi\int r dr[\omega(\rho_3^c,\rho_4^c)-\omega(\rho_3^m,\rho_4^m)], \tag{3}$$

where ρ_q^c and ρ_q^m are the particle densities of the critical and metastable vortices, respectively. It is worth to note that $\Delta\Omega$ is a *finite* quantity: there is no need to introduce any r cutoff as it would have been unavoidable if we had described either configuration separately.

Depending on the situation of the metastable vortex in the P-x plane, there may exist two different kinds of critical configurations. To illustrate it, we show in Fig. 1 the P-x phase diagram at $T=0$ (Ref. 11) and three selected metastable configurations labeled 1 to 3. The grey zone represents the stable region and the dashed line is the demixing line. Configuration 1 is underpressed, configuration 2 is supersaturated, and configuration 3 is both. In all three cases, the metastable configuration corresponds to a rather compact vortex filled with ^{3}He whose radius increases with increasing x. This is not the case for the critical vortex. Indeed, as configuration 2 is in the supersaturated region, the critical vortex may have a large R_c radius if that point is close enough to the demixing line and its core is filled with almost pure ^{3}He. R_c diverges at the demixing line and also $\Delta\Omega$. Since configuration 1 is underpressed and undersaturated, the critical vortex also has a large radius provided point 1 is close to the $P=0$ line, but its core is almost empty, with the surface covered by ^{3}He (Andreev states). R_c diverges at the $P=0$ line, and also $\Delta\Omega$. As configuration 3 is underpressed and supersaturated, it may have two possible critical configurations, one bearing the characteristics of configuration 1 and another bearing those of configuration 2. The one with lower

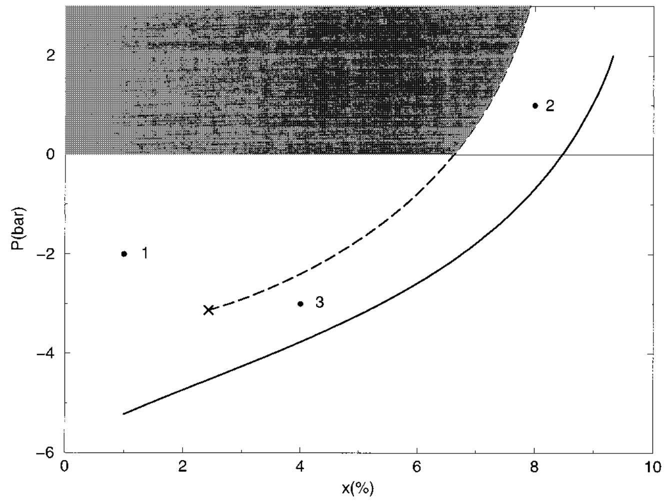

FIG. 1. P-x phase diagram at $T=0$. The grey zone represents the stable region. The dashed curve is the demixing line, which ends at the $P=-3.12$ bar, $x=2.43\%$ (cross), and the solid curve is the vortex spinodal line.

$\Delta\Omega$ will be preferable. It is worth noting that, at negative pressures, irrespective of which kind of critical configuration has a lower barrier and triggers phase separation, for concentrations below $x_{\rm cr}$, the maximum solubility value at $P=0$, the system will eventually evolve towards an homogeneous mixture state at zero pressure,[15] whereas for concentrations above that value, the system will undergo a demixing process that will drive it to a zero pressure state made of a pure ^{3}He phase in equilibrium with a saturated ^{3}He-^{4}He mixture. At positive pressures, metastable vortices in the supersaturated region (point 2, for example) can only cause demixing.

We have plotted in Fig. 2 the critical and metastable density profiles corresponding to a type 1 configuration with $P=-1.66$ bar, $x=1\%$ [Fig. 2(a)], and to a type 2 configuration with $P=0.91$ bar, $x=8\%$ [Fig. 2(b)].

Figure 3 shows the barrier height per unit length as a function of P for $x=1$ to 9 %. For the sake of illustration, we display for $x=4\%$, the barriers corresponding to the two kinds of critical vortices already discussed: the dashed (solid) line is $\Delta\Omega$ for empty- (filled-)core configurations. Notice that these curves have different slopes because they diverge at different pressures, the former at $P=0$, and the later at $P=P_d(x)$. For a given x, the P value at which $\Delta\Omega$ is negligible defines a point along the vortex spinodal curve. That curve is the solid line in Fig. 1.

Figure 4 shows the core radius of the saddle configurations as a function of x (solid line). Following Ref. 6, we have defined that radius as the r value at which the superfluid circulation current $\rho_4^c(r)/r$ has a maximum. We have found that metastable vortices in the mixture have a core radius larger than in pure ^{4}He.[2] This is in agreement with the experimental findings for stable vortices.[1] Also shown in that figure is the radius of the stable vortex at $P=0$ (dashed line), which is actually metastable above $x=6.6\%$. Notice that the vortex spinodal line shown in Fig. 1 may be used to obtain the core radius of the saddle configuration as a function of the pressure instead of x.

The above results have implications on the critical supersaturation degree $\Delta x_{\rm cr}$ of isotopic helium solutions at low

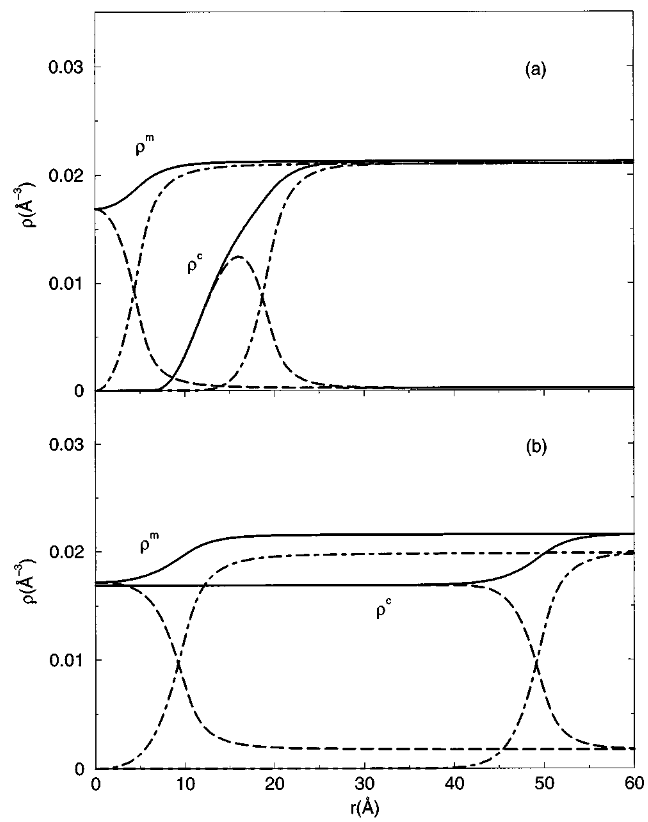

FIG. 2. (a): vortex profiles for $x=1\%$ and $P=-1.66$ bar. (b): vortex profiles for $x=8\%$ and $P=0.91$ bar. The solid lines represent the total particle density and the dash-dotted (dashed) lines, the ρ_4 (ρ_3) densities. Critical (metastable) configurations are denoted as ρ^c (ρ^m).

temperatures. Recent experiments[16,17] have found $\Delta x_{\rm cr}$ below $\sim 1\%$, whereas classical nucleation theory yields $\sim 10\%$.[13,18] The microscopically calculated spinodal line[19] is about the same value. Within the HCM, we have argued[13] that the rather small degree of supersaturation experimentally found could be due to destabilization of ^{3}He-rich vortices.

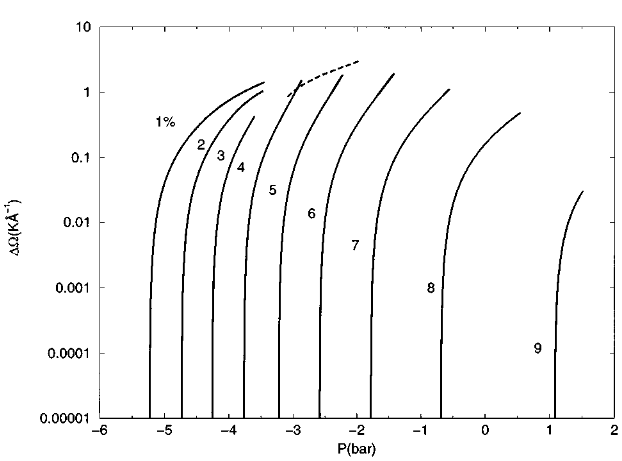

FIG. 3. Barrier height per unit vortex length as a function of P for the indicated ^{3}He concentrations. At $x=4\%$, the dashed line corresponds to empty-core configurations, whereas the solid line corresponds to filled-core ones.

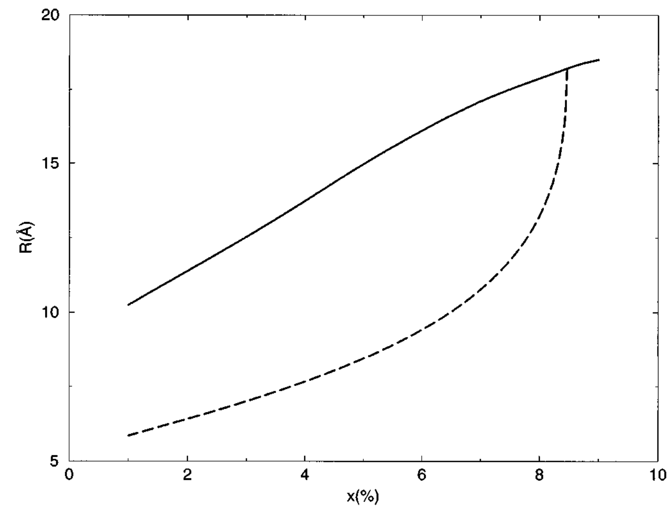

FIG. 4. Solid line, radius of the saddle vortex core. Dashed line, radius of the stable vortex at $P=0$.

The density functional approach yields $\Delta x_{\rm cr}$ values around 2% (we recall that $x_{\rm cr}\sim$ 6.6 %[20]). This is shown in Fig. 5 for $P=0$, 0.5 and 1 bar. A discrepancy with experiment still exists. It is unclear whether considering more realistic vortex geometries, such as vortex rings, could bring theory closer to experiment. Other possibilities to improve on the agreement, such as vortex destabilization due to quantum tunneling through, or to thermal activation over the barrier, seem to be ruled out. The former because of the extremely small quantum-to-thermal crossover temperature[21] and the latter because of the large mass of the critical vortex.[13]

Vortex destabilization at negative pressures may also have implications on the phenomenon of cavitation in isotopic helium solutions.[11,12] Realistic calculations[22] indicate that for a ^{3}He concentration as small as $x\sim 1\%$, cavitation driven either by quantum or by thermal fluctuations triggers phase separation if the system is submitted to a tensile strength of 8.2 bar. If there are metastable vortices in the mixture, the present calculations show that for much smaller tensile strengths, of about 5.2 bar, the solution undergoes phase

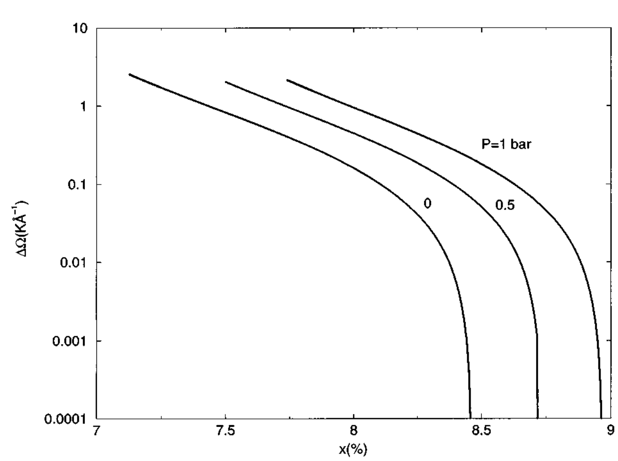

FIG. 5. Barrier height per unit vortex length as a function of x for $P=0$, 0.5 and 1 bar. The corresponding critical x values are 8.46, 8.74, and 8.97 %, respectively.

separation. A delicate question is whether ^{3}He atoms have enough time to diffuse into the vortex core on the time scale of current cavitation experiments[23] when their concentration is too small.

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