Anomalous crossover between thermal and shot noise in macroscopic diffusive conductors

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We predict the existence of an anomalous crossover between thermal and shot noise in macroscopic diffusive conductors. We first show that, besides thermal noise, these systems may also exhibit shot noise due to fluctuations of the total number of carriers in the system. Then we show that at increasing currents the crossover between the two noise behaviors is anomalous, in the sense that the low-frequency current spectral density displays a region with a superlinear dependence on the current up to a cubic law. The anomaly is due to the nontrivial coupling in the presence of the long-range Coulomb interaction among the three time scales relevant to the phenomenon, namely, diffusion, transit, and dielectric relaxation time.

Shot noise and thermal noise are the two prototypes of noise present in nature.[1,2] Thermal noise is displayed by a conductor at or near equilibrium, and is associated with its conductance through Nyquist theorem[3] $S_I^{ther}(0)=4k_BTG$, where $S_I^{ther}(0)$ is the low-frequency current spectral density, k_B the Boltzman constant, T the temperature, and G the conductance. Shot noise is due to the discreteness of the carriers charge, and displays a low-frequency spectral density of current fluctuations in the form $S_I^{shot}(0)=\gamma\ 2q\overline{I}$, where $\overline{I}$ is the average dc current, q the carrier charge, and γ the so called Fano factor. Being an excess noise, it can only be observed under nonequilibrium conditions and provides information not available from linear response coefficients such as conductance. Following Landauer's ideas,[4] these two types of noise are special forms of a more general noise formula representing different manifestations of the same underlying microscopic mechanisms. As a result, for systems displaying shot noise one should expect a continuous and smooth transition between the equilibrium thermal noise and the nonequilibrium shot noise. Two examples of such transitions are provided by the expressions

$$S_I(0)=2q\overline{I}\coth\left(\frac{qV}{2k_BT}\right), \tag{1}$$

and

$$S_I(0)=4k_BTG\left[(1-\gamma)+\gamma\frac{qV}{2k_BT}\coth\left(\frac{qV}{2k_BT}\right)\right], \tag{2}$$

which represent standard transitions for a classical and a quantum system, respectively.[1,4] In previous equations V is the applied voltage. In both cases one obtains $S_I^{ther}(0)$ at or near equilibrium, when $|qV/k_BT|\ll 1$, and $S_I^{shot}(0)$ far from equilibrium, when $|qV/k_BT|\gg 1$.

A variety of classical and quantum physical systems exhibit the above *coth*-like crossover. Among them we note *p-n* junctions,[1] Schottky barrier diodes,[1] tunnel diodes,[1] and mesoscopic diffusive conductors with coherent[5] and semiclassical transport.[6,7] We remark that an essential feature of the above formulas is to predict a monotonic increase of the spectral density with current which never exceeds a linear dependence. Finally, we note that it is a common belief that macroscopic conductors do not display shot noise.[8]

The aim of this article is to prove that macroscopic conductors can display shot noise and that the transition between thermal and shot noise shows a remarkable deviation from the standard *coth*-like behavior. In particular, the region of crossover shows a current spectral density that increases more than linearly with current, up to a cubic dependence.

The system under consideration is a macroscopic homogeneous diffusive conductor of length L (henceforth shortly referred to as *macroscopic diffusive conductor*). The conductor is considered to be macroscopic in the sense that the sample length L satisfies $L\gg l_{in},l_e$, where l_{in} and l_e are the inelastic and elastic mean free paths, respectively. Moreover, homogeneous conditions implies that the stationary electric field and charge density profiles are homogeneous. Although at first sight it seems surprising that macroscopic diffusive conductors are able to display shot noise, see, for instance Ref. 8, it is easy to convince oneself that this is indeed the case. The key argument is provided by the fact that the diffusion of carriers through the sample, a part from velocity fluctuations, also induce *fluctuations of the total number of particles inside the sample*. These number fluctuations are related to the fact that the time a carrier spends to cross the sample depends on the particular succession of scattering events, thus giving rise to fluctuations in the instantaneous value of the total number of particles inside the sample. As a consequence, besides the usual thermal noise associated with velocity fluctuations, we will have an excess noise associated with number fluctuations. Note that existing arguments against the presence of shot noise in macroscopic conductors are always based on the assumption that the number of fluctuations are negligible, what is not always true in macroscopic diffusive conductors, as will be shown below.

That number fluctuations can give rise to shot noise can be seen as follows. The excess noise associated with number fluctuations can be characterized as[9]

$$S_I^{ex}(0)=\left(\frac{\overline{I}}{\overline{N}}\right)^2 S_N(0), \tag{3}$$

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where $S_N(0)=2\int_{-\infty}^{+\infty}dt\,\overline{\delta N(0)\,\delta N(t)}$ is the low-frequency spectral density of number fluctuations and $\overline{N}$ the average number of carriers inside the system. Furthermore, within an exponential model for the decay of number fluctuations one assumes $S_N(0)=\overline{\delta N^2}\tau_N$, where $\overline{\delta N^2}$ is the variance and τ_N the relaxation time for such fluctuations.[10] If the relaxation of number fluctuations takes place on a time scale of the order of the transit time τ_T, then one has $\tau_N\sim\tau_T$. By using in Eq. (3) that the transit time for a homogeneous conductor is $\tau_T=L/v=q\overline{N}/\overline{I}$, where v is the drift velocity, and where we have used $\overline{I}=qA\overline{n}v$, with A being the cross sectional area and $\overline{n}$ the average carrier density, we obtain $S_I^{ex}(0)\sim q(\overline{\delta N^2}/\overline{N})\overline{I}$, which is shot-noise-like.

Therefore, macroscopic diffusive conductors offer a new and simple example in which to investigate in detail the transition between thermal and shot noise. To this purpose, we need an explicit expression for the current spectral density valid, in particular, in the transition region between thermal and shot noise. This explicit expression can be obtained by solving the appropriate equations for the fluctuations. For simplicity the sample is assumed to have a transversal size sufficiently thick to allow a one-dimensional electrostatic treatment in the x direction and to neglect the effects of boundaries in the y and z directions. Furthermore, since we are interested in the low-frequency noise properties (beyond $1/f$ noise), we will neglect the displacement current. Accordingly the standard drift-diffusion Langevin equation for a macroscopic diffusive conductor reads[11]

$$\frac{I(t)}{A}=qn\mu E+qD\frac{dn}{dx}+\frac{\delta I_x(t)}{A}, \tag{4}$$

which after linearization around the stationary homogeneous state gives[12]

$$\frac{\delta I(t)}{A}=q\mu\overline{E}\eta\delta n_x(t)+q\overline{n}\mu\,\delta E_x(t)+qD\frac{d\,\delta n_x(t)}{dx}+\frac{\delta I_x(t)}{A}. \tag{5}$$

Here, $\delta E_x(t)$ and $\delta n_x(t)$ refer to the fluctuations of electric field and number density at point x, respectively, while $\delta I(t)$ refers to the fluctuations of the total current. Moreover, μ is the mobility, $\overline{E}$ the average electric field, D the diffusion coefficient, and the bar denotes a time average. We assume that μ and D may depend on $\overline{n}$, in order to include in the model also degenerate conductors. The numerical factor $\eta=(1+\mu_N'/\mu/\overline{n})$, with $\mu_N'=\partial\mu/\partial\overline{n}$, accounts for the possible dependence of the mobility on the number density, and $\delta I_x(t)$ is a Langevin noise source, which accounts for the fluctuations of current due to the diffusion of carriers inside the sample. It has zero mean and correlation function,

$$\langle\,\delta I_x(t)\,\delta I_{x'}(t')\rangle=\frac{1}{2}K\,\delta(x-x')\,\delta(t-t'), \tag{6}$$

where $K=4qAk_BT\mu\overline{n}$ is the strength of the fluctuations. Equation (5) must be supplemented with the Poisson equation

$$\frac{d\,\delta E_x(t)}{dx}=-\frac{q}{\epsilon}\,\delta n_x(t), \tag{7}$$

where ϵ is the electric permittivity. Generally, Eqs. (5) and (7) are combined into a single equation for the electric field fluctuation of the form

$$\left(\frac{d^2}{dx^2}+\frac{1}{L_E}\frac{d}{dx}-\frac{1}{L_D^2}\right)\delta E_x(t)=\frac{\delta I_x(t)-\delta I(t)}{\epsilon AD}, \tag{8}$$

where $L_E=D/\eta\mu\overline{E}$ and $L_D=(D\epsilon/\mu q\overline{n})^{1/2}$. Here, L_E/L characterizes the ratio between a characteristic carrier energy and the energy supplied by the applied voltage, and L_D is the Debye screening length. The ratio L/L_D constitutes a relevant indicator of the effects of the long-range Coulomb interaction on the current fluctuations, since for $L/L_D\ll 1$, one can neglect the term proportional to $\delta E_x(t)$ in Eq. (5), and the equation for the current fluctuations becomes uncoupled from the Poisson equation. Moreover, since contact effects are negligible we will use as boundary conditions $\delta n_0=\delta n_L=0$, which gives

$$\left.\frac{d\,\delta E_x(t)}{dx}\right|_0=\left.\frac{d\,\delta E_x(t)}{dx}\right|_L=0. \tag{9}$$

Equation (8), together with Eqs. (6) and (9), constitute a complete set of equations to analyze the noise properties of macroscopic diffusive conductors. In the present form, they can be used to describe both degenerate as well as nondegenerate conductors. The fact that the same underlying scattering mechanisms are responsible for the noise properties of the system is reflected by the presence of a unique Langevin source in the model. As Eq. (8) is a second-order differential equation with constant coefficients, its solution can be obtained in a closed analytical form. Hence, from the expression of $\delta E_x(t)$ one can compute the voltage fluctuation under fixed current conditions $\delta_I V(t)=\int_0^L dx\,\delta E_x(t)$ [where one uses $\delta I(t)=0$], from which the current spectral density can be obtained as $S_I(0)=G^2 2\int_{-\infty}^{+\infty}dt\,\overline{\delta_I V(0)\,\delta_I V(t)}$, with $G=qA\mu\overline{n}/L$. After simple but cumbersome algebra, the final result can be written in the form

$$S_I(0)=S_I^{ther}(0)+S_I^{ex}(0), \tag{10}$$

where

$$S_I^{ther}(0)=\frac{K}{L}=4k_BTG, \tag{11}$$

and where

$$S_I^{ex}(0)=K\frac{(\lambda_2^2-\lambda_1^2)}{2L^2\lambda_1^2\lambda_2^2}\frac{(e^{\lambda_1 L}-1)(e^{\lambda_2 L}-1)}{(e^{\lambda_2 L}-e^{\lambda_1 L})^2}[\lambda_2(e^{\lambda_2 L}+1)\times(e^{\lambda_1 L}-1)-\lambda_1(e^{\lambda_1 L}+1)(e^{\lambda_2 L}-1)]. \tag{12}$$

Here, λ_1 and λ_2 are the two eigenvalues of Eq. (8) and are given by

$$\lambda_{1,2}=-\frac{1}{2L_E}\left(1\pm\sqrt{1+4\frac{L_E^2}{L_D^2}}\right). \tag{13}$$

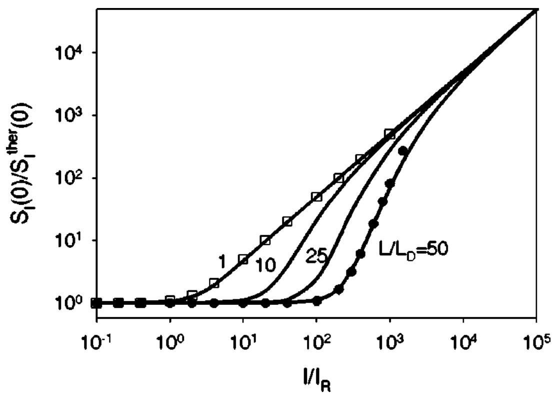

FIG. 1. Normalized current spectral density $S_I(0)/S_I^{ther}(0)$ as a function of the normalized current $\overline{I}/I_R$ for different sample lengths $L/L_D=1,10,25,50$, as obtained from present theory (continuous lines). For $L<L_D$ the curves are indistinguishable from those corresponding to $L/L_D=1$. Also shown for comparison is a standard crossover between thermal and shot noise for a classical system, as given by Eq. (1) (empty squares), and the cubic asymptotic expression of the anomalous crossover, as given in Eq. (16) (filled circles).

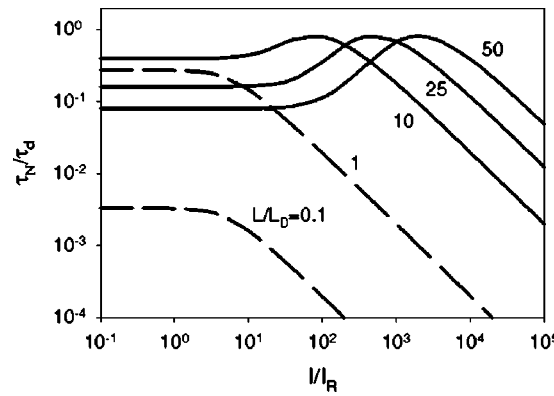

FIG. 2. Characteristic time for number fluctuations normalized to the dielectric relaxation time τ_N/τ_d as a function of the normalized current $\overline{I}/I_R$ for different sample lengths $L/L_D=0.1,1$ (dashed lines) and $L/L_D=10,25,50$ (continuous lines).

Equations (10)–(12) constitute the general expression for the low-frequency current spectral density of a macroscopic diffusive conductor, and represent the main result of the present paper. In Eq. (10) we distinguish two different contributions. The first one, $S_I^{ther}(0)$, corresponds to thermal noise. The second one, $S_I^{ex}(0)$, constitutes an excess noise and it is directly related to carrier number fluctuations. This can be proved directly by computing $S_N(0)$ from the solution of Eq. (8) by considering that the number fluctuations are given through $\delta N(t)=A\int_0^L dx\,\delta n_x=A(\epsilon/q)[\delta E_0(t)-\delta E_L(t)]$. One then obtains the identity

$$S_I^{ex}(0)=\left(\frac{\overline{I}}{\overline{N}}\right)^2\eta^2 S_N(0). \tag{14}$$

Equation (14) is similar to Eq. (3) except for the presence of η, which accounts for the possible dependence of the mobility on carrier density. From Eqs. (12) and (14), it can be shown that when $L_D^2/L_E\gg L\gg L_D$ or $L_D\gg L\gg L_E$ one has

$$S_I^{ex}(0)=2\gamma q\overline{I}, \tag{15}$$

where $\gamma=\eta k_B T\partial\ln\overline{N}/\partial E_F$. This result proves the possibility for macroscopic diffusive conductors to display shot noise. By defining a characteristic time associated with number fluctuations through $\tau_N=S_N(0)/(\overline{\delta N^2})^{eq}$, with $(\overline{\delta N^2})^{eq}=\overline{N}k_B T\partial\ln\overline{N}/\partial E_F$ being the variance of number fluctuation at equilibrium, Eq. (15) corresponds to a situation in which $\tau_N\approx(2/\eta)\tau_T$, thus confirming that when number fluctuations relax on the time scale given by the transit time they give rise to shot noise.

Now we are in a position to investigate the properties of the transition between thermal and shot noise. In Fig. 1 we display the current spectral density for an ohmic conductor obtained from Eqs. (10)–(12), as a function of current for different sample lengths. The current is normalized to $I_R=GV_R$ where $V_R=(\overline{n}/\eta q)\partial E_F/\partial\overline{n}$. In the present units the curves corresponding to $L/L_D<1$ are indistinguishable from the curve corresponding to $L/L_D=1$. In the figure we can easily identify the thermal and shot-noise regimes as the constant and proportional to current behaviors, respectively. Also depicted for comparison is the current spectral density of Eq. (1) that represents the standard transition between thermal and shot noise for a classical system (empty squares). Remarkably, while the transition between thermal and shot noise follows the standard form for $L<L_D$, in the opposite case $L>L_D$ it is anomalous. The anomaly is characterized by a spectral density that at most increases with the third power of the current tending asymptotically to

$$\frac{S_I(0)}{S_I^{ther}(0)}=\left[1+\frac{1}{2}\left(\frac{L_D}{L}\right)^4\left(\frac{\overline{I}}{I_R}\right)^3\right], \tag{16}$$

which holds for $0\leqslant I\lesssim(L/L_D)^2 I_R$ as can be seen in Fig. 1, where the filled circles represent Eq. (16). Since this anomalous crossover is absent for $L<L_D$, i.e., when the long-range Coulomb interaction does not affect the current fluctuations, we conclude that this interaction plays a central role in this unexpected behavior.

To better understand the role of the long-range Coulomb interaction in the origin of this anomaly, we will analyze how the three characteristic times in the system combine to yield τ_N. For the present case the following characteristic times can be identified: the diffusion time $\tau_D=L^2/D$, the dielectric relaxation time $\tau_d=\epsilon/q\overline{n}\mu$ and the already defined transit time τ_T.

In Fig. 2 we plot τ_N as obtained from our theory as a function of current for different sample lengths. Here, we clearly identify two different behaviors for τ_N depending on whether $L/L_D\ll 1$ or $L/L_D\gg 1$. For $L/L_D\ll 1$ we observe a smooth transition between the equilibrium value $\tau_N\approx 1/3\tau_D$ and the far from equilibrium value $\tau_N\approx(2/\eta)\tau_T$. This result shows that when the long-range Coulomb interaction is not effective, only τ_D and τ_T are relevant. As a consequence, near equilibrium we have $\tau_D\ll\tau_T$ and number fluctuations are governed by diffusion, while far from equilibrium we have $\tau_D\gg\tau_T$ and they are governed by the transit time, thus giving rise to shot noise. On the other hand, when $L/L_D\gg 1$ the transition between the equilibrium value $\tau_N\approx 4(\tau_d/\tau_D)^{1/2}\tau_d$ and the far from equilibrium value τ_N

$\approx(2/\eta)\tau_T$ is mediated by a region in which $\tau_N \approx 2\eta\tau_d^2/\tau_T$. The far from equilibrium behavior, being dominated by the transit time, gives rise to shot noise, while in the intermediate region τ_N is proportional to the current, thus giving rise to the cubic dependence of the current spectral density. Notice that the transition between the intermediate and the shot-noise region takes place when $\tau_N \sim \tau_d \sim \tau_T$. From these results we conclude that the origin of the anomalous transition between thermal and shot noise can be found in the nontrivial coupling between the different characteristic times in the presence of long-range Coulomb interaction.

From the previous analysis we argue that there are two possible ways of providing an experimental test of our theory. The first way is an indirect test to be performed at or near equilibrium. It consists in proving the nontrivial coupling of the characteristic times in the presence of the long-range Coulomb interaction. In this case, when $L/L_D \gg 1$, one should obtain a characteristic time for number fluctuations in agreement with the relationship $\tau_N \approx 4(\tau_d/\tau_D)^{1/2}\tau_d = 4(\epsilon/q\mu\overline{n})^{3/2}D^{1/2}/L$. The second way is a direct test, which consists in observing the current dependence of the current spectral density. According to Eq. (16) one should observe the anomalous transition for $L \gg L_D$ when $\overline{I} \gtrsim (L/L_D)^{4/3}I_R$, or analogously for $\overline{V}/L = \overline{E} \gtrsim (L/L_D)^{4/3}V_R/L$. To this end, nondegenerate semiconductor systems offer the best possibilities. For a nondegenerate semiconductor, with typical parameters $\overline{n} \sim 10^{14}\ \mathrm{cm}^{-3}$, $T \sim 300$ K, $\epsilon \sim 10\epsilon_0$, one has $L_D \sim 0.4\ \mu$m and $V_R = k_B T/q = 0.0259$ V. Therefore for $L = 50L_D = 20\ \mu$m one enters the anomalous regime for $\overline{E} \gtrsim 2$ kV/cm. This value of the electric field is experimentally accessible. In addition, when $\overline{I} \gtrsim (L/L_D)^2 I_R$, that is for $\overline{V}/L = \overline{E} \gtrsim (q/\epsilon)\overline{n}L$, one should enter the regime of shot noise. For the parameters chosen above we obtain the condition $\overline{E} \gtrsim 35$ kV/cm, which is still experimentally accessible.[13]

In summary, we have proven that a macroscopic diffusive conductor can display shot noise, and that the transition between thermal and shot noise is anomalous when the length of the sample is much longer than the Debye screening length. The anomaly of the transition consists in a nonlinear dependence of the low-frequency spectral density of current fluctuations upon the current, which can lead up to a cubic behavior. The origin of this unexpected behavior is related to the nontrivial coupling among diffusion, dielectric relaxation, and drift in the presence of the long-range Coulomb interaction.

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[13] By exhibiting linear velocity field characteristics up to several kV/cm, compound semiconductors (for instance, CdTe) constitute the most suitable materials for this type of experiment.