Impedance field and transition from thermal to shot noise in $Cd_{1-x}Zn_xTe$ semi-insulating Ohmic detectors

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Within a drift-diffusion model we investigate the role of the self-consistent electric field in determining the impedance field of a macroscopic Ohmic (linear) resistor made by a compensated semi-insulating semiconductor at arbitrary values of the applied voltage. The presence of long-range Coulomb correlations is found to be responsible for a reshaping of the spatial profile of the impedance field. This reshaping gives a null contribution to the macroscopic impedance but modifies essentially the transition from thermal to shot noise of a macroscopic linear resistor. Theoretical calculations explain a set of noise experiments carried out in semi-insulating CdZnTe detectors.

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I. INTRODUCTION

Shot noise is due to the discreteness of the electrical charge responsible for current. It is evidenced by a frequency independent current spectral density obeying the relation $S_I = \Gamma 2qI$, where q is the absolute value of the charge unit determining the steady current I and Γ is the Fano factor. In the absence of correlation between carriers, $\Gamma = 1$. The presence of attractive or repulsive correlations between carriers leads to the enhancement ($\Gamma > 1$) or suppression ($\Gamma < 1$) from the full value $2qI$ of the shot noise. There is a widespread claim that shot noise can be observed in electronic devices provided that there exists an internal potential energy barrier[1] or that there is no inelastic scattering.[2–4] In other words, it is generally speculated that, in a macroscopic conductor, shot noise is washed out by inelastic interactions, which are unavoidable because of the macroscopic length of the active region.[5] In a previous work,[6] it was noticed that the possibility to observe shot noise in a macroscopic conductor is related to the fact that the instantaneous number of free carriers inside the sample can fluctuate in time.[7] Accordingly, this possibility can be accomplished when the dynamic transit time, $\tau_T = L^2/(\mu V)$, becomes shorter than the dielectric relaxation time of the material, $\tau_d = \epsilon_0 \epsilon_r \rho$ (here ϵ_0 is the vacuum permittivity, ϵ_r is the relative static dielectric constant of the material, ρ is its resistivity, L is the length of the device, μ is the carrier mobility, and V is the applied voltage). Under such a condition, the long-range Coulomb interaction does not induce correlations between current fluctuations; thus, charge neutrality of the device can be violated and shot noise can be observed. Numerical estimates indicate that this constraint is fulfilled when the choice of the sample is limited to highly resistive materials, such as semi-insulating semiconductors. Theoretical predictions, carried out for the case of low-frequency diffusion noise in the presence of Coulomb correlations,[8] were experimentally confirmed in semi-insulating CdTe,[6,9] thus proving that shot noise can be observed also in macroscopic conductors. In particular, the crossover between thermal and shot noise was found to exhibit an anomalous behavior because of the presence of long-range Coulomb interaction and/or of generation-recombination (GR) processes.[8] The case of GR noise alone was investigated in Ref. 10 for the case of low frequency and in Ref. 11 in the full frequency region. However, the joint presence of diffusion and GR noises within a self-consistent approach which includes the role played by Coulomb interaction was not carried out.

The aim of this work is to fill this lack of knowledge by developing a theoretical approach which combines diffusion and GR noise and by investigating how the GR processes can have an indirect effect on the diffusion noise even when the GR noise is not directly observed. Accordingly, we will be in the position to study the transition from thermal to shot noise in semi-insulating linear resistors, usually adopted for room temperature radiation sensor application.[12] For this purpose we make use of a standard impedance field method generalized to include the presence of the diffusion term in the stochastic equation.[13] The impedance field method[14] is a powerful technique to calculate voltage and/or current fluctuations in two terminal devices. Within this scheme, the voltage (current) spectral density is obtained by convolving in a spatial integral the two basic ingredients, namely, the local impedance field and the local noise source. The impedance field is a physical valuable quantity in itself, and a rigorous calculation of this quantity involves the solution of a second-order stochastic differential equation.[13] Here we detail the derivation of the impedance field for the case of a homogeneous resistor within the drift-diffusion model. In particular, we discuss the effect of the presence or the absence of long-range Coulomb interactions and their implications in the calculation of the macroscopic impedance and noise. Theoretical results will be validated with experiments carried out in semi-insulating CdZnTe linear detectors.[15]

The paper is organized as follows. Section II reports the theoretical approach developed for a two terminal linear resistor, where charge transport is determined by a drift-diffusion model under conditions of compensation of shallow impurities and in the presence of GR processes controlled by a deep trapping center for majority carriers. Section III provides an explicit formulation of the impedance field in the presence of long-range Coulomb interaction. Numerical results specialized for the case of semi-insulating CdZnTe are presented and discussed in Sec. IV. Major conclusions are drawn in Sec. V. The Appendix details analytical calculations of number-density fluctuations.

II. THEORY

A. Physical system

We consider the case of a semi-insulating homogeneous semiconductor sample, which is of relevant interest to manufacturing radiation detectors. Accordingly, the system under study is a unipolar semiconductor containing shallow donors and acceptors with densities N_D and N_A, respectively, under compensating conditions and in the presence of traps (for electrons in the present case) with a single energy level E_t and density N_t. The trap level E_t is assumed to be deep in the semiconductor band gap, so that traps are partially ionized and hence contribute (i) to the full ionization of shallow donors and acceptors, (ii) to charge transport (nominally of n type in the present case), and (iii) to the determination of noise properties. For a given temperature, the density of ionized traps is N_t^+. The shallow donors and acceptors contribute to the determination of charge neutrality condition, but they do not participate in the noise processes since they are assumed to be fully ionized. Effects of minority carriers (holes in the present case) are neglected. The sample, of cross-sectional area A, is assumed to be thick enough to justify a one-dimensional electrostatic treatment. For the sake of simplicity we assume that the electron mobility is independent of the electric field and nondegenerate conditions apply, so that Einstein relation for mobility and diffusion holds. The contacts consist of metal plates, assumed to form Ohmic contacts, so that the voltage drop inside them is negligibly small. The semiconductor resistivity is assumed to be high enough so that the metal-semiconductor contacts can be treated within the diffusion approximation.

B. Physical model

The equations describing transport and noise properties of the system under study are (i) the continuity equations for the free charge density and for the fixed charge density, (ii) the current equation within the drift-diffusion approach, and (iii) the Poisson equation. These equations, together with the Langevin sources associated with velocity and GR fluctuations, are written below within a one-dimensional geometry:[16]

$$\frac{\partial n(x,t)}{\partial t} = \frac{1}{q}\frac{\partial J_n(x,t)}{\partial x} - r_n(x,t) + g_n(x,t) + \gamma(x,t), \qquad (1)$$

$$\frac{\partial N_t^+(x,t)}{\partial t} = - r_n(x,t) + g_n(x,t) + \gamma(x,t), \qquad (2)$$

$$J_n(x,t) = q\mu n(x,t)E(x,t) + qD\frac{\partial n(x,t)}{\partial x} + qJ_D(x,t), \qquad (3)$$

$$\frac{\partial E(x,t)}{\partial x} = \frac{q}{\epsilon_0\epsilon_r}[N_D^+ - N_A^- + N_t^+(x,t) - n(x,t)], \qquad (4)$$

where r_n and g_n are the recombination and generation rates of the traps, D is the electron diffusion coefficient, n is the local free-carrier concentration, E is the local electric field, γ is the Langevin source of GR noise, and J_D is the Langevin source of velocity fluctuation (diffusion) noise.

The total current density $J(t)$ is

$$J(t) = J_n(x,t) + \epsilon_0\epsilon_r\frac{\partial E(x,t)}{\partial t}, \qquad (5)$$

which, from the model equations, is independent of the spatial coordinate to satisfy instantaneous charge conservation. The Langevin sources associated with diffusion and GR noises have zero mean and low-frequency spectral density of the standard form[1]

$$S_D(0,x,x') = 2\int_{-\infty}^{+\infty} dt\overline{J_D(x,t)J_D(x',0)} = K_D(x)\delta(x-x'), \qquad (6)$$

$$S_{\rm GR}(0,x,x') = 2\int_{-\infty}^{+\infty} dt\overline{\gamma(x,t)\gamma(x',0)} = K_{\rm GR}(x)\delta(x-x'), \qquad (7)$$

where

$$K_D(x) = \frac{4}{q^2A^2}k_BTq\mu\bar{n}(x), \qquad (8)$$

$$K_{\rm GR}(x) = \frac{2}{A}[\bar{g}_n(x) + \bar{r}_n(x)] \qquad (9)$$

are the strengths of the diffusion and GR noise sources, respectively, k_B is the Boltzmann constant, T is the lattice temperature, and the bar denotes time average.

The boundary conditions for the model are given by

$$\bar{n}_0(t) = \bar{n}_L(t) = n_c, \qquad (10)$$

$$\delta n_0(t) = \delta n_L(t) = 0, \qquad (11)$$

where

$$n_c = N_C\exp\left(-\frac{q\phi_{bn}}{k_BT}\right) \qquad (12)$$

is the carrier density at the contacts independent of time and bias, N_C is the effective density of states in the conduction band, and ϕ_{bn} is the height of the contact barrier.

C. Steady-state solution

We assume that by an appropriate choice of the contact parameters a homogeneous solution exists. This solution will read

$$\bar{n}(x) = \bar{n}, \tag{13}$$

$$\bar{N}_t^+(x) = \bar{N}_t^+, \tag{14}$$

$$\bar{E}(x) = \bar{E}, \tag{15}$$

$$\bar{J}_n(x) = \bar{J}_n = q\mu\bar{n}\,\bar{E}, \tag{16}$$

where the free-carrier and trap ionized densities satisfy

$$g_n(\bar{n},\bar{N}_t^+) = r_n(\bar{n},\bar{N}_t^+), \tag{17}$$

$$\bar{n} = N_D^+ - N_A^- + \bar{N}_t^+. \tag{18}$$

The I-V characteristics corresponding to this homogeneous state are taken to be linear,

$$\bar{I} = A\overline{J_n} = \frac{V}{R}, \tag{19}$$

with the Ohmic resistance R given by

$$R = \frac{L}{Aq\mu\bar{n}}. \tag{20}$$

D. Low-frequency noise

In the present work we are interested in the low-frequency noise properties (the case of a frequency dependent GR noise was considered in Ref. 11). Accordingly, we neglect all time derivatives present in the model, which simplifies the calculations considerably. Therefore, the total current fluctuation can be calculated as

$$\delta I(t) = \frac{A}{L}\int_0^L dx\,\delta J_n(x,t). \tag{21}$$

By linearizing the current equation we obtain

$$\delta J_n(x,t) = q\mu\,\delta n(x,t)\bar{E} + q\mu\bar{n}\,\delta E(x,t) + qD\frac{\partial \delta n(x,t)}{\partial x} + qJ_D(x,t), \tag{22}$$

thus giving for the fluctuation of the total current

$$\delta I(t) = \frac{q\mu\bar{E}}{L}\delta N(t) - \frac{qA\mu\bar{n}}{L}\delta V(t) + \frac{qA}{L}\int_0^L dx J_D(x,t), \tag{23}$$

where we have introduced the fluctuations of the total number of free carriers and of the applied bias,

$$\delta N(t) = A\int_0^L dx\,\delta n(x,t), \tag{24}$$

$$\delta V(t) = \int_0^L dx\,\delta E(x,t), \tag{25}$$

and where we have used the boundary conditions $\delta n_0(t) = \delta n_L(t) = 0$. Under fixed bias conditions $[\delta V(t)=0]$ the current fluctuation simply reads

$$\delta I(t) = \left(\frac{\bar{I}}{\bar{N}}\right)\delta N(t) + \frac{qA}{L}\int_0^L dx J_D(x,t), \tag{26}$$

where we have introduced the average value of the total number of free carriers $\bar{N} = \bar{n}AL$. To evaluate the current fluctuations we then need to compute the fluctuations of the total number of free carriers in the system. This calculation is performed in the Appendix. The final result reads

$$\delta N(t) = \delta N_{\mathrm{GR}}(t) + \delta N_D(t), \tag{27}$$

where the contributions related to GR and diffusion processes are given respectively by

$$\delta N_{\mathrm{GR}}(t) = A\tau\int_0^L dx\tilde{G}(x)\gamma(x,t), \tag{28}$$

$$\delta N_D(t) = -A\tilde{\tau}_D\int_0^L dx\tilde{G}'(x)J_D(x,t), \tag{29}$$

where

$$\tilde{G}(x) = 1 - \frac{e^{\lambda_1 L}(e^{\lambda_2 L} - 1)}{(e^{\lambda_2 L} - e^{\lambda_1 L})}e^{-\lambda_1 x} + \frac{e^{\lambda_2 L}(e^{\lambda_1 L} - 1)}{(e^{\lambda_2 L} - e^{\lambda_1 L})}e^{-\lambda_2 x}, \tag{30}$$

with

$$\lambda_1 = \frac{1}{2L_E}\left(-1 + \sqrt{1 + 4\frac{L_E^2}{\tilde{L}_D^2}}\right),$$

$$\lambda_2 = \frac{1}{2L_E}\left(-1 - \sqrt{1 + 4\frac{L_E^2}{\tilde{L}_D^2}}\right), \tag{31}$$

$$L_E = \frac{k_B T}{q\bar{E}}, \quad L_D = \sqrt{\frac{k_B T\epsilon}{q^2\bar{n}}}, \quad \tilde{L}_D = \sqrt{\frac{\tau}{\tau_t'}}L_D, \tag{32}$$

and the carrier lifetime τ is defined from

$$\frac{1}{\tau} = \frac{1}{\tau_t'} + \frac{1}{\tau_n'}, \tag{33}$$

with

$$\frac{1}{\tau_n'} = \frac{\partial r_n(n,N_t^+)}{\partial n} - \frac{\partial g_n(n,N_t^+)}{\partial n}, \tag{34}$$

$$\frac{1}{\tau_t'} = \frac{\partial r_n(n,N_t^+)}{\partial N_t^+} - \frac{\partial g_n(n,N_t^+)}{\partial N_t^+}. \tag{35}$$

Here τ_n' is the lifetime of the free carriers, and τ_t' is the lifetime of the ionized centers. Note that the Debye screening length in the presence of traps, $\widetilde{L}_D$, appears renormalized with respect to the Debye screening length without traps, L_D. Thus, we can introduce a dimensionless quantity $\alpha \geqslant 1$ which describes the effect of screening on the excess noise due to the presence of ionized traps,

$$\alpha = (L_D/\widetilde{L}_D)^2 = 1 + \tau_t'/\tau_n'. \tag{36}$$

From the expression of number-density fluctuations we arrive at the final expression for current fluctuation,

$$\delta I(t) = \frac{qA}{L}\int_0^L dx J_D(x,t) + \left(\frac{\bar{I}}{\bar{N}}\right)\delta N_{\mathrm{GR}}(t) + \left(\frac{\bar{I}}{\bar{N}}\right)\delta N_D(t). \tag{37}$$

The above expression decomposes the current fluctuation into three terms, the first related directly to diffusion noise and the other two related to number fluctuations (the former associated with GR processes and the latter with diffusion processes). It can be shown that the three terms are uncorrelated (see the Appendix). By computing the correlation functions of current fluctuations we arrive at the following expression for the low-frequency current spectral density:

$$S_I(0) = 2\int_{-\infty}^{\infty} \langle \delta I(0)\delta I(t)\rangle dt = S_I^D(0) + S_I^{\mathrm{GR}}(0), \tag{38}$$

where

$$S_I^D(0) = S_I^{th}(0)[1 + s_I^D(\lambda_1,\lambda_2)] \tag{39}$$

arises from the first and third terms in the right-hand side of Eq. (37) and

$$S_I^{\mathrm{GR}}(0) = S_I^{th}(0)\beta\left[\left(\frac{\widetilde{L}_D}{L}\right)^2\left(\frac{qV}{k_BT}\right)^2 - s_I^{\mathrm{GR}}(\lambda_1,\lambda_2)\right] \tag{40}$$

arises from the second term in the right-hand side of Eq. (37) (in spite of the minus sign in the expression for $S_I^{\mathrm{GR}}(0)$, this magnitude is always positive). Here we have defined the equilibrium thermal noise (Nyquist) as

$$S_I^{th}(0) = \frac{4k_BT}{R} \tag{41}$$

and the dimensionless parameter β, which describes the strength of the GR contribution, as

$$\beta = \frac{\tau^2}{\frac{\bar{n}}{\bar{g}_n}\widetilde{\tau}_D} = \frac{\overline{\Delta N^2}}{\bar{N}}\frac{\tau}{\widetilde{\tau}_D} = \frac{\overline{\Delta N^2}}{\bar{N}}\frac{\tau_t' D}{L_D^2} = \frac{\overline{\Delta N^2}}{\bar{N}}\frac{\tau_t'}{\tau_D}, \tag{42}$$

where we have introduced the variance of the total number of carriers,

$$\overline{\Delta N^2} = AL\bar{g}_n\tau, \tag{43}$$

and a renormalized Debye diffusion time $\widetilde{\tau}_D$,

$$\widetilde{\tau}_D = \frac{\widetilde{L}_D^2}{D} = \frac{\widetilde{L}_D^2}{L_D^2}\tau_D, \tag{44}$$

with τ_D the Debye diffusion time. Moreover,

$$s_I^D(\lambda_1,\lambda_2) = \frac{(e^{\lambda_1 L}-1)(e^{\lambda_2 L}-1)(\lambda_2^2-\lambda_1^2)}{2L\lambda_1^2\lambda_2^2(e^{\lambda_2 L}-e^{\lambda_1 L})^2}[\lambda_2(e^{\lambda_2 L}+1) \times(e^{\lambda_1 L}-1) - \lambda_1(e^{\lambda_1 L}+1)(e^{\lambda_2 L}-1)], \tag{45}$$

$$s_I^{\mathrm{GR}}(\lambda_1,\lambda_2) = \frac{(e^{\lambda_1 L}-1)(e^{\lambda_2 L}-1)(\lambda_2^2-\lambda_1^2)}{2L\lambda_1^2\lambda_2^2(e^{\lambda_2 L}-e^{\lambda_1 L})^2} \times[\lambda_1(1-e^{(\lambda_1+\lambda_2)L}-3e^{\lambda_2 L}+3e^{\lambda_1 L}) - \lambda_2(1-e^{(\lambda_1+\lambda_2)L}-3e^{\lambda_1 L}+3e^{\lambda_2 L})]. \tag{46}$$

Note that $s_I^D(\lambda_1,\lambda_2)$ and $s_I^{\mathrm{GR}}(\lambda_1,\lambda_2)$ are dimensionless and positive definite.

Both the diffusion and GR current spectral densities in Eqs. (39) and (40) display two different contributions. The first contribution corresponds to what one would have obtained in a very long (infinite) sample. This can be shown explicitly since for very long samples it is $\lambda_1 L \to +\infty$, $\lambda_2 L \to -\infty$, from where $s_I^{\mathrm{GR}}(\lambda_1,\lambda_2)$, $s_I^D(\lambda_1,\lambda_2) \to 0$. In this limit the current spectral density would simply read

$$S_I^{\infty}(0) = \frac{4k_BT}{R} + 4\overline{\Delta N^2}\tau\left(\frac{\bar{I}}{\bar{N}}\right)^2. \tag{47}$$

The second contribution, being determined by the terms proportional to $s_I^D(\lambda_1,\lambda_2)$ and $s_I^{\mathrm{GR}}(\lambda_1,\lambda_2)$, is related to the finite size of the system. The most surprising effect concerning these terms is that while the contribution related to diffusion noise source increases the overall noise, that related to GR source decreases the overall noise.[10] In other words, the minimum noise associated with diffusion is the Nyquist noise, while the maximum noise associated with GR is the standard excess noise $4\overline{\Delta N^2}\tau(\bar{I}/\bar{N})^2$. As a conclusive remark on this limit, it is confirmed that shot noise does not appear in infinite systems.[17]

We now consider the case in which the finite size of the sample becomes of relevance. This happens when the length of the sample L does not differ significantly from the effective Debye screening length $\widetilde{L}_D$. To illustrate this case, Figs. 1(a) and 1(b) report the current spectral density normalized to the Nyquist noise as a function of the applied voltage normalized to the thermal one for two values of the ratio $L/\widetilde{L}_D$. In this universal form, the diffusion contribution depends only on $L/\widetilde{L}_D$ and the GR contribution on β. From Figs. 1(a) and 1(b) we note the following important results. (i) The diffusion noise (gray lines) can give rise to an excess noise which at the highest voltages becomes shot noise. The transition from Nyquist to shot noise can be anomalous with respect to the standard behavior $S_I = 2qI\coth(qV/2k_BT)$.[1] Indeed, for $L/\widetilde{L}_D \gg 1$ and $\beta < 0.1$ the crossover region exhibits a cubic dependence on current (or voltage) as found for the case of diffusion noise in the presence of space charge.[8] (ii) The GR noise for low and intermediate applied voltages is in

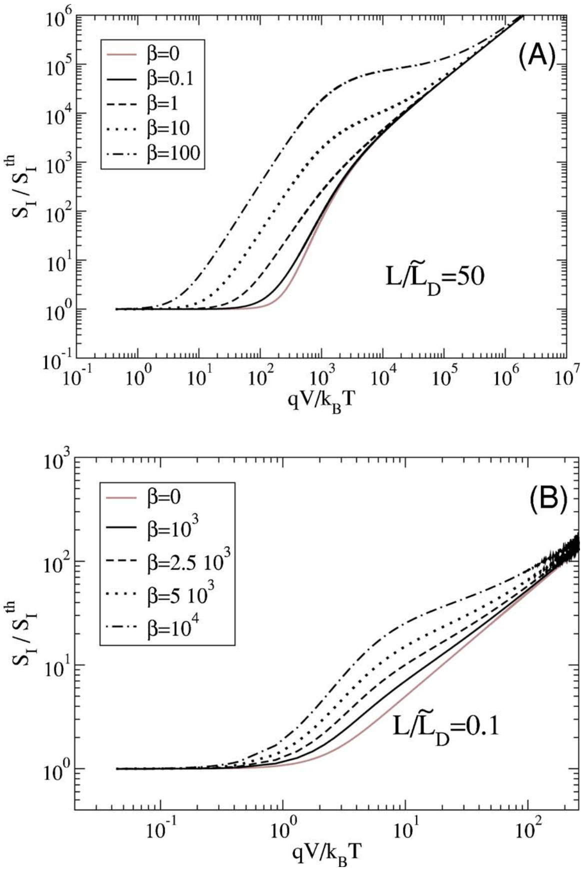

FIG. 1. (Color online) Low-frequency current spectral density normalized to the thermal Nyquist value as a function of the applied bias normalized to the thermal value for several values of the parameter β for the cases (A) $L/\tilde{L}_D=50$ and (B) $L/\tilde{L}_D=0.1$.

all cases proportional to the square voltage. However, at the highest voltages it saturates to a constant value. Accordingly, GR noise does not give rise to shot noise in homogeneous semiconductors, contrary to what is sometimes stated in the literature.[18] (iii) The competitive relevance of diffusion noise against GR noise is controlled by the parameter β. Depending on the value of β, one can find several behaviors of the overall current noise spectral density at increasing voltages [continuous lines in Figs. 1(a) and 1(b)]. In particular, even for negligibly small values of β (i.e., $\beta \leqslant 0.1$), when the direct contribution of GR noise [see Eq. (40)] is negligible, the effect of GR processes can still be detected through the renormalization of the effective Debye screening length controlled by the parameter α. At the highest bias, shot noise is achieved from values below or above the full value $S_I=2qI$.

III. IMPEDANCE FIELD IN THE PRESENCE OF SPACE CHARGE

The impedance field is a relevant physical quantity through which impedance and noise can be easily expressed (for a recent review see Ref. 14). In this section, following the procedure of Ref. 13 the impedance field is explicitly obtained for the sample under investigation. From the standard theory,[14] the impedance field $\nabla Z(x)$ can be extracted from the integral

$$\delta V(t) = \int_0^L \nabla Z(x)\, \delta I_x(t) dx. \tag{48}$$

For the explicit calculation of $\delta V(t)$ we consider the same set of Eqs. (1)–(4) without the Langevin terms, and in Eq. (3) we add a current perturbation at slice x, $\delta I_x(t)$. This set of equations is the same as that for the fluctuations by just taking $\gamma(x,t)=0$ and $\delta I_x(t)=A\,\delta J_D(x,t)$. Accordingly, $\delta V(t)$ is obtained from Eq. (23) by using the following procedure. We impose a constant current condition, i.e., $\delta I(t)=0$, and substitute for $\delta N(t)$ the result reported in the Appendix as Eq. (A25), thus obtaining

$$\delta V(t) = \frac{1}{A\mu\bar{n}} \int_0^L [1 - \mu \bar{E} \tilde{\tau}_D \tilde{G}'(x)] \delta I_x(t) dx$$
$$= \frac{Z}{L} \int_0^L \left[1 - \frac{\lambda_1 + \lambda_2}{\lambda_1 \lambda_2} \tilde{G}(x) \right] \delta I_x(t) dx, \tag{49}$$

where Z is the macroscopic impedance. By comparing Eq. (48) with Eq. (49) we find

$$\nabla Z_{on}(x) = \frac{Z}{L} \left[1 - \frac{\lambda_1 + \lambda_2}{\lambda_1 \lambda_2} \tilde{G}(x) \right]. \tag{50}$$

By using Eq. (A23) of the Appendix, the final result for the impedance field in the presence of space charge, ∇Z_{on}, is

$$\nabla Z_{on}(x) = \frac{Z}{L} y_{on}(x) = \frac{Z}{L} \Bigg[1 - \frac{\lambda_1 + \lambda_2}{\lambda_1 \lambda_2} \Bigg(\lambda_1 \frac{e^{\lambda_1 L}(e^{\lambda_2 L} - 1)}{e^{\lambda_2 L} - e^{\lambda_1 L}} e^{-\lambda_1 x} - \lambda_2 \frac{e^{\lambda_2 L}(e^{\lambda_1 L} - 1)}{e^{\lambda_2 L} - e^{\lambda_1 L}} e^{-\lambda_2 x} \Bigg) \Bigg]. \tag{51}$$

If, by imposing $L/L_D \to 0$, we switch off the Coulomb interaction, then the impedance field in the absence of space charge, ∇Z_{off}, becomes

$$\nabla Z_{off}(x) = \frac{Z}{L} y_{off}(x) = \frac{Z}{L} \left[\frac{L}{L_E} \frac{e^{x/L_E}}{e^{L/L_E} - 1} \right]. \tag{52}$$

In any case, both the above expressions for the impedance field satisfy the sum rule

$$\int_0^L \nabla Z(x) dx = Z. \tag{53}$$

In other words, while the impedance field is sensitive to the presence or absence of long-range Coulomb interaction, the value of the impedance remains the same in both cases, as it should be.

IV. RESULTS AND DISCUSSION

We have applied the above theory to interpret experimental results performed on a CdZnTe homogeneous resistor with 3% of Zn, nominally n type, and with semi-insulating properties. In particular, current voltage (I-V) and current

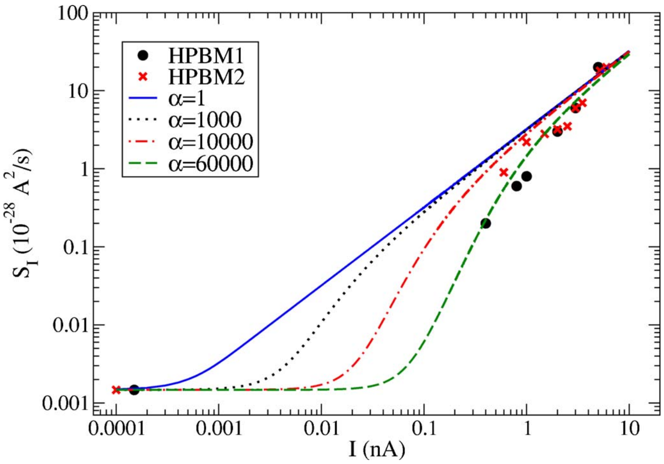

FIG. 2. (Color online) Current spectral density as a function of the current. Experimental data refer to CdZnTe at room temperature and theoretical curves to the fit carried out with the analytic formula for the reported parameters with $\beta < 0.1$. (The experiments at the lowest current are obtained from the Nyquist formula at the measured resistance.)

noise spectra have been measured on two devices grown by high-pressure Bridgman modified (HPBM) technique, named HPBM1 and HPBM2, with a length of 6 mm and a cross section of 6×20 mm^2.[15] Both devices exhibit linear *I-V* characteristics up to voltages of about 200 V (HPBM1) and 1000 V (HPBM2), respectively. In the linear voltage regions, the current noise spectrum at frequency around 10 kHz exhibits a plateau at the value of shot noise softly suppressed at low voltages. From the linear *I-V* characteristics we obtain a resistance of 1.1×10^{11} Ω, which implies a thermal noise level $S_I^{th} = 4K_BT/R = 1.5 \times 10^{-31}$ A^2/s and, by assuming an electron mobility $\mu = 1.0 \times 10^3$ cm^2/(V s), a resistivity of the material $\rho = 2.0 \times 10^{11}$ Ω/cm.

Figure 2 reports the noise-current characteristics of both samples and the theoretical results obtained using $\beta < 0.1$ (i.e., a negligible value of the strength of the GR contribution, which is obtained for a Debye diffusion time $\tau_D = L_D^2/D$ much longer than the lifetime of ionized traps τ_t') and α as fitting parameters. We notice that for the samples considered here $\tau_D = 0.18$ s. In particular, we see that when $\alpha = 1$ the usual hyperbolic cotangent behavior is obtained for the transition between thermal and shot noise. The best fit, for both sets of experimental data, is obtained with $\alpha = 60\,000$, which implies a lifetime of free carriers τ_n' (estimated of 0.1 μs) much shorter than that of ionized traps τ_t'. This means that, in the samples, Coulomb interaction is highly screened by ionized traps, giving rise to a strong anomaly in the transition between thermal and shot noise. A similar anomalous crossover between thermal and shot noise was already predicted and experimentally observed for the case of semi-insulating CdTe.[6,9]

A deeper understanding of the noise characteristic can be achieved by the analysis of the impedance field profiles, calculated as described before. In the plots of Figs. 3–5 we report the normalized impedance field in the presence and in the absence of Coulomb interaction y_{on} and y_{off} as defined in Eqs. (51) and (52), as a function of the normalized position

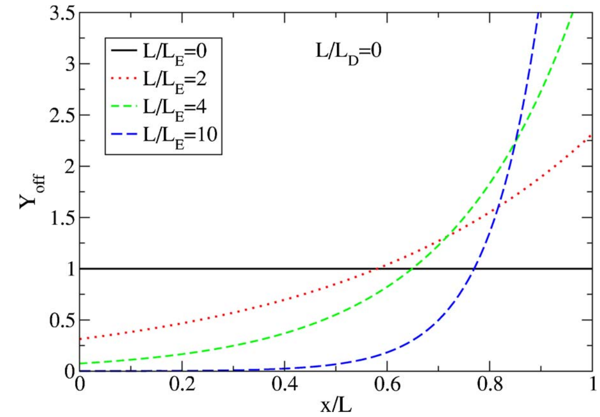

FIG. 3. (Color online) Spatial profile of the normalized impedance field for increasing values of the normalized applied voltage in the absence of long-range Coulomb interaction (i.e., $L/L_D = 0$).

x/L. The dimensionless parameters used are defined as $y_0 = L/L_E$ (which controls the intensity of the applied electric field) and $y_1 = L/\widetilde{L}_D$ (which controls the importance of Coulomb correlations).

Figure 3 reports the profile of the impedance field for increasing values of the normalized electric field y_0 in the absence of Coulomb correlations (i.e., $y_1 = 0$). In the presence of an electric field, the impedance field exhibits a monotonic increase which is maximum at the right contact and tends to vanish at the left contact. This asymmetric shape of the impedance field is associated with the presence of the diffusion current, which is responsible for damping the fluctuations in the number of carriers inside the sample to keep the homogeneous steady-state solution of the system.

Figures 4 and 5, report the profiles of the impedance field at low and high electric fields, respectively, in the presence of increasing intensities of the long-range Coulomb interaction, parametrized by increasing values of y_1. The results show that, at a fixed electric field, the presence of Coulomb interaction tends to flatten the profile of the impedance field. Indeed, the Coulomb interaction is able to move the noise spectral density toward thermal equilibrium conditions, with an effect contrasting that of the applied voltage. This is the

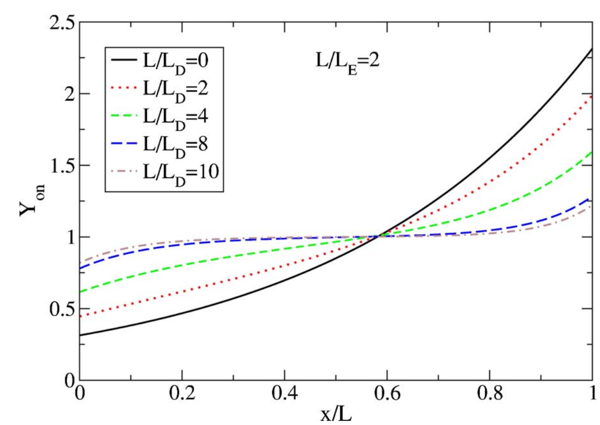

FIG. 4. (Color online) Spatial profile of the normalized impedance field for increasing strengths of long-range Coulomb interaction at a low electric field (i.e., $L/L_E = 2$).

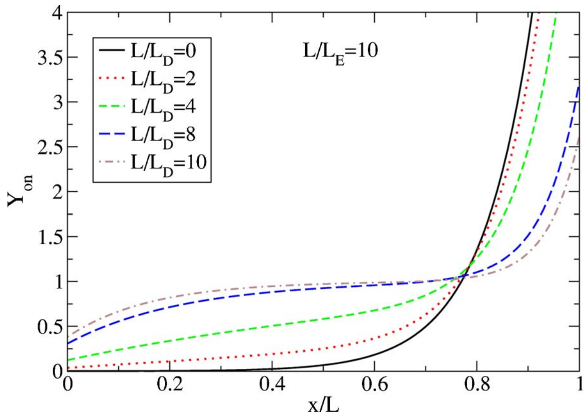

FIG. 5. (Color online) Spatial profile of the normalized impedance field for increasing strengths of long-range Coulomb interaction at a high electric field (i.e., $L/L_E=10$).

reason why, at the increasing Coulomb interaction, the crossover between thermal and shot noise becomes anomalous and occurs at higher voltages with respect to the standard condition $V\simeq 3k_BT/q$. In other words, the fluctuations in charge density are now damped by space charge more effectively than by the diffusion mechanism alone on a time scale which is that of dielectric relaxation. At high electric fields, the dynamic transit time becomes comparable to or shorter than the dielectric relaxation time, the Coulomb interaction loses importance, and the profile of the impedance field becomes again steeper, as shown in Fig. 5.

V. CONCLUSIONS

We have carried out a theoretical investigation of the impedance field and current noise in a macroscopic Ohmic resistor pertaining to semi-insulating semiconductor materials. We have shown that this system can display shot noise at the highest voltages, but that the shot noise is due entirely to the diffusion noise source. The GR contribution can be quadratic on current (voltage) or constant, but in no case proportional to the current. In other words, GR noise cannot be a source of shot noise. Independent results in the same direction obtained by means of numerical resolution of the noise equations have been reported for bipolar systems,[19] thus providing an indirect confirmation of our predictions. We have found that these systems can exhibit an anomalous transition from thermal to shot noise, which is confirmed by the experimental data. Moreover, this transition is physically explained by analyzing the impedance field profiles. We have found that from one side the increase of the applied voltage favors the transition from thermal to shot noise, while from another side the Coulomb screening due to space charge tends to restore the thermal equilibrium (Nyquist) value of the noise. Theory is applied to available experiments carried out in semi-insulating CdZnTe samples nominally of n type at room temperature. The agreement between theory and experiments is found to be quite satisfactory. On this ground, the shot noise measured at increasing applied voltages when the sample exhibits a linear *I-V* characteristic is attributed to the diffusion noise source. In this region of applied voltages we attest to the remarkable and nonintuitive fact that GR noise is of negligible importance apart from the role played by ionized traps in renormalizing the Debye screening length. This result is associated with the wide separation of the characteristic times that describe the relaxation of free carriers to traps, of ionized traps to their neutral state, and of Debye diffusion according to the trend $\tau_n' \ll \tau_t \ll \tau_D$. Present estimates are $\tau_n'=10^{-7}$ s, $\tau_t=6\times 10^{-3}$ s, and $\tau_D=0.2$ s. The renormalization of the Debye screening length implies an anomalous crossover from thermal (Nyquist) to shot noise. Similar features were observed in semi-insulating CdTe (Refs. 6 and 9) and offered evidence of shot noise in a macroscopic resistor. This evidence is here confirmed on a more general ground for the case of semi-insulating CdZnTe.

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APPENDIX: CALCULATION OF FLUCTUATIONS IN TOTAL CARRIER NUMBER

In order to compute $\delta N(t)$ we derive and solve a differential equation for $\delta n(x,t)$. We start by considering the total current equation in the low-frequency limit,

$$\delta J(t) = \delta J_n(x,t), \tag{A1}$$

from which it turns out that $\delta J_n(x,t)$ does not depend on the spatial coordinate. Taking advantage of this property, after linearizing the current equation around the homogeneous steady state, we obtain

$$\frac{\partial \delta J_n(x,t)}{\partial x} = 0 = q\mu_n \frac{\partial \delta n(x,t)}{\partial x}\bar{E} + q\mu\bar{n}\frac{\partial \delta E(x,t)}{\partial x} + qD\frac{\partial^2 \delta n(x,t)}{\partial x^2} + q\frac{\partial J_D(x,t)}{\partial x}. \tag{A2}$$

Moreover, from the linearized Poisson equation it is

$$\frac{\partial \delta E(x,t)}{\partial x} = \frac{q}{\epsilon_0\epsilon_r}[\delta N_t^+(x,t) - \delta n(x,t)], \tag{A3}$$

which also depends on the fluctuations of the number of ionized traps. The fluctuations in the number of ionized traps can be related to the density fluctuations by noting that the continuity equations for both traps and free carriers lead in the low-frequency limit to the equation

$$-\delta r_n(x,t) + \delta g_n(x,t) + \gamma(x,t) = 0. \tag{A4}$$

By applying to the above result a total differentiation, we get

$$-\frac{\partial r_n(x,t)}{\partial n}dn - \frac{\partial r_n(x,t)}{\partial N_t^+}dN_t^+ + \frac{\partial r_n(x,t)}{\partial n}dn + \frac{\partial r_n(x,t)}{\partial N_t^+}dN_t^+ + \gamma(x,t) = 0, \tag{A5}$$

from which, using definitions (34) and (35) the following is obtained:

$$\delta N_t^+(x,t) = -\frac{\tau_t'}{\tau_n'}\delta n(x,t) + \tau_t' \gamma(x,t). \tag{A6}$$

Therefore, the linearized Poisson equation, in the low-frequency limit, can be rewritten as

$$\frac{\partial \delta E(x,t)}{\partial x} = \frac{q}{\epsilon_0 \epsilon_r}\left[\tau_t' \gamma(x,t) - \frac{\tau_t'}{\tau}\delta n(x,t)\right]. \tag{A7}$$

By substituting the linearized Poisson equation into the linearized current equation and by making use of the Einstein relation $D=\mu k_B T/q$, we finally arrive at the following equation for the carrier density fluctuations:

$$\frac{\partial^2 \delta n(x,t)}{\partial x^2} + \frac{1}{L_E}\frac{\partial \delta n(x,t)}{\partial x} - \frac{1}{\tilde{L}_D^2}\delta n(x,t) = -\xi(x,t), \tag{A8}$$

where the Langevin force $\xi(x,t)$ is

$$\xi(x,t) = \frac{1}{\tilde{L}_D^2}\left(\tau \gamma(x,t) + \tilde{\tau}_D \frac{\partial J_D(x,t)}{\partial x}\right), \tag{A9}$$

with the boundary conditions

$$\delta n(0,t) = \delta n(L,t) = 0. \tag{A10}$$

The equation for the density fluctuations is a second-order linear differential stochastic equation with constant coefficients which can be solved in a closed analytical form.[13] The solution reads

$$\delta n(x,t) = A(x,t)e^{\lambda_1 x} + B(x,t)e^{\lambda_2 x}, \tag{A11}$$

where

$$A(x,t) = A_L(t) - \int_x^L dx' \frac{\xi(x',t)e^{-\lambda_1 x'}}{\lambda_2 - \lambda_1}, \tag{A12}$$

$$B(x,t) = B_L(t) - \int_x^L dx' \frac{\xi(x',t)e^{-\lambda_2 x'}}{\lambda_1 - \lambda_2}. \tag{A13}$$

Here, $A_L(t)$ and $B_L(t)$ are integration constants determined by the boundary conditions

$$\delta n_0 = 0 = \left(A_L(t) - \int_0^L dx' \frac{\xi(x',t)e^{-\lambda_1 x'}}{\lambda_2 - \lambda_1}\right) + \left(B_L(t) - \int_0^L dx' \frac{\xi(x',t)e^{-\lambda_2 x'}}{\lambda_1 - \lambda_2}\right), \tag{A14}$$

$$\delta n_L = 0 = A_L(t)e^{\lambda_1 L} + B_L(t)e^{\lambda_2 L}, \tag{A15}$$

which give

$$A_L(t) = \frac{e^{\lambda_2 L}\int_0^L dx \xi(x,t)(e^{-\lambda_1 x} - e^{-\lambda_2 x})}{(\lambda_2 - \lambda_1)(e^{\lambda_2 L} - e^{\lambda_1 L})}, \tag{A16}$$

$$B_L(t) = \frac{e^{\lambda_1 L}\int_0^L dx \xi(x,t)(e^{-\lambda_1 x} - e^{-\lambda_2 x})}{(\lambda_2 - \lambda_1)(e^{\lambda_1 L} - e^{\lambda_2 L})}. \tag{A17}$$

For the sake of completeness we write also the solution found in terms of the low-frequency Green's function of the problem $G(x,x';0)$. In this case it is

$$\delta n(x,t) = \int_0^L dx' G(x,x';0)\xi(x',t), \tag{A18}$$

with

$$G(x,x';0) = \frac{1}{\lambda_2 - \lambda_1}\frac{(e^{-\lambda_1 x'} - e^{-\lambda_2 x'})(e^{\lambda_1 x}e^{\lambda_2 L} - e^{\lambda_2 x}e^{\lambda_1 L})}{(e^{\lambda_2 L} - e^{\lambda_1 L})} - \frac{1}{\lambda_2 - \lambda_1}(e^{\lambda_1(x-x')} - e^{\lambda_2(x-x')})\theta(x-x'), \tag{A19}$$

where $\theta(x-x')$ is the step function. Green's function satisfies the equation for the fluctuations and the boundary conditions

$$G(0,x';0) = G(L,x';0) = G(x,0;0) = G(x,L;0). \tag{A20}$$

Moreover, it is continuous at $x=x'$ but its derivative is discontinuous at this point, as it should be. Now we are in the position to calculate the fluctuation of the total number of carriers,

$$\delta N(t) = A\int_0^L dx \delta n(x,t). \tag{A21}$$

using some algebra we arrive at

$$\delta N(t) = A\int_0^L dx G(x)\xi(x,t), \tag{A22}$$

where

$$G(x) = \int_0^L dx' G(x,x';0) \equiv \tilde{L}_D^2 \tilde{G}(x), \tag{A23}$$

$$\tilde{G}(x) = 1 - \frac{e^{\lambda_1 L}(e^{\lambda_2 L} - 1)}{(e^{\lambda_2 L} - e^{\lambda_1 L})}e^{-\lambda_1 x} + \frac{e^{\lambda_2 L}(e^{\lambda_1 L} - 1)}{(e^{\lambda_2 L} - e^{\lambda_1 L})}e^{-\lambda_2 x}. \tag{A24}$$

Note that $\tilde{G}(0)=\tilde{G}(L)=0$. By substituting the expression for the combined noise source, we finally arrive at

$$\delta N(t) = A\tau\int_0^L dx \tilde{G}(x)\gamma(x,t) + A\tilde{\tau}_D\int_0^L dx \tilde{G}(x)\frac{\partial J_D(x,t)}{\partial x}. \tag{A25}$$

Integrating by parts the second contribution and by using the properties $\tilde{G}(0)=\tilde{G}(L)=0$, we finally obtain

$$\delta N(t) = A\tau\int_0^L dx \tilde{G}(x)\gamma(x,t) - A\tilde{\tau}_D\int_0^L dx \tilde{G}'(x)J_D(x,t) \tag{A26}$$

$$\equiv \delta N_{\mathrm{GR}}(t) + \delta N_D(t). \tag{A27}$$

We note that there are two different contributions to the fluctuations of the total number of carriers, the first one associated with GR processes and the second one associated with diffusion processes. The two contributions are related since the corresponding noise sources are uncorrelated. The equation of current fluctuation then reads

$$\delta I(t) = \frac{qA}{L}\int_0^L dx J_D(x,t) + \left(\frac{\bar{I}}{\bar{N}}\right)\delta N_{\mathrm{GR}}(t) + \left(\frac{\bar{I}}{\bar{N}}\right)\delta N_D(t). \tag{A28}$$

The three contributions to the current fluctuation are uncorrelated. Indeed, since $\gamma(x,t)$ is uncorrelated with $J_D(x,t)$, the second contribution is not correlated with the others. Concerning the first and the third contributions, although their origin is in both cases the diffusion noise, their correlation is proportional to

$$\int_0^L dx\widetilde{G}'(x)\int_0^L dx' 2\int_{-\infty}^{+\infty}\overline{J_D(x,t)J_D(x',t')} = \int_0^L dx\widetilde{G}'(x)K_D(x) = K_D(\widetilde{G}(L)-\widetilde{G}(0)) = 0, \tag{A29}$$

where in the last equality we have used the homogeneous property and the fact that $\widetilde{G}(x)$ vanishes at the boundaries. Hence, these contributions are also not correlated.

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