

A posteriori ratemaking using bivariate Poisson models

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Abstract

In Bermúdez (2009) different bivariate Poisson regression models were used to make an *a priori* ratemaking taking into account the dependence between two type of claims. A natural extension for these papers is to consider *a posteriori* ratemaking (i.e. experience rating models) that also relaxes the independence assumption. We introduce here two bivariate experience rating models that integrate the *a priori* ratemaking based on the bivariate Poisson regression models, extending the existing literature for the univariate case to the bivariate case. These bivariate experience rating models are applied to the same automobile insurance claims data set as used in Bermúdez (2009) to analyse the consequences for posterior premiums when the independence assumption is relaxed. The main finding is that the *a posteriori* risk factors obtained with the bivariate experience rating models are significantly lower than those factors derived under the independence assumption.

Keywords

Bivariate Poisson regression models, Automobile insurance, Experience rating models, Bayesian approach.

1 Introduction and motivation

In Bermúdez (2009) different bivariate Poisson regression models were used to make an *a priori* ratemaking taking into account the dependence between two type of claims. The central idea was that the dependence between two different types of claim must be taken into account to achieve better ratemaking. The papers concluded that even when there are small correlations between the claims, major differences in ratemaking can nevertheless appear. Thus, using a bivariate Poisson regression model results in ratemaking that has larger variances and, hence, larger loadings in premiums than those obtained under the independence assumption.

As it is well-known, all the factors could not be identified, measured and introduced in the *a priori* tariff system. Hence, the idea has arisen of considering individual differences in policies within the same class by using an *a posteriori* mechanism, i.e., fitting an individual premium based on the experience of claims for each insured party. This concept has received the name

of *a posteriori* ratemaking, experience rating or bonus-malus systems. A thorough review of *a posteriori* ratemaking systems for automobile insurance, in the univariate setting, can be found in Denuit et al. (2007). A natural extension for Bermúdez (2009) is to consider *a posteriori* ratemaking, extending classical experience rating models to the bivariate case.

The existing literature in bivariate experience rating models is based on the credibility approach, i.e. multivariate credibility models. For instance, Pinquet (1998), Frees (2003), Bühlmann and Gisler (2005), Denuit et al. (2007), or Englund et al. (2008). They prefer the linear credibility approach since they note that the Bayesian approach turns out to lead to numerical integration. In this paper, our aim is to consider the Bayesian approach to get an experience rating model that integrates the *a priori* ratemaking based on the bivariate Poisson regression models presented in Bermúdez (2009).

The article is organized as follows. First, in Section 2 we introduce two bivariate experience rating models. In Section 3 we discuss the Bayesian methodology used to obtain the *a posteriori* premiums. In Section 4 a numerical application using a database from a Spanish insurance company is described. Finally, we provide concluding remarks in Section 5.

2 Bivariate experience rating models

Let N_{1ij} and N_{2ij} be the random variables representing the number of claims for third-party liability and for the rest of guarantees respectively for individual i at time j . Their values are denoted as n_{1ij} and n_{2ij} respectively. Following Bermúdez (2009), where bivariate Poisson distribution was presented as an instrument that can account for the underlying correlation between two types of claim arising from the same policy, we will denote as $(N_1, N_2) \sim BP(\lambda_1, \lambda_2, \lambda_3)$ to represent that the pair of random variables (N_1, N_2) follows a bivariate Poisson distribution with parameters λ_1, λ_2 and λ_3 .

The bivariate Poisson distribution can be defined with the so-called trivariate reduction method as follows. Let us consider independent random variables X_k ($k = 1, 2, 3$) to be distributed as Poisson with parameters λ_k respectively. Then the random variables $N_1 = X_1 + X_3$ and $N_2 = X_2 + X_3$ follow jointly a bivariate Poisson distribution. The joint probability function for the i -th individual, we remove the subscript j for simplicity, is given by

$$P(n_{1i}, n_{2i}) = P(N_{1i} = n_{1i}, N_{2i} = n_{2i}; \boldsymbol{\lambda}_i)$$

$$= e^{-(\lambda_{1i} + \lambda_{2i} + \lambda_{3i})} \frac{\lambda_{1i}^{n_{1i}}}{n_{1i}!} \frac{\lambda_{2i}^{n_{2i}}}{n_{2i}!} \sum_{s=0}^{\min(n_{1i}, n_{2i})} \binom{n_{1i}}{s} \binom{n_{2i}}{s} s! \left(\frac{\lambda_{3i}}{\lambda_{1i} \lambda_{2i}} \right)^s,$$

where $\lambda_i = (\lambda_{1i}, \lambda_{2i}, \lambda_{3i})$.

This distribution presents several interesting and useful properties. First, it allows for positive dependence between the random variables N_{1i} and N_{2i} . Second, marginally each random variable follows a Poisson distribution with $E(N_{1i}) = \lambda_{1i} + \lambda_{3i}$ and $E(N_{2i}) = \lambda_{2i} + \lambda_{3i}$. Third, $Cov(N_{1i}, N_{2i}) = \lambda_{3i}$, and hence λ_{3i} , is a measure of dependence between the two random variables. In general, if $\lambda_3 = 0$ then the two variables are independent and the bivariate Poisson distribution reduces to the product of two independent Poisson distributions (also known as a double Poisson distribution). For a comprehensive treatment of the bivariate Poisson distribution and its multivariate extensions the reader is referred to Kocherlakota and Kocherlakota (1992) and Johnson et al. (1997). In Bermúdez (2009) *a priori* bivariate ratemaking is addressed by introducing covariates to model λ_1 , λ_2 and λ_3 , obtaining a total *a priori* premium¹ by $\pi_i = E[N_{1i}] + E[N_{2i}] = \lambda_{1i} + \lambda_{2i} + 2\lambda_{3i}$.

Finally, it is important to note that the bivariate Poisson distribution is a convolution closed family, i.e. the sum of two bivariate Poisson vectors is also a bivariate Poisson vector. This property will be useful for the derivations since that emphasize that if we look at more than one period then the distribution of the claims remain in the same family but with changing parameters. The so-called static approach, λ 's remain the same across time for each individual, is usually assumed in experience rating models. Based on the above property of the bivariate Poisson distribution and assuming the static approach, if we use t time periods with total number of claims for the two type of claims $N_{1i\bullet}, N_{2i\bullet}$ respectively, then we have $(N_{1i\bullet}, N_{2i\bullet}) \sim BP(t\lambda_{1i}, t\lambda_{2i}, t\lambda_{3i})$.

In a seminal paper, Dionne and Vanasse (1989) proposed an experience rating model which integrates *a priori* and *a posteriori* information. First, they introduced a regression component into the Poisson counting model in order to use all available information in the estimation of claim frequency. Second, the unexplained heterogeneity was modelled by the introduction of a latent variable (Θ_i) representing the influence of hidden or unknown policy characteristics. Assuming this random effect to be gamma-distributed yields the negative binomial model for the number of claims. Extending this model to the bivariate case, we propose a bivariate Poisson

¹Assuming the amount of the expected claims equals one monetary unit, for both type of claims.

regression model to use the *a priori* available information and then we also introduce a random effect to account for the unobserved heterogeneity. However, in the bivariate setting, we have different ways to do this.

2.1 Model A

The first model proposed here considers that the unobserved heterogeneity that can be addressed through experience rating is similar for both type of claims. In other words, the random effect is the same for each type of claim. The model, assuming the static approach, is based on the following assumptions:

(A1) Adding a common random effect for each parameter of the bivariate Poisson distribution:

$$(N_{1i\bullet}, N_{2i\bullet}) \sim BP(t\lambda_{1i}\Theta_i, t\lambda_{2i}\Theta_i, t\lambda_{3i}\Theta_i).$$

(A2) Following the Bayesian paradigm, $\Theta_i \sim \text{Gamma}(\alpha, \alpha)$.

Note that we have forced Θ_i to have unit mean, and variance $1/\alpha$, to ensure that the *a priori* risk evaluation is thus correct on average. For N_{1i} , $E(N_{1i}) = (\lambda_{1i} + \lambda_{3i})E(\Theta_i) = \lambda_{1i} + \lambda_{3i}$.

Finally, assuming a quadratic loss function in the Bayesian procedure summarized in next section, the posterior premium for individual i and type of claim k results:

$$\Pi_{ki}(n_{1i\bullet}, n_{2i\bullet}) = (\lambda_{ki} + \lambda_{3i}) \cdot E[\Theta_i | n_{1i\bullet}, n_{2i\bullet}], \quad k = 1, 2.$$

And the total posterior premium for individual i results:

$$\Pi_i(n_{1i\bullet}, n_{2i\bullet}) = (\lambda_{1i} + \lambda_{2i} + 2\lambda_{3i}) \cdot E(\Theta_i | n_{1i\bullet}, n_{2i\bullet})$$

where $\lambda_{1i} + \lambda_{2i} + 2\lambda_{3i}$ is the prior premium for the policyholder i and $E(\Theta_i | n_{1i\bullet}, n_{2i\bullet})$ the posterior expected value of the random effect Θ_i , i.e. the *a posteriori* risk factor that corrects the *a priori* premium of policyholder i given its past claim history.

2.2 Model B

Another way to introduce random effects in the bivariate setting is to consider that the hidden features that are revealed by the number of claims reported by the policyholders are different for each type of claims. Hence, the hidden features are modeled by different random effects

according to each type of claim. It is assumed that random effects are independent. This way may be summarised with the following assumptions:

(B1) Adding a different random effect for each parameter of the bivariate Poisson distribution:

$$(N_{1i\bullet}, N_{2i\bullet}) \sim BP(t\lambda_{1i}\Theta_{1i}, t\lambda_{2i}\Theta_{2i}, t\lambda_{3i}\Theta_{3i}).$$

(B2) Following the Bayesian paradigm, $\Theta_{ki} \sim \text{Gamma}(\alpha_k, \alpha_k)$, $k = 1, 2, 3$.

Finally, assuming the quadratic loss function as in the previous model, the posterior premium for individual i and type of claim k results:

$$\Pi_{ki}(n_{1i\bullet}, n_{2i\bullet}) = \lambda_{ki} \cdot E[\Theta_{ki} | n_{1i\bullet}, n_{2i\bullet}] + \lambda_{3i} \cdot E[\Theta_{3i} | n_{1i\bullet}, n_{2i\bullet}], \quad k = 1, 2.$$

And the total posterior premium for individual i results:

$$\Pi_i(n_{1i\bullet}, n_{2i\bullet}) = \lambda_1 \cdot E(\Theta_{1i} | n_{1i\bullet}, n_{2i\bullet}) + \lambda_2 \cdot E(\Theta_{2i} | n_{1i\bullet}, n_{2i\bullet}) + 2\lambda_3 \cdot E(\Theta_{3i} | n_{1i\bullet}, n_{2i\bullet}).$$

Note that in this model, there is no a common *a posteriori* risk factor in the sense explained above. Each parameter of the bivariate Poisson distribution is corrected by the number of claims, for both type of claims, reported by the policyholder.

3 Bayesian procedure

3.1 Model A

Removing the i -subscript to help the notation, model A assumes $(N_{1\bullet}, N_{2\bullet}) \sim BP(t\lambda_1\Theta, t\lambda_2\Theta, t\lambda_3\Theta)$ and $\Theta \sim \text{Gamma}(\alpha, \alpha)$. Then tedious calculations show that the unconditional distribution has probability mass function given by

$$\begin{aligned} P(n_{1\bullet}, n_{2\bullet}) &= \int P(n_{1\bullet}, n_{2\bullet}; \Theta) f(\Theta) d\Theta = \\ &= \sum_{s=0}^{\min(n_{1\bullet}, n_{2\bullet})} \frac{\binom{n_{1\bullet}}{s} \binom{n_{2\bullet}}{s} s! t^{n_{1\bullet}+n_{2\bullet}-s} \lambda_1^{n_{1\bullet}-s} \lambda_2^{n_{2\bullet}-s} \lambda_3^s \alpha^\alpha \Gamma(n_{1\bullet} + n_{2\bullet} + \alpha - s)}{n_{1\bullet}! n_{2\bullet}! \Gamma(\alpha) (\alpha + \Lambda)^{n_{1\bullet}+n_{2\bullet}+\alpha-s}} \end{aligned}$$

where $\Lambda = t(\lambda_1 + \lambda_2 + \lambda_3)$.

This distribution can be also derived via the trivariate reduction scheme by using negative binomial instead of Poisson random variables. Note that the marginals are negative binomial. The distribution has been described in Subrahmaniam (1966).

Then posterior distribution for Θ can be deduced in the usual way. We obtain that

$$f(\Theta | n_{1\bullet}, n_{2\bullet}) = \frac{P(n_{1\bullet}, n_{2\bullet}; \Theta)f(\Theta)}{P(n_{1\bullet}, n_{2\bullet})}$$

which in our case turn to be

$$f(\Theta | n_{1\bullet}, n_{2\bullet}) \propto \sum_{s=0}^{\min(n_{1\bullet}, n_{2\bullet})} \frac{\binom{n_{1\bullet}}{s} \binom{n_{2\bullet}}{s} s! (t\lambda_1)^{n_{1\bullet}-s} (t\lambda_2)^{n_{2\bullet}-s} (t\lambda_3)^s \alpha^\alpha}{n_{1\bullet}! n_{2\bullet}! \Gamma(\alpha)} \Theta^{n_{1\bullet}+n_{2\bullet}+\alpha-s} \exp(-\Theta(\alpha+\Lambda))$$

This can be recognized as a finite mixture of Gamma densities, i.e. of the form

$$f(\Theta | n_{1\bullet}, n_{2\bullet}) = \sum_{s=0}^{\min(n_{1\bullet}, n_{2\bullet})} w_s \text{Gamma}(\Theta; \alpha + n_{1\bullet} + n_{2\bullet} - s, \alpha + \Lambda)$$

where w_s are given as:

$$w_s \propto \binom{n_{1\bullet}}{s} \binom{n_{2\bullet}}{s} s! \left(\frac{\lambda_3}{t\lambda_1\lambda_2} \right)^s \frac{\Gamma(n_{1\bullet} + n_{2\bullet} + \alpha - s)}{(\alpha + \Lambda)^{n_{1\bullet}+n_{2\bullet}+\alpha-s}}$$

for $s = 0, \dots, \min\{n_{1\bullet}, n_{2\bullet}\}$. So the number of components in the mixture depends on the observed values and specifically by their minimum. The normalizing constant is easy to be found since s has finite support and the weights shall sum to one.

In addition posterior moments are easy to derive since it is a finite mixture:

$$E(\Theta | n_{1\bullet}, n_{2\bullet}) = \sum_{s=0}^{\min(n_{1\bullet}, n_{2\bullet})} w_s \frac{\alpha + n_{1\bullet} + n_{2\bullet} - s}{\alpha + \Lambda}.$$

The key issue is the selection of the value for the parameter α . One may derive by fitting via maximum likelihood method the bivariate negative binomial mentioned above.

3.2 Model B

Removing the i subscript, model B assumes $(N_{1\bullet}, N_{2\bullet}) \sim BP(t\lambda_1\Theta_1, t\lambda_2\Theta_2, t\lambda_3\Theta_3)$ and $\Theta_k \sim \text{Gamma}(\alpha_k, \alpha_k)$ with $k = 1, 2, 3$. Then the unconditional distribution has probability mass function given by

$$\begin{aligned} P(n_{1\bullet}, n_{2\bullet}) &= \int \int \int P(n_{1\bullet}, n_{2\bullet}; \theta) f(\Theta_1) f(\Theta_2) f(\Theta_3) d\Theta_3 d\Theta_2 d\Theta_1 \\ &= \sum_{s=0}^{\min(n_{1\bullet}, n_{2\bullet})} \frac{(t\lambda_1)^{n_{1\bullet}-s} (t\lambda_2)^{n_{2\bullet}-s} (t\lambda_3)^s}{(n_{1\bullet}-s)! (n_{2\bullet}-s)! s!} \frac{\alpha_1^{\alpha_1} \alpha_2^{\alpha_2} \alpha_3^{\alpha_3}}{\Gamma(\alpha_1) \Gamma(\alpha_2) \Gamma(\alpha_3)} \\ &\times \frac{\Gamma(n_{1\bullet} - s + \alpha_1) \Gamma(n_{2\bullet} - s + \alpha_2) \Gamma(s + \alpha_3)}{(\alpha_1 + t\lambda_1)^{n_{1\bullet}-s+\alpha_1} (\alpha_2 + t\lambda_2)^{n_{2\bullet}-s+\alpha_2} (\alpha_3 + t\lambda_3)^{s+\alpha_3}}. \end{aligned}$$

This can be easily recognized as arising from a trivariate reduction where three negative binomial variates are used. Importantly the marginals are not any more negative binomial since the convolution of two negative binomial variates is not necessarily a negative binomial.

The posterior distributions for Θ 's can be seen again to be a finite mixture of Gammas. Namely we obtain that

$$f(\Theta_k | n_{1\bullet}, n_{2\bullet}) = \sum_{s=0}^{\min(n_{1\bullet}, n_{2\bullet})} \tau_s \text{Gamma}(\Theta_k; \alpha_k + n_{1\bullet} - s, \alpha_k + t\lambda_k), \quad k = 1, 2$$

$$f(\Theta_3 | n_{1\bullet}, n_{2\bullet}) = \sum_{s=0}^{\min(n_{1\bullet}, n_{2\bullet})} \tau_s \text{Gamma}(\Theta_3; \alpha_3 + s, \alpha_3 + t\lambda_3)$$

where τ_s are given as

$$\tau_s \propto \binom{n_{1\bullet}}{s} \binom{n_{2\bullet}}{s} s! \left(\frac{\lambda_3}{t\lambda_1\lambda_2} \right)^s \frac{\Gamma(n_{1\bullet} + \alpha_1 - s)}{(\alpha_1 + t\lambda_1)^{n_{1\bullet} + \alpha_1 - s}} \frac{\Gamma(n_{2\bullet} + \alpha_2 - s)}{(\alpha_2 + t\lambda_2)^{n_{2\bullet} + \alpha_2 - s}} \frac{\Gamma(\alpha_3 + s)}{(\alpha_3 + t\lambda_3)^{\alpha_3 + s}}$$

for $s = 0, \dots, \min\{n_{1\bullet}, n_{2\bullet}\}$. So the number of components in the mixture depends on the observed values and specifically by their minimum.

The posterior expectation needed for the calculations are then easy to derive, being just weighted expectations from a gamma distribution. Namely we have

$$E(\Theta_k | n_{1\bullet}, n_{2\bullet}) = \sum_{s=0}^{\min(n_{1\bullet}, n_{2\bullet})} \tau_s \frac{n_{1\bullet} + \alpha_k - s}{\alpha_k + t\lambda_k}, \quad k = 1, 2$$

$$E(\Theta_3 | n_{1\bullet}, n_{2\bullet}) = \sum_{s=0}^{\min(n_{1\bullet}, n_{2\bullet})} \tau_s \frac{\alpha_3 + s}{\alpha_3 + t\lambda_3}$$

For more details on the derivations one can see Karlis and Tsiamyrtzis (2008). Again in order to estimate α 's, we resort to maximum likelihood method.

4 Numerical application

4.1 The database and the *a priori* ratemaking

We used the same automobile insurance claims data set as used in Bermúdez (2009). For our purpose here, we have only selected policyholders with full coverage, i.e. policies including third-party liability (claimed and counted as N_1 type), a set of basic guarantees such as emergency roadside assistance or legal and medical assistance (claimed and counted as N_2 type) and, finally,

comprehensive coverage (damage to one’s vehicle caused by any unknown party, for example, damage resulting from theft, flood or fire) and collision coverage (damage resulting from a collision with another vehicle or object when the policyholder is at fault), also claimed and counted as N_2 type. For each policy, the initial information at the beginning of the period and the total number of claims (for the two types of claim) from policyholders at fault were reported within this yearly period. Nine exogenous variables, as in Bermúdez (2009), were considered for prior ratemaking purposes. The cross-tabulation for the number of claims for third-party liability (N_1) and number of claims for the rest of guarantees (N_2) is shown in Table 1.

For comparative purposes, three different profiles were selected from the portfolio. The first can be classified as the best profile since it presents the lowest total mean score. The second, a profile with a mean lying very close to average for the portfolio. Finally, a profile classified as the worst driver since it presents the highest total mean score. Table 2 shows the results in the *a priori* ratemaking, through the mean (*a priori* premium) and the variance of the number of claims per year, for the three profiles and the two models considered, the independence Poisson model and the bivariate Poisson model. From these results, the main consequence of accounting for dependence is that bivariate Poisson regression model presents larger variances and, hence, larger loadings than those obtained under the independence assumption.

4.2 Results

In order to compare the consequences for posterior premiums when the independence assumption is relaxed, we have calculated the *a posteriori* risk factors, as the proportion between total posterior premiums and total prior premiums, for model A and model B and also for the independence case. Previously, the prior parameters should be estimated. Resorting on the maximum likelihood method, in Table 3 the prior parameter estimations for each model are shown. From this results, we may conclude that the random variable Θ_1 representing the unknown risk characteristics of the policyholder for third-party liability claims presents larger variance than the variance for the corresponding random variable for the rest of claims, Θ_2 . This fact may lead to a larger *a posteriori* risk factors for the third-party liability claims.

The *a posteriori* risk factors, calculated as mentioned, for three different profiles (Best, Average and Worst) and $t = 1$ are compared in Table 4 for Independent model, model A and model B. Putting aside model A, and focusing on model B as opposed to the independence

case, the *a posteriori* risk factors located at the first row ($N_1 = 0$) or at the first column ($N_2 = 0$) do not diverge significantly between model B and the independence case. However, the *a posteriori* risk factors corresponding to policyholders that report at least one claim of each type are significantly different. In particular, relaxing the independence assumption leads to a lower *a posteriori* risk factors. For model A, we can observe the same behaviour even though its specific symmetrical nature. Concentrating on the differences between the three profiles, the bivariate experience rating models behave in the same manner as univariate models, that is, producing larger *a posteriori* risk factors for those policyholders with a lower prior premium.

To show the behaviour of the *a posteriori* risk factors over time, Table 5 displays the results for twelve years in the case where a policyholder has reported one claim of each type during the first year and no more claims are ever reported by him. For model B, we have found that the difference mentioned above for $t = 1$ between the *a posteriori* risk factors of model B and the corresponding factors of the independence case is attenuated over time in all cases. For model A, this conduct is not equal for the three profiles.

5 Conclusions

We introduce here two different bivariate experience rating models that integrate the *a priori* ratemaking based on the bivariate Poisson regression model presented in Bermúdez (2009) to account for the dependence between two type of claims. Using the standard Bayesian procedure, we extend the existing literature for the univariate case to the bivariate case obtaining posterior premiums and *a posteriori* risk factors for the bivariate experience rating models proposed here.

The two bivariate models proposed here may respond to different market needs. As model A uses a common random effect, this model can be used when the insurer is not interested in differentiating the posterior ratemaking for the two type of claims, but taking into account the dependence between them. If, on the other hand, the insurer prefers different posterior ratemaking for each type of claim, model B will be the best option.

When the independence assumption is relaxed, the main consequence for posterior ratemaking is that the *a posteriori* risk factors corresponding to policyholders that report at least one claim of each type are significantly reduced compared with those factors derived from the independence case. This is particularly true with regard to those policyholders with lower *a priori*

premiums (good drivers) since they will present larger *a posteriori* risk factors than policyholders classified as bad drivers. From the numerical application is also derived that this reduction in *a posteriori* risk factors is attenuated over time.

It is therefore worth giving careful consideration to this issue. The idea behind *a posteriori* ratemaking is fitting an individual premium based on the claim experience for each insured party on the assumption that the unknown risk characteristics (i.e. driving ability, obedience of traffic regulations) are revealed by the number of claims reported by the policyholders. It is reasonable to believe that some of these characteristics simultaneously affect the two type of claims. We interpret the reduction in *a posteriori* risk factors when the independence assumption is relaxed as the effect of taking into account this latter fact. That is, the bivariate experience rating models do not recharge twice for the same unknown risk characteristic.

Finally, we would like to mention various ways in which this paper might be extended. First, zero-inflated bivariate Poisson models presented in Bermúdez (2009) can be also used and derived. Second, finite mixtures models presented in Bermúdez and Karlis (2012) can be also used and derived but they will be much more complicated, but still tractable. Third, although in the present paper we limit our analysis to the bivariate case, it could be extended to include larger dimensions, following the general model presented by Bermúdez and Karlis (2011). Fourth, in model B, instead of independent random effects, we can also assume some dependence and proceed. Finally, we could replace the Gamma assumption with an inverse Gaussian one (or any other mixing distribution) easily.

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Table 1: Cross-tabulation of data

N_1	N_2						
	0	1	2	3	4	5	6
0	24408	1916	296	69	12	6	0
1	1068	317	61	21	6	2	2
2	203	71	18	6	2	1	1
3	49	14	8	3	3	1	0
4	11	6	2	0	1	0	0
5	2	0	0	0	0	0	1
6	1	0	0	1	0	0	0
8	0	0	1	0	0	0	0

N_1 : number of claims for third-party liability.
 N_2 : number of claims for the rest of guarantees.

Table 2: Comparison of *a priori* ratemaking

Model	Best		Average		Worst	
	Mean	Variance	Mean	Variance	Mean	Variance
DP	0.1624	0.1624	0.2106	0.2106	0.3634	0.3634
BP	0.1570	0.1883	0.2078	0.2391	0.3525	0.3838

DP: Independence Poisson; BP: Bivariate Poisson.

Table 3: Prior parameter estimation

Model	Prior distribution	Parameter estimation
Indep.	$\Theta_1 \sim \text{Gamma}(a, a)$	$\hat{a} = 0.1629$
	$\Theta_2 \sim \text{Gamma}(b, b)$	$\hat{b} = 0.3288$
A	$\Theta \sim \text{Gamma}(\alpha, \alpha)$	$\hat{\alpha} = 0.3598$
B	$\Theta_1 \sim \text{Gamma}(\alpha_1, \alpha_1)$	$\hat{\alpha}_1 = 0.1309$
	$\Theta_2 \sim \text{Gamma}(\alpha_2, \alpha_2)$	$\hat{\alpha}_2 = 0.3101$
	$\Theta_3 \sim \text{Gamma}(\alpha_3, \alpha_3)$	$\hat{\alpha}_3 = 0.0555$

Table 4: Comparison of *a posteriori* ratemaking

Model		t=1											
		Best			Average			Worst					
	$N_1 \setminus N_2$	0	1	2	$N_1 \setminus N_2$	0	1	2	$N_1 \setminus N_2$	0	1	2	
Indep.	0	0.72	1.82	2.91	0	0.70	2.01	3.33	0	0.57	1.76	2.94	
	1	2.93	4.03	5.12	1	2.32	3.64	4.95	1	1.72	2.90	4.08	
	2	5.14	6.24	7.33	2	3.94	5.26	6.57	2	2.86	4.04	5.22	
Model A	0	0.72	2.71	4.71	0	0.65	2.46	4.28	0	0.52	1.95	3.39	
	1	2.71	3.52	5.44	1	2.46	3.44	5.18	1	1.95	3.00	4.39	
	2	4.71	5.44	6.69	2	4.28	5.18	6.52	2	3.39	4.39	5.62	
Model B	0	0.74	1.69	2.64	0	0.72	1.97	3.22	0	0.60	1.80	3.00	
	1	2.99	3.60	4.60	1	2.36	3.01	4.38	1	1.78	2.26	3.67	
	2	5.24	5.97	6.37	2	3.99	4.90	5.09	2	2.96	3.84	3.98	

Table 5: *A posteriori* risk factors for policyholders reporting one claim of each type

	Best			Average			Worst		
	Ind.	A	B	Ind.	A	B	Ind.	A	B
$t = 1$	4.03	3.52	3.60	3.64	3.44	3.01	2.90	3.00	2.26
$t = 2$	3.13	2.91	2.94	2.78	2.68	2.44	2.04	2.07	1.72
$t = 3$	2.57	2.45	2.48	2.25	2.17	2.04	1.57	1.58	1.38
$t = 4$	2.19	2.11	2.14	1.89	1.82	1.75	1.28	1.27	1.15
$t = 5$	1.91	1.85	1.89	1.63	1.57	1.53	1.07	1.07	0.99
$t = 6$	1.69	1.64	1.69	1.43	1.37	1.36	0.93	0.92	0.87
$t = 7$	1.52	1.48	1.53	1.28	1.22	1.23	0.82	0.80	0.78
$t = 8$	1.38	1.34	1.40	1.15	1.10	1.12	0.73	0.72	0.70
$t = 9$	1.26	1.23	1.28	1.05	1.00	1.02	0.66	0.65	0.64
$t = 10$	1.17	1.14	1.19	0.96	0.92	0.95	0.60	0.59	0.59
$t = 11$	1.08	1.05	1.11	0.89	0.85	0.88	0.55	0.54	0.54
$t = 12$	1.01	0.98	1.04	0.83	0.79	0.82	0.51	0.50	0.50