Optimal tax enforcement under prospect theory

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Abstract

Prospect Theory (PT) has become the most credited alternative to Expected Utility Theory (EUT) as a theory of decision under uncertainty. This paper characterises the optimal income tax and audit schemes under tax evasion, when taxpayers behave as predicted by PT. Under reasonable assumptions on the reference income and on the utility function of taxpayers, we show that the optimal audit probability function is non-increasing and the optimal tax function is non-decreasing and concave. The conditions under which those results hold for PT are weaker than the corresponding one for EUT.

JEL classification codes: D81; H26; K42

Keywords: Tax evasion; Optimal Income Tax; Prospect theory; Audit

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*The project was started at Toulouse School of Economics: we thank all members for their suggestions, help and comments. For their comments and discussions, we are particularly indebted to A. Al-Nowaili, M. Bernasconi, T. Gajdos, M. Jeleva, M. Le Breton, J. Pirttilä, M. Rablen, M. Tuomala and A. Trannoy. Piolatto acknowledges financial support from IVIE, the Spanish Ministry of Science and Innovation (grant ECO/ 2012-37131), and the Government of Catalonia (grant 2009 SGR 102). Amedeo Piolatto, piolatto[at]ub.edu; c/ Tinent Coronel Valenzuela, 1-11, 08034 Barcelona, Spain; Gwenola Trotin, gwenola.trotin[at]univ-amu.fr
1 Introduction

Tax administrations rely generally on income and wealth self-reports from taxpayers. This occurs for self-employed workers but also, up to a point, for all citizens with side sources of revenue, that should be integrated in the declaration of taxable wealth. Furthermore, citizens can reduce their tax liability by eluding taxes and inflating the amount of deductions they are entitled to. Taxpayers have a clear incentive to misreport their income/wealth, in order to reduce their liability. Losses to public budgets from tax evasion are indeed significant: the US Internal Revenue Service (IRS), for example, estimates the tax gap in 2001 at USD 345 billion, i.e., almost 16% of the total tax revenue (IRS, 2006).\(^1\) The main tool in the hand of the tax administration to limit possible misbehaviour is to audit taxpayers and verify the information provided. Audits being costly,\(^2\) the tax administration generally selects the reports to be audited. Assessed misreporting may result in penalties and fines; the setting up of penalty schemes also satisfies an objective of both horizontal and vertical equity among taxpayers. On top of tax rates, an optimal tax policy includes therefore an audit strategy and a scheme of penalties.

This gives rise to interesting questions, that we aim at analysing, about the optimal audit strategies and penalty schemes, and the nature of interactions between tax rates and audit strategies. For that, we need to take into account taxpayers' attitude toward risk and uncertainty. Allingham and Sandmo (1972) are the first to analyse tax evasion as a decision under uncertainty. Since then, several authors studied this risky decision in the framework of Expected Utility Theory (EUT).\(^3\) Although EUT has been considered for long time the most convenient framework (mainly because of its tractability and nice mathematical properties), there is a growing consensus about the need of an alternative theory of the agents' behaviour under uncertainty.\(^4\)

Pioneered by nobel laureate D. Kahneman (Kahneman and Tversky, 1979), Prospect Theory (PT) has become one of the most prominent alternatives to EUT, and it is widely used both in theoretical and in empirical research.\(^5\) According to PT, agents well-being depends on the distance between the final income and a predetermined reference income. The reference income is the point for which the utility of an agent is equal to zero. For an income lower than this point, the utility is negative. For a larger one, it is positive. Agents think of gains and losses relative to this reference point. This phenomenon is known in cognitive sciences as reference dependence.\(^6\)

According to PT, the payoff function differs from the standard EUT one in three dimensions. Besides reference dependence, it shows diminishing sensitivity, meaning that agents marginal utilit y decreases in the distance from the reference. Consequently, the payoff function is concave for gains (above the reference) and convex for losses (below it). Furthermore, in incorporates the loss aversion phenomenon: individuals care generally more about potential losses than potential gains, hence the disutility of a loss is larger than the utility of a gain of the same magnitude. Cumulative Prospect Theory introduces

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1The field experiment in Kleven et al. (2011) shows that tax evasion in Denmark mostly concern self-reported income. However, third-party employed workers may still evade or avoid part of their tax allowance, through deductible expenses, or when declaring their wealth or their investments. For more on tax compliance and on the shadow economy, see Schneider (2005), Slemrod (2007), Buehn and Schneider (2012) and Alm (2012).
2Wages, the formation of the tax administration agents, and the incentive problems related to their corruptibility are amongst the main source of cost (Hindriks et al., 1999).
3See Andreoni et al. (1998), Slemrod and Yitzhaki (2002), Sandmo (2012).
4See, for instance, Mirrlees (1997).
5See Vaní (1999), Camerer (2000), Camerer and Loewenstein (2003), or Rablen (2010). Hashimzade et al. (2012) and Barberis (2013) provide interesting surveys on PT. See Bruhin et al. (2010) and Conte et al. (2011) provide empirical evidence supporting that a majority of economic agents behaviour is better explained by PT than EUT. See Neilson and Stowe (2002) for some limitations of PT.
a further element of distinction from EUT: *probability weighting*. According to it, agents assign weights to objective probabilities transforming them, in the words of Prelec (1998), into “decision weights”. Prospect theory is nowadays commonly used in cognitive sciences and has become one of the standards in the behavioural economics literature.\(^7\)

The recent literature on taxation highlights problems in using the Expected Utility Theory setting for tax evasion decision issues, because it contradicts the empirical evidence in several ways. In particular, with a reasonable degree of risk aversion, it overestimates the willingness of agents to misreport their income, therefore, it predicts more tax evasion than what really occurs. Subjective probabilities under Prospect Theory allow us to easily overcome this issue.\(^8\) Furthermore, under the assumption of Decreasing Absolute Risk Aversion (DARA), Expected Utility Theory predicts that an increase in the tax rate leads to a decrease in tax evasion (the so called “Yitzhaki paradox”).\(^9\) As a consequence, we observe a growing interest for Prospect Theory within the taxation literature. Kanbur et al. (2008) study the optimal non-linear taxation under Prospect Theory, and show that the standard Mirrlees (1974) results are modified in several interesting ways. Dhami and Al-Nawaihi (2007) apply Prospect Theory to the taxpayers’ decision to evade taxes, and show that predictions are both quantitatively and qualitatively more in line with the empirical evidence than under Expected Utility Theory. In Dhami and Al-Nawaihi (2010) the tax rate is endogenous: one main finding is that the best description of the data is obtained by combining taxpayers behaving according to Prospect Theory and the government acting as predicted by EUT.

To the best of our knowledge, ours is the first attempt to analyse the optimal audit scheme, assuming that agents behave according to PT. For the case of EUT, this was analysed in Chander and Wilde (1998), Chander (2004, 2007), and Cremer and Gahvari (1996). The latter focus on the moral hazard problem occurring when the labour supply choice is endogenous. In Chander and Wilde (1998), the authors characterise the optimal tax schedule in the presence of enforcement costs and clarify the nature of the interplay between optimal tax rates, audit probabilities and penalties for misreporting. In particular, under the assumption of risk neutral expected-utility-maximiser taxpayers, they show that the optimal tax function must generally be increasing and concave. This is because a progressive tax function implies stronger incentives to misreport and thus it calls for larger audit probabilities. Chander (2004, 2007) studies the same issues for the case of risk averse taxpayers, when the incentive to misreport is weaker. By introducing a measure of aversion to large risks, he shows that the optimal tax function is increasing and concave if the taxpayer’s aversion to such large risks is decreasing with income.

Our paper extends the optimal tax enforcement literature, considering agents that behave according to PT. We show that the second best solution for the tax authority is to enforce a regressive tax system, since any other system would be incentive compatible. We do that using a mechanism design setting in which, through the *revelation principle*, we derive the optimal (second best) audit scheme in a truth-telling mechanism. Clearly, this need not to be the unique way to reach a second best optimum, but by the revelation principle we can ensure that it cannot exist a not truth-telling mechanism such that the authority can implement a better scheme. In our equilibrium, the tax agency audits agents which in equilibrium are always declaring all their income. This may seem odd at first, but it’s a standard consequence of the truth-telling mechanisms. Audit is used as a deterrent, and the agency needs to audit in order to guarantee truth-telling reports.\(^10\)

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\(^7\) See, for instance, Frank (1997), List (2003, 2004) and Post et al. (2008).

\(^8\) See Hashimzade et al. (2012).

\(^9\) For more on the paradox, see Yitzhaki (1974), Trannoy and Trotin (2010), and Hashimzade et al. (2012).

\(^10\) Notice that the way we construct and solve the model does not guarantee that the equilibrium will be unique, indeed, the tax authority can induce several equilibria, and those may depend on whether we consider pure strategy or mixed strategy equilibria. What we find is the second-best solution that
Reference dependence is a crucial element in PT. To define a general reference income, the most natural choice is to restrict the attention, setting the legal income (i.e., the after-tax disposable income, under no tax evasion) as a lower-bound and the pre-tax income as an upper-bound. We show that the optimal audit probability function is always non-increasing. Concerning the optimal tax function, we show that it is always non-decreasing and concave when the pre-tax income is used as a reference. Nevertheless, for the same result to hold when the reference income is the legal one, we need to impose a further restriction. We show that a sufficient condition is to have Decreasing Absolute Risk Aversion (DARA). This is in line with the results in Chander and Wilde (1998) and Chander (2004) for the EUT case, but if agents behave according to PT, these properties hold under a set of less restrictive assumptions.

The paper is organised as follows. The next section describes a general model of income tax enforcement under Prospect Theory and introduces the definition of an optimal tax and audit scheme. Sections 3 and 4 solve the model using as the reference income respectively the legal income and the pre-tax income. Section 5 concludes.

2 The model

Taxpayers’ income \( w \) is a random variable with distribution function \( g \), defined over the interval \([0, \bar{w}]\), with \( \bar{w} > 0 \). The tax administration knows \( g \) but not \( w \). Following Prospect Theory, a taxpayer considers possible outcomes relative to a certain reference point \(^{11}\) when sending a message \( x \in [0, \bar{w}] \), (i.e., declaring income) to the tax administration.\(^{12}\) For a reference income \( R \), a taxpayer considers larger outcomes as gains and lower as losses. The payoff function \( u \) is:

i. continuous on \( \mathbb{R} \), twice continuously differentiable on \( \mathbb{R}^* \setminus \{0\} \) and equal to zero in zero: \( u(0) = 0 \),

ii. increasing, convex for losses and concave for gains: \( u' > 0 \) on \( \mathbb{R}^* \), \( u'' > 0 \) on \( \mathbb{R}^* \) and \( u'' < 0 \) on \( \mathbb{R}^*_+ \) (Diminishing sensitivity),

iii. steeper for losses than for gains: \( u'(-k) > u'(k) \) for \( k \in \mathbb{R}^*_+ \) (Loss aversion).

Figure 1 represents a typical payoff function.

The tax administration sets up a mechanism, consisting of a set \( X \subset [0, \bar{w}] \) of messages (i.e., income declarations), a tax function \( t : X \to \mathbb{R}_+ \), twice continuously differentiable, an audit probability function \( p : X \to [0, 1] \), and a penalty function \( f : [0, \bar{w}] \times X \to \mathbb{R}_+ \). A taxpayer with initial income \( w \) and sending the message \( x \in X \), is audited with probability \( p(x) \) and pays \( t(x) \in \mathbb{R}_+ \) if no audit occurs, or \( f(w, x) \in [t(x), w] \) if an audit occurs.\(^{13}\) The associated payment function for taxpayers is defined by:

\[
    r(w, x) = (1 - p(x))t(x) + p(x)f(w, x), \text{ for all } (w, x) \in [0, \bar{w}] \times X. \tag{1}
\]

Audits are assumed to be costly, \( c \) being the cost for an audit. Ceteris paribus, the tax administration then prefers smaller audit probabilities to reduce audit costs.

The reference income, to which the taxpayer compares his final income, is a function of his initial income. The shape of \( R \) may depend on i) the price the taxpayer is willing can be achieved by the authority by designing a truth-telling mechanism. We can ensure that no other mechanism can be better for the authority, but there may be other (not truth-telling) mechanisms yielding the same tax proceeds, in which agents behave differently. We are indebted to Ali Al-Nowaihi for the very interesting discussion about that and the possibility that some mixed-strategy equilibria may exist.

\(^{11}\) On the opposite, what matters in EUT is only the absolute value of the outcome.

\(^{12}\) See, for example, Kahneman and Tversky (2000).

\(^{13}\) It is assumed that if an audit occurs, the actual income of the taxpayer is discovered without error.
to pay for public goods (which can be expressed as a function of the tax rate), and ii) the characteristics of the cheating game to which he subjects himself by not declaring his entire income (i.e., the probability of audit and the penalty function):

\[ R = R_{t,p,f}(w) \in [0, w]. \]

(2)

Taxpayers have limited liability: they cannot pay more than their true income. This rules out full-information optima with large penalties. A feasible mechanism \((X, t, p, f)\) is such that for any message that a taxpayer can send, the payment is never larger than his initial income. The mechanisms that satisfy the following requirements are called direct revelation mechanisms:

i. First feasibility requirement: For all \(w \in [0, \bar{w}]\), the set of feasible messages \(X(w) = \{x \in [0, w], t(x) \leq w\}\) contains at least one element and for all \(x \in X(w)\), \(f(w, x) \leq w\).

ii. Second feasibility requirement: The maximisation problem of the taxpayer,

\[ \max_{x \in X(w)} [V(x)], \]

where the value function \(V(x)\) is defined as \(V(x) = (1 - p(x))u(w - t(x) - R(w)) + p(x)u(w - f(w, x) - R(w))\), has a solution for all \(w \in [0, \bar{w}]\).

A direct revelation mechanism is said to be incentive compatible if it is optimal for each taxpayer to report his income truthfully. The revelation principle applies to this setting. The principle guarantees the existence of a solution for the maximisation problem as defined in the second feasibility requirement, and it allows to confine the attention to the incentive compatible direct revelation mechanisms.

**Proposition 1.** For each direct revelation mechanism \((X, t, p, f)\), there exists a scheme \((t', p', f')\) such that \((X, t', p', f')\) is an incentive compatible direct revelation mechanism, and the two are equivalent from the point of view of both the tax administration and each taxpayer.

**Proof.** See the Appendix.
Without loss of generality, our attention can be confined to mechanisms in which taxpayers declare their income and they are provided with sufficient incentives to be rational for them to report truthfully. Taking into account the feasibility requirements mentioned above, the incentive compatible (IC) direct revelation mechanism is a scheme \((t, p, f)\) such that for all \(w \in [0, \bar{w}]\):

**IC1.** \(t(w) \leq w\),

**IC2.** \(f(w, x) \leq w\), for all \(x \in X(w)\),

**IC3.** \((1 - p(w))u(w - t(w) - R(w)) + p(w)u(w - f(w, w) - R(w)) \geq (1 - p(x))u(w - t(x) - R(w)) + p(x)u(w - f(w, x) - R(w))\), for all \(x \in X(w)\).

The third condition, henceforth incentive constraint, says that the value function of the taxpayer is maximised when reporting income truthfully.

With an incentive compatible direct revelation mechanisms, the payment function for the taxpayers is defined by:

\[
r(w) = (1 - p(w))t(w) + p(w)f(w, w), \text{ for all } w \in [0, \bar{w}].
\]

We assume that the objective of the tax administration is to maximise tax proceeds net of audit costs:\footnote{Although it is reasonable to assume that a tax administration may be only interested in maximising tax proceeds, it is fair to admit that the government may have a different objective function. For tractability reasons, we cannot solve the problem for a generic social welfare function, but our results are robust to changes in the administration objective. For more on that, see Border and Sobel (1987) and Chander and Wilde (1995, 1998).}

\[
\max_{r, p} \left[ \int_0^{\bar{w}} r(w)g(w)dw - c \int_0^{\bar{w}} p(w)g(w)dw \right].
\]

Denote Chander and Wilde (1998), a scheme \((t, p, f)\) is efficient in \(F\) if there is no other scheme \((t', p', f') \in F\) such that \(p' \leq p, r' \geq r\) and \(r' \neq r\) or \(p' \neq p\), where \(r\) and \(r'\) are the payment functions corresponding to \((t, p, f)\) and \((t', p', f')\). This means that we cannot reduce a taxpayer’s audit probability without either increasing someone else’s audit probability or negatively affecting tax proceeds. Also, we cannot raise the payment at some income level without either lowering the payment at some other levels of income or increasing some audit probabilities.

Notice that an optimal scheme maximises the tax administration’s total proceeds, net of audit cost, and by definition it is efficient.

### 3 When the reference income is the legal income

Through this section, the reference income is assumed to be the legal after-tax income: \(R(w) = w - t(w)\). Therefore, the taxpayer is in the domain of gains as soon as he pays less than the amount of tax initially planned for him, and in the domain of losses as soon as he pays more. Under this assumption, propositions 2 and 3 identify the characteristics of an efficient scheme \((t, p)\).

As a specificity of the use of Prospect Theory, when maximising the value function of the taxpayer \(V(x)\), with \(x \in X(w)\), the second order condition may not hold. However, conditions under which the solution is interior can be highlighted. Indeed, since \(V(w) = 0\), the following conditions are sufficient for an interior solution:

\[
V(0) \geq 0 \iff (1 - p(0))u(t(w) - t(0)) + p(0)u(t(w) - w) \geq 0
\]
and \( V'(0) > 0 \leftrightarrow p'(0) [u(t(w) - w) - u(t(w) - t(0))] \)
\[
- t'(0)(1-p(0))u'(t(w) - t(0)) > 0
\] (6)

With the present reference income, the incentive constraint defined in IC3 is:
\[
p(w)u(t(w) - f(w, w)) \geq (1 - p(x))u(t(w) - t(x)) + p(x)u(t(w) - f(w, x)),
\]
for all \( x \in X(w) \). (7)

**Lemma 1.** This incentive constraint can be rewritten as
\[
(1 - p(x))u(t(w) - t(x)) + p(x)u(t(w) - w) \leq 0, \text{ for all } x \in X(w).
\]

*Proof.* See the Appendix. □

It can be noticed in particular that \( f(w, w) = t(w) \) and \( f(w, x) = w \) if \( x \neq w \). \( F \) is the set of all schemes \((t, p)\) that satisfy conditions IC1 and IC2 for an incentive compatible direct revelation mechanism and the new incentive constraint (8). An efficient scheme in \( F \) is now a scheme \((t', p')\) for which there is no other scheme \((t', p') \in F\) such that \( t' \geq t \), \( p' \leq p \) and \( t' \neq t \) or \( p' \neq p \).

The following monotonicity and concavity results hold for any efficient (henceforth optimal) scheme.

**Lemma 2.** A scheme \((t, p) \in F\) is efficient in \( F \) only if the incentive constraint for each income level \( w \in [0, \bar{w}] \) is binding at some \( x \in X(w) \).

*Proof.* See the Appendix. □

**Proposition 2.** A scheme \((t, p) \in F\) is efficient in \( F \) only if \( t \) is non-decreasing and \( p \) is non-increasing.

*Proof.* See the Appendix. □

**Lemma 3.** If for all \( \hat{w} \in [0, \bar{w}] \), there exists an affine function \( l_{\hat{w}} \) on \( [0, \hat{w}] \) such that for all \( w \in [0, \bar{w}] \), \( l_{\hat{w}}(w) \geq t(w) \) and \( l_{\hat{w}}(\hat{w}) = t(\hat{w}) \), then \( t \) is concave.

*Proof.* See the Appendix. □

The first claim of this proposition (i.e., \( t \) is non-decreasing) is in line with what we would expect. The tax to be paid should at least not decrease with income, otherwise it would generate an incentive to over-declare income. As for the second one, the idea is that the probability of audit should not increase with the declared income. This result implies that taxpayers with a lower income \( w \) are more likely to be audited, but this is not the aim of the scheme. The intuition for \( p \) to be non-increasing is that the tax authority is willing to audit more the taxpayers declaring lower incomes and not those earning less. Hence, the administration observes agents who are identical in all the observable characteristics, and it expects that a fraud is more likely to occur when the reported income is low.

Proposition 3 further characterises an efficient scheme \((t, p)\). For that we need first to define risk aversion in Prospect Theory. The incentive constraint (8), when binding, requires the utility of an agent to be the same when declaring all his income and when not. This can be seen as a lottery with an expected utility equal to zero, where gambling the legal income \( w - t(w) \) against the gap between the two tax levels \( t(w) - t(x) \). To measure risk aversion we use the standard Arrow-Pratt absolute risk aversion measure:
\[
r_A(k) = \frac{u''(k)}{u'(k)}, \text{ for all } k \in \mathbb{R}, \]
(9)
Because agents utility is increasing in its argument (i.e., the payoff function is increasing in income), the sign of $r_A(k)$ depends on the sign of the second derivative. Indeed, while the standard assumption in EUT is that the second derivative is negative (agents are risk-averse at all levels of income), under Prospect Theory $u''$ takes negative values in the domain of gains (i.e., $u$ is concave at any point above the reference) and positive values in the domain of losses (i.e., $u$ is convex at any point below the reference).

**Proposition 3.** If $u$ satisfies DARA, then a scheme $(t,p) \in F$ is efficient in $F$ only if $t$ is concave.

**Proof.** See the Appendix.

Proposition 3 states that under decreasing absolute risk aversion, a necessary condition of all efficient schemes is that the tax function is not only non-decreasing (as required by proposition 2) but also concave. This result can be seen as a result of the standard rent enjoyed by the top type in mechanism design. For the incentive compatibility constraint to be satisfied, agents with higher income enjoy a rent. Indeed, keeping the tax rate relatively high for low incomes (or relatively low for high incomes) reduces the incentives of high income agents to misreport their true income. Therefore, it reduces the incentives of high income taxpayers to misreport, and it allows the tax administration to reduce their auditing expenditure.

Define by $y = w - t(x) - R(w)$ the after tax income, net of the Reference income, in case the tax payer is not audited, and define $-z = w - f(w, x) - R(w)$ the after tax income, net of the Reference income, in case of audit. Under prospect theory, the payoff function $u$ satisfies decreasing absolute risk aversion (DARA) in $p \in [0, 1[, \mathbb{R}^+_*$, if $z$ is increasing in $y \in \mathbb{R}^+_*$ at a non-decreasing rate, where $z$ is implicitly defined by equation (10):

$$(1 - p)u(y) + pu(-z) = 0. \quad (10)$$

Notice that, in the setting of Prospect Theory, $z$ is always increasing with $y$. Furthermore, the concavity condition generally holds. This is the case, for instance, of the power utility function in Tversky and Kahneman (1992) used to describe the behaviour of individuals under risk:

$$u(k) = \begin{cases} k^\alpha & \text{if } k \geq 0, \\ -\mu(-k)^\alpha & \text{if } k < 0, \end{cases} \quad (11)$$

where $0 < \alpha < 1$, and $\mu > 1$ because of loss aversion.

The concavity condition in Equation 9 is equivalent to:

$$(1 - p) \left| \frac{r_A(-z)}{u(-z)} \right| \geq p \left| \frac{r_A(y)}{u(y)} \right|, \quad (12)$$

where $y$ and $z$ are defined by (10). This means that, weighted by probability coefficients, the risk loving behaviour in case of loss must be larger than the risk aversion in case of gain. This condition is very weak and easily holds, because $p$ is usually very close to zero.

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15 This measure can also be defined on 0 in this manner: $r_A(0_-) = -\frac{u''(0_-)}{u'(0_-)}$ and $r_A(0_+) = -\frac{u''(0_+)}{u'(0_+)}$.

16 An analogous result is obtained in Chander and Wilde (1998) for the case of Expected Utility Theory.

17 From the experiment in Tversky and Kahneman (1992), the authors suggest, for instance, that reasonable values would be $\alpha = 0.88$ and $\mu = 2.25$. 

7
4 When the reference income is the initial income

In this section we consider the initial income as the reference income: \( R(w) = w \). This case corresponds to an extremely tax-averse taxpayer. This implies that any payment to the tax administration always lies in the loss domain, therefore the taxpayer is risk-averse.

Even though the conditions for an interior solution can not be extended to this extreme case, the continuity of the model suggests that for all \( R \) between the legal income and the initial income, such conditions hold. The present case must be considered as the extreme case among all possible.

The incentive constraints with this reference income becomes:

\[
(1 - p(w))u(-t(w)) + p(w)u(-f(w, w)) \geq (1 - p(x))u(-t(x)) + p(x)u(-f(w, x)),
\]

for all \( x \in X(w) \). \( (13) \)

Similarly to what we did for the previous case, the incentive constraint can be rewritten.

**Lemma 4.** The incentive constraint is equivalent to:

\[
u(-t(w)) \geq (1 - p(x))u(-t(x)) + p(x)u(-w), \quad \text{for all } x \in X(w). \]

\( (14) \)

**Proof.** See the Appendix.

Here again, \( f(w, w) = t(w) \) and \( f(w, x) = w \) if \( x \neq w \). \( F \) is now the set of all schemes \((t, p)\) that satisfies conditions IC1. and IC2. for an incentive compatible direct mechanism, and the incentive constraint \((14)\). The notion of efficiency is the same as before.

Then, optimal schemes are characterised by the following monotonicity and concavity results.

**Lemma 5.** A scheme \((t, p) \in F\) is efficient in \( F\) only if the incentive constraint for each income level \( w \in [0, \bar{w}] \) is binding at some \( x \in X(w) \).

**Proof.** The proof is similar to the one for Lemma 2.

**Proposition 4.** A scheme \((t, p) \in F\) is efficient in \( F\) only if \( t \) is non-decreasing, \( p \) is non-increasing and \( t \) is concave.

**Proof.** See the Appendix.

Results are very similar to those of Section 3 and the same intuitions hold. However, under the current framework, we do not need additional assumptions about the shape of the payoff function to ensure that the tax function of a revenue maximising scheme is concave. This comes directly from the concavity of the utility function, all its arguments being negative.

A priori, the most natural restriction for the reference income is to be not lower than the legal one and not higher than the initial one \((w - t(w) \leq R(w) \leq w)\). Indeed, this corresponds to the case of a taxpayer whose final income will exceed the legal income while remaining below the initial one. The taxpayer derives an obvious disutility from paying the legal tax. Following the reasonings for the legal income and the initial income, it can be proved that, in a revenue maximising framework, i) the probability function is non-increasing, ii) the tax function is non-decreasing, and iii) the interval on which the utility function is convex is larger when the reference income increases. Therefore, the conditions for the tax function to be concave become less restrictive.
5 Conclusions

Following the growing interest in behavioural models that are able to solve the paradoxes in the standard EUT taxation literature, we characterise the optimal income tax and audit schemes when tax evasion decisions of taxpayers follow Prospect Theory. Our results are in line with the basic intuition in Mirrlees (1971) about tax progressivity.\footnote{See Tuomala (2010) for a discussion of Mirrlees (1971) and its drawbacks.}

Regardless of the desire, in democratic societies, for progressive taxation, the tax administration has to be cautious when choosing the tax function. The limited-information framework in which the administration operates, together with the cost of audit, limits the action space of the administration. Taxpayers enjoy an informational rent, which appears in the form of a concave tax function, because this incentivises taxpayers to declare all their income, and therefore the cost of audit shrinks. This result appears, for the case of Expected Utility Theory, in Chander and Wilde (1998) at least when agents utility shows Decreasing Absolute Risk Aversion.

The fact that Prospect theory seems to predict a different behaviour of taxpayers that seems more consistent with empirical observation, induced us to wonder if the previous results in the literature concerning tax progressivity still hold. Actually, they are even reinforced, since the conditions under which the optimal tax function has to be concave are weakened. This is because reference dependence reduces the difference in the optimal behaviour of agents with different incomes. In other words, when taxpayers’ payoff depends on the distance from a reference point, the incentives that the tax administration can provide are less effective (it is harder to target a subset of taxpayers). As a consequence, the rent that must be left to wealthier agents not to evade is larger, that is, it is more likely that the tax function will need to be concave.

Similarly to what is found in the previous theoretical literature, larger penalties for misreporting can be an alternative and effective instrument to reduce misreporting by deterrence instead of audit.\footnote{For more on deterrence, see DeAngelo and Charness (2012).} Higher penalties may, nevertheless, increase the incentives for corruption and generate larger costs of collecting fines, trials and convictions, therefore it may even be suboptimal to increase punishment.\footnote{We are grateful to Ali Al-Nowaihi for pointing this out. For more on the trade off between the audit probability and the fine rate, see RABLEN (Forthcoming).} In here, we disregard these possible extra costs, hence, to avoid the equilibrium in which the punishment is sufficiently large to deter evasion, we invoked the usual "limited-liability" argument (which goes together with the "punishment fitting the crime" one), and assume a binding cap to the penalty that can be inflicted. In our model, any reduction in the fine (below the cap) would increase the incentives for misreporting, hence, to compensate, the optimal tax scheme should be even more progressive.

A Appendix

Proof of Proposition 1. Denote by $\chi$ the function defined by the second feasibility requirement:

$$
\chi(w) = \arg\max_{x \in X(w)} [(1 - p(x))u(w - t(x) - R(w)) + p(x)u(w - f(w, x) - R(w))].
$$

Then, at each $w \in [0, \bar{w}]$, the scheme $(t, p, f)$ associates $(t(\chi(w)), p(\chi(w)), f(w, \chi(w)))$.

Denote by $(t', p', f')$ the scheme such that for all $w \in [0, \bar{w}]$,

$$(t'(w), p'(w), f'(w, w)) = (t(\chi(w)), p(\chi(w)), f(w, \chi(w))).$$

Using the definition of $\chi$, we have:

$$(1 - p'(w))u(w - t'(w) - R(w)) + p'(w)u(w - f'(w, w) - R(w)) \geq$$

\begin{align*}
\text{(15)}
\end{align*}

\[(1 - p'(x))u(w - t'(x) - R(w)) + p'(x)u(w - f'(w,x) - R(w)), \text{ for all } x \in X(w).\]

\((X,t',p',f')\) is an incentive compatible direct revelation mechanism and \((t',p',f')\) is equivalent because the utility of each agent is maximised.

**Proof of Lemma 1.** We can weaken the constraint by rising \(f(w,x)\). This is possible as long as \(f(w,x) < w\), and up to \(f(w,x) = w\), for which equations (7) and (8) are equivalent.

In addition, the payment function is \(r(w) = (1 - p(w))t(w) + p(w)f(w,w)\), the constraint is then equivalent to:

\[p(w)u\left(\frac{t(w) - r(w)}{p(w)}\right) - (1 - p(x))u(t(w) - t(x)) - p(x)u(t(w) - w) \geq 0.\]

The function \(\phi(t) = p(w)u\left(\frac{t - r(w)}{p(w)}\right) - (1 - p(x))u(t - t(x)) - p(x)u(t - w)\) is increasing with \(t\) when \(t\) is smaller but sufficiently close to \(r\), for all \(r > 0\). Then, the constraint can be weakened by rising \(t(w)\), while keeping constant the payment \(r(w)\). That is, the constraint can be weakened by decreasing \(f(w,w)\) and rising \(t(w)\), while keeping constant the payment \(r(w)\), as long as \(f(w,w) > t(w)\). The conditions in (7) and (8) are then equivalent when \(f(w,w) = t(w) = r(w)\).

Notice that, as specified at page 3, \(f : [0,\bar{w}] \times X \to \mathbb{R}_+\), meaning that the fine is always weakly positive (the public authority does not distribute money to audited taxpayers who declared their real income).

**Proof ofLemma 2.** Suppose that \(w \in [0,\bar{w}]\) exists such that for all \(x \in X(w)\), the following inequality holds:

\[(1 - p(x))u(t(w) - t(x)) + p(x)u(t(w) - w) < 0.\]

Because \(u\) is increasing, \(t\) such that \(t(w) > t(w), t(x) = t(x)\) for all \(x \in X(w) \setminus \{w\}\) and

\[(1 - p(x))u(t'(w) - t'(x)) + p(x)u(t'(w) - w) < 0,

can then be considered. This contradicts the efficiency of \((t,p)\) in \(F\).

**Proof of Proposition 2.**

- Suppose that there exists \(w, w' \in [0,\bar{w}]\) such that \(w < w'\) and \(t\) is decreasing on \([w,w']\). According to Lemma 2, there exists \(x' \in X(w')\) such that the incentive constraint (8) for \(w'\) is binding at \(x'\). By the incentive constraint (8) for \(w\),

\[(1 - p(x'))u(t(w) - t(x')) + p(x')u(t(w) - w) \leq 0,
\]

and, because \(u\) is increasing,

\[(1 - p(x'))u(t(w') - t(x')) + p(x')u(t(w') - w') < 0.
\]

This contradicts the fact that for \(w', (8)\) is binding at \(x'\).

- According to (8), for all \(x \in X\), for all \(w \in [0,\bar{w}]\) such that \(x \in X(w)\),

\[p(x) \geq \frac{u(t(w) - t(x))}{u(t(w) - t(x)) - u(t(w) - w)}.\]

Then, \((t,p)\) being efficient,

\[p(x) = \sup_{w > t(x)} \frac{u(t(w) - t(x))}{u(t(w) - t(x)) - u(t(w) - w)}.\]

\(t\) is non-decreasing, \(p\) is thus non-increasing. If there exists \(x \in X\) which does not belong to any \(X(w), w \in [0,\bar{w}]\), then, according to (8), \(p(z) = 0\) for all \(z \geq x\).
Proof of Lemma 3. Let there be some \( \hat{w} \in [0, \bar{w}] \).

- The slope of \( l_{\hat{w}} \) is \( t'(\hat{w}) \). Indeed, for all \( w \in [0, \hat{w}] \), the first order Taylor expansion of \( t \) near \( \hat{w} \) is:

\[
t(w) = t(\hat{w}) + t'(\hat{w})(w - \hat{w}) + r_1(w), \text{ with } r_1(w) \ll w - \hat{w} \text{ (when } w \to \hat{w}).
\]

Since \( l_{\hat{w}} \) is an affine function which crosses \( t \) in \( \hat{w} \), \( l_{\hat{w}}(w) = t(\hat{w}) + \lambda(w - \hat{w}) \). For all \( w \in [0, \hat{w}] \), \( l_{\hat{w}}(w) \geq t(w) \), then:

\[
\lambda(w - \hat{w}) \geq t'(\hat{w})(w - \hat{w}) + r_1(w).
\]

For all \( w > \hat{w}, \lambda \geq t'(\hat{w}) + r_0(w) \), with \( r_0(w) \ll 1 \), then \( \lambda \geq t'(\hat{w}) \), for all \( w < \hat{w}, \lambda \leq t'(\hat{w}) + r_0(w) \), with \( r_0(w) \ll 1 \), then \( \lambda \leq t'(\hat{w}) \), then \( \lambda = t'(\hat{w}) \).

- For all \( w \in [0, \hat{w}] \), \( l_{\hat{w}}(w) = t(\hat{w}) + t'(\hat{w})(w - \hat{w}) \). In addition, the second order Taylor expansion of \( t \) near \( \hat{w} \) is:

\[
t(w) = t(\hat{w}) + t'(\hat{w})(w - \hat{w}) + t''(\hat{w})\frac{(w - \hat{w})^2}{2} + r_2(w),
\]

with \( r_2(w) \ll (w - \hat{w})^2 \).

Since \( l_{\hat{w}}(w) \geq t(w) \), then \( t''(\hat{w})\frac{(w - \hat{w})^2}{2} + r_2(w) \leq 0 \), then \( t''(\hat{w}) \leq 0 \). This is verified for all \( \hat{w} \in [0, \bar{w}] \), \( t \) is then concave on \( [0, \bar{w}] \).

\[\square\]

Proof of Proposition 3. Let there be some \( \hat{w} \in [0, \bar{w}] \). Since \((t, p)\) is efficient, according to Lemma 2, it exists some \( \hat{x} \in [0, \bar{w}] \) such that \( t(\hat{x}) \leq \hat{w} \) and \( (1 - p(\hat{x}))u(t(\hat{x}) - t(\hat{w})) + p(\hat{x})u(t(\hat{w}) - \hat{w}) = 0 \). Three cases arise from the value of \( p(\hat{x}) \).

- First case: \( p(\hat{x}) = 0 \), then \( u(t(\hat{w}) - t(\hat{x})) = 0 \), then \( t(\hat{w}) = t(\hat{x}) \). In addition, according to the incentive constraints (8), for all \( w \in [0, \bar{w}] \), \( u(t(w) - t(\hat{x})) \leq 0 \), then \( t(w) \leq t(\hat{x}) \). The (constant) affine function \( l_{\hat{w}}(w) = t(\hat{x}) \) satisfies the assumptions of Lemma 3.

- Second case: \( p(\hat{x}) = 1 \), then \( u(t(\hat{w}) - \hat{w}) = 0 \), then \( t(\hat{w}) = \hat{w} \). Then, since \( t(w) \leq w \), for all \( w \in [0, \bar{w}] \), the affine function \( l_{\hat{w}}(w) = w \) satisfies the assumptions of Lemma 3.

- Third case: \( 0 < p(\hat{x}) < 1 \), since \( u \) satisfies DARA, the curve \( C_{p(\hat{x})} \) defined by:

\[
(1 - p(\hat{x}))u(y) + p(\hat{x})u(-z) = 0
\]

is increasing and convex in the coordinate system \((0, y, z)\). Denote by \( \hat{\Phi} \) the associated function and let there be some \( \hat{z} \in [0, \bar{w}] \). Denote by \( \hat{y} \) the real number such that \( \hat{\Phi}(\hat{y}) = \hat{z} \). The tangent to \( C_{p(\hat{x})} \) at \( \hat{y} \) in \((0, y, z)\) is below itself. Denote by \( \hat{k} \) the function associated to the tangent:

\[
\hat{k}(y) = a(y - \hat{y}), \text{ with } \hat{k}(\hat{y}) = \hat{\Phi}(\hat{y}) = \hat{z}, \text{ } \hat{y} \in [0, \bar{w}], \text{ and } a > 0.
\]

\[\square\]
For all $z \in [0, \hat{w}]$ such that $(1 - p(\hat{x}))u(y) + p(\hat{x})u(-z) \leq 0$, $\hat{k}(y) \leq \hat{\Phi}(y) \leq z$, because $u$ is increasing.

Consider $z = w - t(w)$, $\hat{z} = \hat{w} - t(\hat{w})$, $\hat{y} = t(\hat{w}) - t(\hat{x})$ and $y = t(w) - t(\hat{x})$, $(1 - p(\hat{x}))u(y) + p(\hat{x})u(-\hat{z}) \leq 0$ according to (8) and $(1 - p(\hat{x}))u(\hat{y}) + p(\hat{x})u(-\hat{z}) = 0$, then the affine function:

$$l_w(w) = \frac{w + a(t(\hat{x}) + \hat{y})}{a + 1}$$

satisfies the assumptions of Lemma 3.

This is verified for all $\hat{w} \in [0, \hat{w}]$, $t$ is then concave on $[0, \hat{w}]$, according to Lemma 3. □

Proof of Lemma 4. We can weaken the constraint by rising $f(w, x)$. This is possible as long as $f(w, x) < w$, and up to $f(w, x) = w$, for which equations (13) and (14) are equivalent.

In addition, the payment function is $r(w) = (1 - p(w))t(w) + p(w)f(w, w)$, the constraint is then equivalent to:

$$(1 - p(w))u(-t(w)) + p(w)u \left( \frac{r(w) - (1 - p(w))t(w)}{p(w)} \right) - (1 - p(x))u(-t(x)) - p(x)u(-w) \geq 0.$$

The function $\psi(t) = (1 - p(w))u(-t) + p(w)u \left( \frac{r(w) - (1 - p(w))t}{p(w)} \right) - (1 - p(x))u(-t(x)) - p(x)u(-w)$ is increasing with $t$ when $t$ is smaller but sufficiently close to $x$, for all $r > 0$.

Then, the constraint can be weakened by rising $t(w)$, while keeping constant the payment $r(w)$. That is, the constraint can be weakened by decreasing $f(w, w)$ and rising $t(w)$, while keeping constant the payment $r(w)$, as long as $f(w, w) > t(w)$. The conditions in (13) and (14) are then equivalent when $f(w, w) = t(w) = r(w)$. □

Proof of Proposition 4.

- Let us suppose that there exists $w, w' \in [0, \hat{w}]$ such that $w < w'$ and $t$ is decreasing on $[w, w']$. According to Lemma 5, there exists $x' \in X(w')$ such that the incentive constraints (14) for $w'$ are binding at $x'$. But according to the incentive constraints (14) for $w$,

$$u(-t(w)) \geq (1 - p(x'))u(-t(x')) + p(x')u(-w).$$

$u$ being increasing and $t$ being decreasing on $[w, w']$, the following function is increasing on $[w, w']$:

$$\psi(v) = u(-t(v)) - (1 - p(x'))u(-t(x')) - p(x')u(-v).$$

Then, $\psi(w') > \psi(w) > 0$, which contradicts the fact that the constraints (14) for $w'$ are binding at $x'$.

- According to (14), for all $x \in X$, for all $w \in [0, \hat{w}]$ such that $x \in X(w)$,

$$p(x) \geq \frac{u(-t(x)) - u(-t(w))}{u(-t(x)) - u(-w)}.$$ 

Then, $(t, p)$ being efficient,

$$p(x) = \sup_{u(t(x))} \frac{u(-t(x)) - u(-t(w))}{u(-t(x)) - u(-w)}.$$ 

t is non-decreasing, $p$ is thus non-increasing. If there exists $x \in X$ which does not belong to any $X(w), w \in [0, \hat{w}]$, then, according to (14), $p(z) = 0$ for all $z \geq x$.  

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As for the concavity, let there be some \( \hat{w} \in [0, \bar{w}] \). Since \((t,p)\) is efficient, according to Lemma 5, it exists some \( \hat{x} \in [0, \bar{w}] \) such that \( t(\hat{x}) \leq \hat{w} \) and \( u(-t(\hat{w})) = (1 - p(\hat{x}))u(-t(\hat{x})) + p(\hat{x})u(-\hat{w}) \). Three cases arise from the value of \( p(\hat{x}) \).

- **First case:** \( p(\hat{x}) = 0 \), then \( u(-t(\hat{w})) = u(t(\hat{x})) \), then \( t(\hat{w}) = t(\hat{x}) \). In addition, according to (14), for all \( w \in [0, \bar{w}] \), \( u(-t(w)) \geq u(-t(\hat{x})) \), then \( t(w) \leq t(\hat{x}) \). The (constant) affine function \( l_\hat{w}(w) = t(\hat{x}) \) satisfies the assumptions of Lemma 3.

- **Second case:** \( p(\hat{x}) = 1 \), then \( u(-t(\hat{w})) = u(-\hat{w}) \), then \( t(\hat{w}) = \hat{w} \). Then, since \( t(w) \leq w \), for all \( w \in [0, \bar{w}] \), the affine function \( l_\hat{w}(w) = w \) satisfies the assumptions of Lemma 3.

- **Third case:** \( 0 < p(\hat{x}) < 1 \), then, \( u \) being convex on \( \mathbb{R}^*_+ \), for all \( w \in [0, \bar{w}] \),

\[
  u(-t(w)) \geq (1 - p(\hat{x}))u(-t(\hat{x})) + p(\hat{x})u(-w) \geq u(-(1 - p(\hat{x}))t(\hat{x}) - p(\hat{x})w),
\]

then, \( t(w) \leq l_\hat{w}(w) \), where \( l_\hat{w} \) is the affine function defined by \( l_\hat{w}(w) = (1 - p(\hat{x}))t(\hat{x}) + p(\hat{x})w \).

In addition, following the incentive constraints (14), the expected utility for the initial income \( \hat{w} \) is maximised by \( \hat{x} \). The payment when declaring \( \hat{x} \) is then lower than the one when declaring truthfully, that is:

\[
  r(\hat{w}, \hat{x}) = (1 - p(\hat{x}))t(\hat{x}) + p(\hat{x})\hat{w} \leq r(\hat{w}) = t(\hat{w}),
\]

then \( t(\hat{w}) = l_\hat{w}(\hat{w}) \).

This is verified for all \( \hat{w} \in [0, \bar{w}] \), \( t \) is then concave on \( [0, \bar{w}] \), according to Lemma 3. \( \square \)

**References**


