Estimation of temperature effects on fission barriers

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An accurate mass formula at finite temperature has been used to obtain a more precise estimation of temperature effects on fission barriers calculated within the liquid drop model.

[NUCLEAR REACTIONS, FISSION Finite temperature effects.]

It is well known at present time that fission barriers are lowered by the effect of temperature or, equivalently, by nuclear excitation.[1,2] When temperature increases, nuclear deformations and shell effects are washed out.[3] Thus fission becomes symmetric and we may use, more confidently, semiclassical models to study nuclear fission.

A direct way to get an estimation of such temperature effects relies on liquid drop model calculations of the barrier height (BH). Within this model, the potential energy E of a deformed drop relative to the spherical drop (at zero temperature) reads[4]:

$$E=[B_s-1+2\chi(B_C-1)]E_s^{(0)} \quad , \tag{1}$$

where χ is the so-called fissility parameter

$$\chi\equiv E_C^{(0)}/2E_s^{(0)} \quad . \tag{2}$$

$E_C^{(0)}$ and $E_s^{(0)}$ are the Coulomb and surface energies of the spherical drop, and B_s and B_C account for the total surface and Coulomb energies (in units of $E_s^{(0)}$ and $E_C^{(0)}$, respectively) at a given deformation. Functions B_s and B_C are temperature independent by construction and have been tabulated in Ref. 4.

The generalization of Eq. (1) to finite temperature is straightforward and gives the following expression for the free energy of deformation[1,2]:

$$F_{\text{def}}[y,\chi(T)]=\{B_s(y)-1+2\chi(T)[B_C(y)-1]\}F_s(T) \quad . \tag{3}$$

$F_s(T)$ stands for the surface free energy of the spherical drop at T MeV temperature and

$$\chi(T)=E_C(T)/2F_s(T) \quad ; \tag{4}$$

y is the deformation coordinate as introduced in Ref. 4; $y=0$ corresponds to a sphere and $y=1$ to two tangent spheres.

Previous calculations of BH at finite temperature,[1,2] using the liquid drop model expression (3), have too serious limitations to be realistic. In Ref. 1, the surface symmetry energy contribution to E_s was neglected; moreover, temperature effects on isolated nuclei were not treated self-consistently. In Ref. 2, the effect of temperature on the surface energy was underestimated. We refer the reader to Ref. 5 for more details concerning the finite temperature formulation.

In this Brief Report we want to undertake the calculation of fission barriers at finite T, overcoming the above indicated shortcomings of Refs. 1 and 2. Our starting point is a mass formula at finite temperature developed in Ref. 6. In order to set up such a formula we have calculated the most stable nucleus for values of Z ranging from $Z=20$ to 96, up to 4 MeV temperature, using a hot Thomas-Fermi model.[7]

Most of the coefficients entering the formula have been obtained through a leptodermous expansion of the energy functional[6,7] and the rest of them from a fit to the calculated energies. This procedure allows us to obtain an accurate determination of these coefficients. The mass formula in its final expression reads

$$\begin{aligned}\frac{B}{A}=&a_v+\alpha_vT^2+(a_s+\alpha_sT^2)A^{-1/3}\\&+(a_{\text{sym}}+\alpha_{\text{sym}}T^2)\frac{I^2}{A^2}\\&-\left[\frac{3}{5}\frac{e^2}{r_0}\left(1-\frac{a_{\text{Coul}}}{A^{2/3}}\right)+\alpha_{\text{Coul}}T^2\right]\frac{Z^2}{A^{4/3}}\\&+(a_{ss}+\alpha_{ss}T^2)\frac{I^2}{A^{7/3}} \quad ,\end{aligned} \tag{5}$$

where $I=N-Z$, and the constants are

$r_0=1.12, \quad e^2=1.44 \quad ,$

$a_v=16., \quad \alpha_v=-0.064 \quad ,$

$a_s=-20.8, \quad \alpha_s=-0.2238 \quad ,$

$a_{\text{sym}}=-33.96, \quad \alpha_{\text{sym}}=7.15\times10^{-3} \quad ,$

$a_{\text{Coul}}=3.1445, \quad \alpha_{\text{Coul}}=-1.02\times10^{-3} \quad ,$

$a_{ss}=71.54, \quad \alpha_{ss}=0.8184 \quad .$

TABLE I. Barrier height BH (in MeV) and deformation y for selected nuclei.

Nucleus	$^{232}_{90}Th$		$^{238}_{92}U$		$^{242}_{94}Pu$		$^{246}_{96}Cm$	
T (MeV)	BH	y	BH	y	BH	y	BH	y
0	7.65	0.25	6.19	0.23	4.81	0.21	3.67	0.19
1	6.55	0.24	5.55	0.22	4.31	0.20	3.28	0.19
2	4.61	0.22	3.54	0.20	2.80	0.18	2.03	0.16
3	2.20	0.17	1.57	0.16	1.04	0.14	0.63	0.12

If we rewrite (5) as

$$\frac{B}{A}=\frac{B}{A}(T=0)+\alpha T^2 , \qquad (6)$$

then $-\alpha$ is the level density parameter.[6]

The values of our $T=0$ parameters lie within the range of the currently accepted ones, though a_{ss} is as much as 1.5 times the value obtained in Ref. 8. However, we want to point out that we are primarily concerned with the evolution of the BH, so that we do not expect this discrepancy will affect our conclusions very much.

It is worth mentioning that the BH obtained by using (1) or (3) is very sensitive to χ, for B_s-1 and B_C-1 are $\lesssim 10^{-2}-10^{-1}$ in those cases of practical interest. Also, B_s and B_C are not independent from the values of $E_s^{(0)}$ and $E_C^{(0)}$ used to determine them. Hence, in order to have realistic deformation paths at $T=0$, we have actually taken *for* $\chi(T=0)$ the expression used by Nix in Ref. 4.

At finite temperature, the T dependence of χ arises from our expressions for $E_C(T)$ and $F_s(T)$. There are reasons to believe that there is no inconsistency in this procedure. First, Coulomb energy is quite insensitive to T and the value of α_C obtained by different authors is $\lesssim 10^{-3}$,[2,6,9] whatever way used to determine it. Second, α_s is almost force independent, as shown in Ref. 5, *provided* the nuclear surface is well described; α_s is ≈ -0.22 for all the Skyrme-like forces (see Table I in Ref. 6).

Following the outlined procedure, we have performed the calculation of BH and deformation potential energy $F_{\rm def}[y,\chi(T)]$. Table I collects BH values of several nuclei in the vicinity of ^{238}U. The comparison between our $F_{\rm def}$ curves (Fig. 1) and their homologous curves in Refs. 1 and 2 clearly shows that temperature effects on fission barriers have been underestimated in these earlier calculations.

At high temperature, the reduction in BH arises essentially from the fact that $\chi(T)$ increases because $F_s(T)$ decreases. For this reason our barriers are lower than those obtained in Ref. 2, though our $F_s(T)$ is bigger in the range of temperatures we are

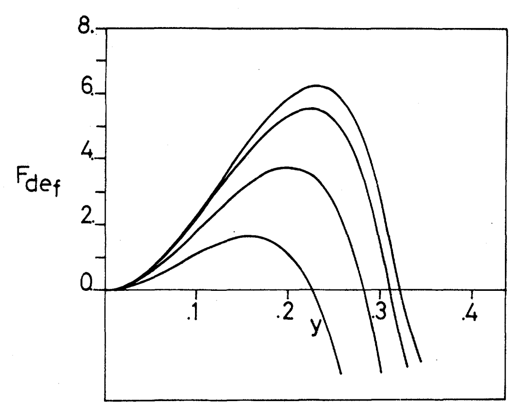

FIG. 1. Deformation energy (in MeV) of ^{238}U vs y (see text). The curves, starting from the top, refer to $T=0$, 1, 2, and 3 MeV temperature.

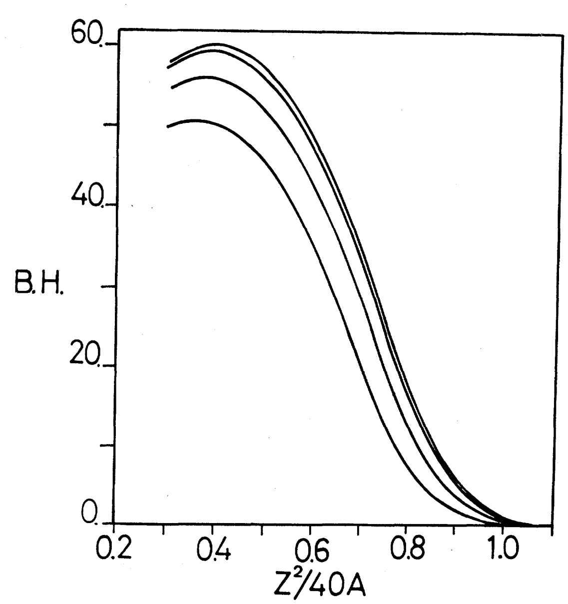

FIG. 2. BH (in MeV) versus $Z^2/40A$. The curves, starting from the top, refer to $T=0$, 1, 2, and 3 MeV temperature.

interested in. One may also see from Table I that, when temperature increases, fission takes place at lower deformation y.

Figure 2 shows the BH as function of the quantity $Z^2/40A$. This figure has only a qualitative value because the whole formalism breaks down for medium and light nuclei.

Finally, we have checked the sensitivity of the results to α_{ss} since this parameter is the more difficult one to be accurately determined.[6] Setting $\alpha_{ss}=0$ and $T=2$ MeV, we have computed the BH of ^{238}U and obtained 3.20 MeV, compared with 3.54 MeV when $\alpha_{ss}=0.8144$.

We have presented here the most straightforward application of the mass formula obtained in Ref. 6. We expect that in the future the formula itself and the level density parameter associated with it may become a useful tool in the study of phenomena involving high energy nuclear excitations.

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