

$\Lambda N \rightarrow NN$ weak interaction in effective-field theoryAssumpta Parreño,¹ Cornelius Bennhold,² and Barry R. Holstein^{3,4}¹*Departament ECM, Facultat de Física, Universitat de Barcelona, E-08028, Barcelona, Spain*²*Center of Nuclear Studies, Department of Physics, The George Washington University, Washington, DC 20052, USA*³*Department of Physics-LGRT, University of Massachusetts, Amherst, Massachusetts 01003, USA*⁴*Thomas Jefferson National Accelerator Facility, Theory Group, Newport News, Virginia 23606, USA*

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The nonleptonic weak $|\Delta S|=1$ ΛN interaction, responsible for the dominant nonmesonic decay of all but the lightest hypernuclei, is studied in the framework of an effective-field theory. The long-range physics is described through tree-level exchange of the SU(3) Goldstone bosons, while the short-range potential is parametrized in terms of the lowest-order contact terms. We obtain reasonable fits to available weak hypernuclear decay rates and quote the values for the parity-violating asymmetry as predicted by the present effective-field theory.

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For the past 50 years, Λ hypernuclei, systems of one or more Λ hyperons bound to a core nucleus, have been used to extend our knowledge of both the strong and the weak baryon-baryon interaction from the NN case into the SU(3) sector. The nonmesonic hypernuclear weak decay, facilitated via the $|\Delta S|=1$ four-fermion interaction, thus complements the weak $\Delta S=0$ NN case, which allows the study of only the parity-violating amplitudes.

In analogy to NN phenomenology, the nonmesonic hypernuclear decay has traditionally been modeled using a meson-exchange approach [1]. The long-range part of the interaction is naturally explained by one-pion exchange, which could approximately reproduce the total (one-nucleon-induced) nonmesonic decay rate, $\Lambda N \rightarrow NN$, but not the partial rates, the proton-induced $\Lambda p \rightarrow np$ rate Γ_p , and the neutron-induced $\Lambda n \rightarrow nn$ rate Γ_n [2]. Due to the ΛN mass difference, the process $\Lambda N \rightarrow NN$ produces nucleons with momenta around ≈ 420 MeV, suggesting that the short-range part of the interaction cannot be neglected. These contributions have been described either through the exchange of vector mesons [3,4], whose production thresholds are too high for the free Λ decay, or through direct quark exchange [5].

In contrast to previous theoretical studies, we present an exploratory study in order to determine the possible efficacy of the use of effective-field theory (EFT) methods in hypernuclear decay. Studies in this direction have already begun in Ref. [6], where a Fermi ($V-A$) interaction was added to the one-pion-exchange (OPE) mechanism to describe the weak $\Lambda N \rightarrow NN$ transition. The present approach is motivated by the remarkable success of EFT techniques based on chiral expansions in the (nonstrange) SU(2) sector [7–10], which suggests its extension to the SU(3) realm, even though stability of the chiral expansion is less clear for the SU(3) sector, due to the significant degree of SU(3) symmetry breaking. A well-known example of the problems facing the SU(3) chiral perturbation theory has been the prediction [11–14] of the four parity-conserving (PC) amplitudes in the weak nonleptonic decays of octet baryons, $Y \rightarrow N\pi$, with $Y = \Lambda, \Sigma$, or Ξ . In particular, Refs. [14] and [15] studied the contributions

from negative-parity intermediate states and demonstrated their potential to resolve this long-standing issue. For our purpose, these higher-order effects are beyond the simple lowest-order analysis considered here.

The EFT approach is based on the existence of well-separated scales in the physical process under study. Formally, the high-momentum (short-distance) modes in the four-baryon interaction Lagrangian are replaced by contact operators, compatible with the underlying physical symmetries, of increasing dimension. Built in such a way, the Lagrangian will contain an infinite number of terms, and a consistent power-counting scheme is needed in order to truncate such an expansion to a given order. The coefficients appearing in the Lagrangian are then fitted to reproduce the available data in the low-energy regime. Whether an EFT will succeed to describe a particular process or not is directly related to the success in obtaining a controlled, systematic expansion in terms of a small parameter. In contrast to the NN case, however, the $\Lambda N \rightarrow NN$ transition corresponds to an energy release ≈ 177 MeV ($|\mathbf{p}| \approx 417$ MeV) at threshold. It is therefore not at all clear if low-energy expansions can be successfully carried out. In light of this energy release at threshold, it is reasonable to include the pion ($m_\pi \approx 138$ MeV) and the kaon ($m_K \approx 494$ MeV) as dynamical fields. Working within SU(3) also supports treating the pion and kaon on equal footing. The last member of the SU(3) Goldstone boson octet, the η , is usually not included, since the strong ηNN coupling is an order of magnitude smaller than the strong πNN and $K\Lambda N$ couplings [16]. Thus, following a power-counting scheme based on the engineering dimensions of the operators, at leading order (order unity in the external momentum, p^0) the present study of the weak $\Lambda N \rightarrow NN$ transition includes the contribution of the long-ranged pion and kaon exchanges, and a short-range contribution given by leading nonderivative contact terms (leading-order parity-conserving, LO PC, terms). Our study also includes next-to-leading-order (NLO) terms in the momentum expansion (or equivalently, leading-order parity-violating pieces, LO PV). Not considered here are contributions from intermediate-range 2π exchange. Such a

TABLE I. $\Lambda N \rightarrow NN$ partial waves.

Partial wave	Operator	Order	I
$^1S_0 \rightarrow ^1S_0$	$\hat{1}, \vec{\sigma}_1 \vec{\sigma}_2$	1	1
$^1S_0 \rightarrow ^3P_0$	$(\vec{\sigma}_1 - \vec{\sigma}_2) \vec{q}, (\vec{\sigma}_1 \times \vec{\sigma}_2) \vec{q}$	q/M_N	1
$^3S_1 \rightarrow ^3S_1$	$\hat{1}, \vec{\sigma}_1 \vec{\sigma}_2$	1	0
$^3S_1 \rightarrow ^1P_1$	$(\vec{\sigma}_1 - \vec{\sigma}_2) \vec{q}, (\vec{\sigma}_1 \times \vec{\sigma}_2) \vec{q}$	q/M_N	0
$^3S_1 \rightarrow ^3P_1$	$(\vec{\sigma}_1 + \vec{\sigma}_2) \vec{q}$	q/M_N	1
$^3S_1 \rightarrow ^3D_1$	$(\vec{\sigma}_1 \times \vec{q})(\vec{\sigma}_2 \times \vec{q})$	$(q/M_N)^2$	0

component is two orders higher in the chiral expansion than the corresponding single-pion exchange piece. Also, previous studies [17] found such contributions to be small due to significant cancellations between the correlated and uncorrelated 2π exchanges.

The weak and strong Lagrangians for pion and kaon exchanges are the same as in Ref. [18], and we use experimental values for the couplings at the strong ($g_{NN\pi}=13.16$) and weak ($\Lambda N\pi=1.05$ for the parity-violating amplitude and -7.15 for the parity-conserving one, in units of $G_F m_\pi^2 = 2.21 \times 10^{-7}$) baryon-baryon-pion vertices. Like the $NN\pi$ coupling, the ΛNK and ΣNK coupling constants represent a fundamental input into our calculation. Unlike the $NN\pi$ coupling, however, their values are considerably less well known, with $g_{\Lambda NK}=13-17$ and $g_{\Sigma NK}=3-6$. Here, we choose the values given by the Nijmegen Soft-Core NSC97f interaction model [19], $g_{\Lambda NK}=-17.66$ and $g_{\Sigma NK}=-5.38$. Of course, the weak nucleon-nucleon-kaon coupling constants are not accessible experimentally, so we obtain numerical values by making use of SU(3) and chiral algebra considerations [3,4]. In addition, those potentials will be regularized by using monopole form factors at each vertex [20]. We note that all the strong model-dependent ingredients used in the present calculation (such as cut-off parameters or strong couplings) have been taken from the NSC97f model [19].

If no model is assumed, the low-energy $\Lambda N \rightarrow NN$ process can be parametrized through the six partial waves listed in Table I [21]. The $^1S_0 \rightarrow ^1S_0$ and $^3S_1 \rightarrow ^3S_1$ transitions can only be produced by the $\hat{1} \cdot \delta^3(\vec{r})$ and $\vec{\sigma}_1 \cdot \vec{\sigma}_2 \cdot \delta^3(\vec{r})$ operators, where $\delta^3(\vec{r})$ represents the contact interaction. The $^1S_0 \rightarrow ^3P_0$ and $^3S_1 \rightarrow ^1P_1$ transitions proceed through the combination of the spin-nonconserving operators: $(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \{\vec{p}_1 - \vec{p}_2, \delta^3(\vec{r})\}$, $(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot [\vec{p}_1 - \vec{p}_2, \delta^3(\vec{r})]$, $i(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \{\vec{p}_1 - \vec{p}_2, \delta^3(\vec{r})\}$, and $i(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot [\vec{p}_1 - \vec{p}_2, \delta^3(\vec{r})]$, where \vec{p}_i is the derivative operator acting on the “*i*th” particle [22]. The $^3S_1 \rightarrow ^3P_1$ transition is allowed by the combination of the spin-conserving operators $(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \{\vec{p}_1 - \vec{p}_2, \delta^3(\vec{r})\}$ and $(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot [\vec{p}_1 - \vec{p}_2, \delta^3(\vec{r})]$, while only two-derivative operators can produce the last (tensor) transition.

Using power counting we can discard operators of order q^2/M_N^2 , where the momentum transferred is defined by $\vec{q} = \vec{p}_\Lambda - \vec{p}_{N_1} = \vec{p}_{N_2} - \vec{p}_N$. The remaining (lowest-order) operators lead to the following four-fermion interaction in \vec{r} space (in units of $G_F = 1.166 \times 10^{-11}$ MeV $^{-2}$):

$$V_{4P}(\vec{r}) = \left\{ C_S^0 + C_S^1 \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \frac{2r}{\delta^2} \left[C_P^0 \frac{\vec{\sigma}_1 \cdot \hat{r}}{2\tilde{M}} + C_P^1 \frac{\vec{\sigma}_2 \cdot \hat{r}}{2M} + C_P^2 \frac{(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \hat{r}}{2\tilde{M}} \right] \right\} \times \frac{e^{-r^2/\delta^2}}{\delta^3 \pi^{3/2}} [C_{IS} \hat{1} + C_{IV} \vec{\tau}_1 \cdot \vec{\tau}_2], \quad (1)$$

where the last factor represents the $\Delta I=1/2$ isospin part of the $4P$ interaction. Note that the $\delta^3(\vec{r})$ functions have been smeared by using a Gaussian form with a typical vector-meson (ρ) range, $\delta \sim \sqrt{2} m_\rho^{-1} \approx 0.36$ fm. Here, $\tilde{M} = (3M + M_\Lambda)/4$ is a weighted average of N, Λ masses while C_S^j and C_P^j are the j th low-energy coefficients (LEC) at zeroth and first order, respectively. Although the form of the contact terms is model independent, the size of these LEC's depends upon how the theory is formulated, and they are expected to be of order of the other couplings in the problem. These couplings provide a very simple representation of the short-distance contributions to the process at hand. In a complete model, they would be represented by specific dynamical contributions, such as ρ, ω , etc. exchange. However, we eschew the temptation to be more specific—in fact, this generality is one of the strengths of our approach. We evaluate the coefficients purely phenomenologically and leave theoretical interpretation of the pieces to future investigations. Of course, the specific size of such coefficients depends upon the chiral order to which we are working. However, if the expansion is convergent, then the values of these effective couplings should be relatively stable as NLO or higher effects are included.

It is well known that the high momentum transferred in the $\Lambda N \rightarrow NN$ reaction makes this process sensitive to the short-range physics, which is characterized by our contact coefficients. Moreover, since the $|\Delta S|=1$ reaction takes place in a finite nucleus, the extraction of reliable information of the elementary weak two-body interaction requires a careful investigation of the many-body nuclear effects present in the hypernucleus. In the present calculation, we use a shell model for the initial hypernucleus, where the single-particle Λ and N orbits are taken to be solutions of harmonic-oscillator mean-field potentials with parameters (b_Λ and b_N) adjusted to experimental separation energies and charge form factor (respectively) of the hypernucleus under study. For $^{12}_\Lambda\text{C}$ and $^{11}_\Lambda\text{B}$, $b_N=1.64$ and $b_\Lambda=1.87$ fm, while for $^5_\Lambda\text{He}$ they take the values $b_N=1.4$ and $b_\Lambda=1.85$ fm. The strong YN interaction at short distances, absent in mean-field models, is accounted for by replacing the mean-field two-particle ΛN wave function by a correlated [23] one obtained from a microscopic finite-nucleus G -matrix calculation [24] using the soft-core and hard-core Nijmegen models [25]. The NN wave function is obtained from the Lippmann-Schwinger (T -matrix) equation with the input of the Nijmegen soft-core potential model; details of the calculation can be found in the appendix of Ref. [18], which presented a detailed study of different approaches to final-state interactions (FSI) in the decay process.

TABLE II. LEC coefficients corresponding to the LO calculation. The values in parentheses have been obtained including $\alpha({}_\Lambda^5\text{He})$ in the fit.

	+LO (PC)	+LO (PC+PV)
C_S^0	-1.54 ± 0.39	$-1.31 \pm 0.41(-1.04 \pm 0.33)$
C_S^1	-0.87 ± 0.24	$-0.70 \pm 0.35(-0.57 \pm 0.27)$
C_P^0		$-5.82 \pm 5.31(-4.49 \pm 1.57)$
C_P^1		$2.47 \pm 3.13(1.84 \pm 1.93)$
C_P^2		$-5.68 \pm 3.13(-4.47 \pm 2.31)$
C_{IS}	5.01 ± 1.26	$4.68 \pm 0.67(5.97 \pm 0.86)$
C_{IV}	1.45 ± 0.38	$1.22 \pm 0.20(1.56 \pm 0.26)$

We begin the discussion of our results with a remark on the data. One might wonder if there can be only three independent data points in the nonmesonic decay: the proton-induced and neutron-induced rates Γ_p and Γ_n , and the asymmetry \mathcal{A} (associated with the proton-induced decay), relating observables from one hypernucleus to another through hypernuclear structure coefficients. While one may indeed expect measurements from different p -shell hypernuclei, say, $A=12$ and 16 , to provide the same constraint, the situation is different when including data from s -shell hypernuclei like $A=5$. For the latter, the initial ΛN pair can only be in a relative s state, while for the former, relative p states are allowed as well. We therefore include data from the $A=5$, 11 , and 12 hypernuclei in our fits.

Note that we do not attempt to perform a final, quantitative fit to all hypernuclear data. Rather, we are exploring whether an EFT can be used to reproduce various reasonable subsets of the hypernuclear decay data in order to verify the validity of such an expansion. We note that the presently large experimental error bars in hypernuclear decay observables puts strong limitations in any EFT approach to the decay process. In this sense, the values for the parameters we are presenting have to be taken with caution. A quantitatively more rigorous understanding awaits better data, as well as a theory that includes higher-order effects. Only the more recent measurements from the last 12 years were used, excluding, however, those Γ_n/Γ_p (hereafter= n/p) ratio data whose error bars were larger than 100%. A more detailed discussion of the minimization calculation will be presented elsewhere [26,27].

We also have taken the data at face value and have not applied any corrections due to, e.g., the two-nucleon-induced mechanism, which has been estimated to amount up to 25% of the total decay rate for p -shell hypernuclei [28,29]. Ideally, exclusive experiments would separate this mechanism from the measured total decay rate, permitting a fit to observables that are not contaminated with multinucleon effects. At the present time, given the sizable error bars of the data, this omission would not change our conclusions.

No parameters were fitted for the results with only π and K exchange, shown in Table III. As has been known for a long time, π exchange alone reasonably well describes the observed total rates, while dramatically underestimating the n/p ratio. The tensor PC channel dominates the proton-

induced rate while it is absent in the $L=0$ neutron-induced one. Incorporation of kaon exchange yields a destructive interference between both mechanisms (OPE and OKE) in the PC amplitudes, while the interference is constructive in their PV counterparts. As a consequence, n/p is enhanced by about a factor of 5, within reach of the lower bounds of the experimental measurements, while the total rate underpredicts the observed value by about a factor of 2. This π - K interference also leads to values for the asymmetry that are close to experiment for the p -shell hypernuclei, but far off for $A=5$.¹ Since the contributions of both η exchange and two-pion exchange are negligible, these discrepancies illustrate the need for short-range physics.

Allowing contact terms of order unity (leading-order PC operators) to contribute leads to four free parameters, C_S^0 , C_S^1 , C_{IS} , and C_{IV} . Data on the total and partial decay rates for all three hypernuclear systems are included in the fit, but asymmetry measurements are not. The inclusion of the contact terms roughly doubles the values for the total decay rates, thus restoring agreement with experiment. The impact on the n/p ratio is noteworthy: the value for ${}_\Lambda^5\text{He}$ increases by 10% while the n/p ratios for ${}_\Lambda^{11}\text{B}$ and ${}_\Lambda^{12}\text{C}$ almost double. This is an example of the differing impact certain operators can have for s - and p -shell hypernuclei. The effect on the asymmetry is opposite, almost no change for $A=11$ and 12 , but a 30% change for $A=5$. The magnitudes of the parameters, C_S^0 , C_S^1 , C_{IV} , listed in Table II, are each around their natural size of unity, while C_{IS} is a factor of 3 or so larger. Note the substantial error bars on all the parameters, reflecting the uncertainties in the measurements.

Three new parameters are admitted when we allow the leading-order PV terms (of order q/M_N) to contribute with the coefficients C_P^0 , C_P^1 , and C_P^2 . As shown in Table II, the parameters for the PV contact terms are larger than the ones for the PC terms, and in fact, compatible with zero. Including the three new parameters does not substantially alter the previously fitted ones, thus supporting the validity of our expansion. Regarding their impact on the observables, the PV contact terms barely modify the total and partial rates but significantly affect the asymmetry, as one would expect for an observable defined by the interference between PV and PC amplitudes. The calculated asymmetry considerably decreases in size for all three hypernuclei, giving p -shell values close to zero, but still negative. In order to further understand this behavior, we have performed a number of fits including the asymmetry data points of either ${}_\Lambda^5\text{He}$ or ${}_\Lambda^{11}\text{B}$ or both. Tables II and III display the result of one of those fits. Inclusion of the ${}_\Lambda^5\text{He}$ asymmetry helps in constraining the values of two of the LO PV parameters. We find that the two present experimental values for $A=5$ and $A=11$ cannot be fitted simultaneously with this set of contact terms. Future experiments will have to settle this issue.

¹Note that for ${}_\Lambda^5\text{He}$ we quote the value of the intrinsic Λ asymmetry parameter, a_Λ , which is experimentally accessible, while for p -shell hypernuclei the accessible quantity is \mathcal{A} , the difference between the number of protons coming parallel and antiparallel with respect to the polarization axis. This quantity can be related to a_Λ through the relation $\mathcal{A}=p_\Lambda a_\Lambda$, where p_Λ is the Λ polarization, to be extracted from theoretical models.

TABLE III. Results obtained for the weak decay observables, when a fit to the Γ and n/p for ${}^5_{\Lambda}\text{He}$, ${}^{11}_{\Lambda}\text{B}$, and ${}^{12}_{\Lambda}\text{C}$ is performed. The values in parentheses have been obtained including $\alpha_{\Lambda}({}^5_{\Lambda}\text{He})$ in the fit.

	π	+K	+LO PC	+LO PC+PV	EXP:
$\Gamma({}^5_{\Lambda}\text{He})$	0.42	0.23	0.43	0.44 (0.44)	0.41 ± 0.14 [30], 0.50 ± 0.07 [31]
$n/p({}^5_{\Lambda}\text{He})$	0.09	0.50	0.56	0.55 (0.55)	0.93 ± 0.55 [30], 0.50 ± 0.10 [32]
$\alpha_{\Lambda}({}^5_{\Lambda}\text{He})$	-0.25	-0.60	-0.80	-0.33 (0.24)	0.24 ± 0.22 [33]
$\Gamma({}^{11}_{\Lambda}\text{B})$	0.62	0.36	0.87	0.88 (0.88)	0.95 ± 0.14 [31]
$n/p({}^{11}_{\Lambda}\text{B})$	0.10	0.43	0.84	0.92 (0.92)	$1.04^{+0.59}_{-0.48}$ [30]
$\mathcal{A}({}^{11}_{\Lambda}\text{B})$	-0.09	-0.22	-0.22	-0.02 (0.08)	-0.20 ± 0.10 [34]
$\Gamma({}^{12}_{\Lambda}\text{C})$	0.74	0.41	0.95	0.93 (0.93)	1.14 ± 0.2 [30], 0.89 ± 0.15 [31], 0.83 ± 0.11 [35]
$n/p({}^{12}_{\Lambda}\text{C})$	0.08	0.35	0.67	0.77 (0.77)	0.87 ± 0.23 [36]
$\mathcal{A}({}^{12}_{\Lambda}\text{C})$	-0.03	-0.06	-0.05	-0.01 (0.02)	-0.01 ± 0.10 [34]
χ^2			0.93	1.54 (1.15)	

We have also performed fits allowing a contribution from an isospin $\Delta I=3/2$ transition operator. The resulting fit, with $\chi^2 \sim 1.4$, shifts strength from the isoscalar contribution to the new $\Delta I=3/2$ one, leaving the other parameters unchanged. However, as shown in Table II, we can clearly get good fit to all observables without such transitions while obtaining couplings of reasonable size. In addition, we have checked that our conclusions are independent of the strong interaction model used to describe FSI in the transition. Employing NN wave functions that are obtained with either the NSC97f or the NSC97a models in the fit leaves the observables almost unchanged, with the exception of the asymmetry parameter, which can change up to 50%. The obtained couplings can easily absorb the changes but remain compatible within their error bars. Similarly, we performed a study of the sensitivity of the calculated observables to the smearing function in Eq. (1). For values of δ going from 0.3 to 0.4 fm, the results are remarkably insensitive, except again in the case of the asymmetry.

In conclusion, we have studied the nonmesonic weak decay using an effective-field theory framework for the weak interaction. The long-range components were described with

pion and kaon exchange, while the short-range part is parametrized in leading-order PV and PC contact terms. We find coefficients of natural size with significant error bars, reflecting the level of experimental uncertainty. The largest contact term corresponds to an isoscalar, spin-independent central operator. There is no indication of any contact terms violating the $\Delta I=1/2$ rule. In this study we have not speculated as to the dynamical origin of these contact contributions. Rather, our aim was to ascertain their basic magnitude and to establish the validity of the EFT framework for the weak decay. The next generation of data from recent high-precision weak decay experiments currently under analysis holds the promise to provide much improved constraints for studies of this nature.

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