Endogenous Technology and Climate Change

Antonio Haro Bañón

University of Barcelona
MSc in Economics Thesis

Abstract

This paper studies the implications of the event of climate change for an economy that is characterized by an endogenous growth framework. The technological change is driven by the accumulation of knowledge that increases the efficiency of a renewable energy sector, leading to a reduction of the alternate fossil fuel energy use. A decentralized equilibrium is defined, and by comparing with the social planner solution an optimal carbon tax is obtained. This environmental policy accounts for the externality imposed by a rising temperature that is the result of the generation of pollution from the fossil fuel consumption, and that damages the output of the overall economy.

Keywords: Climate change, Energy, Endogenous technological change, Carbon Tax.
JEL Classification: O33, H23, Q54, Q58

NIUB: 15355572

Advisors: Xavier Raurich Puigdevall & Fernando Sánchez Losada

Deadline: 20^{th} June, 2017

UNIVERSITAT DE BARCELONA
SCHOOL OF ECONOMICS

1 Introduction

Global warming has been a main topic of discussion and is in the spotlight of international policy. It has been called the greatest externality for mankind (Stern, 2007). The long-term scope of this event and consequences support their analysis through models that fall under this perspective. The approach taken so far has followed two strands with regard to the driver of technological change. The main purpose of this paper is to continue with the path marked by the endogenous growth literature by providing a simple general equilibrium framework in which to analyze the effects of climate change on a economy with two sources of energy whose use affect in different ways the environment; and to obtain the optimal policy that leads to the maximum welfare.

The treatment and analysis of climate change took a turning point after the publication of the Stern Review (Stern, 2007) and the spread of integrated models linking economic and climate dynamics with the popular example of the Dynamic Integrated model of Climate and the Economy (DICE model) by Nordhaus (2008). Particularly, this model attempts to determine an efficient strategy for coping with the threat of global warming by using the tools of modern economics. The argument behind this study is that societies should undertake environmental policies only when cost-benefit analysis yield an excess of their benefits over their costs and that the level of environmental control should be at that point where the incremental benefits of additional controls no longer exceed the incremental costs. All these models asked for a global price of carbon, either through carbon taxes or global market for tradable CO2 permits (Golosov et al, 2014).

There is a growing evidence that environmental policy influences the direction of technological change, however few climate change models consider to feature this link directly. The literature that deals with this issue includes innovation typically one of two ways. One of the is the so-called "bottom-up" models, which include a detailed specification of energy systems, usually both traditional fossil fuels and alternative energy technologies. However, they normally do not include a modelization of the overall economy, and consider the technological change driven by a learning-by-doing framework, in which the costs of various technologies decrease with experience. Some examples are Grübler and Messner (1998) and Manne and Richels (2002).

In comparison, "top-down" models focus on the links between environmental policy and macroeconomic performance by including accumulated investment in research and development (R&D) as the source of endogenous technological change. Some examples include Popp (2004), Buonanno et al. (2003) and Goulder and Schneider (1999) and the more recent Bosetti et al. (2009), Edenhofer et al. (2006), Gerlagh (2006). One of the main models in this field is the ENdogenous Technological change Integrated model of Climate and the Economy with Backstop R&D (the ENTICE-BR model) by Popp (2006), which is a modified version of the DICE model that includes endogenous links between climate policy and energy innovation. In general, such models focus on comparing optimal trajectories with business-as-usual scenarios, some of them avoiding a general equilibrium analysis. Therefore, it is helpful to work with general equilibrium frameworks that enable to check different combinations of instruments, as in Grimaud et al. (2011).

In line with the "top-down" approach and based on the DICE and ENTICE-BR models, I develop an endogenous growth model in which energy services can be produced from a polluting non-renewable resource as well as a clean backstop that contributes to the production function and does not imply a pollutant by-product, nor supposes a direct mitigation for reduction of the CO2 stock. The accumulated CO2 stock increases the atmospheric CO2 and affects the climate system by rising the sensitivity of the temperature function and thus, leading to an accelerated increase in temperature.

At this point, it is important to remark the way we deal with R&D sectors in the decentralized framework. In the standard endogenous growth theory, such as Aghion and Howitt (1992) and Romer (1990), the production of an innovation is associated with a particular intermediate good. Research is funded by the monopoly profits of intermediate producers who benefit from an exclusive right, like a patent, for the production and the sale of these goods. In this paper, to simplify the analysis, I do not explicitly introduce tangible intermediate goods in research sectors, as it is done for instance by Gerlagh and Lise (2005), Edenhofer et al. (2006) and Popp (2006). Then, I adopt the shortcut proposed by Grimaud and Rougé (2008) in the case of growth models with polluting resources and environmental concerns, and consider that research is performed by some research firms that use a certain amount of labor to generate the stock of knowledge required to improve the efficiency of the sector of renewable energy.

The endogenous technological change is defined by an R&D sector that increases the overall productivity of the renewable energy. The agents of the economy choose how to allocate their spending over the inputs of fossil and renewable energy, and over the research effort to make to boost the amount of knowledge and innovation about clean energy. Furthermore, because of the externality introduced by pollution, there is a carbon tax imposed over the amount of pollution emitted by the usage of fossil fuel energy in order to disincentivate it and limit the increase in the growth rate of temperature that shall harm the economy.

The paper is structured as follows. Section 1 introduces while section 2 presents the assumptions and setup of the model. The decentralized equilibrium is characterized in section 3. Section 4 displays the social planner solution, the first-best optimum and the optimal policy is obtained. In section 5 it is discussed the existence of a steady state of the model, while section 6 concludes. An annex detailing all the optimization calculations is included as well.

2 The Dynamic Climate-Economy Model

The model is mainly based on ENTICE-BR (Popp, 2006). Consider a worldwide economy containing four production sectors: final output, energy services, fossil fuel and carbon-free backstop. The fossil fuel combustion process releases CO2 flows which accumulate into the atmosphere, inducing a rise of the average temperatures. Feedbacks on the economy are captured by a damage function measuring the continuous and gradual losses in terms of final output due to global warming. Moreover, an atmospheric carbon concentration cap can be eventually introduced to take into account the high levels of uncertainty and irreversibility that are generally avoided by the standard damage function. The production of backstop or renewable energy require specific knowledge provided by a R&D sector (in the sense of Acemoglu, 2002). We assume that all sectors are perfectly competitive. Finally, in order to correct the distortion involving pollution we introduce an environmental tax on the fossil fuel use. The following subsection derives the individual behaviors.

2.1 Behavior of the agents

2.1.1 The final and energy goods sectors

The production of a quantity Y_t of final goods depends on three endogenous elements: capital K_t, energy services E_t, and a scaling factor Ω_t that accounts for climate-related damages, as discussed below. It also depends on exogeneous inputs: the total factor productivity A_t and the population level L_Y, which are constant and grow at a rate equal to zero.

$$Y(\Omega_t, A, L, K_t, E_t) = \Omega_t A K_t^{\beta} E_t^{\gamma} L_Y^{1-\beta-\gamma} \tag{1}$$

The production function is assumed to feature the standard properties, that is, increasing and concave in each argument. It is assumed that $0 < \beta, \gamma < 1$, hence Y_t shows constant returns to scale. Normalizing to one the price of the final output and denoting by $p_{E,t}$, w_t, r_t and δ_K the price of energy services, the real wage, the interest rate[1] and the depreciation of capital, respectively, the instantenous profit of producers is expressed as $\Pi_t^Y = Y_t - p_{E,t}E_t - w_t L_Y - (r_t + \delta_K)K_t$. Maximization of this profit function with respect to K_t, L_Y and K_t, leads to the following first-order conditions[2]:

$$\frac{\partial \Pi_t^Y}{\partial K_t} = Y_K - (r_t + \delta_K) = 0 \tag{2}$$

$$\frac{\partial \Pi_t^Y}{\partial L_Y} = Y_{L_Y} - w_t = 0 \tag{3}$$

$$\frac{\partial \Pi_t^Y}{\partial E_t} = Y_E - p_{E,t} = 0 \tag{4}$$

Market clearing for labor requires labor demand, composed by the amount of workers in the final goods sector and in the research sector, to be less than total labor supply, which is normalized to 1, i.e.,

$$L_{Y,t} + L_{H,t} = 1 \tag{5}$$

At each time t, the amount E_t of energy services is produced from two primary energies: a fossil fuel F_t and a backstop energy source B_t. The energy supply is defined by equation (6), where E(.) is increasing and concave in each argument as $0 < \phi < 1$.

$$E_t = E(F_t, B_t) = F_t^{\phi} B_t^{1-\phi} \tag{6}$$

The fossil fuel input of energy is extracted from the stock Z_t of remaining nonrenewable natural resource, which must remain nonnegative; and its rate of change is the flow F of resource extraction:

$$Z_t = Z_0 - \int_0^t F_s ds \iff \dot{Z}_t = -F_t \tag{7}$$

Its extraction is subject to a unitary cost that depends on the remaining stock. This unitary cost is non-decreasing, and tends to infinity as the stock of resources is depleted; and is defined as:

$$c(Z_t) = \frac{1}{Z_t} \tag{8}$$

[1] Assume that the representative household holds the capital and rents it to firms at a rental price R_t. Standard arbitrage conditions imply $R_t = r_t + \delta$.

[2] For the sake of clarity and without loss of generality, J_x represents the partial derivative of a function J with respect to variable x.

Denoting by $p_{F,t}$ and $p_{B,t}$ the fossil fuel and renewables prices, and by τ the unitary carbon tax on the flow of carbon emissions ψF_t, the energy producer chooses F_t and B_t that maximizes its instantaneous profit $\Pi_t^E = p_{E,t}E_t - p_{F,t}F_t - p_{B,t}B_t - \tau\psi F_t$. The first-order conditions are:

$$\frac{\partial \Pi_t^E}{\partial F_t} = p_{E,t}E_F - p_{F,t} - \tau\psi = 0 \tag{9}$$

$$\frac{\partial \Pi_t^E}{\partial B_t} = p_{E,t}E_B - p_{B,t} = 0 \tag{10}$$

Because the fossil fuel extraction is bound to a certain amount of resource, the profits arising from this activity are to be maximized in the following way: $\Pi_t^F = p_{F,t}F_t - \frac{1}{Z_t}F_t$. The first-order condition of this maximization is $\partial\Pi_t^F/\partial F_t = p_{F,t} - 1/Z_t = 0$, hence the price of the fossil fuel reflects the negative impact of the depletion of the resource in such a way that, as more of it is consumed, its cost and therefore its price increases and it becomes more costly to continue the activity.

2.1.2 The renewable energy and R&D sectors

The renewable or backstop energy production function $B(.)$ is defined by equation (11), which is increasing and concave in the specific spending $I_{B,t}$ and in the stock of knowledge H_t[3].

$$B(I_{B,t}, H_t) = a_B I_{B,t}^{\eta} H_t^{1-\eta} \tag{11}$$

where $I_{B,t}$ is the instantaneous investment in renewables given by the following equation, which expresses that it is to be equal to the variation with respect to the previous period and the depreciated loss change:

$$I_{B,t} = \dot{I_B} + \delta_B I_{B,t} \tag{12}$$

The efficiency in the industry is determined by the amount of knowledge stock, which is accumulated according to the innovation function defined by equation (13). It depends on the existing stock of efficiency knowledge in the sector and the effort exert by the share of labor $L_H = 1 - L_Y$. The parameter a_H reflects both the size and likelihood of a new innovation that increases the productivity of the sector.

$$\dot{H} = H(L_H, H_t) = a_H H_t L_H = a_H H_t (1 - L_Y) \tag{13}$$

At each time t, the flow of profits of the research firms that generate this stock of knowledge can be expressed by the function $\Pi_t^H = p_{H,t}H_t - w_t L_H$. Profits are obtained by providing the stock of efficiency to the renewables industry at price $p_{H,t}$ net of the labor cost, and the maximization is performed over current and future profits, i.e., the present value of the returns,

[3] This production function can be interpreted as consumig intermediate goods to produce the backstop energy. In such a way alike to the Romer model of intermediate goods, the output function can be transformed to $B_t = a_B \int_0^1 x(i)^{\eta} H(i) di$, being $H(i)$ an efficiency-related parameter that measures the quality of the current productivity in each of the intermediate inputs production.Hence, each intermediate good is produced according to the constant returns function $x(i) = I_B(i)/H(i)$ that depends on the amount of capital needed to produce each input $I_{B,t}$, and on the efficiency stock $H(i)$. The maximization of the backstop production function solves to obtain a Cobb-Douglas specification.

as current production of knowledge determines the amount of the next period. Hence, the maximization by the producers is of the form:

$$\max_{L_H, H_t} \int_t^{\infty} (p_{H,s} H_s - w_s L_H) e^{-r(s-t)} ds \quad s.t. \quad \dot{H} = a_H H_t L_H \tag{14}$$

By solving this problem it can obtained that the optimal price is $p_{H,t} = (\rho w_t - \dot{w}_t)/a_H H_t$. Substituting por $p_{H,s}$ into the value function $V_t = \int_t^{\infty} \Pi_t^H e^{-r(s-t)} ds$ we get the inventor's net present value of profit at time t. Differentiating with respect to time V_t we get

$$r_t = \frac{\dot{V}_t + \Pi_t^H}{V_t}$$

which says that the rate of return to the investment in capital K_t and in renewables $I_{B,t}$, r_t, equals the rate of return to investing in R&D. The R&D rate of return equals the profit rate, Π_t^H / V_t, plus the rate of capital gain or loss derived from the change in the value of the research firm, $\dot{V}_t / V_t$.

In the market equilibrium the renewable energy producer chooses the amount in which to invest $I_{B,t}$ and the amount H_t to acquire from the research firms that maximizes its instantaneous profit $\Pi_t^B = p_{B,t} B_t - (r_t + \delta_{I_B}) I_{B,t} - p_{H,t} H_t$. The first-order conditions are:

$$\frac{\partial \Pi_t^B}{\partial I_{B,t}} = p_{B,t} B_{I_B} - (r_t + \delta_{I_B}) = 0 \tag{15}$$

$$\frac{\partial \Pi_t^B}{\partial H_t} = p_{B,t} B_H - p_{H,t} = 0 \tag{16}$$

2.1.3 Households

The balance equation of the final output writes in such a way that the final output is used for aggregate consumption, fossil fuel use and renewables production, and capital accumulation:

$$Y_t = C_t + I_{B,t} + I_{K,t} + c(Z_t) F_t \tag{17}$$

where $I_{K,t}$ is the instantaneous investment in capital given by:

$$I_{K,t} = \dot{K}_t + \delta_K K_t \tag{18}$$

Denoting by C_t the consumption at time t, by $U(.)$ the instantaneous utility function and by ρ the pure rate of time preferences, the welfare function to maximize is $W = \int_0^{\infty} U(C_t) e^{-\rho t} dt$. It is assumed that the utility function fulfills the standard properties and the Inada conditions by $U(.)$ adopting a logarithmic form. The household maximizes this function subject to the dynamic budget constraint given by (19).

$$\dot{I_{B,t}} + \dot{K}_t = r_t (K_t + I_{B,t}) + w_t L_t + (\Pi_t^Y + \Pi_t^E + \Pi_t^F + \Pi_t^B) + r_t V_t - C_t \tag{19}$$

This budget constraint expresses that the change in total assets, i.e., both investments in capital K_t and in renewables $I_{B,t}$, is constrained by the amount the household can contribute from the perceived returns to investment, the returns from labor, the profits arising from each production sector and the value change in the research firm[4], after consuming the final goods.

[4]Note that $r_t V_t = \dot{V}_t + \Pi_t^H$

2.2 The environment and damages

The climate dynamics are captured by equation (20). The variation in temperature depends direct, positively on the extraction of fossil fuel. Hence, in this model it has been assumed that the mechanisms by which fossil fuel extraction generates pollution that increases the stock of atmospheric CO_2, which ultimately boosts a rising temperature, are captured by the equation (20).

$$\dot{T} = \psi F_t - mT_t \tag{20}$$

Parameter ψ captures not only the amount of fossil fuel use that is transformed into pollution, but it can also be interpreted as the sensitivity by which pollution affects the variations in global mean temperatures. For instance, a larger ψ would imply that a given pollution level would generate larger changes in temperature, reflecting identified features such as the mechanisms by which global warming contributes to the melting of ice sheets and the release of methane stored within the permafrost, further increasing the mean temperature, as studied in Archer (2007) and Schaefer et al. (2011). So as for parameter m, it can be interpreted as the natural regeneration factor of the environment that contributes to stabilize temperature to a certain degree.

The damages from global warming are captured by equation (21). It measures the final output losses when the global mean temperature increases. The function is decreasing and concave in T_t and captures the incremental and non-catastrophic economic damages incurred throughout the entire optimization time frame.

$$\Omega_t = 2 - exp^{(T_t - T_0)} \tag{21}$$

3 Decentralized equilibrium and welfare analysis

3.1 Characterization of the decentralized equilibrium

From the previous analysis of individual behaviors, we can now study the set of equilibria. A particular equilibrium is defined as a vector of quantity trajectories $\{Y_t, K_t, E_t, ...\}_{t=0}^{\infty}$ and a vector of price profiles $\{r_t, p_{E,t}, ...\}_{t=0}^{\infty}$ such that: i) firms maximize profits, ii) the representative household maximizes utility, iii) markets are perfectly competitive and cleared. Such an equilibrium is characterized by the set of equations given by Proposition 1 below. Clearly, as analyzed in the following subsection, if the policy tool is set to its optimal levels, these equations also characterize the first-best optimum together with the system of prices that implements it.

Proposition 1. *At each time t, the equilibrium in the decentralized economy is characterized by the following equation system, which has the associated system of prices* $\{r_t, w_t, p_{E,t}, p_{F,t}, p_{B,t}, p_{H,t}\}_{t=0}^{\infty}$ *obtained from equations* (2), (3), (4), (9), (15) *and* (16), *respectively for a given policy tool* τ*:*

$$Y_K - \delta_K = \frac{\dot{C}_t}{C_t} + \rho \tag{22}$$

$$Y_E E_B B_{I_B} - \delta_{Ib} = r_t \tag{23}$$

$$\tau_t = \frac{1}{\psi}\left(Y_E E_F - \frac{1}{Z_t}\right) \tag{24}$$

$$\frac{\dot{Y_{L_Y}}}{Y_{L_Y}} = \rho - \frac{Y_H}{Y_{L_Y}} a_H L_H \tag{25}$$

Proof. See Appendix 8.2. □

The first equation characterizes the standard trade-off between capital K_t and consumption C_t, while the second one links this relation with the net marginal productivity of the investment in renewable energy. The third equation relates the carbon tax with the net marginal benefit from using fossil fuel resources. Finally, the last equation characterizes the trade-off between the amount of work dedicated to final goods and the one assigned to the generation of the stock of renewable energy knowledge.

3.2 Mazimization form of the decentralized equilibrium

In order to solve numerically the market outcome, it is possible to transform the decentralized problem described above into a single maximization program. The results can be read in fact as the welfare maximization program of a representative agent who would own all firms (final sector, energy, fossil fuel, renewables and R&D) and who would face the same incentive policies (carbon tax) than firms in the decentralized economy. This approach is the same than the one followed by Sinclair (1994) who also writes the market equilibrium under maximization form. The main difference is that he assumes an exogenous rate of Hicks-neutral technical change.

Proposition 2. *The maximization of utility $\int_0^\infty ln(C_t)e^{-\rho t}dt$ with respect to the control variables $C_t, L_{Y,t}, F_t, b$ subject to the following state equations leads to the same system of equations as in Proposition 1:*

$$\dot{K}_t = Y(\Omega, K_t, L_Y, E_t) - C_t - \delta_K K_t - \delta_{I_B} I_{B,t} - b - c(Z_t)F_t - \tau\psi F_t \tag{26}$$

$$\dot{I_{B,t}} = b \tag{27}$$

$$\dot{H}_t = a_H H_t L_{H,t} \tag{28}$$

$$\dot{Z}_t = -F_t \tag{29}$$

Proof. See Appendix 8.3. □

The first state equation expresses the dynamic constraint of stock of capital and renewables investment, whose variation is composed by the amount of output left after consumption, the depreciation losses of capital and renewables investment, and total cost of fossil fuel use, i.e., the cost of extraction plus the tax imposed on pollution emission. Second state equation is introduced in order to solve the system of equations. Third and fourth equations are the dynamics on efficiency knowledge generation and the natural resource extraction, respectively.

4 First-best optimum and implementation

This section characterizes the optimum solution by the social planner, which consists in choosing $\{C_t, L_Y, F_t, b\}_{t=0}^\infty$ that maximizes the social welfare W, subject to the energy sectors production and technological constraints, the output allocation constraint (17), the state equations (7), (12), (13), (18) and (20). The details of the optimization are written in annex 8.4. The Hamiltonian of the program is:

$$\begin{aligned}\mathcal{H} = ln(C_t) + \lambda_K[Y(\Omega, K_t, L_Y, E_t) - C_t - \delta_K K_t - \delta_{I_B} I_{B,t} - b - c(Z_t)F_t] + \\ + \lambda_{I_B} b + \lambda_H[a_H H_t(1 - L_Y)] + \lambda_Z(-F_t) + \lambda_T(\psi F_t - mT_t)\end{aligned} \tag{30}$$

Proposition 3. *At each time t, an optimal solution is characterized by the following system of equations:*

$$Y_K - \delta_K = \frac{\dot{C}_t}{C_t} + \rho \tag{31}$$

$$Y_E E_B B_{I_B} - \delta_{Ib} = r_t \tag{32}$$

$$Y_F - \frac{1}{Z_t} = \frac{\psi}{r_t}(mC_t\lambda_T - Y_T) \tag{33}$$

$$\frac{\dot{Y_{L_Y}}}{Y_{L_Y}} = \rho - \frac{Y_H}{Y_{L_Y}} a_H L_H \tag{34}$$

Proof. See Appendix 8.4. □

The interpretation of these conditions is quite alike to that of the characterizing conditions of the decentralized equilibrium, however these are optimally solved for a carbon tax policy that considers the externality introduced by the damages from a change in temperature. This tax instrument will be given by the third characterizing condition of this equation system.

By comparing equations (24) and (33) it is possible to obtain the optimal carbon tax that internalizes the externality introduced by the climate change dynamic. This carbon tax will be optimal if and only if it is given by

$$\tau_t = \frac{1}{r_t}(mC_t\lambda_T - Y_T) \tag{35}$$

The optimal trajectory of the carbon tax is defined by equation (35). Since the marginal effect of the temperature on production is negative, i.e., $Y_T < 0$, it implies that $\tau_t \geq 0$ for any $t \geq 0$. This carbon tax can be expressed as the sum of two components. By solving the differential equation of λ_T[5], its solution equals the discounted sum of marginal damages from the current point to the future in terms of utility. The first component is then the ratio of this discounted amount and the marginal utility of consumption. The second term refers to the marginal impact of an increase in temperature at the moment t.

5 Discussion

Having defined and characterized the decentralized equilibrium and the optimal solution of the social planner, this section argues the existence of a steady-state point to which the economy converges. To do so, I state a relevant although quite restricting assumption:

Assumption 1. *Agents take the values of r_t and w_t as given, such that $r_t = r^*$ and $w_t = w^*$.*

By taking this assumption it is possible to discuss that the growth rates of the variables $Y_t, K_t, L_{Y,t}$ and $I_{B,t}$ is zero and so there exists a steady-state solution.

First, from the Euler condition, as the interest cannot be changed the marginal productivity of the investments in both capital and renewables must be constant as well, i.e., $\beta Y/K = \gamma\eta(1-\phi)Y/I_B = r^*$. By log-differentiating with respect to time, this implies that the production of the final goods and capital and investment in renewables must vary at the same rate, i.e., $\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{I_B}}{I_B}$. Furthermore, the growth rate of consumption should hold constant as it is implied

[5] Solution of such differential equation is provided in Annex 8.4.

from the constant marginal productivity of capital.

Second, the returns to labor are taken as given as well by the agents, and it is assumed that the wages are the same for both workers of final goods production and research sector. This consideration implies two features: first, that the marginal productivity of the workers in the final goods sector shall remain constant, so that considering the specification given by equation (1), production and the amount of workers in this sector must vary at the same rate, i.e., $\frac{\dot{Y}_t}{Y_t} = \frac{\dot{L_Y}}{L_Y}$. Second, considering the maximization of the profits by the research firms given by equation (14) it resulted in the equilibrium price $p_{H,t} = \frac{\rho w_t - \dot{w}_t}{a_H H_t}$. Then solving for a constant wage w^* it is obtained that the price of selling the efficiency input to the renewables is decreasing with the growth rate of knowledge accumulation: $\frac{\dot{p_{H,t}}}{p_{H,t}} = \rho - \frac{\dot{H}_t}{H_t} = \rho - a_H L_H$.

This result implies the following. Each period some amount of knowledge is accumulated and sold by the research firms, which is used to increase the production of renewable energy, contributing as well to an increase in the production of final goods, which uses it as an input. This increase in final output rises the marginal productivity of labor in this sector, and more workers are hired by the final goods industry out of the research sector. Because the amount of labor that is dedicated to accumulate knowledge decreases, the efficiency stock grows at a lower rate, which leads the research firms to rise the price at which it is sold to the renewables sector. In view of the fact that less knowledge stock is generated, output grows at a lower rate, and workers are relocated to the research sector in order to keep wages constant, and the previous shift is compensated. Henceforth, there exists a certain amount of workers L_Y^* that can be obtained from equation (50) that is kept constant and that makes output grow at a rate of zero to keep productivities constant.

Therefore, not only do output and workers hold a constant level at the steady state, but so do capital K_t and I_B due to a constant interest rate. By log-differentiating with respect to time equation (1), and substituting the growth rates of K_t, $L_{Y,t}$ and $I_{B,t}$ with zero, the growth rate of final output can be expressed as follows. Equation (36) expresses that the variations in the damage function resulting from changes in temperature are to be compensated by the use of fossil fuel as an energy source, and the accumulation of knowledge regarding the renewables sector:

$$\frac{\dot{Y}}{Y} = \frac{1}{\gamma(1-\eta(1-\phi))}\left[\frac{\dot{\Omega}}{\Omega} + \phi\gamma\frac{\dot{F}}{F} + \gamma(1-\phi)(1-\eta)\frac{\dot{H}}{H}\right] \tag{36}$$

Because of the previous discussion, then the amount of labor dedicated to research L_H^* should hold constant as well, so equation (36) can be transformed, using the decentralized equilibrium framework, as:

$$\frac{\dot{\Omega}}{\Omega} + \phi\gamma\left[\frac{r^* F_t}{\phi\gamma Y^*}\left(\frac{1}{Z_t} + \tau\psi\right) - r^*\right] + \gamma(1-\phi)(1-\eta)a_H L_H^* = 0 \tag{37}$$

The decentralized equilibrium implies that $\psi\tau = Y_F - 1/Z_t$, as given by the third characterizing condition from Proposition 1. As a result, equation (37) can be solved as:

$$\gamma(1-\phi)(1-\eta)a_H L_H^* = \frac{e^{T_t - T_0}}{2 - e^{T_t - T_0}}(\psi F_t - mT_t) \tag{38}$$

This expression implies that the growth rate of the stock of knowledge (left-hand side of the equation) must be set in such a way that it compensates the marginal increase in the damage function (right-hand side) that result from the use of a pollutant resource.

6 Concluding Remarks

In this article I introduced endogenous technical change in a growth model with environmental constraints and limited resources. The analysis primarily consisted in decentralizing the "top-down" ENTICE-BR model (Popp, 2006) in order to characterize the full set of equilibria. First, I provided a characterization of this set of equilibria (Proposition 1). Second, I showed that we can obtain any decentralized equilibrium as the solution of a maximization program (Proposition 2). Third, I characterized the first-best optimum (Proposition 3) and I showed that there exists an optimal carbon tax that accounts for the the externality imposed by the climate change. This optimal environmental policy considers all prospect marginal damages adjusted for the marginal utility. Finally, under certain restricting assumptions, there exists a steady-state equilibrium in which the efficiency knowledge in the renewables sector must grow in such a way that it compensates the growth rate of the damages from climatic change.

A prospect simulation of the model should yield a time path of the optimal carbon tax that is generally non-monotonic over time and follows an inverted U-shaped time-path, which should be in line with Goulder and Mathai (2000) and Ulph and Ulph (1994). In addition, a comparison between the business-as-usual and the optimal policy scenarios should provide a graphical illustration of the responses and behavior of the agents of the model, together with a sensitivity analysis for the initial valus and parameters.

Future research can be undertaken in several directions. First, because of the likely pressence of externalities from the R&D sector involving the mismatch between the private and the social value of innovation or of the research effort, an environmental policy that adds research subsidies to the policy set should reflect reality in a more accurate way. Second, it would be useful to to analyze the impact of abrupt climate changes in presence of endogenous technological change, that is, the relevance that investing in R&D can acquire if there exists some temperature threshold beyond which the damages from climate change are irreversible. Third, a multicountry model with endogenous technology and environmental constraints, which can be used to discuss issues of global policy coordination and the degree to which international trade should be linked to environmental policies. Finally, an interesting question would be to incorporate environmental risk into this framework, such as considering the ex-ante uncertainty on future damage costs to production.

7 References

1. Acemoglu, D. (2002). Directed technical change. *The Review of Economic Studies*, 69(4), 781-809.
2. Aghion, P., & Howitt, P. (1992). A model of growth through creative destruction. *Econometrica*, 60(2), 323-351. Archer, D. (2007). Methane hydrate stability and anthropogenic climate change. *Biogeosciences Discussions*, 4(2), 993-1057.
3. Bosetti, V., Carraro, C., & Galeotti, M. (2006). The dynamics of carbon and energy intensity in a model of endogenous technical change. *The Energy Journal*, 27, 191-206.
4. Buonanno, P., Carraro, C., & Galeotti, M. (2003). Endogenous induced technical change and the costs of Kyoto. *Resource and Energy economics*, 25(1), 11-34.
5. Edenhofer, O., Lessmann, K., & Bauer, N. (2006). Mitigation Strategies and Costs of Climate Protection: The Effects of ETC in the Hybrid Model MIND. *The Energy Journal*, 207-222.
6. Gerlagh, R. (2006). ITC in a global growth-climate model with CCS: The value of induced technical change for climate stabilization. *The Energy Journal*, 223-240.
7. Gerlagh, R., & Lise, W. (2005). Carbon taxes: a drop in the ocean, or a drop that erodes the stone? The effect of carbon taxes on technological change. *Ecological Economics*, 54(2), 241-260.
8. Golosov, M., Hassler, J., Krusell, P., & Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1), 41-88.
9. Goulder, L. H., & Mathai, K. (2000). Optimal CO2 abatement in the presence of induced technological change. *Journal of Environmental Economics and management*, 39(1), 1-38.
10. Goulder, L. H., & Schneider, S. H. (1999). Induced technological change and the attractiveness of CO 2 abatement policies. *Resource and energy economics*, 21(3), 211-253.
11. Grimaud, A., Lafforgue, G., & Magné, B. (2011). Climate change mitigation options and directed technical change: A decentralized equilibrium analysis. *Resource and Energy Economics*, 33(4), 938-962.
12. Grimaud, A., & Rouge, L. (2008). Environment, directed technical change and economic policy. *Environmental and Resource Economics*, 41(4), 439-463.
13. Grübler, A., & Messner, S. (1998). Technological change and the timing of mitigation measures. *Energy economics*, 20(5), 495-512.
14. Manne, A. S., & Richels, R. G. (2005). MERGE: an integrated assessment model for global climate change. *Energy and environment* (pp. 175-189). Springer US.
15. Nordhaus, W. D. (1992). The'dice'model: Background and structure of a dynamic integrated climate-economy model of the economics of global warming (No. 1009). Cowles Foundation for Research in Economics, Yale University.
16. Nordhaus, W. D. (2008). A question of balance: economic modeling of global warming. Yale University Press.
17. Popp, D. (2004). ENTICE: endogenous technological change in the DICE model of global warming. *Journal of Environmental Economics and management*, 48(1), 742-768.

18. Popp, D. (2006). ENTICE-BR: The effects of backstop technology R&D on climate policy models. *Energy Economics*, 28(2), 188-222.
19. Romer, P. M. (1990). Endogenous technological change. *Journal of political Economy*, 98(5, Part 2), S71-S102.
20. Schaefer, K., Zhang, T., Bruhwiler, L., & Barrett, A. P. (2011). Amount and timing of permafrost carbon release in response to climate warming. *Tellus B*, 63(2), 165-180.
21. Sinclair, P. J. (1994). On the optimum trend of fossil fuel taxation. *Oxford Economic Papers*, 869-877.
22. Stern, N. H. (2007). The economics of climate change: the Stern review. Cambridge University press.
23. Ulph, A., & Ulph, D. (1994). The optimal time path of a carbon tax. *Oxford Economic Papers*, 857-868.

8 Annex

8.1 Solution of knowledge sector maximization problem

$$\begin{aligned} \mathcal{H} &= p_{H,t}H_t - w_t L_H + \lambda_\xi a_H H_t L_H \\ \frac{\partial \mathcal{H}}{\partial L_H} &: \quad -w_t + \lambda_\xi a_H H_t = 0 \\ \frac{\partial \mathcal{H}}{\partial H_t} &: \quad p_{H,t} + \lambda_\xi a_H L_H = -\dot{\lambda_\xi} + \rho \lambda_\xi \end{aligned}$$

Hence $\lambda_\xi = w_t / a_H H_t$, which log-differentiated with respect to time yields $\dot{\lambda_\xi}/\lambda_\xi + \dot{H_t}/H_t = \dot{w}_t / w_t$. Plugged into the second first-order condition and using the expression for λ_ξ, it yields $p_{H,t} = \frac{\rho w_t - \dot{w}_t}{a_H H_t}$.

8.2 Decentralized equilibrium: proof of Proposition 1

The first characterizing condition is obtained from the maximization of the households welfare subject to their dynamic budget constraint (19) which leads to the standard Ramsey rule, i.e., $\rho + \frac{\dot{C}_t}{C_t} = r_t$and by using equation (2). Combining $p_{E,t}$ and $p_{B,t}$ from equations (4) and (10) into equation (15) yields as a result the second characterizing condition involving the marginal productivity of renewable energy. The third condition is obtained by using equation (4) into equation (9) and the result for $p_{F,t}$ from the fossil fuel profit maximization problem. Finally, the last characterizing condition can be got using $p_{B,t}$ from equation (10) into equation (16), and substituting the price of knowledge stock $p_{H,t}$ by the result that is achieved from the profit maximization of the research firms of equation (14).

8.3 Decentralized equilibrium optimization problem

Let $\mathcal{H}^m$ be the discounted value of the Hamiltonian of the maximization program of the decentralized equilibrium.

$$\begin{aligned} \mathcal{H}^m = & ln(C_t) + \lambda_K [Y(\Omega, K_t, L_Y, E_t) - C_t - \delta_K K_t - \delta_{I_B} I_{B,t} - b - c(Z_t)F_t - \tau \psi F_t] + \\ & + \lambda_{I_B} b + \lambda_H [a_H H_t (1 - L_Y)] + \lambda_Z(-F_t) \end{aligned} \tag{39}$$

The first order conditions $\partial \mathcal{H}^m / \partial C = 0$, $\partial \mathcal{H}^m / \partial L_Y = 0$, $\partial \mathcal{H}^m / \partial F = 0$ and $\partial \mathcal{H}^m / \partial b = 0$ yield, respectively,

$$\frac{1}{C_t} = \lambda_K \tag{40}$$

$$\lambda_K Y_{L_Y} = \lambda_H a_H H_t \tag{41}$$

$$\lambda_K [Y_E E_F - c(Z_t) - \tau \psi] - \lambda_Z = 0 \tag{42}$$

$$-\lambda_K + \lambda_{I_B} = 0 \tag{43}$$

Moreover, $\partial \mathcal{H}^m / \partial K = -\dot{\lambda_K} + \rho \lambda_K$, $\partial \mathcal{H}^m / \partial I_B = -\dot{\lambda_{I_B}} + \rho \lambda_{I_B}$, $\partial \mathcal{H}^m / \partial H = -\dot{\lambda_H} + \rho \lambda_H$ and $\partial \mathcal{H}^m / \partial Z = -\dot{\lambda_Z} + \rho \lambda_Z$ yield

$$\lambda_K(Y_K - \delta_K) = -\dot{\lambda_K} + \rho\lambda_K \tag{44}$$
$$\lambda_K(Y_E E_B B_{I_B} - \delta_{I_b}) = -\dot{\lambda_{I_B}} + \rho\lambda_{I_B} \tag{45}$$
$$\lambda_K Y_H + \lambda_H[a_H(1 - L_Y)] = -\dot{\lambda_H} + \rho\lambda_H \tag{46}$$
$$\lambda_K \frac{F_t}{Z_t^2} = -\dot{\lambda_Z} + \rho\lambda_Z \tag{47}$$

Log-differentiating (40) with respect to time yields $g_{\lambda_K} = \dot{\lambda_K}/\lambda_K = -\dot{C}_t/C_t$ and replacing it into (44) yields the standard Euler condition characterizing the trade-off between capital K_t and consumption C_t.

$$g_C = \frac{\dot{C}_t}{C_t} = (Y_K - \delta) - \rho = \left(\beta\frac{Y}{K} - \delta\right) - \rho = r_t - \rho \tag{48}$$

By using $\lambda_K = \lambda_{I_B}$ into equation (45), it shows that the marginal productivity of the investment in renewables net of depreciation equals the interest rate and therefore the marginal productivity of capital. This result is equivalent to the first two equations of Proposition 1.

$$Y_E E_B B_{I_B} - \delta_{I_B} = r_t = Y_K - \delta_K \tag{49}$$

Let's assume that both stocks of capital depreciate at the same amount $\delta_K = \delta_{I_B} = \delta$. Then it can be obtained from equation (49) the ratio $\frac{K_t}{I_{B,t}} = \frac{\beta}{\gamma\eta(1-\phi)}$, implying that both stocks of capital must grow at the same rate to keep the ratio constant, i.e., $g_{I_B} = g_K$.

From 41, it can be rewritten as $\lambda_K/\lambda_H = a_H H_t/Y_L$. Dividing (46) by λ_H and replacing the expression for λ_K/λ_H yields the following equation.

$$\frac{\dot{\lambda_H}}{\lambda_H} = g_{\lambda_H} = \rho - a_H\left(\frac{L_Y\gamma(1-\eta)(1-\phi) + (1-L_Y)(1-\beta-\gamma)}{1-\beta-\gamma}\right)$$

Also, log-differentiating with respect to time equation (41) it can be obtained $g_{\lambda_K} + g_{Y_{L_Y}} = g_{\lambda_H} + g_H$. Since it can be shown that $Y_{L_Y} = (1-\beta-\gamma)Y/L_Y$, then $g_{Y_{L_Y}} = g_Y - g_{L_Y}$. Using this, equation (48) and the previous equation, then

$$-g_C + g_Y - g_{L_Y} = \rho - a_H\left(\frac{L_Y\gamma(1-\eta)(1-\phi) + (1-L_Y)(1-\beta-\gamma)}{1-\beta-\gamma}\right) + a_H(1-L_Y)$$
$$-g_C + g_Y - g_{L_Y} = \rho - a_H\left(\frac{L_Y\gamma(1-\eta)(1-\phi)}{1-\beta-\gamma}\right)$$

which is equivalent to the fourth equation of proposition 1. It can be isolated an expression for the growth rate of the labor workforce in the final goods sector:

$$g_{L_Y} = g_Y - \left(\beta\frac{Y_t}{K_t} - \delta\right) + a_H\left(\frac{L_Y\gamma(1-\eta)(1-\phi)}{1-\beta-\gamma}\right) \tag{50}$$

By differentiating with respect to time equation (42) we get

$$\dot{\lambda_K}\left(Y_F - \frac{1}{Z} - \tau\psi\right) + \lambda_K\left(Y_F - \frac{\dot{1}}{Z} - \tau\psi\right) = \dot{\lambda_Z} \iff -\frac{\dot{C}_t}{C_t^2}\left(Y_F - \frac{1}{Z} - \tau\psi\right) + \frac{1}{C_t}\left(\dot{Y_F} + \frac{\dot{Z}_t}{Z^2}\right) = \dot{\lambda_Z}$$

Considering that $Y_F = \phi\gamma Y_t/F_t$ and substituting $\dot{\lambda_Z}$ from (47), it can be obtained that

$$\frac{1}{C_t}\left[\phi\gamma\frac{Y_t}{F_t}(g_Y - g_F - g_C - \rho) + \frac{g_C + \rho}{Z}\right] = \rho\frac{1}{C_t}\left(\phi\gamma\frac{Y_t}{F_t} - \frac{1}{Z_t} - \tau\psi\right) - \frac{F_t}{C_t Z_t^2}$$

As explained in section 5 the growth rate of final output and that of fossil fuel use is nil, and due to the fact that it is assumed that the interest rate takes a non-zero steady-state value, it can be obtained as a result that a constant carbon tax is set to be equal to the net marginal benefit of the fossil fuel use, as such is the third characterizing equation of Proposition 1.

8.4 Social planner's optimization problem

Let $\mathcal{H}^S$ be the discounted value of the Hamiltonian of the maximization program of the social planner

$$\mathcal{H}^S = ln(C_t)e^{-\rho t} + \lambda_K\left[(2 - e^{T_t - T_0})AK_t^\beta L_Y^{1-\beta-\gamma}\left(F_t^\phi(a_B I_{B,t}^\eta H_t^{1-\eta})^{1-\phi}\right)^\gamma - C_t - \delta_K K_t - \delta_{I_B} I_{B,t} - b - \frac{F_t}{Z_t}\right] +$$
$$+ \lambda_{I_B} b + \lambda_H\left[a_H H_t(1 - L_Y)\right] + \lambda_Z(-F_t) + \lambda_T(\psi F_t - mT_t)$$

The associated first-order conditions are:

$$\frac{\partial\mathcal{H}^S}{\partial C_t}: \quad \frac{1}{C_t} = \lambda_K$$

$$\frac{\partial\mathcal{H}^S}{\partial L_Y}: \quad \lambda_K\left\{(1-\beta-\gamma)(2 - e^{T_t - T_0})AK_t^\beta L_Y^{-\beta-\gamma}\left(F_t^\phi(a_B I_{B,t}^\eta H_t^{1-\eta})^{1-\phi}\right)^\gamma\right\} - \lambda_H a_H H_t = 0$$

$$\frac{\partial\mathcal{H}^S}{\partial F_t}: \quad \lambda_K\left\{\phi\gamma(2 - e^{T_t - T_0})AK_t^\beta L_Y^{1-\beta-\gamma}F_t^{\phi\gamma-1}(a_B I_{B,t}^\eta H_t^{1-\eta})^{(1-\phi)\gamma} - \frac{1}{Z_t}\right\} - \lambda_Z + \lambda_T\psi = 0$$

$$\frac{\partial\mathcal{H}^S}{\partial b}: \quad -\lambda_K + \lambda_{I_B} = 0$$

$$\frac{\partial\mathcal{H}^S}{\partial K_t}: \quad \lambda_K\left\{\beta(2 - e^{T_t - T_0})AK_t^{\beta-1}L_Y^{1-\beta-\gamma}\left(F_t^\phi(a_B I_{B,t}^\eta H_t^{1-\eta})^{1-\phi}\right)^\gamma - \delta\right\} = -\dot{\lambda_K} + \rho\lambda_K$$

$$\frac{\partial\mathcal{H}^S}{\partial I_{B,t}}: \quad \lambda_K\left\{\eta(1-\phi)\gamma(2 - e^{T_t - T_0})AK_t^\beta L_Y^{1-\beta-\gamma}F_t^{\phi\gamma}(a_B H_t^{1-\eta})^{(1-\phi)\gamma}I_{B,t}^{\eta(1-\phi)\gamma-1} - \delta_{I_B}\right\} = -\dot{\lambda_{I_B}} + \rho\lambda_{I_B}$$

$$\frac{\partial\mathcal{H}^S}{\partial H_t}: \quad \lambda_K\left\{(1-\eta)(1-\phi)\gamma(2 - e^{T_t - T_0})AK_t^\beta L_Y^{1-\beta-\gamma}F_t^{\phi\gamma}(a_B I_{B,t}^\eta)^{(1-\phi)\gamma}H_t^{(1-\eta)(1-\phi)\gamma-1}\right\} =$$
$$= -\dot{\lambda_H} + \rho\lambda_H - \lambda_H\left\{a_H(1 - L_Y)\right\}$$

$$\frac{\partial\mathcal{H}^S}{\partial Z_t}: \quad \lambda_K\frac{F_t}{Z_t^2} = -\dot{\lambda_Z} + \rho\lambda_Z$$

$$\frac{\partial \mathcal{H}^S}{\partial T_t}: \quad \lambda_K \left\{ (-e^{T_t - T_0}) A K_t^{\beta} L_Y^{1-\beta-\gamma} \left(F_t^{\phi} (a_B I_{B,t}^{\eta} H_t^{1-\eta})^{1-\phi} \right)^{\gamma} \right\} - m\lambda_T = -\dot{\lambda_T} + \rho\lambda_T$$

Log-differentiating $\partial\mathcal{H}^S/\partial C_t$ with respect to time yields $g_{\lambda_K} = \dot{\lambda_K}/\lambda_K = -\dot{C}_t/C_t$ and replacing it into $\partial\mathcal{H}^S/\partial K_t$ yields the standard Euler condition characterizing the trade-off between capital K_t and consumption C_t.

$$g_C = \frac{\dot{C}_t}{C_t} = (Y_K - \delta) - \rho = \left(\beta\frac{Y}{K} - \delta\right) - \rho = r_t - \rho$$

From $\partial\mathcal{H}^S/\partial L_{Y,t}$, it can be rewritten as $\lambda_K/\lambda_H = a_H H_t/Y_L$. Dividing $\partial\mathcal{H}^S/\partial H_t$ by λ_H and replacing the expression for λ_K/λ_H yields the following equation.

$$\frac{\dot{\lambda_H}}{\lambda_H} = g_{\lambda_H} = \rho - a_H \left(\frac{L_Y\gamma(1-\eta)(1-\phi) + (1-L_Y)(1-\beta-\gamma)}{1-\beta-\gamma} \right)$$

Then, log-differentiating with respect to time equation $\partial\mathcal{H}^S/\partial L_{Y,t}$ it can be obtained $g_{\lambda_K} + g_{Y_L} = g_{\lambda_H} + g_H$. Since it can be shown that $Y_{L_Y} = (1-\beta-\gamma)Y/L_Y$, then $g_{Y_L} = g_Y - g_{L_Y}$. Using this, the previous equation and $\partial\mathcal{H}^S/\partial C_t$, then it is possible to obtain the fourth characterizing condition in Proposition 3:

$$-g_C + g_Y - g_{L_Y} = \rho - a_H \left(\frac{L_Y\gamma(1-\eta)(1-\phi) + (1-L_Y)(1-\beta-\gamma)}{1-\beta-\gamma} \right) + a_H(1-L_Y)$$

$$-g_C + g_Y - g_{L_Y} = \rho - a_H \left(\frac{L_Y\gamma(1-\eta)(1-\phi)}{1-\beta-\gamma} \right)$$

$$g_{L_Y} = g_Y - \left(\beta\frac{Y_t}{K_t} - \delta\right) + a_H \left(\frac{L_Y\gamma(1-\eta)(1-\phi)}{1-\beta-\gamma} \right)$$

By using $\lambda_K = \lambda_{I_B}$ into equation $\partial\mathcal{H}^S/\partial I_{B,t}$, it shows that following the Euler condition the marginal productivity of the investment in renewables net of depreciation equals the interest rate.

$$Y_E E_B B_{I_B} - \delta_{I_B} = r_t = Y_K - \delta_K$$

Finally, by differentiating $\partial\mathcal{H}^S/\partial F_t$ and considering that $Y_F = \phi\gamma Y_t/F_t$, differentiating this we obtain that

$$\dot{\lambda_K}\left(Y_F - \frac{1}{Z}\right) + \lambda_K \left(\dot{\overline{Y_F - \frac{1}{Z}}}\right) = \dot{\lambda_Z} - \psi\dot{\lambda_T} \iff -\frac{\dot{C}_t}{C_t^2}\left(Y_F - \frac{1}{Z}\right) + \frac{1}{C_t}\left(\dot{Y_F} + \frac{\dot{Z}_t}{Z^2}\right) = \dot{\lambda_Z} - \psi\dot{\lambda_T}$$

Using conditions $\partial\mathcal{H}^S/\partial Z_t$ and $\partial\mathcal{H}^S/\partial T_t$ to substitute for $\dot{\lambda_Z}$ and $\dot{\lambda_T}$, and rearranging we obtain

$$\frac{1}{C_t}\left[\phi\gamma\frac{Y_t}{F_t}(g_Y - g_F - g_C - \rho) + \frac{g_C + \rho}{Z} - \psi Y_T\right] = -m\psi\lambda_T \tag{51}$$

The value of λ_T can be obtained by solving the differential equation that comes from condition $\partial\mathcal{H}^S/\partial T_t$. The solution of such a differential equation is:

$$\lambda_{T,t} = e^{(\rho+m)t}\left(\lambda_{T,0} - \int_0^t e^{-(\rho+m)s}\frac{Y_{T,s}}{C_s}ds\right)$$

By rearranging equation (51) it can be obtained the condition governing the net marginal benefit of fossil fuel use, the third one from Proposition 3.