Review Article
Non-Gaussianity from Large-Scale Structure Surveys

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Received 27 January 2010; Accepted 18 March 2010

Academic Editor: Dragan Huterer

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With the advent of galaxy surveys which provide large samples of galaxies or galaxy clusters over a volume comparable to the horizon size (SDSS-III, HETDEX, Euclid, JDEM, LSST, Pan-STARRS, CIP, etc.) or mass-selected large cluster samples over a large fraction of the extra-galactic sky (Planck, SPT, ACT, CMBPol, B-Pol), it is timely to investigate what constraints these surveys can impose on primordial non-Gaussianity. I illustrate here three different approaches: higher-order correlations of the three-dimensional galaxy distribution, abundance of rare objects (extrema of the density distribution), and the large-scale clustering of halos (peaks of the density distribution). Each of these avenues has its own advantages, but, more importantly, these approaches are highly complementary under many respects.

1. Introduction

The recent advances in the understanding of the origin and evolution of the Universe have been driven by the advent of high-quality data, in unprecedented amount (just think of WMAP and SDSS, e.g.). Despite this, most of the information about cosmological parameters come from the analysis of a massive compression of the data: the power spectrum of their statistical fluctuations over the mean. The power spectrum is a complete statistical description of a random field only if it is Gaussian. Even the simplest inflationary models predict deviations from Gaussian initial conditions. These deviations are expected to be small, although “small” in some models may be “detectable.” For a thorough review of inflationary non-Gaussianity see [1]; for our purpose it will be sufficient to say that to describe inflation-motivated departures from Gaussian initial conditions many write [2–5]

$$\Phi = \phi + f_{\text{NL}} (\phi^2 - \langle \phi^2 \rangle).$$

Here $\phi$ denotes a gaussian field and $\Phi$ denotes Bardeen’s gauge-invariant potential, which, on sub-Hubble scales reduces to the usual Newtonian peculiar gravitational potential, up to a minus sign. In the literature, there are two conventions for (1): the large-scale structure (LSS) and the Cosmic Microwave Background (CMB) one. In the LSS convention $\Phi$ is linearly extrapolated at $z = 0$; in the CMB convention $\Phi$ is instead primordial: thus $f_{\text{NL}}^{\text{LSS}} = g(z = \infty) / g(0) f_{\text{NL}}^{\text{CMB}} \sim 1.3 f_{\text{NL}}^{\text{CMB}}$, where $g(z)$ denotes the linear growth suppression factor relative to an Einstein-de-Sitter Universes. In the past few years it has become customary to always report $f_{\text{NL}}^{\text{CMB}}$ values even if, for simplicity as it will be clear below, one carries out the calculations with $f_{\text{NL}}^{\text{LSS}}$.

While for simplicity one may just assume $f_{\text{NL}}$ in (1) to be a constant (yielding the so-called local model or local-type) in reality the expression is more complicated and $f_{\text{NL}}$ is scale and configuration dependent. Even if the bispectrum does not completely specify non-Gaussianity, in most practical applications, the non-Gaussianity is set by writing down the bispectrum of $\Phi$. For example, one can see that for the local model the bispectrum is

$$B_{\Phi}(k_1, k_2, k_3) = 2 f_{\text{NL}} P_{\phi}(k_1) P_{\phi}(k_2) + 2 \text{cyc},$$

where $P$ denotes the power spectrum and it is often assumed that $P_\phi = P_\Phi$; “cyc.” denotes two cyclic terms over $k_1, k_2, k_3$.

It has been shown [6] that for non-Gaussianity of the local type the bispectrum is dominated by the so-called squeezed configurations, triangles where one wave-vector length is much smaller than the other two. Models such as the curvaton for example, have a non-Gaussianity of...
The most widespread technique for testing Gaussianity in the CMB is to use the CMB bispectrum:

$$B(k_1, k_2, k_3) = 6 f_{\text{NL}} \left\{ -P(k_1)P(k_2) + 2\text{cyc.} 
- 2[P(k_1)P(k_2)P(k_3)]^{2/3} 
+ P^{1/3}(k_1)P^{2/3}(k_2)P(k_3) + 5\text{cyc.} \right\}. \quad (3)$$

The $f_m^m$ are the coefficients of the spherical harmonic expansion of the CMB temperature fluctuation: $\Delta T/T = \sum_m a_m^m Y_m^m$ and the presence of the 3-J symbol ensures that the bispectrum is defined if $l_1 + l_2 + l_3 = \text{even}$, $l_1 + l_2 \geq l_3 \geq |l_1 - l_2|$ (triangle rule), and that $m_1 + m_2 + m_3 = 0$. It should be however clear that secondary CMB anisotropies and foregrounds also induce a CMB bispectrum which can mask or partially mimic the signal seen, for example [5, 12–17] and references therein.

In the last few years, this area of research has received an impulse, motivated by the recent full sky CMB data from WMAP. In particular it has been shown that the constraints can be greatly improved by effectively “reconstructing” the potential $\Phi$ from CMB temperature and polarization data rather than simply using the temperature bispectrum alone [18, 19]. This technique would yield constraints on non-Gaussianity of the local type of $\Delta f_{\text{NL}} \sim 1$ for an ideal experiment and $\Delta f_{\text{NL}} \sim 3$ for the Planck satellite. This is particularly promising as $f_{\text{NL}}$ of order unity or larger is produced by broad classes of inflationary models (see e.g., [1] and references therein).

Currently, the most stringent constraints for the local type are $27 < f_{\text{NL}} < 147$ at the 95% confidence (central value 87) from WMAP 3 years data [20]; and from the WMAP 5 years data, $-9 < f_{\text{NL}} < 111$ at the 95% confidence level (central value 55) [10] and $-4 < f_{\text{NL}} < 80$ [21]. Despite the heated debate on whether $f_{\text{NL}} = 0$ is ruled out or not, the two measurements are not necessarily in conflict: the two central values differ by only about 1σ, different, although not independent, data sets were used with different galactic cuts, and the maximum multipole considered in the analyses is also different. What makes the subject very interesting, is that, if the central value for $f_{\text{NL}}$ is truly around 60, forthcoming data will yield a highly-significant detection.

### 2. Higher-Order Correlations

Theoretical considerations (see discussion in e.g., [11] and references therein) lead us to define primordial non-Gaussianity by its bispectrum. While in principle there may be types of non-Gaussianity which would be more directly related to higher-order correlations (e.g., [22] and references therein), and while a full description of a non-Gaussian distribution would require the specification of all the higher-order correlations, it is clear that quantities such as the bispectrum enclose information about the phase correlation between $k$-modes. In the Gaussian case, different Fourier modes are uncorrelated (by definition of Gaussian random phases) and a statistic like the power spectrum does not carry information about phases. The bispectrum is the lowest-order correlation with zero expectation value in a Gaussian random field. But, even if the initial conditions were Gaussian, nonlinear evolution due to gravitational instability generates a nonzero bispectrum. In particular, gravitational
instability has its own “signature” bispectrum, at least in the next-to-leading order in cosmological perturbation theory [23]:

\[ B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2P(k_1)P(k_2)J(k_1, k_2) + 2\text{cyc}, \]

where \( J(k_1, k_2) \) is the gravitational instability “kernel” which depends very weakly on cosmology and for an Einstein-de-Sitter Universe is

\[ J(k_1, k_2) = \frac{5}{7} + \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{2k_1k_2} + 2\left(\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1 k_2}\right)^2. \]  

(6)

In the highly nonlinear regime the detailed form of the kernel changes, but it is something that could be computed and calibrated by extending perturbation theory beyond the next-to-leading order and by comparing with numerical N-body simulations (see other contributions in this issue). It was recognized a decade ago [4] that this signal is quite large compared to any expected primordial non-Gaussianity and that the primordial signal “redshifts away” compared to the gravitational signal. In fact, a primordial signal given by a local type of non-Gaussianity parameterized by a given \( f_{\text{NL}} \), would affect the late-time dark matter density bispectrum with a contribution of the form

\[ B_{\text{NL, local}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2f_{\text{NL}}P(k_1)P(k_2) D(z)/D(z = 0) + 2\text{cyc}, \]

(7)

where \( D(z) \) is the linear growth function which in an Einstein-de-Sitter Universe goes like \( (1 + z)^{-1} \) and

\[ F = \frac{\mathcal{M}(k_1)}{\mathcal{M}(k_1)\mathcal{M}(k_2)} \quad \text{and} \quad \mathcal{M}(k) = \frac{2}{3} \frac{k^2 T(k)}{H_0^2 \Omega_{m,0}}, \]

(8)

\( T(k) \) denoting the transfer function, \( H_0 \) the Hubble parameter, and \( \Omega_{m,0} \) the matter density parameter. Clearly the two contributions have different scale and redshift dependence and the two kernel shapes in configuration space are different, thus, making the two components, at least in principle and for high signal-to-noise, separable.

Unfortunately, with galaxy surveys, one does not observe the dark matter distribution directly. Dark matter halos are believed to be hosts for galaxy formation, and different galaxies at different redshifts populate halos following different prescriptions. In large-scale structure studies, often the assumption of linear scale independent bias is made. A linear bias will not introduce a nonzero bispectrum in a Gaussian field and its effect on a field with a nonzero bispectrum is only to rescale its bispectrum amplitude. This is, however, an approximation, possibly roughly valid at large scales for dark matter halos, and when looking at the power spectrum, but unlikely to be true in detail. To go beyond the linear bias assumption, often the assumption of quadratic bias is made, where the relation between dark matter overdensity field and galaxy field is specified by two parameters: \( b_1 \) and \( b_2 \), \( \delta_g(x) = b_1 \delta_{\text{DM}}(x) + b_2(\delta_{\text{DM}}^2 - \langle \delta_{\text{DM}}^2 \rangle) \); \( b_1 \) and \( b_2 \) are assumed to be scale-independent (although this assumption must break down at some point) but they can vary with redshift. Clearly, a quadratic bias will introduce non-Gaussianity even on an initially Gaussian field. In summary, for local non-Gaussianity and scale-independent quadratic bias we have

\[ B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, z) = 2P(k_1)P(k_2)b_1(z)^3 \times \left[ f_{\text{NL}} \frac{F(k_1, k_2)}{D(z)} + J(k_1, k_2) + \frac{b_2(z)}{2b_1(z)} \right] + \text{cyc}. \]

(9)

Before the above expression can be compared to observations it needs to be further complicated by redshift space distortions (and shot noise). Realistic surveys use the redshift as a proxy for distance, but gravitationally induced peculiar velocities distort the redshift-space galaxy distribution. We will not go into these details here as including redshift space distortions (and shot noise) will not change the gist of the message.

From a practical point of view, it is important to note that photometric surveys, although in general can cover larger volumes that spectroscopic ones, are not suited for this analysis: the projection effects due to the photo-z smearing along the line-of-sight is expected to suppress significantly the sensitivity of the measured bispectrum to the shape of the primordial one (see e.g., [25, 26]).

Reference [4] concluded that “CMB is likely to provide a better probe of such (local) non-Gaussianity.” Much more recently, reference [27] revisited the issue and found that, assuming a given known redshift dependence of the \( (b_1, b_2) \) bias parameters and an all sky survey from \( z = 0 \) to \( z = 5 \) with a galaxy number density of at least \( 5 \times 10^{-4} \) kpc\(^{-3}\), the bispectrum can provide constraints on the \( f_{\text{NL}} \) parameter competitive with CMB. However, for all planned surveys, the forecasted errors are much larger than Planck forecasted errors. This holds qualitatively also for the equilateral case.

While the gravitationally induced non-Gaussian signal in the bispectrum has been detected to high statistical significance (see [28] and references therein, see also other contributions to this issue), the nonlinear bias signature is not uncontroversial, and there have been so far no detection of any extra (primordial) bispectrum contributions.

Of course one could also consider higher-order correlations. One of the advantages of considering, for example, the trispectrum is that, contrary to the bispectrum, it has very weak nonlinear growth [29], but has the disadvantage that the signal is delocalized: the number of possible contributions is that, contrary to the bispectrum, it has very weak nonlinear growth [29], but has the disadvantage that the signal is delocalized: the number of possible configurations grows fast with the dimensionality \( n \) of the \( n \)-point function!

In summary, higher-order correlations as observed in the CMB or in the evolved Universe, can be used to determine the bispectrum shape. The two approaches should be seen as complementary as they are affected by different systematic effects and probe different scales. The next two probes we
3. The Mass Function

The abundance of collapsed objects (dark matter halos as traced, e.g., by galaxies and galaxy clusters) contains important information about the properties of initial conditions on galaxy and clusters scales. The Gaussian assumption plays a central role in analytical predictions for the abundance and statistical properties of the first objects to collapse in the Universe. In this context, the formalism proposed by Press and Schechter [30], with its later extensions and improvements, has become the standard lore for predicting the number of collapsed dark matter halos as a function of redshift. However, even a small deviation from Gaussianity can have a deep impact on those statistics which probe improvements, has become the standard lore for predicting the number of collapsed dark matter halos as a function of redshift. However, even a small deviation from Gaussianity can have a deep impact on those statistics which probe the tails of the distribution. This is indeed the case for the abundance of high-redshift objects like galaxies and clusters at \( z \geq 1 \) which correspond to high peaks, that is, rare events, in the underlying dark matter density field. Therefore, even small deviations from Gaussianity might be potentially detectable by looking at the statistics of high-redshift systems. Before proceeding let us introduce some definitions.

We are interested in predictions for rare objects, that is, the collapsed objects that form in extreme peaks of the density field \( \delta(x) = \delta \rho / \rho \). The statistics of collapsed objects can be described by the statistics of the density perturbation smoothed on some length scale \( R \) (or equivalently a mass scale \( M = 4\pi R^3 \rho \), \( \delta_R \).

To incorporate non-Gaussian initial conditions into predictions for the smoothed density field, we need an expression for the probability distribution function (PDF) for \( \delta_R \). For a particular real-space expansion like (1), one may make a formal change of variable in the Gaussian PDF to generate a normalized distribution [31]. However, this may not be possible in general and the change of variables does not work for the smoothed cumulants of the density field. In general, the PDF for a generic non-Gaussian distribution can be written exactly as a function of the cumulant’s generating function \( W_R \) for the smoothed density field:

\[
P(\delta_R) d\delta_R = \frac{d\lambda}{2\pi} \exp[-i\lambda \delta_R + W_R(\lambda)] d\delta_R \tag{10}
\]

with

\[
W_R(\lambda) = \sum_{n=2}^{\infty} \frac{(i\lambda)^n}{n!} \mu_{n,R},
\]

where \( \mu_{n,R} \) denote the cumulants and, for example, \( \mu_{2,R} = \sigma_R^2 = \langle \delta^2 \rangle_R \) and the skewness \( \mu_{3,R} \) is related to the normalized skewness of the smoothed density field \( S_{3,R} = \mu_{3,R} / \mu_{2,R} \).

It is useful to define a “skewness per \( f_{NL} \) unit” \( S_{3,R}^{NL} \) so that \( S_{3,R} = f_{NL} S_{3,R}^{NL} \). The skewness \( \mu_{3,R} \) is related to the underlying bispectrum by

\[
\mu_{3,R} = \int \frac{d^3 \vec{k}_1 d^3 \vec{k}_2 d^3 \vec{k}_3}{(2\pi)^6} B_{\delta R}(|\vec{k}_1|,|\vec{k}_2|,|\vec{k}_3|) \delta^D_R \chi_{\vec{k}_1,\vec{k}_2,\vec{k}_3}(\vec{k}), \tag{12}
\]

where \( \delta^D \) denotes the Dirac delta function and \( B_{\delta R} \) denotes the bispectrum of the \( \delta \) overdensity field smoothed on scale \( R \). It is related to the potential one trivially by remembering the Poisson equation: \( \delta_R(k) = M(k) W_R(k) \phi(k) \). Here, \( W_R(k) \) denotes the smoothing kernel, usually taken to be the Fourier transform of the top hat window. In any practical application the dimensionality of the integration can be reduced by collapsing the expressing \( \delta_R \) as a function of \( \vec{k}_1 \) and \( \vec{k}_2 \).

It is important at this point to make a small digression to specify definitions of key quantities. Even in linear theory, the normalized skewness of the density field depends on redshift; however in the Press and Schechter framework one should always use linearly extrapolated quantities at \( z = 0 \). In this context therefore, when writing \( S_{3,R} = f_{NL} S_{3,R}^{NL} \), if \( S_{3,R}^{NL} \) is such that if the density field extrapolated linearly at \( z = 0 \) then \( f_{NL} \) must be the LSS one and not the CMB one.

To compute the abundance of collapsed objects from the PDF one will then follow the Press and Schechter swindle: first compute

\[
\mathcal{P}(>\delta_c | z, R) = \int_{\delta_c(z)}^{\infty} d\delta_R \mathcal{P}(\delta_R), \tag{13}
\]

where \( \delta_c \) denotes the critical threshold for collapse; then the number of collapsed objects is

\[
n(M,z)dM = 2 \frac{3H_0^2 \Omega_m}{8\pi G M^2} \frac{d\mathcal{P}(>\delta_c | z, R)}{dM} \bigg|_{M}. \tag{14}
\]

Note that the redshift dependence is usually enclosed only in \( \delta_c : \sigma_M \) is computed on the field linearly extrapolated at \( z = 0 \), and \( \delta_c(z) = \Delta_c(z) D(z = 0) / D(z) \) and \( \Delta_c(z) \) depends very weakly on redshift and \( \Delta_c(z) \sim 1.68 \).

Equation (14) however cannot be computed analytically and exactly starting from (10): some approximations need to be done in order to obtain an analytically manageable expression. Two approaches have been taken so far in the literature which we will briefly review below.

3.1. MVJ Approach. The authors of [31] proceed by first performing the integration over \( \delta_R \) to obtain an exact expression for \( \mathcal{P}(>\delta_c | z, M) \). At this point they expand the generating functional to the desired order, for example, keeping only terms up to the skewness, then perform a Wick rotation to change variables and finally a saddle-point approximation to evaluate the remaining integral. The saddle point approximation is very good for large thresholds \( \delta_c / \sigma_M \gg 1 \), thus for rare and massive peaks. For the final expression for the mass function they obtain

\[
\hat{n}(M,z) = 2 \frac{3H_0^2 \Omega_m}{8\pi G M^2} \frac{1}{\sqrt{2\pi} \sigma_M} \exp \left[ -\frac{\delta_c^2}{2\sigma_M^2} \right] \times \sqrt{\frac{1}{1 - S_{3,M}\delta_c / 3}} \frac{dS_{3,M}}{d \ln M} + \frac{\delta_*}{\sigma_M} \frac{d\sigma_M}{d \ln M}, \tag{15}
\]

where \( \sigma_M \) denotes the rms value of the density field, the subscript \( M \) denotes that the density field has been smoothed.
on a scale $R(M)$ corresponding to $R(M) = \left(M^3/(4\pi)\right)^{1/3}$, and $\delta_c = \delta_c(\sqrt{1 - \delta_c}S_{1,M}/3)$. This derivation shows that the mass function in principle depends on all cumulants, but that if non-Gaussianity is small (and non-Gaussianity is small) but for rare events (the tails of the distribution) $\delta_c(z)/\sigma_M$ is large. One thus expects this approximation to break down for large masses, high redshift, and high $f_{\text{NL}}$. Reference [9] quantified the range of validity of their approximation by assuming that when terms proportional to $S_3$ become important is no longer valid to neglect terms proportional to higher-order cumulants. Then they define the validity regime of their mass function to be where corrections from the $S_3$ are unimportant. They find, as expected, that for very massive objects the approximation breaks down and that the upper mass limit for applicability of the mass function decreases with redshift and $f_{\text{NL}}$. But for low masses, redshifts and $f_{\text{NL}}$ their formula is better than the MVJ. On the other hand the MVJ range of validity extends to higher masses, redshifts and $f_{\text{NL}}$ values, as expected, as MVJ applied the saddle point approximation to $P(>\delta_c | M, z)$ which is an increasingly good approximation for rare objects.

Of course, the natural observable to apply this method to are not only galaxy surveys (and the clusters found there), but, especially suited, are the mass-selected large clusters surveys offered by on-going Sunyaev-Zeldovich experiments (e.g., Planck, ACT, SPT). A detailed comparison with N-body simulations is the next logical step to pursue.

3.3. Comparison with N-Body Simulations. Before we proceed we should consider that the Press and Schechter formulation of the mass function even in the Gaussian initial conditions case, can be significantly improved see for example, [34–36]. Much improved expressions have been extensively calibrated on Gaussian initial conditions N-body simulations. The major limitations in both the MVJ and LMSV derivations (since they follow the classic Press and Schechter formulation) are the assumption of spherical collapse and the sharp $k$-space filtering. In addition, the excursion set improvement on the original Press and Schechter swindle relies on the random-phase hypothesis, which is not satisfied for non-Gaussian initial conditions. Since these improvements of the mass function have not yet been generalized to generic non-Gaussian initial conditions (but work is on-going, see other contributions in this issue) the analytical results above should be used to model fractional corrections to the Gaussian case.

Thus the non-Gaussian mass function, $n_{\text{NG}}(M, z)$ can be written as a function of a Gaussian one, $n_G(M, z)$ (accurately calibrated on N-body simulations) with a non-Gaussian correction factor $R$ (see e.g., [9, 37]):

$$n_{\text{NG}}(M, z) = n_G(M, z)R(S_3, M, z),$$

(17)

where

$$R(S_3, M, z) = \frac{\hat{n}(M, z, f_{\text{NL}})}{\hat{n}(M, z, f_{\text{NL}} = 0)}$$

(18)

and $\hat{n}$ is given by the MVJ or LMSV approximation. The correction $R$ can then be calibrated on N-body simulations.

Reference [33] argues that the same correction that is in the Gaussian case modifies the collapse threshold, $\delta_c$, to

![Figure 1: Skewness $S_{1, R}$ of the density field at $z = 0$ as a function of the smoothing scale $R$ for different types of non-Gaussianity. Figure reproduced from [32].](image-url)
improve over the original Press and Schechter formulation, may apply to the non-Gaussian correction. The detailed physical interpretation of this is still matter of debate in the literature [38–40]. In summary, reference [33] proposes to write the non-Gaussian correction factor for the MVJ [31] case as

$$R_{NG}(M, z, f_{NL}) = \exp \left[ \frac{\delta_{c}^{3} S_{3,M}}{6 \sigma_{M}} \right] \times \left[ \frac{1}{6} \frac{\delta_{c}}{\sigma_{M}} \frac{dS_{3,M}}{d \ln \sigma_{M}} + \sqrt{1 - \frac{\delta_{c}^{2} S_{3,M}}{3}} \right],$$

(19)

and for the LMSV [9] case:

$$R_{NG}(M, z, f_{NL}) = 1 + \frac{1}{6} \frac{\sigma_{M}^{2}}{\delta_{c}} \times \left[ S_{3,M} \left( \frac{\delta_{c}^{4} S_{3,M}}{\sigma_{M}^{2}} - 2 \frac{\delta_{c}^{2} S_{3,M}}{\sigma_{M}} - 1 \right) + \frac{dS_{3,M}}{d \ln \sigma_{M}} \left( \frac{\delta_{c}^{2} S_{3,M}}{\sigma_{M}^{2}} - 1 \right) \right],$$

(20)

where $\delta_{c}$ denotes the modified critical density for collapse, which for high peaks is $\delta_{c} \sim \sqrt{q}$. Reference [33] calibrated these expressions on N-body simulations to find $q = 0.75$. We anticipate here that the validity of this extrapolation (i.e., in terms of a correction to the critical collapse threshold) can be tested independently on the large-scale non-Gaussian halo bias as described in Section 4. Note that, in both cases, in the limit of small non-Gaussianity the correction factors reduce to

$$R = 1 + S_{3,M} \frac{\delta_{c}^{3}}{6 \sigma_{M}^{2}}.$$  

(21)

Non-Gaussian mass functions have been computed from simulations and compared with different theoretical predictions in several works [41–45]. In the past, conflicting results were reported, but the issue seems to have been settled, there is agreement among mass function measured from different non-Gaussian simulations performed by three different groups as shown for example in Figure 2. As expected both MVJ and LMSV prescriptions for the non-Gaussian correction to the mass function agree with the simulation results, provided one makes the substitution $\delta_{c} \rightarrow \delta_{c}$, with some tentative indication that MVJ may be better for very massive objects while LMSV performs better for less rare events. This is shown in Figure 3 where the points represent measurements from N-body simulations presented in [33].

3.4. Voids. While galaxy clusters form at the highest over-densities of the primordial density field and probe the high-density tail of the PDF, voids form in the low-density regions and thus probe the low-density tail of the PDF. Most of the volume of the evolved universe is underdense, so it seems interesting to pay attention to the distribution of underdense regions. A void distribution function can be derived in an

Figure 2: Correction to the Gaussian mass function as measured in different non-Gaussian simulations. There is now agreement between different simulations. The $y$ axis should be interpreted as $\log_{10} R(M, z, f_{NL})$. For negative values of $f_{NL}$ the absolute value of the non-Gaussian correction is considered. Reproduced from [33, Figures 4 and 5].
analogous way to the Press Schechter mass function by realizing that negative density fluctuations grow into voids [32], a critical underdensity \( \delta_v \) is necessary for producing a void and this plays the role of the critical overdensity \( \delta_c \) for producing bound objects (halos). The more underdense a void is the more negative \( \delta_v \) becomes. The precise value of \( \delta_v(z) \) depends on the precise definition of a void (and may depend on the observables used to find voids); realistic values of \( \delta_v(z = 0) \) are expected to be \( \gtrsim 1 \). In the absence of a better prescription, here, following [32], \( \delta_v \) is treated as a phenomenological parameter and results are shown for a range of \( \delta_v \) values. To derive the non-Gaussian void probability function one proceeds as above with the only subtlety that \( \delta_v \) is negative and that \( P(< \delta) = 1 - P(> \delta) \) thus \( |dP(< \delta)/d\delta| = |dP(> \delta)/d\delta| \). Thus the void PDF as a function of \( |\delta_v| \) can be obtained from the PDF of MVJ [31] or LMSV [9], provided one keeps track of the sign of each term. For example in the LMSV approximation the void distribution function becomes [32]

\[
\hat{n}(R,z,f_{\text{NL}}) = \frac{9}{2\pi^2} \sqrt{\frac{\pi}{2}} \frac{1}{R^4} e^{-\delta_v^2/2\delta_c^2} \times \left\{ \frac{d\ln \sigma_M}{d\ln M} \right\} \times \left[ \frac{|\delta_v| S_{\text{NL}} \sigma_M}{\sigma_M} 6 \left( \frac{\delta_v^4}{\sigma_M^4} - 2 \frac{\delta_v^2}{\sigma_M^2} - 1 \right) \right]
\]

\[
+ \frac{1}{6} \frac{dS_3}{dM} \left( \frac{\delta_v^2}{\sigma_M^2} - 1 \right),
\]

where the expression is reported as a function of the smoothing radius rather than the mass, since a void Lagrangian radius is probably easier to determine than its mass.

Note that while a positive skewness (\( f_{\text{NL}} > 0 \)) boosts the number of halos at the high-mass end (and slightly suppress the number of low-mass halos), it is a negative skewness that will increase the voids size distribution at the largest voids end (and slightly decrease it for small void sizes). Reference [32] concluded that the abundance of voids is sensitive to non-Gaussianity, \( |\delta_v| \) is expected to be smaller than \( \delta_c \) by a factor 2 to 3. If voids probe the same scales as halos then they should provide constraints on \( f_{\text{NL}} \) to 3 times worse. However voids may probe slightly larger scales than halos, in many non-Gaussian models, \( S_3 \) increases with scales (see e.g., Figure 1), compensating for the threshold.

The approach reviewed here provides a rough estimate of the fractional change in abundance due to primordial non-Gaussianity but will not provide reliably the abundance itself. It is important to stress here that rigorously quantitative results will need to be calibrated on cosmological simulations and mock survey catalogs.

### 4. Effects on the Halo Power Spectrum

Recently, reference [43, 46] showed that primordial non-Gaussianity affects the clustering of dark matter halos (i.e., density extrema) inducing a scale-dependent bias for halos on large scales. This can be seen for example by considering halos as regions where the (smoothed) linear density field exceeds a suitable threshold. All correlations and peaks considered in the section are those of the initial density field (linearly extrapolated to the present time). Thus for example in the Gaussian case [47–49] for high peaks we would have the following relation between the correlation function of halos of mass \( M \), \( \xi_{h,M} \) (and that of the dark matter distribution smoothed on scale \( R \), corresponding to mass \( M \), \( \xi_{h,R} \)):

\[
\xi_{h,M}(r) = \left( b_{h,L} \right)^2 \xi_{h,R}(r),
\]

where \( b_{h,L}^2 = \delta_c / \sigma_R^2 \) denotes the Lagrangian halo bias (in the Gaussian case), although more refined expressions can be found in for example, [50, 51].

The Lagrangian bias appears here because correlations and peaks are those of the initial density field (linearly extrapolated). Making the standard assumptions that halos move coherently with the underlying dark matter, one can obtain the final Eulerian bias as \( b_{h,E} = 1 + b_L \), using the techniques outlined in [50–53]. Below we will omit the subscript \( E \) for Eulerian bias.

The two-point correlation function of regions above a high threshold has been obtained, for the general non-Gaussian case, in [54–56]:

\[
\xi_{h,M}(|x_1 - x_2|) = -1 + \exp[X],
\]

where

\[
X = \sum_{N=2}^{N-1} \frac{r^N}{N!} \sum_{j=1}^N \frac{r^{N-j}}{(N-j)!} \frac{1}{j!} \frac{f_{NL}}{f_{NL}^0} \frac{\sigma_R^2}{\sigma_R^0} \frac{\xi_{NL}}{\xi_{NL}^0} \times \left( \frac{N-j}{N} \right) \times \left( \frac{N-j}{N} \right),
\]

Figure 3: The points show the non-Gaussian correction to the mass function as measured in the N-body simulations of [33]. Blue corresponds to \( f_{\text{NL}} = 200 \) and red to \( f_{\text{NL}} = -200 \). the dashed lines correspond to the MVJ formulation and the dot-dashed lined to the LMSV formulation. In both cases the substitution \( \delta_v \rightarrow \delta_v \) has been performed. The y axis should be interpreted as \( R(M,z,f_{\text{NL}} = 200) \). Reproduced from Figure 7 [33].
where $\nu = \delta_0 \sigma_R$. For large separations the exponential can be expanded to first order. This is what we will do in what follows but we will comment on this choice below.

For small non-Gaussianities (Effectively that is for values of $f_{NL}$ consistent with observations.), we can keep terms up to the three-point correlation function $\xi^{(3)}$, obtaining that the correction to the halo correlation function, $\Delta \xi_h$ due to a non-zero three-point function is given by:

$$
\Delta \xi_h = \frac{\sigma^3_R}{\sigma^2_R} \left[ \xi^{(3)}(x_1, x_2, x_3) + \xi^{(3)}(x_1, x_1, x_3) \right] \tag{26}
$$

For a general bispectrum $B(k_1, k_2, k_3)$ this yields a correction to the power spectrum (see [46] for steps in the derivation):

$$
\Delta P = \frac{\delta^3(z)}{\mathcal{M}(k)} \left[ \frac{1}{4\pi^2 \sigma^2_R} \int dk_1 k_1^2 \mathcal{M}_R(k_1) \right] \times \int \left[ d\mu \mathcal{M}_R(\sqrt{\alpha}) \right] \beta_R(k_1, \sqrt{\alpha}, k) \frac{P_{\phi}(k)}{P_{\phi}(k)} \tag{27}
$$

where we have made the substitution $\alpha = k_1^2 + k_2^2 + 2k_3 \mu$. Here $\mathcal{M}_R = W_R \mathcal{M}$ and $W_R$ and $\mathcal{M}$ were introduced in Section 3. The effect on the halo bias is $\Delta b_h^f/b_h^L = (1/2)(\Delta P/P)$ and thus

$$
b_h^f = b_h^L + \frac{\Delta b_h}{\sigma^2_D(z)} \left[ 1 + \delta(z) \beta_{NL}(k) \right], \tag{28}
$$

where the expression for $\beta$ can be obtained by comparing to (27). The term $\Delta \xi(z)/|\sigma^2_D(z)| = b^G - 1$ can be recognized as the Gaussian Lagrangian halo bias.

So far the derivation is generic for all types of non-Gaussianity specified by a given bispectrum. We can then consider specific cases. In particular for local non-Gaussianity we obtain

$$
\beta_R(k) = \frac{2 f_{NL}}{8 \pi^2 \sigma^2_R \mathcal{M}_R(k)} \int dk_1 k_1^2 \mathcal{M}_R(k_1) P_{\phi}(k_1) \times \int d\mu \mathcal{M}_R(\sqrt{\alpha}) \left[ \frac{P_{\phi}(\sqrt{\alpha})}{P_{\phi}(k)} + 2 \right] \tag{29}
$$

Thus $\Delta b_h^f/b_h^L = 2 f_{NL}$ times a redshift-dependent factor $\Delta_b(z)/D(z) = \delta(z)$, times a $k$- and mass-dependent factor. The function $\beta_R(k)$ is shown as the dashed line in Figure 4. This result for the local non-Gaussianity has been derived in at least three other ways. Reference [43] generalizes the Kaiser [47] argument of high peak bias for the local non-Gaussianity. Starting from $\nabla^2 \phi = -2 f_{NL} |\nabla \phi|^{2}$ where near peaks $|\nabla \phi|^2$ is negligible they obtain $\delta = \delta_{NL} + [1 + 2 f_{NL} \phi]$. The Poisson equation to convert $\phi$ in $\delta$ then gives the scale-dependence. More details are presented elsewhere in this issue.

Reference [58] works in the peak-background split. This approach is especially useful to understand that it is the coupling between very large and small scales introduced by local (squeezed-configurations) non-Gaussianity to boost (or suppress) the peaks clustering. In this approach, the density field can be written as $\rho(x) = p(1 + \delta_l + \delta_2)$ where $\delta_l$ denotes long wavelength fluctuations and $\delta_2$ short wavelength fluctuations. $\delta_l$ is the one responsible for modulating halo formation (i.e., to boost peaks above the threshold for collapse), so the halo number density is $n = n(1 + b_{NL} \delta_l)$ and $b_{NL} = \mathcal{M}^{-1} \partial n/\partial \delta_l$.

In the local non-Gaussian case they decompose the Gaussian field $\phi$ as a combination of long and short wavelength fluctuations $\phi = \phi_l + \phi_s$, thus $\mathcal{M} = \phi_l + f_{NL} \phi_s + (1 + 2 f_{NL} \phi) \phi_s + f_{NL} \phi_s^2 + \text{const}$. Also in this non-Gaussian case one can split the density field $\delta_l$ and $\delta_2$ and relate this to $f_{NL}$ (it is easier to work in Fourier space): $\delta(k) = a(k) \phi(k)$ and $\delta_l = a(k)(1 + 2 f_{NL} \phi) \phi_l + f_{NL} \phi_s^2 \approx a(k) |X_1 \phi + X_2 \phi_s|$, the last equality giving the definition of $X_1$ and $X_2$. Note that $\delta_l$ cannot be ignored here because $\delta_l$ enters in $X_1$, in other words, local non-Gaussianity couples long and short wavelength modes. The local halo number density is now function of $\delta_l$, $X_1$, and $X_2$ yielding the following result for the Lagrangian halo bias:

$$
b_{NL} = \mathcal{M}^{-1} \left[ \frac{\partial n}{\partial \delta_l} + 2 f_{NL} \frac{\partial \phi_l}{\partial \delta_l} \frac{\partial n}{\partial X_1} \right] \tag{30}
$$

where $a(k)$ encloses the scale-dependence of the effect. Reference [59] redervies the ellipsoidal collapse for small deviations from Gaussianity of the local type. They find that a non-zero $f_{NL}$ modifies the threshold for collapse, the modification is proportional to $f_{NL}$. This should sound familiar from Section 3. They then use the definition $b_{NL} = \mathcal{M}^{-1} \partial n/\partial \delta_l$, keeping track of the fact that $\delta$ is “modulated” by $f_{NL}$. 

![Figure 4: The scale-dependence of the large-scale halo bias induced by a nonzero bispectrum for different types of non-Gaussianity. The dashed line corresponds to the local type and the dot-dot-dashed to equilateral type. Figure reproduced from [57].](image-url)
where $\beta_R(k) = f_{\text{NL}} \beta_{NL}^{NL=1}(k)$, can be improved in several ways.

First of all, we have not made any distinction between the redshift at which the object is being observed ($z_o$) and that at which is being formed ($z_f$). Except for the rarest events this should be accounted for. The Gaussian Lagrangian bias expression used so far is an approximation, a more accurate expression is [50, 52, 53]

$$b_{hL}^G(z_o, M, z_f) = \frac{1}{D(z_o)} \left[ \frac{\delta_c(z_f)}{\sigma_M^2} - \frac{1}{\delta_c(z_f)} \right].$$

Then, the halo bias expressions are derived within the “classical” Press and Schechter theory, as we have seen in Section 3, subsequent improvements on the mass function can be seen as a correction to the collapse threshold. In the expression for the Gaussian halo bias $b_{NL}^G = \pi^{-1} \partial \sigma^G/\partial \delta_c$, one can consider mass functions that are better fit to simulations than the standard Press and Schechter one obtaining:

$$b_{hL}^G(z_o, M, z_f) = \frac{1}{D(z_o)} \left[ \frac{q \delta_c(z_f)}{\sigma_M^2} - \frac{1}{\delta_c(z_f)} \right] + \frac{2p}{\delta_c(z_f) D(z_o)} \left[ 1 + \left( \frac{q \delta_c(z_f)}{\sigma_M^2} \right)^p \right]^{-1}.$$ (33)

The parameters $q$ and $p$ account for nonspherical collapse and fit to numerical simulations yield $q \sim 0.75, p = 0.3$ for example, [34]. In this expression the term in the second line is usually subdominant. The term “$−1/\delta_c$” in the first line is known as “antibias”, and it becomes negligible for old halos $z_f \gg z_o$. Note that by including the antibias correction in $b_{NL}^G$ of (31) one recovers the “recent mergers” approximation of [58].

The same correction should also apply to the non-Gaussian correction to the halo bias:

$$\Delta b = f_{\text{NL}} \delta_c \left( b_{NL}^G - 1 \right) \beta_{NL}^{NL=1}(k),$$ (34)

where $q'$ should coincide with $q$ above; it can be calibrated to N-body simulations and is found indeed to be $q' = 0.75$ [33]. The non-Gaussian halo bias prediction and results from N-body simulations with local non-Gaussianity are shown in Figure 6.

Finally one may note that for $f_{\text{NL}}$ large and negative, (27) and (28) would formally yield $b_{NL}^{NL}$ and $P_{NL}(k)$ negative on large enough scales. This is a manifestation of the breakdown of the approximations made: (a) all correlations of higher order than the bispectrum were neglected, for large NG this truncation may not hold; (b) The exponential in (24) was expanded to linear order. This however could be easily corrected for, remembering that the $P(k)$ obtained from (27) in reality the Fourier transform of $X$, the argument of the exponential. One would then compute the halo correlation function using (24) and Fourier transforming back to obtain the halo power-spectrum.
So far we have concentrated on local non-Gaussianity, but the expression of (27) and (28) is more general. Using this formulation, reference [57] computed the quantity \( \beta_{R}^{b_{NL}}(k) \) for several types of non-Gaussianity (equilateral, local, and enfolded); this is shown in Figure 4. It is clear that the non-Gaussian halo-bias effect has some sensitivity to the bispectrum shape, for example the effect for the equilateral type of non-Gaussianity is suppressed by orders of magnitude compared to the local-type and the flattened case is somewhere in the middle. Figure 4 also shows a type of non-Gaussianity arising from General-relativistic (GR) corrections on scales comparable to the Hubble radius. Note that perturbations on super-Hubble scales are initially needed in order to “feed” the GR correction terms. In this respect the significance of this contribution is analogous to the well-known large-scale anticorrelation between CMB

Figure 6: The quantity \( \Delta b/b \) as function of \( k \), for simulation snapshots at \( z = 0.44, 1.02, \) and 1.54. Simulation outputs and theory lines are shown for \( f_{NL} = \pm 100 \) and \( f_{NL} = \pm 200 \). Figure reproduced from [33]. The \( f_{NL} \) values reported in this figure legend should be interpreted as \( f_{NL}^{LSS} \).
temperature and E-mode polarization, it is a consequence of the properties of the inflationary mechanism to lay down the primordial perturbations. This effect has the same magnitude as a local non-Gaussianity with \( f_{NL} \). The next logical step is then to ask how well present or forthcoming data could constrain non-Gaussianity using the halo-bias effect. It is interesting to note that surveys that aim at measuring Baryon Acoustic Oscillations (BAO) in the galaxy distribution to constrain dark energy are well suited to also probe non-Gaussianity; they cover large volumes and their galaxy number density is well suited so that on the scale of interest (both for BAO and non-Gaussianity) shot noise does not dominate the signal. Photometric surveys are also well suited: as the non-Gaussian signal is localized at very large scales and is a smooth function of \( k \), the photometric surveys effects are unimportant.

The theory developed so far describes the clustering of halos while we observe galaxies. Different galaxy populations occupy dark matter halos following different prescriptions. If we think in the halo-model framework (e.g., [60] and references therein) at very large scales only the "two-halo" contribution matters and the details of the halo occupation distribution (the "one-halo" term) become unimportant.

What is important to keep in mind is that the effect of the non-Gaussianity parameter one wants to measure, \( f_{NL} \), is fully degenerate with the value of the Gaussian (small scales) halo bias. Figure 7 shows the dependence of the non-Gaussian correction on the Gaussian bias.

Thus highly biased tracers will show a larger non-Gaussian effect for the same \( f_{NL} \) value. Of course for a given cosmological model the Gaussian bias can be measured accurately by comparing the predicted dark matter power spectrum with the observed one. Alternatively, two differently biased tracers can be used in tandem to disentangle the two effects [61, 62]. Since clustering amplitude may depend on the entire halo history, it becomes then interesting to model in details the dependence of the effect on the halo merger tree (Reid et al., in preparation).

**4.1. Outlook for the Future.** How well can this method do to constrain primordial non-Gaussianity compared with the other techniques presented here? The Integrated Sachs Wolfe (ISW) effect offers a window to probe clustering on the largest scales (where the signal is large); on the other hand, a measurement of clustering of tracers of dark matter halos is a very direct window into this effect. A Fisher matrix approach [58, 59, 63] shows that the ISW signal is weighted at relatively low redshift (where dark energy starts dominating) while the non-Gaussian signal grows with redshift, thus making the shape of the halo power spectrum a more promising tool.

An overview of current constraints from different approaches can be found in Table 1 and future forecasts in Table 2, for non-Gaussianity of the local type. Large, mass-selected cluster samples as produced by SZ-based experiments will provide a optimally suited data-set for this technique (see e.g., [64]).

While for a given \( f_{NL} \) model such as the local one, methods that exploit the non-Gaussian bias seem to yield the smallest error-bars for large-scale structure, it should be
kept in mind that the bispectrum can be used to investigate the full configuration dependence of $f_{\text{NL}}$ and thus is a very powerful tool to discriminate between different type of non-Gaussianity. In addition CMB-bispectrum and halo bias test non-Gaussianity on very large scales while the large scale structure bispectrum mostly probes mildly nonlinear scales. As primordial non-Gaussianity may be scale-dependent, all these techniques are highly complementary.

The above estimates assume that the underlying cosmological model is known. The large-scale shape of the power spectrum can be affected by cosmology. Carbone et al. (in prep.) explore possible degeneracies between $f_{\text{NL}}$ and cosmological parameters. They find that the parameters that are most strongly correlated with $f_{\text{NL}}$ are parameters describing dark energy clustering, neutrino mass and running of the primordial power spectrum spectral slope. For surveys that cover a broad redshift range the error on $f_{\text{NL}}$ degrade little when marginalizing over these extra parameters; the peculiar redshift dependence of the non-Gaussian signal lifts the degeneracy.

5. Conclusions

A natural question to ask at this point may be “what observable will have better chances to constrain primordial non-Gaussianity?”

In principle the abundance of rare events is a very powerful probe of non-Gaussianity; however, in practice, it is limited by the practical difficulty of determining the mass of the observed objects and its corresponding large uncertainty in the determination. This point is stressed for example, in [9]. With the advent of high-precision measurements of gravitational lensing by massive clusters, the mass uncertainty, at least for small to moderate size clusters samples can be greatly reduced. Forthcoming Sunyaev-Zeldovich experiments will provide large samples of mass-selected clusters which could then be followed up by lensing mass measurements (see e.g., [65, 66]). So far there is only one very high redshift ($z = 1.4$) very massive $M \approx 8 \times 10^{14} M_\odot$ with high-precision mass determination via gravitational lensing [67]. Reference [68] pointed out that this object is extremely rare, for Gaussian initial conditions there should be 0.002 such objects or less in the surveyed area, which is uncomfortably low probability. But the cluster mass is very well determined; a non-Gaussianity still compatible with CMB constraints could bring the probability of observing of the object to more comfortable values. This result should be interpreted as a “proof of principle” showing that this a potentially powerful avenue to pursue.

The measurement of the three-point correlation function allows one to map directly the shape-dependence of the bispectrum. For large-scale structures the limiting factors are the large non-Gaussian contribution induced by gravitational evolution and the uncertainty of the nonlinear behavior of galaxy bias.

The halo-bias approach can yield highly competitive constraints, but it is less sensitive to the bispectrum shape. Still, the big difference in the magnitude and shape of the scale-dependent biasing factor between different non-Gaussian models implies that the halo bias can become a useful tool to study shapes when combined with for example, measurements of the CMB bispectrum. Table 3 highlights this complementarity. For example, one could envision different scenarios.

If non-Gaussianity is local with negative $f_{\text{NL}}$ and CMB obtains a detection, then the halo bias approach should also give a high-significance detection (GR correction and primordial contributions add up), while if it is local but with positive $f_{\text{NL}}$, the halo-bias approach could give a lower statistical significance for small $f_{\text{NL}}$ as the GR correction contribution has the opposite sign.

If CMB detects $f_{\text{NL}}$ at the level of $\sim 10$ and of a form that is close to local, but halo bias does not detect it, then the CMB bispectrum is given by secondary effects.

If CMB detects non-Gaussianity but is not of the local type, then halo bias can help discriminate between equilateral and enfolded shapes; if halo bias sees a signal it indicates enfolded type; if halo bias does not see a signal it indicates equilateral type. Thus even a nondetection of the halo-bias effect, in combination with CMB constraints can have an important discriminatory power.

In any case, if the simplest inflationary scenario holds, for surveys like Euclid and LSST, the halo-bias approach is expected to detect a non-Gaussian signal very similar to the local type signal with an amplitude of $f_{\text{NL}} \sim -1.5$ which is due to large-scales GR corrections to the Poisson equation. This effect should leave no imprint in the CMB; once again the combination of the two observable can help enormously to discriminate among models for the origin of cosmological structures.

In addition we should bear in mind that non-Gaussianity may be scale-dependent. In fact for models like DBI inflation it is expected to be scale-dependent. A proposed parameterization of the scale-dependence of non-Gaussianity is given by

$$B_{k_1 k_2 k_3} = f_{\text{NL}} \left( \frac{K}{k_p} \right)^{n_{\text{NG}}} F(k_1, k_2, k_3),$$

where $K = (k_1 k_2 k_3)^{1/3}$ [70], $k_p$ denotes the pivot and $n_{\text{NG}}$ the slope or running of non-Gaussianity, although other authors prefer to use $K = (k_1 + k_2 + k_3)/3$ [9, 59] as for

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<th>Table 3: Forecasted non-Gaussianity constraints: (A) [20] (B) [63] (C) [69, 70] (E) [57] (F) e.g., [15].</th>
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<td><strong>CMB Bispectrum</strong></td>
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squeezed configurations $K \neq 0$. It is still an open issue which parameterization is better in practice.

In any case different observables probe different scales (see Figure 8) and their complementary means that “the combination is more than the sum of the parts.”

What is clear, however, is that the thorny systematic effects that enter in all these approaches will require that a variety of complementary avenues be taken to establish a robust detection of primordial non-Gaussianity.

Acknowledgments

This work is supported by MICCIN Grant AYA2008-03531 and FP7-IDEAS-Phys.LSS 240117. The author would like to thank her closest collaborators in many of the articles reviewed here: Carmelita Carbone, Klaus Dolag, Margherita Grossi, Alan Heavens, Raul Jimenez, Marc Kamionkowski, Marilena LoVerde, Sabino Matarrese, Lauro Moscardini, Sarah Shandera, and her collaborators for the reported work-in-progress: Carmelita Carbone, Olga Mena, Beth Reid. She would also like to thank the referee for a careful review of the manuscript.

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