Neutrino oscillations

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The following paper is a simple summary of the nowadays knowledge of properties and physics of neutrino oscillations. We will deal with the simplest cases and compare them with real data collected with the main experiments that have been done.

I. INTRODUCTION

Neutrino is a particle introduced by W. Pauli in 1930 [1] in order to secure the energy, momentum and spin conservations in nuclear beta decays. Neutrinos are one of the most abundant particles in the Universe, they are particles hard to detect, thus their study requires complex and innovative experimental methods. Neutrino oscillations were already predicted theoretically back in 1957 by Bruno Pontecorvo [2]. Several experiments have been carried out since then. The first hint was found back in 1960 when Raymond Davis and collaborators constructed the first solar neutrinos detector in the Homestake gold mine [3] [1]. More experiments continued showing evidence for neutrino oscillations [5]. A neutrino oscillates because the neutrino mass eigenstates are not interaction eigenstates (flavours). Then a neutrino produced in a certain flavor can be detected with a different flavor.

II. NEUTRINO PHENOMENOLOGY

Neutrino flavor eigenstates $|\nu_\alpha\rangle$ can be thought as a superposition of mass eigenstates $|\nu_m\rangle$[2]. Since they span the same space we can relate them with an unitary matrix:

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U^*_{\alpha i} |\nu_i\rangle \qquad (1)$$

This unitary matrix is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. The parametrization of the matrix is quite arbitrary, we choose the parametrization given in Particle Data Group (PDG) [6]:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{12}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ij} = \cos(\theta_{ij}), \quad s_{ij} = \sin(\theta_{ij})$$

In the experiments the mixing angles θ_{ij} and the squared mass differences $\Delta m^2_{ij} \equiv m^2_j - m^2_j$ are measured. If we have 3 neutrinos, only 2 these parameters are independent. Mass eigenstastes are labelled in mass order, i.e. $m_1 < m_2 < m_3$. If $\Delta m^2_{12} \ll \Delta m^2_{13}$ then there are two light and one heavy neutrinos, this is called normal hierarchy (NH) whereas on the opposite $\Delta m^2_{12} \gg \Delta m^2_{13}$, there are one light and two heavy neutrinos and it is called inverted hierarchy (IH) [6].

As we will see in the following section, experiments in the vacuum are not sensible to the sign of Δm^2_{ij}, so one cannot distinguish between (NH) or (IH). On the other hand, neutrinos traveling through matter acquire an effective mass due to a coherent forward interaction produced by the surrounding particles. Since matter, normally, contains electrons, but not muons or tauons, there is a flavor dependence on this phenomenon. In other words, electron neutrino will have different interaction in matter than muon or tau neutrino. Since the probability depends on the mass squared differences, matter effects will be shown when dealing with electron neutrinos. This is called MSW effect. This effect makes it possible to distinguish between (NH) and (IH) [7]. According to this, Wolfstein potentials in neutral non-polarized matter are given by [7]:

$$\begin{aligned} (CC) \qquad & V_c(x) = \sqrt{2} G_F N_e(x) \\ (NC) \qquad & V_n(x) = -\frac{1}{\sqrt{2}} G_F N_n(x) \end{aligned} \qquad (2)$$

Where N_e, N_n stand for the electron and neutron number densities respectively and G_F is the Fermi Constant. (CC) stand for *charged-current* interaction, one charged lepton absorbs a W bosson and then it is converted to a neutrino of the interacting lepton flavor. (NC) stand for *neutral-current* interaction, lepton or quark absorbs

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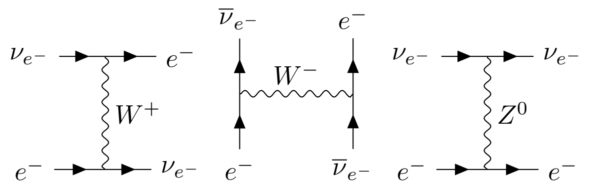
[1] For an overview of solar neutrino experiments see e.g. [4] and references tehrein.

[2] We use Greek letters to refer to neutrino flavors eigenstates and Latin letters to refer to neutrino mass eigenstates.

a neutral Z bosson [8]. For example:



III. MATHEMATICAL DESCRIPTION

A. Neutrinos in vacuum

All the mathematical development is done in natural units ($\hbar = c = 1$). We label as $|\Psi(\vec{x},t)\rangle$ the neutrinos state. At $x = 0,\ t = 0$ a α flavor neutrino is created. At this point the neutrino state is given by:

$$|\Psi(0,0)\rangle = |\nu_\alpha\rangle = \sum_{i=1}^{3} U^*_{\alpha i}\,|\nu_i\rangle \tag{3}$$

In the Schrödinger picture and in the vacuum ($H = cte$) the time evolution of the flavour eigenstates comes from the evolution of the mass eigenstates. We approximate neutrino mass eigenstates to a plane wave, with fixed momentum and energy:

$$|\Psi(\vec{x},t)\rangle = \sum_{i=1}^{3} U^*_{\alpha i} e^{i(\vec{p_i}\vec{x}-E_i t)}\,|\nu_i\rangle \tag{4}$$

Since neutrino velocities are near $\sim c$, we can do the ultrarelativistic approximation $x \sim t$ and:

$$\vec{p}\cdot\vec{x} - Et \simeq t\,(p - E_i) \simeq t\left(E\left(1 - \frac{1}{2}\frac{m^2}{E^2}\right) - E\right) \simeq \frac{1}{2}\frac{m^2}{E}t$$

Accepting this approximation equation (4) becomes:

$$|\Psi(\vec{x},t)\rangle = \sum_{i=1}^{3} U^*_{\alpha i} e^{-i\frac{m_i^2 t}{2E_i}}\,|\nu_i\rangle \tag{5}$$

The probability amplitude to detect a ν_β neutrino is given by:

$$\begin{aligned} A_{\Psi,\nu_\beta} &= \langle\nu_\beta|\Psi(\vec{x},t)\rangle = \sum_{j=1}^{3} \langle\nu_j|\,U_{\beta j} \sum_{i=1}^{3} U^*_{\alpha i} e^{-i\frac{m_i^2 t}{2E_i}}\,|\nu_i\rangle \\ &= \sum_{i=1}^{3} U_{\beta i} U^*_{\alpha i} e^{-i\frac{m_i^2 t}{2E_i}} \end{aligned}$$

Where we used that $\langle\nu_i|\nu_j\rangle = \delta_{ij}$. The probability of detection at a distance $L \sim t$ is thus:

$$\begin{aligned} P_{\Psi,\nu_\beta} &= |\,\langle\nu_\beta|\Psi(\vec{x},t)\rangle\,|^2 \\ &= \sum_{j=1}^{3} U^*_{\beta j} U_{\alpha j} e^{i\frac{m_j^2 t}{2E_j}} \sum_{i=1}^{3} U_{\beta i} U^*_{\alpha i} e^{-i\frac{m_i^2 t}{2E_i}} \\ &= \sum_{i,j=1}^{3} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} e^{i\xi_{ij}} \end{aligned}$$

Where $\xi_{ij} = \dfrac{\Delta m_{ij}^2}{2E}L$. We considered that, since neutrino masses are much more smaller than its momentum $E_{i,j} \simeq |p|$, so $Ei \simeq E_j \simeq E$. We proceed splitting the sum from the probability expression:

$$\begin{aligned} P_{\Psi,\nu_\beta} &= \sum_{i=1}^{3} U_{\beta i} U^*_{\alpha i} U^*_{\beta i} U_{\alpha i} + \sum_{\substack{i=1 \\ j\neq i}}^{3} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} e^{i\xi_{ij}} \\ &= \sum_{i=1}^{3} |U_{\beta i}|^2 |U_{\alpha i}|^2 + \sum_{\substack{i=1 \\ j\neq i}}^{3} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} \\ &\quad -2\sum_{\substack{i=1 \\ j\neq i}}^{3} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} \sin^2\left(\frac{\xi_{ij}}{2}\right) \\ &\quad +i\sum_{\substack{i=1 \\ j\neq i}}^{3} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} \sin(\xi_{ij}) \end{aligned}$$

In the previous step we have developed the exponential and done some trigonometric identities. For the first two terms we have:

$$\begin{aligned} (1,2) &= \sum_{i=1}^{3} |U_{\beta i}|^2 |U_{\alpha i}|^2 + \sum_{\substack{i=1 \\ j\neq i}}^{3} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} \\ &= \sum_{i,j=1}^{3} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} = \sum_{i=1}^{3} U_{\beta i} U^*_{\alpha i} \sum_{j=1}^{3} U^*_{\beta j} U_{\alpha j} = \delta_{\alpha\beta} \end{aligned}$$

We have used that $\sum_i U_{\alpha i} U^*_{\beta i} = \delta_{\alpha\beta}$. The third term can be also split:

$$\begin{aligned} (3) &= \sum_{i>j} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} \sin^2\left(\frac{\xi_{ij}}{2}\right) \\ &\quad + \sum_{i<j}^{3} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} \sin^2\left(\frac{\xi_{ij}}{2}\right) \end{aligned}$$

We recast i, j in the second term from so we obtain the conjugate of the first term:

$$\begin{aligned} (3) &= \sum_{i>j}\Big(U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} + U_{\beta j} U^*_{\alpha j} U^*_{\beta i} U_{\alpha i}\Big)\sin^2\left(\frac{\xi_{ij}}{2}\right) \\ &= 2\sum_{i>j} Re\Big(U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j}\Big)\sin^2\left(\frac{\xi_{ij}}{2}\right) \end{aligned}$$

We do the same thing with the fourth term but being careful with a change of sign coming from recasting i, j from $\sin\left(\dfrac{\xi_{ij}}{2}\right)$, which will give us the imaginary part instead:

$$(4) = 2\sum_{i>j} Im\Big(U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j}\Big)\sin\left(\frac{\xi_{ij}}{2}\right)$$

Altogether gives us the complete expression:

$$P_{\Psi,\nu_\beta} = \delta_{\alpha\beta} - 4\sum_{i>j} Re\Big(U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}\Big)\sin^2\left(\frac{\xi_{ij}}{2}\right) + 2\sum_{i>j} Im\Big(U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}\Big)\sin\left(\frac{\xi_{ij}}{2}\right) \tag{6}$$

B. 2 neutrino case in vacuum

A good approximation is to deal only with 2 neutrinos if only a transition between 2 flavors is observed. There will only be one rotation angle in the PMNS matrix(i.e. the other rotation angles are 0):

$$U^*_{\alpha i} = \begin{pmatrix} \cos(\theta_{ij}) & \sin(\theta_{ij}) \\ -\sin(\theta_{ij}) & \cos(\theta_{ij}) \end{pmatrix}$$

PMNS matrix becomes a rotation in two dimensions. Expresion (6) becomes:

$$P_{\Psi,\nu_\beta} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2_{ij}L}{2E}\right) \tag{7}$$

We are given the parameters inside (7) in natural units. However in experimental setups usually $\Delta m^2_{ij}(eV)$, E(GeV) and $L(km)$ so it is convenient to reinstate the powers of $\hbar$ and c, and deal with easy-to-work units:

$$\frac{\Delta m^2_{ij}[kg^2]L[m]}{2E[J]}\cdot\frac{c^3}{\hbar} \to \frac{1.27\Delta m^2_{ij}[eV^2]L[km]}{E[GeV]}$$

Neutrino mixing may be observable if at least $\Delta m^2_{ij}[eV^2] \geq \dfrac{E[GeV]}{L[km]}$ (so $\xi_{ij} \gg 1$, otherwise $\sin \ll 1$), so this expression also gives the sensitivity to Δm^2_{ij}. Since equation (7) is an even function of Δm^2_{ij}, the measurement of neutrino oscillations in this case cannot determine its sign. When neutrinos travel in matter the potentials (2) provide an extra effective mass to neutrinos, providing a probability expression which breaks the sign degeneracy [6, 7].

IV. NEUTRINO SOURCES

Neutrino experiments are carried with neutrinos coming mainly from four different sources: the atmosphere, the Sun, nuclear reactors and accelerators. It is important to discuss the properties from each source in order to understand how to develop a correct experimental process. Given the simple approach nature of this article, the following discussion will only deal with the results in (NH) and will not be taking into account matter effects.

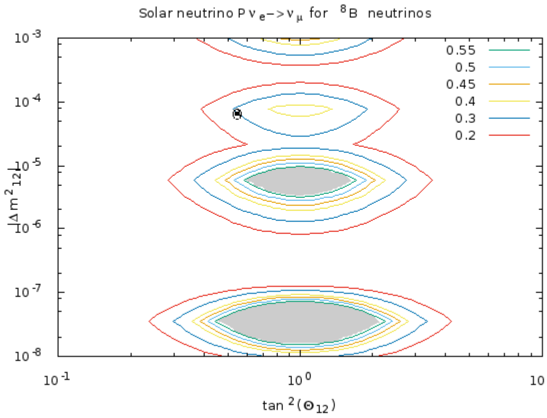


FIG. 1: Contourplot with the 2 neutrinos's vacuum model. Isolines represent the probability to oscillate P_{ν_e,ν_μ} on the $\Delta m^2_{12}/\tan^2(\theta_{12})$ plane for solar ^8B neutrinos travelling in the vacuum and detected at the Earth ($E = 9\ MeV$, and mean Earth-Sun distance $L = 149,6\cdot 10^6 km$). The gray zones represent the allowed regions. Also shown is the present world average [6].

A. Solar neutrinos

There are several nuclear reactions inside the Sun which produce neutrinos. The different reactions emit neutrinos with different flux and energy [9]. Nonetheless, the Sun only emits electron neutrinos, and in the Earth electron neutrinos coming from the Sun are detected. We tested the simplified 2 neutrino model in the vacuum with real data. For the sake of simplicity, we consider only the flux and energy coming from ^8B reaction. The energy and flux expected from neutrinos coming from this reaction are, respectively, using the BPS08(GS) solar model $E_{^8B} \sim 9MeV$, $\phi_{^8B} = 5.94\cdot(1\pm 0.11)\cdot 10^6 cm^{-2}s^{-2}$ [10]. We have taken an energy value near the maximum value of flux. Figure 1 shows a contour plot of the transition probability P_{ν_e,ν_μ} on the $\Delta m^2_{12}/\tan^2(\theta_{12})$ plane for solar ^8B neutrinos travelling in the vacuum and detected at the Earth ($E = 9\ MeV$, and mean Earth-Sun distance $L = 149,6\cdot 10^6 km$).
In order to know the allowed regions from the plot we use that

$$P_{\nu_e,\nu_\mu} = \frac{\phi_{exp}}{\phi_{theo}}$$

where $\phi_{exp} = 3.26\ cm^{-2}s^{-1}$, $\phi_{theo} = \phi_{^8B}$[10]. That gives us P_{ν_e,ν_μ}. This means that the regions allowed are the ones inside the region limited by the isoline with value 0.55 in figure 1.
With the simple 2 model neutrino we get two results: $\Delta m^2_{12} \sim 6.0\cdot(1\pm 2.0)\cdot 10^{-6}eV^2$, $\tan^2(\theta_{12}) \sim 1\pm 0.5$ and $\Delta m^2_{12} \sim 3.5\cdot(1\pm 1.5)\cdot 10^{-8}eV^2$, $\tan^2(\theta_{12}) \sim 1\pm 0.6$. The present world average, also added in Figure 1, is $\Delta m^2_{12} = 7.37^{+0.2}_{-0.22}\cdot 10^{-5}eV^2$, $\tan^2(\theta_{12}) = 0.452^{+0.035}_{-0.033}$ [6].

We see that a simple model, although with a result far from the one known nowadays, can give an evidence of neutrino oscillations.

B. Atmospheric neutrinos

From the cosmic rays passing through the atmosphere and colliding with its particles, mainly nitrogen and oxygen atoms, kaons, muons and pions are produced. These hadrons's decay produce the so called atmospheric neutrinos, mainly through the reactions:

$$\pi^{\pm}(K^{\pm}) \rightarrow \mu^{\pm} + \nu_\mu(\overline{\nu}_\mu)$$
$$\mu^{\pm} \rightarrow e^{\pm} + \nu_e(\overline{\nu}_e) + \overline{\nu}_\mu(\nu_\mu)$$

For energies lower than 1 GeV this decay is produced before particles reach Earth's ground, so the process can be studied as if neutrinos were in the vacuum. If energy is higher than 1 GeV due to Lorentz space contraction the hadrons will not decay before reaching the ground and matter effects have to be taken into account [11]. Since we do not consider matter effects, we will only deal with neutrinos not passing the Earth.
The problem with atmospheric neutrinos comes again due to a mismatch between predictions and observations. For instance, one can measure the ratio of electron neutrino flux to muon neutrino flux, and compare with the prediction, the ratio:

$$R = \frac{\left(\phi_{(\nu_e+\overline{\nu}_e)}/\phi_{(\nu_\mu+\overline{\nu}_\mu)}\right)_{DATA}}{\left(\phi_{(\nu_e+\overline{\nu}_e)}/\phi_{(\nu_\mu+\overline{\nu}_\mu)}\right)_{PREDICTED}}$$

should be 1 if predictions agreed with data, but according to the first experiments from Kamiokande detectors $R = 0.63 \pm 0.07$ [12]. According to the experiment this result is due mainly to a deficit of muon neutrinos, since the expected flux for electronic neutrinos shows almost no fluctuations from the prediction. So muon neutrinos are oscillating into tau neutrinos.
We can again see how the 2 neutrino in the vacuum model works, in this case with atmospheric neutrinos not crossing the Earth. For this case $E_{\nu_\mu} \sim 1GeV$ [6], and we took a mean traveled neutrino distance of $L \sim 9900\ km$. Figure 2 shows a contour plot of the transition probability P_{ν_μ,ν_τ} on the $\Delta m^2_{23}/\sin^2(\theta_{23})$ plane for atmospheric neutrinos traveling in the vacuum ($E = 1\ GeV$ and mean distance $L = 9900\ km$). In this case $P_{\nu_\mu,\nu_\tau} \sim R = 0.63 \pm 0.07$ the allowed regions will be inside the isoline with value ~ 0.65. With the simple 2 neutrino model we get the following results: $\Delta m^2_{23} \sim 0.4 \cdot (1 \pm 0.5)\ eV^2$, $\sin^2(\theta_{23}) \sim 0.5 \pm 0.2$, $\Delta m^2_{23} \sim 5.5 \cdot (1 \pm 1.7) \cdot 10^{-2}\ eV^2$, $\sin^2(\theta_{23}) \sim 0.5 \pm 0.3$ and $\Delta m^2_{23} \sim 2.0 \cdot (1 \pm 0.3) \cdot 10^{-3}\ eV^2$, $\sin^2(\theta_{23}) \sim 0.5 \pm 0.2$. In this case the present world average is [6] $\Delta m^2_{23} = 2.66^{+0.15}_{-0.40} \cdot 10^{-3} eV^2$, $\sin^2(\theta) = 0.425^{+0.194}_{-0.034}$. We see in this case the 2 neutrino model fits better than for the solar neutrinos. That is because atmospheric neutrinos are most likely to be in the vacuum than a solar neutrino, which have to travel a path inside the Sun.

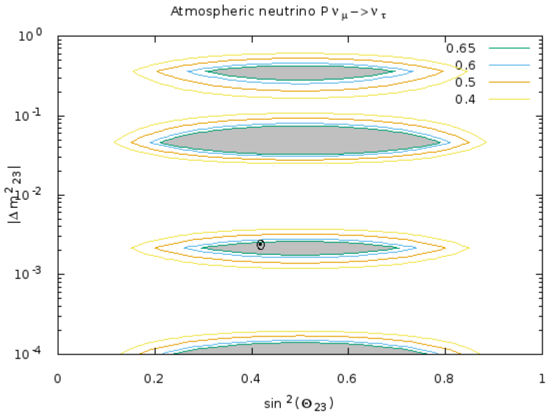


FIG. 2: Contourplot with the 2 neutrinos's vacuum model. Isolines represent the probability to oscillate P_{ν_μ,ν_τ} on the $\Delta m^2_{23}/\sin^2(\theta_{23})$ plane for atmospheric neutrinos traveling in the vacuum ($E = 1\ GeV$ and mean distance $L = 9900\ km$). The gray zones represent the allowed regions. Also shown is the present world average [6].

C. Reactor neutrinos

Nuclear reactions that are being carried out in the Earth generate power by the nuclear fission mainly of ^{235}U, ^{238}U, ^{239}P, ^{241}Pu. Neutrinos then are produced by the products of the fission β-decays so electron antineutrinos are produced [13]. The average energy freed by a fission is $\sim 200MeV$.
In order to detect this neutrinos, inverse β-decays are generated near the detector $p+\overline{\nu}_e \rightarrow n+e^+$. The positron will eventually get annihilated by electrons in matter producing two photons of energy $E_\gamma \sim 0.511MeV$ each that will be detected. The neutron also is "thermalized", captured by a proton, giving away aproximately $\sim 2.2\ MeV$ [14].
Since in this experiments the neutrino energies are low, they need a long distance to completely oscillate into tau or muon neutrino. Long baseline experiments are nowadays being done, with distances $L \sim 200\ km$ (KamLAND [15]) and $L \sim 1\ km$ (Chooz [16]). The aim for this experiments is to contribute to the data to the atmospheric anomalies and find the θ_{13} mixing angle.

D. Accelerator neutrinos

Neutrinos can be (artificially) prepared using accelerators. The beam of new created neutrinos can be directed to a desired detector in order to study the proprieties of the beam once it has travelled a certain distance. For instance K2K [17] experiment was carried out from 1999 to 2004 using muon neutrinos from an accelerator in KEK which were pointed to Kamiokande detector, 250 km away from the accelerator. In this accelerator, protons are accelerated to $0.99c$. Using superconducting magnets protons are pointed to Kamioka and fired to a target made of graphite, which causes the generation of pions. Pions enter to the decay volume where they decay producing muon neutrinos. A detector 280 m from the exit of the accelerator is situated in order to compare the newborn muon neutrinos with the neutrinos detected afterwards in Super-Kamiokande [17]. Longer-baseline experiments have been developed. Working examples nowadays are the T2K experiment in Japan with $L = 295\ km$, which is the continuation from the K2K experiment [18] or the CERN to Gran-Sasso experiment in Europe with $L = 730\ km$ with its two detectors OPERA and ICARUS [19, 20].

V. CONCLUSIONS

We have given a short overview of neutrino oscillation knowledge. We have reviewed mainly the experimental situation of solar and atmospheric neutrinos, and briefly reviewed other kind of neutrino experiments. We have commented on the possibility of breaking the sign degeneracy by making use of matter effects. Our review is based on a general three neutrino oscillation theory (6). We have also made a simplified model with 2 neutrino oscillation theory (7) and used it in a numerical application to predict the disappearance for both solar and atmospheric neutrinos. We have taken the experimental values of the transition probabilities, which has given us a very loose measurement of the neutrino oscillation parameters for both solar and atmospheric neutrinos. We have compared our results and found that their order of magnitude is comparable to the present world averages [6], our nearest results to the world averages are:

$$\Delta m^2_{12} \sim 6.0 \cdot (1 \pm 2.0) \cdot 10^{-6} eV^2, \quad \tan^2(\theta_{12}) \sim 1 \pm 0.5$$
$$\Delta m^2_{23} \sim 2.0 \cdot (1 \pm 0.3) \cdot 10^{-3}\ eV^2, \quad \sin^2(\theta_{23}) \sim 0.5 \pm 0.2$$

We might not expect a better result due to the simplifications used in the present computation.

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