Models of dynamical dark energy in the expanding Universe

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Abstract: The most common explanation for the dark energy corresponds to the standard ΛCDM model but it has some huge problems such as the Cosmological Constant Problem [1]. Thus, it is reasonable to search for alternative dark energy models. For this purpose, in this work we study the properties of a set of dynamical dark energy models different from the traditional scalar field approach (Quintessence and the like). Based on our results, and also on previous studies [2], we find that the ΛCDM hypothesis $\Lambda = const$ might not be the best description of the observational data.

I. INTRODUCTION

A century ago, in 1917, Albert Einstein introduced a new term in the General Relativity fundamental equations. He assumed that the Universe was static and thus there was a need to add a positive cosmological constant Λ to balance the gravitational attraction of all the cosmological objects [3]. These equations, known as the Einstein's field equations can be written as:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{1}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, R is the Ricci scalar curvature, $T_{\mu\nu}$ is the energy-momentum tensor, $g_{\mu\nu}$ is the metric tensor and G is the Newton's gravitational constant.

Later, in 1922, Alexander Friedmann proved that static solutions to the Einstein's field equations are unstable and also he established that in general these equations admit dynamical solutions [4]. The Einstein static Universe was finally discredited in 1929, when Edwin Hubble showed that the Universe is expanding [5].

In the late nineties, the study of type Ia supernovae enabled the teams led by A. G. Riess and S. Perlmutter [6, 7] to discover that the expansion of the late Universe has been accelerated. This acceleration is attributed to a gravitational repulsive energy form known as dark energy [8]. The most common interpretation for the dark energy is the Λ Cold Dark Matter (ΛCDM) model that assumes Λ to be constant and positive [8]. Thus, although Λ has not the purpose designed by Einstein, the ΛCDM model also introduces it in the Einstein's equations.

The ΛCDM has been able to fit most of cosmological observations and has led to valuable predictions [9].

Nevertheless, the quantum field theory (QFT) and string theory have predicted a cosmological constant value that is many orders of magnitude greater than the value obtained from the observations. This is called the Cosmological Constant Problem [1] and shows that the ΛCDM can be improved.

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Hence, this paper studies alternative cosmological models that try to reproduce the success of ΛCDM and at the same time better fit the cosmological observations.

II. DYNAMICAL DARK ENERGY MODELS

The Einstein's field equations can also be expressed as:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G\left(T_{\mu\nu} + \frac{\Lambda}{8\pi G}g_{\mu\nu}\right), \tag{2}$$

where $G_{\mu\nu}$ is the Einstein tensor, and $\rho_\Lambda = \frac{\Lambda}{8\pi G}$ is the vacuum energy density.

To propose alternative models we will replace ρ_Λ in Eq. (2) by a dark energy density ρ_D that depends on the Hubble function (H) and its derivatives. Thus, the dark energy density that we propose can be written as:

$$\rho_D(H) = a_0 + a_1 \cdot H + b_1 \cdot \dot{H} + a_2 \cdot H^2 + \ldots$$

The higher H orders would only be remarkable in the early Universe, for instance, to study the inflationary epoch [1]. As this is not the object of the present work, we will focus on a dependency on order H^2 at most. Moreover, the coefficients a_i and b_i will be rewritten in order to express ρ_D in terms of dimensionless parameters.

The models have been labelled as "dynamical models" because their Equation of State (EoS) is dynamic, i.e. it evolves in terms of the scale factor: $w_D \equiv w_D(a)$, where $p_D = w_D\rho_D$. This contrasts with the ΛCDM model since it assumes that the dark energy nature can be described as a vacuum energy and therefore the EoS of the dark energy is $w_\Lambda = -1$. On the other hand, in this work we will not consider the traditional dynamical dark energy models based on scalar fields [8].

Since the Hubble function is defined in terms of the scale factor ($H(a) = \frac{\dot{a}}{a}$), ρ_D also depends on the scale factor. Thus, to characterize the models we will find the evolution of the Hubble function, the dark energy density and the EoS in terms of the scale factor or the redshift ($z = \frac{1}{a} - 1$). Finally, we will obtain the deceleration parameter, that is a measure of the expansion rate of the Universe, in order to find the epoch when the Universe proceeds from a decelerated expansion to an accelerated expansion, known as the transition point.

III. $\mathcal{D}A$ MODELS

The first class of dynamical dark energy models that we are going to study is named $\mathcal{D}A$ (or type-A dynamical models) and its density is given by:

$$\mathcal{D}A: \qquad \rho_D(H) = \frac{3}{8\pi G}\left(c_0 + \nu H^2 + \frac{2}{3}\alpha\dot{H}\right). \quad (3)$$

We may suppose that the dimensionless parameters α and ν are small ($|\alpha|, |\nu| \ll 1$) since the model should not significantly differ from the ΛCDM model.

In a Universe with a spatially flat FLRW metric, the gravitational field equations in the presence of a dynamical dark energy are formally identical to the usual ones [1]:

$$3H^2 = 8\pi G(\rho_m + \rho_r + \rho_D), \quad (4)$$

$$2\dot{H} + 3H^2 = -8\pi G(p_m + p_r + p_D), \quad (5)$$

where the overdot refers to the derivative with respect the cosmic time t and ρ_m and ρ_r are the matter and radiation densities. Notice that we are not attempting to derive this generalization of the field equations from an effective action, this goes beyond the scope of the present work.

From Eq. (4) and (5) we can obtain:

$$\begin{aligned} 0 = \dot{\rho_m} + \dot{\rho_r} + \dot{\rho_D} + \\ + 3H[(\rho_m + p_m) + (\rho_r + p_r) + (\rho_D + p_D)]. \end{aligned} \quad (6)$$

Assuming that there is no transfer of energy between the non-relativistic matter and the radiation and also assuming that the dark energy density is self conserved, equation (6) can be read as three independent equations: $(0 = \dot{\rho}_i + 3H(\rho_i + p_i) = \dot{\rho}_i + 3H\rho_i(1 + w_i))$. We can solve this equation for the cold matter and the radiation using that their EoS are $w_m = 0$ and $w_r = \frac{1}{3}$, respectively. Thus, we find:

$$\rho_m(a) = \rho_m^0 a^{-3}; \qquad \rho_r(a) = \rho_r^0 a^{-4}, \quad (7)$$

where we have used the relation between the derivative with respect to the cosmic time and the derivative with respect to the scale factor: $\frac{\mathrm{d}}{\mathrm{d}t} = aH\frac{\mathrm{d}}{\mathrm{d}a}$. Notice also that the superscript 0 refers to the current values.

Inserting (3) and (7) into Eq. (4) and using the definition of the critical density $\rho_c = \frac{3H}{8\pi G}$ and the normalized densities $\Omega_i = \frac{\rho_i}{\rho_c}$ we find:

$$3(1-\nu)H^2 = 3H_0^2(\Omega_r^0 a^{-4} + \Omega_m^0 a^{-3}) + 3c_0 + \alpha a\frac{\mathrm{d}H^2}{\mathrm{d}a}, \quad (8)$$

where H_0 is the current value of the Hubble function.

Integrating Eq. (8) we can find H^2 in terms of the scale factor:

$$\begin{aligned} H^2(a) = H_0^2\left[\frac{c_0}{(1-\nu)H_0^2} + \left(\frac{\Omega_m^0}{1-\nu+\alpha}\right)a^{-3}\right] + \\ + H_0^2\left[-\eta a^{3\beta} + \left(\frac{\Omega_r^0}{1-\nu+\frac{4}{3}\alpha}\right)a^{-4}\right], \end{aligned} \quad (9)$$

where $\eta = \frac{c_0}{(1-\nu)H_0^2} + \frac{\Omega_m^0}{1-\nu+\alpha} + \frac{\Omega_r^0}{1-\nu+\frac{4}{3}\alpha} - 1$, and $\beta = \frac{1-\nu}{\alpha}$.

We use this expression in (3) to obtain the dark energy density in terms of the redshift.

$$\begin{aligned} \rho_D(z) = \rho_c^0\left[\frac{c_0}{H_0^2(1-\nu)} + \left(\frac{\nu-\alpha}{1-\nu+\alpha}\right)\Omega_m^0(1+z)^3\right] + \\ + \rho_c^0\left[\left(\frac{-\frac{4}{3}\alpha+\nu}{1-\nu+\frac{4}{3}\alpha}\right)\Omega_r^0(1+z)^4 - \eta(1+z)^{-3\beta}\right]. \end{aligned} \quad (10)$$

Now we can find the EoS of the dark energy solving the equation $0 = \dot{\rho_D} + 3H\rho_D(1 + w_D)$, as we have previously done for matter and radiation. The current value of the EoS can be compared with a recent result obtained from cosmological observations (see section V) and therefore it can contribute to judge the models validity. Thus, we will focus on a matter-dominated Universe ignoring the radiation term of equations (9) and (10). This reasoning will be also used to obtain the deceleration parameter.

Then, the EoS in terms of the redshift is given by:

$$w_D(z) = \frac{\rho_c^0}{\rho_D(z)}\left[\frac{\eta(1+\beta)}{(1+z)^{3\beta}} - \frac{c_0}{(1-\nu)H_0^2}\right]. \quad (11)$$

By imposing $w_D(z=0) = w_D^0$ we can determine the expression of the c_0 coefficient:

$$c_0 = H_0^2[\Omega_D^0 - \nu + \alpha(1 + w_D^0\Omega_D^0)]. \quad (12)$$

To obtain the current value of the EoS we should simplify the expression (11). For this purpose we have to analyse (8). If $\alpha < 0$, the last $r.h.s$ term could become arbitrary large and negative since

$$\lim_{a\to+\infty} a\frac{\mathrm{d}H^2}{\mathrm{d}a} = +\infty$$

Thus, according to (8), H^2 could become negative. Consequently, $\alpha \geq 0$ and $a^{3\beta} = (1+z)^{-(1-\nu)/\alpha} \simeq 0$ if $z \neq 0$ ($a \neq 1$). Using this result:

$$w_D(z) = -\frac{1}{1 + \frac{H_0^2(1-\nu)}{c_0}\Omega_m^0\left(\frac{\nu-\alpha}{1-\nu+\alpha}\right)(1+z)^3}. \quad (13)$$

In addition, for low values of the redshift we can expand $w_D(z)$ linearly in ν and α (since $|\nu|, |\alpha| \ll 1$):

$$w_D(z) \approx -1 + \left(\frac{\Omega_m^0}{1-\Omega_m^0}\right)(\nu-\alpha)(1+z)^3. \quad (14)$$

Finally, the deceleration parameter can be expressed as $q(z) = -1 + \frac{(1+z)}{2H^2}\frac{\mathrm{d}H^2}{\mathrm{d}z}$ [10]. To obtain the transition point we use that q must vanish in it, and we neglect again the $(1+z)^{-3\beta}$ term. Thus:

$$q(z) = \frac{3H_0^2}{2H^2(z)}\left(\frac{\Omega_m^0(1+z)^3}{1-\nu+\alpha} + \eta\beta(1+z)^{-3\beta}\right) - 1, \quad (15)$$

$$z_{tr} = \frac{1}{a_{tr}} - 1 = \left(\frac{2(1-\nu+\alpha)c_0}{H_0^2\Omega_m^0(1-\nu)}\right)^{\frac{1}{3}} - 1. \quad (16)$$

Based on the dark energy density of Eq. (3) we can consider three different sub-models:

$$\mathcal{D}A1: \qquad \rho_D(H) = \frac{3}{8\pi G}\left(c_0 + \nu H^2\right), \tag{17}$$

$$\mathcal{D}A2: \qquad \rho_D(H) = \frac{3}{8\pi G}\left(c_0 + \nu H^2 + \frac{2}{3}\alpha \dot{H}\right), \tag{18}$$

$$\mathcal{D}A3: \qquad \rho_D(H) = \frac{3}{8\pi G}\left(c_0 + \frac{2}{3}\alpha \dot{H}\right). \tag{19}$$

Thus, we have been studying the model $\mathcal{D}A2$. We can obtain the results for the model $\mathcal{D}A1$ by taking the limit $\alpha \to 0^+$ and for the model $\mathcal{D}A3$ by setting $\nu = 0$.

IV. $\mathcal{D}C$ MODELS

Up to now we have focused on models whose dark energy density hardly differs from the ΛCDM vacuum density ($\rho_\Lambda = \frac{3\Lambda}{8\pi G}$) since $|\nu|, |\alpha| \ll 1$.

However, in this section we are going to analyse dark energy densities with $c_0 = 0$, the $\mathcal{D}C$ models. The first model under study will be the following:

$$\mathcal{D}C2: \qquad \rho_D(H) = \frac{3}{8\pi G}\left(\nu H^2 + \frac{2}{3}\alpha \dot{H}\right). \tag{20}$$

In fact, the Hubble function, the dark energy density ($\rho_D(z)$) and the EoS for this model can be obtained from equations (9), (10) and (11) by setting $c_0 = 0$. Thus, focusing on a matter-dominated Universe and on the current Universe:

$$H^2(a) = H_0^2\left[a^{3\beta} + \left(\frac{\Omega_m^0}{1-\nu+\alpha}\right)\left(a^{-3} - a^{3\beta}\right)\right], \tag{21}$$

$$\rho_D(z) = \rho_c^0\left(\frac{\Omega_m^0}{1-\nu+\alpha}\right)(\nu-\alpha)(1+z)^3 + \\ + \rho_c^0\left(1 - \frac{\Omega_m^0}{1-\nu+\alpha}\right)(1+z)^{-3\beta}, \tag{22}$$

$$w_D(z) = -\frac{\rho_c^0}{\rho_D(z)}\left(\frac{1-\nu+\alpha-\Omega_m^0}{\alpha}\right)(1+z)^{-3\beta}. \tag{23}$$

Furthermore, from Eq. (12) we obtain the following constraint (as $c_0 = 0$): $\Omega_D^0 - \nu + \alpha(1 + w_D^0\Omega_D^0) = 0$. Therefore, we can not suppose that ν and α are both small.

Moreover, this constraint is satisfied if $w_D^0 = -1$ and $\nu = \alpha = 1$. In fact, for $\nu = \alpha = 1$, in addition to recovering the ΛCDM EoS we also recover the standard Hubble function ($H^2(a) = H_0^2\left[\Omega_\Lambda^0 + \Omega_m^0 a^{-3}\right]$) and the dark energy density becomes the vacuum density ($\rho_\Lambda = \rho_c^0\Omega_\Lambda$). Notice that neglecting the radiation, $\Omega_\Lambda = 1 - \Omega_m$ in the ΛCDM context.

As the standard ΛCDM model properly reproduces the current state of these functions, it might be a good assumption to fix $w_D^0 = -1$.

Thus, we can choose ν as the unique undetermined parameter of the model $\mathcal{D}C2$ and use the constraint to find α. We also expect $\nu, \alpha \sim 1$ to reproduce the ΛCDM behaviour.

We conclude the analysis of this model determining the deceleration parameter and the transition point:

$$q(z) = \frac{\Omega_m^0(1+z)^{3(1+\beta)} - (3\beta+2)(1-\nu+\alpha-\Omega_m^0)}{2(1-\nu+\alpha-\Omega_m^0) + 2\Omega_m^0(1+z)^{3(1+\beta)}}, \tag{24}$$

$$z_{tr} = \left[\frac{(2\alpha+3-3\nu)(1-\nu+\alpha-\Omega_m^0)}{\alpha\Omega_m^0}\right]^{\frac{\alpha}{3(1-\nu+\alpha)}} - 1. \tag{25}$$

Finally, we are going to study the model $\mathcal{D}C1$:

$$\mathcal{D}C1: \qquad \rho_D(H) = \frac{3}{8\pi G}\left(\varepsilon H_0 H + \nu H^2\right). \tag{26}$$

Unlike the previous models, this dark energy density includes the linear term H. As we have done for $\mathcal{D}A$ models (and also $\mathcal{D}C2$), we insert (7) and (26) into Eq. (4) in order to obtain the Hubble function:

$$H^2 = 3H_0^2(\Omega_m^0 a^{-3} + \Omega_r^0 a^{-4}) + \varepsilon H_0 H + \nu H^2.$$

Solving the quadratic equation and choosing the positive sign since $H(a) > 0$:

$$H(a) = H_0\left[\frac{\varepsilon + \sqrt{\varepsilon^2 + 4(1-\nu)(\Omega_m^0 a^{-3} + \Omega_r^0 a^{-4})}}{2(1-\nu)}\right]. \tag{27}$$

By imposing $H(a=1) = H_0$ we obtain $\nu = \Omega_D^0 - \varepsilon$ and therefore $\mathcal{D}C1$ has only one undetermined parameter, ν.

Substituting Eq. (27) into (26) would allow us to obtain the dark energy density in terms of the scale factor (or the redshift). To find the EoS, the deceleration parameter and the transition point, we neglect once again the radiation term:

$$w_D(z) = -1 + \frac{\Omega_m^0 H_0^2(1+z)^3(\varepsilon H_0 + 2\nu H)}{H(\varepsilon H_0 + \nu H)(2(1-\nu)H - \varepsilon H_0)}, \tag{28}$$

$$q(z) = -1 + \frac{3\Omega_m^0 H_0^2(1+z)^3}{H(2(1-\nu)H - \varepsilon H_0)}, \tag{29}$$

$$z_{tr} = \left(\frac{2(\Omega_D^0 - \nu)^2}{\Omega_m^0(1-\nu)}\right)^{\frac{1}{3}} - 1. \tag{30}$$

V. RESULTS

In this section we are going to compare the current value of the EoS ($w_D^0 \equiv w_D(0)$) and the transition point for all the $\mathcal{D}A$ and $\mathcal{D}C$ models with the ΛCDM results. All these values have been computed using the Table I.

The current EoS value will be also compared with the result obtained by the Planck Collaboration in 2015 [9]:

$$w_D^0 = -1.006 \pm 0.045.$$

Model	Ω_m^0	ν_{eff}
ΛCDM	$0.291^{+0.008}_{-0.007}$	-
$\mathcal{D}A1$	$0.286^{+0.012}_{-0.011}$	-0.024 ± 0.018
$\mathcal{D}A2$	0.286 ± 0.011	-0.024 ± 0.018
$\mathcal{D}A3$	0.287 ± 0.011	$-0.023^{+0.017}_{-0.018}$
$\mathcal{D}C1$	0.286 ± 0.014	-0.64 ± 0.13
$\mathcal{D}C2$	0.285 ± 0.013	$1.03^{+0.09}_{-0.06}$

TABLE I: Values fitted to the expansion history data (SNIa+ BAO$_A$+BAO$_{d_z}$) and the CMB shift-parameter data [10]. For $\mathcal{D}A$ models $\nu_{eff} = \nu - \alpha$ and for $\mathcal{D}A2$ we have also chosen the possible configuration: $\nu = -\alpha$. For $\mathcal{D}C$ models, $\nu_{eff} = \nu$.

We have also plotted the normalized dark energy density, the EoS and the deceleration parameter in terms of the redshift for the $\mathcal{D}A$ and $\mathcal{D}C$ models, using the values in Table I.

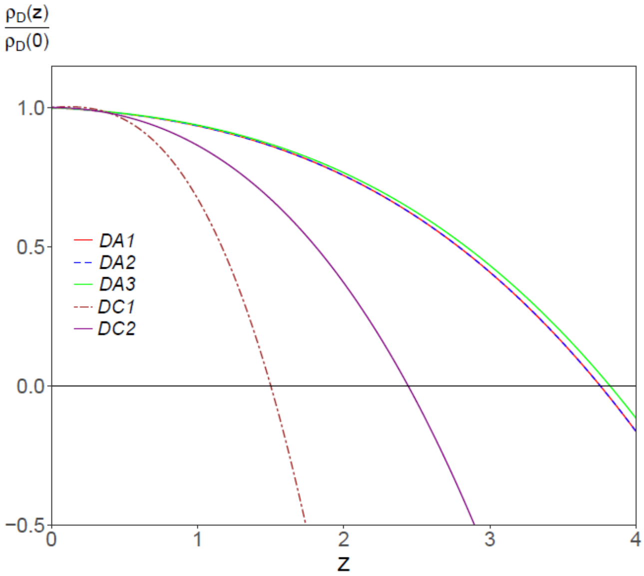

FIG. 1: Normalized dark energy density for $\mathcal{D}A$ (10), $\mathcal{D}C1$ and $\mathcal{D}C2$ (22) in terms of the redshift. To obtain the $\mathcal{D}C1$ density we have substituted (27) into (26).

To find the current EoS results we use (14) for $\mathcal{D}A$, and substituting (27) into (28) we get the $\mathcal{D}C1$ expression:

$$w^0_{D,\mathcal{D}A1} = -1.010 \pm 0.007; \qquad w^0_{D,\mathcal{D}A2} = -1.010 \pm 0.007;$$

$$w^0_{D,\mathcal{D}A3} = -1.009 \pm 0.007;$$

$$w^0_{D,\mathcal{D}C1} = -\frac{\varepsilon}{(1-\Omega^0_m)(\varepsilon + 2\Omega^0_m)} = -0.98 \pm 0.03.$$

Thus, all these EoS values are compatible with the Planck Collaboration result and also with the ΛCDM value $w_\Lambda = -1$.

Remember that we have set $w^0_{D,\mathcal{D}C2} = -1$ to mimic the ΛCDM behaviour, in order to be also compatible with the observations.

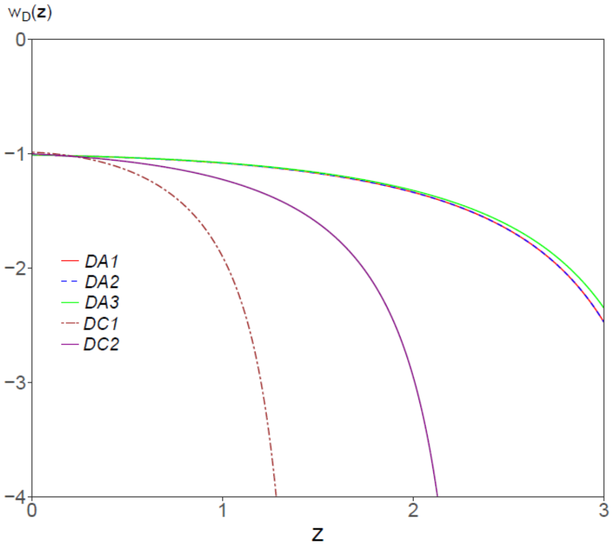

FIG. 2: EoS for $\mathcal{D}A$ (11), $\mathcal{D}C1$ (28) and $\mathcal{D}C2$ (23) in terms of the redshift.

Using (16) for $\mathcal{D}A$ models, (30) for $\mathcal{D}C1$ and (25) for $\mathcal{D}C2$ we obtain:

$$z_{tr}^{\mathcal{D}A1} = 0.73 \pm 0.04; \qquad z_{tr}^{\mathcal{D}A2} = 0.73 \pm 0.03;$$

$$z_{tr}^{\mathcal{D}A3} = 0.72 \pm 0.03;$$

$$z_{tr}^{\mathcal{D}C1} = 0.98 \pm 0.07; \qquad z_{tr}^{\mathcal{D}C2} = 0.77^{+0.19}_{-0.13}$$

Thus, the transition from the decelerated to the accelerated expansion for all the models is earlier than in the ΛCDM model:

$$z_{tr}^{\Lambda\mathrm{CDM}} = \left(\frac{2(1-\Omega^0_m)}{\Omega^0_m}\right)^{\frac{1}{3}} - 1 = 0.70 \pm 0.02.$$

The only transition point that is not compatible with the ΛCDM value is the one corresponding to $\mathcal{D}C1$.

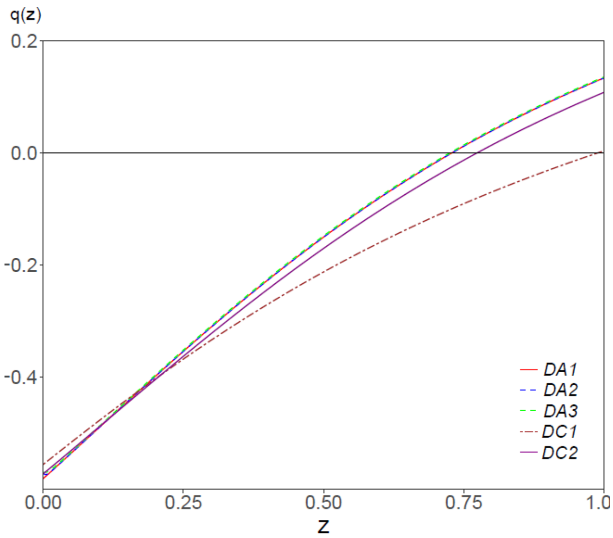

FIG. 3: Deceleration parameter for $\mathcal{D}A$ (15), $\mathcal{D}C1$ (29) and $\mathcal{D}C2$ (24) in terms of the redshift.

In addition, as expected, we can recover the ΛCDM solution for both the EoS and the transition point for the $\mathcal{D}A$ models (setting $\nu = \alpha = 0$) and for $\mathcal{D}C2$ (if we set $\nu = \alpha = 1$).

VI. PHANTOM AND QUINTESSENCE

The explanation for the accelerated expansion of the Universe has been previously attempted in terms of the traditional Quintessence and Phantom models [8, 11]. These models assume the existence of a scalar field with an energy density and pressure given by [11]:

$$\rho_\Phi = \frac{1}{2}\varepsilon\dot{\Phi}^2 + V(\Phi), \qquad p_\Phi = \frac{1}{2}\varepsilon\dot{\Phi}^2 - V(\Phi), \qquad (31)$$

where $V(\Phi)$ is the potential of the field. Notice that for canonical fields, $\varepsilon = 1$. Thus, from (31) the EoS is:

$$w_\Phi = \frac{p_\Phi}{\rho_\Phi} = \frac{\frac{1}{2}\varepsilon\dot{\Phi}^2 - V(\Phi)}{\frac{1}{2}\varepsilon\dot{\Phi}^2 + V(\Phi)} = -1 + \frac{2\varepsilon\dot{\Phi}^2}{\varepsilon\dot{\Phi}^2 + 2V(\Phi)}. \qquad (32)$$

When $-1 < w_\Phi < -\frac{1}{3}$ the models are called Quintessence and when $w_\Phi < -1$ they are called Phantom. From Eq. (32), the value $w_\Phi < -1$ can only be obtained assuming that the scalar field is approaching to an equilibrium ($\dot{\Phi}^2 \ll V(\Phi)$) and also with a negative kinetic energy corresponding to $\varepsilon < 0$.

The Phantom description of the dark energy has another dramatic property: the energy density becomes eventually infinite with the expansion and therefore, the corresponding gravitational repulsion rips every cosmic structure. This scenario is called Big Rip [12].

As we have previously found, for all the $\mathcal{D}A$ models $w_D^0 \lesssim -1$ and for $\mathcal{D}C1$, $w_D^0 \gtrsim -1$. Nevertheless, these results do not imply that $\mathcal{D}A$ models are actually Phantom and that $\mathcal{D}C1$ is Quintessence, since we are not describing dark energy by means of a scalar field.

In fact, for $\mathcal{D}A$ models the density has an upper (finite) limit given by equation (10) when we take $z \to -1$:

$$\lim_{z\to -1} \rho_D(z) = \rho_c^0 \left[\frac{\Omega_D^0 - \nu + \alpha(1 + w_D^0\Omega_D^0)}{1-\nu}\right].$$

VII. CONCLUSIONS

In this work we have considered dynamical dark energy models different from the traditional ones, based on scalar fields [8], and we have found that some of these models describe consistently the observation data.

The $\mathcal{D}C1$ model has not a ΛCDM limit for any value of its unique parameter (ν) and, accordingly, the transition point for this model is not compatible with the ΛCDM result. Furthermore, in [10] it has been proved that $\mathcal{D}C1$ is not able to reproduce the structure formation data.

On the other hand, we recover the ΛCDM values for $\nu = \alpha = 1$ in the $\mathcal{D}C2$ model and, as the best fitted parameters (Table I) are $\nu = 1.030$ and $\alpha = 1.105$ we expect this model to have a behaviour similar to ΛCDM. Nevertheless, more detailed studies [10] prove that this situation is only certain for low redshifts. Thus, although the results obtained in this work are compatible with the ΛCDM values, the $\mathcal{D}C2$ model do not successfully describe the radiation dominated epoch.

The $\mathcal{D}A$ models do not present these problems. Their parameters are small ($|\nu|, |\alpha| \ll 1$) and consequently the EoS and the transition point are both compatible to those corresponding to the ΛCDM model and the cosmological observations. In addition, recent studies [2, 10, 13] have proved that dynamical dark energy models of the type considered here reproduce significantly better the overall cosmological observations (based on supernovae, the baryonic acoustic oscillations, the large scale structure and the cosmic microwave background anisotropies) than the standard model.

Hence, while the $\mathcal{D}C$ models can be discarded to explain the dark energy nature, the $\mathcal{D}A$ models reinforce the possibility that ΛCDM description of the cosmological data might be improved by allowing the dark energy of the Universe to be slightly dynamical.

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