MARKET STRUCTURE, THE FUNCTIONAL FORM OF DEMAND AND THE SENSITIVITY OF THE VERTICAL REACTION FUNCTION

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Tax Systems Analysis
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ABSTRACT: Tax incidence and tax competition have largely been studied separately. Models assessing the incidence of excise taxes do not consider strategic interaction and exclusively assess the pass-through of taxes to prices. These settings focus on imperfectly competitive markets where prices can react more (less) than proportionally to a variation in tax rates. On the other hand, tax competition models focus on the strategic interactions arising because of a shared tax base but assume producer prices to be constant. Hence, the pass-through of taxes is restricted to be fully on consumers. This paper extends Keen (1997) by relaxing this assumption and, thus, by allowing local governments to internalize the possibility that taxes are over-shifted (undershifted). Interestingly, market structure (that was absent in previous settings), turns out to be one of the determinants of the vertical reaction function in this model; particularly determining the sensitivity of local tax setters to a variation of higher-tier taxes.

JEL Codes: H22, H70, D43, L13
Keywords: Vertical tax competition, pass-through, market structure, excise taxes, tax incidence

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1 Introduction

The tax incidence literature is unconcerned about tax competition issues. Indeed, traditional tax incidence models usually consider tax rates to be exogenously determined and focus on the pass-through of taxes to consumer prices without taking the tax setting-process into account.\(^1\) On the other hand, several strands of literature such as optimal taxation and tax competition do focus on the tax setting decision but do not consider tax incidence. Standard tax competition models, for example, consider the strategic tax setting by different levels of government (or different jurisdictions belonging to the same tier of government) sharing the tax base but implicitly assume that the incidence of taxes is fully on consumers by assuming producer prices to be constant.\(^2\) Therefore, the potential under-/overreaction of prices to a variation in tax rates is ruled out.

Interestingly though, both strands of literature do have some common features. The functional form of demand, for instance, plays a key role in both frameworks. In a tax incidence setting, a linear demand function implies undershifting and an iso-elastic demand function results in overshifting of taxes.\(^3\) Similarly, in a vertical tax competition framework, the sign of the reaction function is determined by the curvature of demand. Keen (1997) is among the first to show the importance of the functional form of demand to determine the sign of the vertical reaction function. As it is standard in theoretical tax competition studies, the interpretation of the results goes through the analysis of two special cases: linear and iso-elastic demand functions. Whilst the former implies that taxes are strategic substitutes (i.e., the vertical reaction function is negative), the latter suggests strategic complementarity (i.e., the vertical reaction function is positive). Devereux et al. (2007) extend Keen’s setting by allowing for horizontal competition (introduced by cross-border shopping). The authors show that, in the symmetric case, the horizontal reaction function between two states (\(i\) and \(j\)) is always positive i.e., \(\frac{d t_i}{d t_j} = \frac{d t_j}{d t_i} > 0\). Moreover, even in the presence of individual’s preferences heterogeneity, sufficient conditions for this result to hold are not too strong. More recently, Agrawal (2015) goes one step forward by introducing multiple competing federal governments and, thus, by allowing for diagonal externalities i.e., fiscal externalities between neighboring jurisdictions that are of a different level of government. The author finds that diagonal interactions have the same sign as horizontal ones, but are smaller in magnitude.\(^4\) All these papers assume

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\(^1\)Krzyzaniak and Musgrave (1963) were among the first to suggest that, in oligopolistic and monopolistic markets, taxes could be overshifted to final prices. Katz and Rosen (1985), Seade (1985) and Besley (1989) are among the main theoretical references having taken up their point. For a detailed review on tax incidence literature, see Fullerton and Metcalf (2002).

\(^2\)See, for instance, Keen (1997) and Devereux et al. (2007).

\(^3\)For the sake of correctness, two parameters define the under/overshifting condition in tax incidence models: the functional form of demand and cost functions. Though, as described by Seade (1985) and Besley (1989), under fairly standard assumptions and without too much loss of generality, the tax incidence condition is uniquely determined by the curvature of the demand function.

\(^4\)Other papers explore different aspects of tax competition. Esteller-Moré et al. (2012), for instance, consider a similar setting to the one in this paper i.e., a federation with two layers of government, in which Leviathan policy makers levy excise taxes on a consumption good and that is produced in an imperfectly competitive market. Nevertheless, their focus is on the negative externality produced by the good and,
producer prices to be constant. Hence, the pass-through of taxes is restricted to be fully on consumers and, therefore, the possibility of an overreaction of prices to taxes is ruled out by construction. This is particularly striking because excise taxes are usually levied in oligopolistic, highly-concentrated markets such as those for cigarettes, gasoline or alcohol beverages. As it is well known from tax incidence literature, under imperfectly competitive markets prices can react more (less) than proportionally to a variation in tax rates i.e., taxes can be overshifted (undershifted). In other words, by assuming producer prices to be constant, previous tax competition models constraint taxes to be fully shifted to consumers and, thus, rule out any potential impact coming from tax incidence features.

To sum up, this paper extends the simplest model of tax competition in excise taxes that was set in Keen (1997), where only vertical interactions are considered, by relaxing the assumption that producer prices are constant and, therefore, by explicitly introducing the tax incidence features that were ruled out. By allowing prices to under/overreact to a variation in tax rates, market structure turns out to have a key role in determining the vertical reaction function. The sign of the reaction function (and, thus, the complementarity/substitutability condition) is not modified but, interestingly, the number of firms in the industry has now an impact on the sensitivity of the vertical reaction. In other words, the level of market concentration determines how reactive local taxes are to a variation in higher-tier tax rates.

The rest of the paper is structured as follows. In next section the model is set up and solved backwards. Section 3 derives the vertical reaction function in both settings. First, \( \frac{dt}{dT} \) is derived by constraining taxes to be fully-shifted to prices as in Keen (1997) and, afterwards, without imposing any restriction on the shifting of taxes. Finally, Section 4 provides some concluding remarks.

2 The model

2.1 Setting

This model extends a vertical tax competition model à la Keen (1997) by adding a traditional Cournot setting as the one developed in Tirole (1988) as the second stage of the game. Moreover, given the scope of the paper, an excise tax \( (\tau = t + T) \) is added to the profit function of the firm. While \( t \) is the excise tax rate applied by the local government, \( T \) is the one applied by the higher-tier or federal government. Firms react in the same way no matter the level of government levying the tax.\(^5\)

The goal of the model is to endogenize the (local) tax setting decision by assuming that

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\(^5\) A few empirical papers such as Chouinard and Perloff (2007) and Marion and Muehlegger (2011) assess the differences in the pass-through depending on the level of government levying the tax. Nevertheless, this remains an empirical issue and, as it is standard in theoretical tax incidence models this distinction is not taken into account in this model.
lower-tier policy makers anticipate the reaction of firms to $\tau$ and adjust their decisions in consequence. In other words, local governments take firm’s under/overshifting of taxes into account when setting the tax rates $t$.

There are three different agents in the economy: policy makers (federal and lower-tier governments), producers and consumers.\(^6\) The federal government sets a tax rate $T$ that is assumed exogenous for the rest of the agents. Then, in the first stage of the game, local governments in each state $j = 1, \ldots, S$ play Nash with respect to the federal government and react to $T$ by setting $t_j$ in order to maximize revenue $R_j = t_j X_j$, where $X_j$ is the tax base. There are $i = 1, \ldots, N$ profit-maximizing firms competing à la Cournot by choosing their level of output $q_i$ so that $\sum_{i=1}^{N} q_i = Q$. Finally, following Keen (1997), I characterize consumers’ preferences by the indirect utility function $\nu(P) + \Gamma(g, G)$, where $\nu_i(\cdot)$ and $\Gamma_i(\cdot)$ are strictly concave; $g$ and $G$ are the quantities of local and federal public goods, respectively and $P$ is the consumer price of the taxed good. I assume additivity in $(\cdot)$ in order to assure that the demand for the taxed good, $x(P) = -\nu'(P)$ (by Roy’s identity), is independent of public expenditure.

The model is set as a two-stage game. In the first stage, the tax policy $t$ is determined. In the latter one, firms maximize profits given the tax rates set in the previous stage and the equilibrium is determined. The model is solved backwards.

### 2.2 Backwards solution

#### 2.2.1 Second stage

Firms play a Cournot-Nash game in which they compete by choosing their level of output $q_i$ conditional on the expectations of their competitors’ output levels. Let firm $i$’s profit function be:

$$\pi_i = P(q_i + Q_{-i})q_i - c(q_i) - \tau q_i, \quad (1)$$

where $q_i$ is the level of output of firm $i$, $Q_{-i}$ is the output of all other firms in the industry and $P(Q)$ is the inverse demand function for market demand $Q$. Finally, $c(\cdot)$ is the cost function that is assumed identical for each firm. Indeed, since the model focuses on symmetric equilibria, firms are assumed to be symmetric. Thus,

$$Nq = Q. \quad (2)$$

Hence, subscripts are dropped and Equation (1) is re-expressed as follows:

$$\pi = P(Nq)q - c(q) - \tau q. \quad (1.1)$$

\(^6\)The lower tier of government will be called local or state government throughout the paper.
The first- and second-order conditions for a given firm are as follows:  

\[
\frac{d\pi}{dq} = \frac{dP}{dq} q + P - \frac{dc}{dq} - \tau = 0
\]  

(3)  

\[
\frac{d^2\pi}{dq^2} = \frac{d^2P}{dq^2} q + 2 \frac{dP}{dq} - \frac{d^2c}{dq^2} < 0.
\]  

(4)  

Finally, solving Equation (3) for \( q \), one gets the following equilibrium expression for the firms’ output:  

\[
\hat{q} = \frac{\frac{dc}{dq} + \tau - P}{\frac{dP}{dq}}.
\]  

(5)  

2.2.2 First stage

As it was mentioned above, local tax rates are set in the first stage of the game. As the focus of the paper is on vertical interactions, states are assumed to be symmetric \((t_j = t)\); each consisting of a single representative consumer. In addition, the tax base is assumed to be completely immobile across states. These two assumptions considerably simplify the model. First, by ruling out horizontal competition. Second, by imposing a single consumer one can define the tax base for each state equal to the individual demand of the single consumer living in that jurisdiction i.e., \( X = x(P) \). Indeed, using the equilibrium condition \((X = Q)\), one can define the tax base in each state as \( X = x(P) = Q \).

Local governments are Leviathans and, thus, aim at maximizing revenue \( R = tX = tQ \) by setting \( t \). Therefore, the first- and second-order conditions of the revenue maximizing problem are the following ones:  

\[
\frac{dR}{dt} = Q + t \frac{dQ}{dt} = 0
\]  

(6)  

\[
\frac{d^2R}{dt^2} = 2 \frac{dQ}{dt} + t \frac{d^2Q}{dt^2} < 0.
\]  

(7)  

Now, solving Equation (6) for \( t \) by using the chain rule (i.e., \( \frac{dQ}{dt} = \frac{dQ}{dP} \frac{dP}{dt} \)), one obtains the following expression for the equilibrium tax rate:  

\[
\hat{t} = -\frac{Q}{\frac{dQ}{dP} \frac{dP}{dt}}.
\]  

(8)  

---

\(^7\)Note that tax incidence models usually use a slightly different notation than the one used in this stage of the game. For example, in Equation (3) most of these studies would have used \( \pi' \) to indicate "the derivative of the profit function with respect to \( q' \) rather than \( \frac{d\pi}{dq} \). The reason to use a different notation is simply that this is the one used in tax competition settings and, thus, in the first stage of this model. In other words, given that tax incidence and tax competition models use different notations, I decided to stick to the one used in tax competition settings by adjusting the notation used in this stage of the game.

\(^8\)The second-order condition is assumed to hold.
Finally, Equation (8) can be re-expressed in \textit{ad-valorem} terms as follows:

\[
\frac{\hat{t}}{P} = -\frac{1}{\frac{dQ}{dP} \frac{dP}{dQ} \frac{dQ}{dt}} = \frac{1}{\epsilon \frac{dP}{dt}},
\]

where \(\epsilon = -\frac{dQ}{dP} \frac{dP}{Q} > 0\) is the elasticity of the aggregate demand function.

Traditional vertical competition models assume producer prices to be constant (sometimes they are even normalized to zero). Therefore, consumer prices are given by \(P = \tau = t + T\) and, therefore \(\frac{dP}{d\tau} = \frac{dP}{dt} = 1\). Hence, as expected, by assuming \(\frac{dP}{d\tau} = 1\) (and, thus, \(\frac{dP}{dt} = 1\)), one is back to the standard formula present in previous tax competition settings indicating that the optimal tax rate (in \textit{ad-valorem} terms) is inversely proportional to the elasticity of the tax base.

By relaxing this assumption, this model explicitly allows policy makers to internalize tax incidence features when setting their tax rates. In other words, local governments recognize that taxes are not necessarily fully-passed to consumer prices but can also be under/overshifted and take this into consideration when setting \(t\).

Next section shows how the vertical competition reaction function is modified once there is no constraint on \(\frac{dP}{d\tau}\). The main outcome of the model (i.e., the comparability/substitutability condition) is not modified but, interestingly, market structure turns out to play a crucial role in determining the sensitivity of local tax rates \((t)\) to a variation in \(T\).

3 The vertical reaction function

One of the key features of the first stage of the game is given by the two tiers of government taxing the same good and, thus, sharing the tax base. From a tax competition perspective, the focus is on the strategic interactions arising because of this. In other words, one would like to assess how a variation in \(T\) affects the state’s choice of \(t\).

3.1 The vertical reaction function when \(\frac{dP}{d\tau} = 1\) is imposed

As shown in Section 2 previous tax competition models obtain the standard expression that the tax rate (in \textit{ad-valorem} terms) is inversely proportional to the elasticity of demand i.e., Equation (9) is simplified as follows:

\[
\frac{\hat{t}}{P} = \frac{1}{\epsilon},
\]

Now, the goal of this paper is to assess the vertical reaction function i.e., the reaction of state governments to a variation in the federal tax rate. To say it differently, one would like to know how the equality in Equation (9.1) will be affected by a variation in \(T\) and, particularly, how \(t\) will adjust in order to restore it.

\[9\] See, for example, Keen (1997) and Devereux et al. (2007).
Unfortunately, it is impossible to solve tax competition models (i.e., to sign $\frac{dt}{dT}$) for the general case. Nevertheless, one can illustrate the main outcome of these models through the two most studied cases in the literature: iso-elastic and linear demand functions.

When demand is of the iso-elastic type ($\bar{\epsilon}$), an increase in $T$ will rise the consumer price $P$ and, thus, reduce the left hand side of Equation (9.1). Given that the elasticity of demand is constant, $t$ has to increase in order to restore the equality in Equation (9.1). Thus, $\frac{dt}{dT} > 0$ and taxes are strategic complements.

If demand is of the linear type, Equation (9.1) can be re-expressed as follows:

$$\hat{t} = \frac{1}{-\left(\frac{Q'}{Q}\right)}.$$  

(9.2)

An increase in $T$ will rise the consumer price $P$ and, thus, $Q$ will decrease. Given that the elasticity of demand is linear, the slope of the demand curve is constant ($\bar{Q}'$) and, therefore, the equality in Equation (9.2) can only be restored by decreasing $t$. Thus, in the linear demand/cost case, $\frac{dt}{dT} < 0$ and taxes are strategic substitutes.

In other words, one can summarize the main outcome of previous vertical tax competition studies by the following proposition:

**Proposition 1** The vertical reaction function $\frac{dt}{dT}$ depends on the functional form of the demand function. First, if $x(P) = \bar{x}$ so that individual demand is inelastic, then in the neighborhood of Nash equilibrium, $\frac{dt}{dT} = 0$. Second, in the symmetric case, once the individual demand function is allowed to be elastic, the sign of the reaction function is undefined for the general case. Indeed, the sign of $\frac{dt}{dT}$ depends on the elasticity of the demand function and, interestingly, the two most analyzed cases in the literature have the following implications:

- **If demand is iso-elastic**, $\frac{dt}{dT} > 0$ (t and T are strategic complements)
- **If demand is linear**, $\frac{dt}{dT} < 0$ (t and T are strategic substitutes)

### 3.1.1 Correspondence between $\frac{dP}{dt}$ and $\frac{dt}{dT}$.

Before showing how the tax competition setting is affected by allowing $\frac{dP}{dt} \neq 1$, it is worth highlighting the following feature shared by both tax incidence and tax competition models: the functional form of demand is the key parameter determining the main output in both settings.

Proposition 1 clearly shows how, from a tax competition perspective, the strategic complementarity/substitutability condition depends on the curvature of the demand function. A similar result is obtained by recalling the following condition, that was derived and discussed in previous tax incidence studies such as Fullerton and Metcalf (2002) or
Jametti et al. (2013):

\[
\frac{dP}{d\tau} = \frac{N}{N + (\eta + k)} \begin{cases} 
< 1 & \Rightarrow \text{undershifting} \\
= 1 & \Rightarrow \text{full shifting,} \\
> 1 & \Rightarrow \text{overshifting}
\end{cases}
\] (10)

where \(\eta = Q \left(\frac{d^2P}{dq^2}\right)\) is the elasticity of the slope of the inverse demand function and \(k = 1 - \left(\frac{d^2c}{dq^2}\right)\) measures the relative slopes of the demand and marginal cost curves.

Equation (10) is a well known result in tax incidence literature and has some implications that will be useful for the rest of the paper. First, market structure \((N)\) does not determine tax incidence condition, in equilibrium. Note that, counter-intuitively, even highly concentrated markets may undershift taxes under non-cooperative profit maximization if \((\eta + k) < 0\). Second, market structure does, nevertheless, determine the degree of the tax shifting. Independently of whether taxes are under or overshifted, the absolute value of \(\frac{dP}{d\tau}\) is always the furthest away from one (full shifting) when \(N = 1\), and approaches full shifting as \(N\) tends to infinity (perfect competition).

Moreover, note that if costs are linear \((\frac{d^2P}{dq^2} = 0\) and, thus, \(k = 1\)), a necessary and sufficient condition for taxes to be overshifted is that \(\eta < -1\). If demand is of the constant elasticity type, this is always the case because a demand elasticity \(\epsilon > 0\) implies that \(\eta = -\frac{1+\epsilon}{\epsilon} < -1\) for all \(\epsilon\). In fact, in this case, Equation (10) can be re-expressed as follows:

\[
\frac{dP}{d\tau} = \frac{N}{N - \frac{1+\epsilon}{\epsilon} + 1} > 1.
\] (11)

Thus, in the linear cost/iso-elastic demand case there is always overshifting.

Similarly, with linear costs and a linear demand function \((\eta = 0)\), Equation (10) is simplified as follows:

\[
\frac{dP}{d\tau} = \frac{N}{N + 1} < 1.
\] (12)

Thus, in the linear cost/linear demand case, taxes are always undershifted.

To sum up, there exists a correspondence between the outputs of tax incidence and tax competition models that is illustrated in Table 1 and formalized by the following conjecture:

**Conjecture 1** In the symmetric case, at symmetric Nash equilibrium, the following correspondence between the pass through of taxes to consumer prices and the vertical reaction function arises:

- \(\frac{dP}{d\tau} < 1 \iff \frac{d^c}{d^T} < 0\)
- \(\frac{dP}{d\tau} > 1 \iff \frac{d^c}{d^T} > 0\)

\(^{10}\)Equation (10) can be derived from the first stage of the model. The formal derivation is shown in the Appendix.
Despite this correspondence, these two strands of literature have not been studied in a comprehensive setting. Hence, the formalization of this result is a first contribution of this model to the public finance literature.

3.2 The vertical reaction function when no constraint on \( \frac{dP}{d\tau} \) is imposed

As one can observe from Equation (10), the \( \frac{dP}{d\tau} = 1 \) condition is verified if and only if \( \eta + k = 0 \) or under perfect competition \( (N \rightarrow \infty) \). Assuming either of these two conditions to hold is quite a strong statement that deserves, at least, some comments. First, note that \( \eta + k = 0 \) is far from being the general rule or even from representing the most important cases in the literature. Indeed, under the two most analyzed scenarios given by linear costs \( (k = 1) \) and, either linear \( (\eta = 0) \) or iso-elastic \( (\eta = -\frac{1+\epsilon}{\epsilon}) \) demand functions, \( \eta + k \neq 0 \). Second, it would be even more striking to justify \( \frac{dP}{d\tau} = 1 \) by assuming perfect competition. In particular, because most of these tax competition models focus on excise taxes that are usually levied in highly concentrated industries such as gasoline, cigarette or alcohol beverages markets. In other words, assuming constant producer prices and, thus, imposing \( \frac{dP}{d\tau} = 1 \) considerably simplifies the model. Nevertheless, this seem to be a strong and hardly justifiable assumption.

Now, the crucial question is how the vertical reaction function is modified when no constraint on \( \frac{dP}{d\tau} \) is imposed.

Plugging the tax incidence condition for the iso-elastic case given by Equation (11) in Equation (9), I obtain:

\[
\hat{t} = \frac{1}{\epsilon} \left( 1 - \frac{1}{\epsilon N} \right). \tag{9.3}
\]

An increase in \( T \) will rise the consumer price \( P \) and, thus, reduce the left hand side of Equation (9.3). Given that the elasticity of demand is constant \( (\bar{\epsilon}) \), the only way to restore the equality (in the fixed-\( N \) case) is through an increase in \( t \). Thus, in the iso-elastic demand/linear cost case, \( \frac{dt}{dT} > 0 \) and taxes are strategic complements.

Similarly, in the linear demand case the expression for the equilibrium tax rate is given by Equation (9):

\[
\hat{t} = \frac{1}{\epsilon} \frac{dP}{dt}. \tag{9}
\]

Once again, for the sake of simplicity, one can re-express Equation (9) as follows:

\[
\hat{t} = \frac{1}{-\left(\frac{Q'}{Q}\right)} \frac{dP}{dt}. \tag{9.4}
\]

Plugging the tax incidence condition for the linear demand case given by Equation (12) in Equation (9.4), one gets:

\[
\hat{t} = \frac{1}{-\left(\frac{Q'}{Q}\right)} \frac{N}{N+1}. \tag{9.5}
\]
An increase in $T$ will rise the consumer price $P$ and, thus, $Q$ will decrease. Given that the elasticity of demand is linear, the slope of the demand curve is constant ($\bar{Q}'$) and, therefore, the equality in Equation (9.5) can only be restored by decreasing $t$. Thus, in the linear demand/cost case, $\frac{dt}{dT} < 0$ and taxes are strategic substitutes.

### 3.3 Discussion of results

By comparing equations (9.3) and (9.5) to equations (9.1) and (9.2), respectively, one can already identify the main difference between previous models and this setting. Whereas the number of firms ($N$) was absent in previous tax competition models, it is now explicitly introduced as a determinant of the vertical reaction function. Note that the number of firms ($N$) does not define the sign of the vertical reaction function but determines how reactive local governments are to a variation in the federal tax rate. In other words, $N$ does not modify the complementarity/subsitutability condition (that is exactly the same as the one found in previous models) but defines the sensitivity of $t$ to a variation in $T$. This is the main result of the paper.

A simple numerical application of the linear cost/iso-elastic demand case (based on the U.S. cigarette market) will help the reader by nicely illustrating the main result of the model discussed above. The model is calibrated by plugging the values for $T$, $P$ and $\epsilon$ in order to show how the sensitivity of $t$ to a 1% increase in $T$ varies for different values of $N$. Prices and tax rates figures are taken from Orzechowski and Walker (2012).\footnote{The annual compendium on tobacco revenue and industry statistics known as The Tax Burden on Tobacco is produced by the economic consulting firm Orzechowski and Walker and published by the Federation of Tax Administrators.}

Regarding the price elasticity of cigarette demand, an important variance among the different estimates is observed in the literature. The estimates seem to vary considerably depending, for instance, on the different methodologies and samples considered. As one could expect, the estimates seem to be highly dependent on the target group as well. Different studies focus on particular groups in order to cluster by age, sex, socioeconomic situation, educational attainment or even particular characteristics such as pregnancy.\footnote{For a complete review on the estimates of the price elasticity of cigarette demand, see, Surgeon’s General Report (2000).}

The model is calibrated using the estimates of Ding (2003) and Hana and Chaloupka (2004) by setting an elasticity of demand equal to 1.4.$^{13}$

Figure 1 illustrates the reaction functions when $\frac{dP}{dT} = 1$ is imposed (red-dashed line) and when there is no restriction on the shifting condition (blue-solid line).\footnote{This is an arbitrary choice. Nevertheless, as already discussed throughout the paper, the results of the model are robust and hold for any $\epsilon > 0$.} As one can observe, in both cases $\frac{dt}{dT} > 0$. This reflects the complementarity condition that holds under both frameworks. Now, the straight-dashed line indicates that the reaction function is constant ($\frac{dt}{dT} = 0.1789$) and does not vary with $N$. On the other hand, the solid line shows how, in this setting, the sensitivity of $t$ to a variation in $T$ decreases with the

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\textsuperscript{11}The annual compendium on tobacco revenue and industry statistics known as The Tax Burden on Tobacco is produced by the economic consulting firm Orzechowski and Walker and published by the Federation of Tax Administrators.

\textsuperscript{12}For a complete review on the estimates of the price elasticity of cigarette demand, see, Surgeon’s General Report (2000).

\textsuperscript{13}This is an arbitrary choice. Nevertheless, as already discussed throughout the paper, the results of the model are robust and hold for any $\epsilon > 0$.

\textsuperscript{14}Figure 2 shows the results for the linear cost/linear demand case.
number of firms in the industry. Indeed, note that the highest value of $\frac{dt}{dT}$ equals 0.6261 for the monopoly case and it decreases as $N$ increases. In the limit, when $N \to \infty$, it converges to 0.1789. This makes sense given that under perfect competition $\frac{dP}{dT} = 1$, which turns out to be the assumption made by previous tax competition studies.

To put it differently, in this setting, a 10% increase in $T$ would be followed by a reaction of local governments roughly lying between 6.3% and 1.8%, depending on the level of concentration of the industry. Previous settings, would only consider the lowest bound (1.8%) which might be particularly misleading given the highest level of concentration observed in those industries where excise taxes are levied. For instance, the 2002 economic census published by the U.S. Department of Commerce shows that the largest four companies in the cigarette industry accounted for 95.3% of total shipments.\footnote{www.census.gov/prod/ec02/ec0231sr1.pdf} In such a concentrated industry, one could expect the vertical reaction function to be very sensible and, thus, closer to the highest bound. In other words, unlike previous settings, this model not only allows the federal policy maker to know whether local governments would increase or decrease their tax rates after a variation of the federal tax rate, but also how important this reaction would be.

### 4 Conclusion

This model brings together tax incidence and tax competition, two strands of literature that in spite of several points in common have only been studied separately. In both cases the curvature of the demand function is the key parameter determining the main result of the model. From a tax incidence perspective, taxes are under-, fully, or over-shifted depending on $\eta$, a parameter accounting for the elasticity of the slope of the inverse demand function. Similarly, the elasticity of the demand function determines the complementarity/substitutability condition in a tax competition framework. Merging these two frameworks in a comprehensive setting and formalizing the existence correspondence between their main outputs is a nice contribution to public finance literature per se.

Nevertheless, the main contribution concerns market structure and its impact on the vertical reaction function $\frac{dt}{dT}$. This model extends a classical vertical tax competition model by allowing local governments to internalize the possibility that taxes are over-/undershifted. In order to do this, the assumption that producer prices are constant is explicitly relaxed. This assumption (standard in previous tax competition settings) implicitly constrains taxes to be fully-passed to consumers and, thus, rules out the impact of any tax incidence feature from the tax setting decision.

To put it differently, by relaxing the assumption that producer prices are constant, consumer prices are allowed to under/overreact to a variation of tax rates and, hence, tax incidence features are internalized into the tax setting process. As a consequence, market structure now plays a key role on the tax setting decision. In particular, it turns out to be one of the determinants of the vertical reaction function. Interestingly, even if the number of firms in the industry does not modify the strategic complementarity/substitutability condition from previous settings, it determines the sensitivity of local tax rates ($t$) to a
variation in higher-level tax rates ($T$). This is a crucial piece of information, with potential relevant policy implications, that was absent in previous tax competition models.
Appendix. Derivation of the tax incidence condition

Following the standard notation in tax incidence models that has been first defined in Seade (1980), one can rewrite the second-order condition in Equation (4) as follows:

\[
\frac{d}{dN} \left( \frac{dP}{dq} \right) (\eta + N + Nk) < 0, \tag{A.1}
\]

where \( \eta = Q \left( \frac{d^2 P}{dq^2} \right) \) is the elasticity of the slope of the inverse demand function and \( k = 1 - \left( \frac{d^2 c}{dq^2} \right) \) measures the relative slopes of the demand and marginal cost curves.

Note that since \( \frac{dP}{dq} < 0 \), \( \eta + N + Nk > 0 \) is necessary and sufficient for the second-order condition to hold.

Now, using this notation, one can differentiate Equation (3) to get:

\[
\frac{dq}{d\tau} = \frac{1}{\frac{dP}{dq} (\eta + N + k)}. \tag{A.2}
\]

Thus,

\[
\frac{dQ}{d\tau} = \frac{N}{\frac{dP}{dq} (\eta + N + k)} \tag{A.3}
\]

and, therefore,

\[
\frac{dP}{d\tau} = \frac{dP}{dq} \frac{dQ}{d\tau} = \frac{N}{N + \eta + k}. \tag{A.4}
\]

The standard tax incidence condition previously shown in Equation (10) is directly derived from Equation (A.4):

\[
\frac{dP}{d\tau} = \frac{N}{N + (\eta + k)} \begin{cases} < 1 \Rightarrow \text{undershifting} \\ = 1 \Rightarrow \text{full shifting} \\ > 1 \Rightarrow \text{overshifting} \end{cases}
\]
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Table 1: Correspondence: the vertical reaction function and the tax incidence condition

<table>
<thead>
<tr>
<th>Tax incidence</th>
<th>Tax competition</th>
<th>Strategic substitutes ($\frac{dt}{dP} &lt; 0$)</th>
<th>Strategic complements ($\frac{dt}{dP} &gt; 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undershifting</td>
<td>$(\frac{dP}{dP} &lt; 1)$</td>
<td>√</td>
<td>Not possible</td>
</tr>
<tr>
<td>Overshifting</td>
<td>$(\frac{dP}{dP} &gt; 1)$</td>
<td>Not possible</td>
<td>√</td>
</tr>
</tbody>
</table>

Figure 1: Sensitivity of the vertical reaction function (Iso-elastic demand)
Figure 2: Sensitivity of the vertical reaction function (Linear demand)
2011/1, Oppedisano, V; Turati, G.: “What are the causes of educational inequalities and of their evolution over time in Europe? Evidence from PISA”
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