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The Economics of Fishing: Intertemporal Equilibria in a Two-Country Transboundary Resource

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Abstract

For the problem of two-country fisheries sharing a common resource we calculate the pre-commitment solutions and address the time consistent ones. We analyse different initial states in order to obtain the optimal paths to reach a steady state. The two-agent problem is determine by the efficiency of each country and their discount rate, different levels of efficiency and discounting are set in order to find their relative consequences between both players. We find that the optimal path is depending on the other country behaviour, and that in the long run the final state may be different than the initial. The results of the model implementation may be extended to other type of renewable resources. This study is providing new insights in the two-player equilibria for the case of two agents transboundary fishing resource that may be used by policy makers facing fisheries management.

JEL Classification:

C62, C71, D90, Q01, Q22

Keywords:

Fisheries Management, Bioeconomic Modelling, Renewable resources, Time inconsistency, Heterogeneous Discount Rate.

Contents

1 Introduction

In what extent is it possible to find an optimal time-consistent management for a renewable resource shared by two countries in the context of different discount rates, costs, and harvesting efforts? This question is developed for the case of the fisheries. In many regions of the world fish stocks are overfished, the fishing harvests are not sustainable. United Nations identifies fisheries as a critical environmental stock (Ekeland et al., 2015). Overfishing may be biological, i.e., fish are harvested at a high rate and the biomass has a negative growth. Currently is estimated that more than 30% of the commercial fishing incurs in biological overfishing (FAO, 2016). On the other hand overfishing may be economical; economic overfishing refers to the case in which the resource of the rent is not maximised; the causes of overfishing are normally related to failures of fisheries management (Beddington et al., 2007). Fisheries are a wealth source for many nations since they give direct employment in processing, fishing, and ancillary services accounting more than US$ 220 billion annually (Dyck and Sumaila, 2010). Fish-based food provide more than 3.0 Billion people with 15% of their animal protein needs, when it is included the post-catch work places and workers' dependants, fisheries represent the 8% world population family sustenance (Ekeland et al., 2015).

Fisheries economics and resource economics can be understood in capital-theory terms, the biomass or fish population may be represented as the stock of capital, and it would yield a sustainable consumption flow through the time. As with capital, the current consumption decision has an impact in the stock level and it will have an impact in the future consumption decisions and stock levels.

Fisheries management have to deal with many difficulties, the fish stocks are often managed as a common pool resource. Anyone can use the resource and the harvesting of one user is diminishing the stock of the resource available for the other users (Ostrom et al., 1999). Moreover the harvest levels are usually over the sustainable levels due to the search for short-term benefits and the management does not respond to an holistic ecosystem interests (Botsford, 1997) neither to long-term maximum benefits.

The particular nature of fish stocks present difficulties in terms of management because the resource is 'transboundary', it implies that the resource is crossing across frontiers usually between two countries, the policy adopted by one country is affecting the other and vice versa. The complexity arises because there is no reason for both countries to have the same policy towards the resource, if both have different economical situations, implying that the social discount rate will not coincide, one country will be definitely more impatient at the time of extracting the resource. Moreover, differences in technology, population, priorities, culture, etc. May affect the willingness of one country to put a certain amount of effort in extract the resource, hence the effort and the harvest rates should also differ.

It is assumed that each country is sovereign over their seas and the management policy over fisheries is a matter for each social manager and for the fisheries that operate over transboundary populations, the objective is to maximise their own country benefits from the fisheries. There is no reason to believe that two countries are going to have the same policy and therefore the same appreciation of the social discount rate, on the contrary it is expected that they are going to differ in their objectives. Conflicts in management of the resource are likely to appear as a consequence of the aforementioned differences, the resolution of conflict stories involves establishing strategies that maximise the joint benefit. Although the natural framework for most economic problems is to assume that agents compete among each other, in some models (international trade agreements, environmental economics...) is necessary to look for mechanisms that introduce cooperation among the different economic agents.

1.1 Problem Formulation

The problem of fish resources has been widely analysed during the last fifty years (Clark, 2010). The transboundary dimension aggregated to the discounting variance between two countries may be analysed using mathematical modelling. The problem of shared renewable resources lies in the main characteristic of stock conservation, and the extraction agreements have to be properly studied. This problem is interesting because fishing wild resources are usually transboundary, therefore in regions where political conditions are proper to made agreements those

must be reached. Nowadays fishing resources are changing those biological behaviours due to climate reasons, for example, water temperature changes. In the economic side, each country has its own characteristics (capital availability, technology sources, labour force...), moreover these characteristics are dynamic over time, which implies the necessity of the continuous planning over time horizon.

To solve the problem we have embedded a classical fishery model into a two-player game setting on a infinite time T horizon. Following the application made by Munro (1979) of the Maximum Principle we obtain the optimal fish biomass stock (x^*) and the corresponding optimal harvesting rate ($h^*(t)$), the solutions are characterised for different cases in which different parameters are considered over time (harvest, discount rate, share of harvesting) as control or exogenous variables. We focus on the two-player game structure, but it is still remaining to analyse the case of 3 or more (N) agents as well as the case of some parameters (as price, maximum stock, catchability...) varying over time.

1.2 Limitations

The study of the two-player approach is limited to very specific cases, many fishing areas are bounded by more than two countries, at the same time, the transboundary resources may involve more than one player. The analysis is also limited by the simplification of some variables as the vessel size or the efficiency of it. Some variables are time varying at different rates and the consideration of this variance/non-variance is an implicit limitation of the results depending each case.

Real data may be also be considered in order to contrast the theoretical approach, but this type of analysis is beyond this study.

1.3 Structure of the Report

The rest of the report is structured as follows. Section 2 is a review of the relevant literature concerning fishery economics on game theory and discounting analysis for the multi-player setting, the section 3 is an explanation of the model that gives a basic intuition of their main aspects, explaining briefly what are the main variables involved, the dynamics of conservation

and the capital theory equivalence. Sections 4 and 5 shows the contributions of this work. First the Pre-commitment solution extended to a general case, and secondly, in the section 5 the Time-Consistent solution is briefly treated. Section 6 shows briefly some numerical simulations. Section 7 shows a discussion of the obtained results and the conclusion derived from them. Finally the appendices are showing the detailed calculus procedures.

2 Literature Survey

The initial attempts to deal with the problem of common access resource economics were at the 1950's. The foundational paper on the theory of fishery economics is attributed to Gordon (1954). In a similar approach Scott (1955) struggled with the problem of the fisheries sole ownership and the problem of the fishery management understood in terms of capital theory. The most extended bioeconmical model was introduced by Schaefer (1957) in a well-known dynamic model which is still currently used by policy makers and Multilateral organisations as the FAO (2016).

Fishing Models

Different models of fishing in the existing literature can be divided in two main fields. The first group is the one who deals with the Open access and sole ownership while the second one meets the Game-Theoretic models. In terms of the former, economic studies have to deal with the principal characteristic of the fisheries: their common property nature. Gordon's first analysis (Gordon, 1954) was extended by Hannesson (1993) dealing with the political economy of fishing regulations. They treated the common property characteristic which is directly related with the open access and the right bounds to the fishery (i.e. a review (Bjorndal, 1992) of the social planners and the open access equilibrium outcomes pointed out the inefficiencies of open access exploitation). Other analyses previously published in fisheries economics (Christy and Scott, 1965; Plourde, 1971; Smith, 1969) have focused on the types of property rights (full rights and no rights). Each system of rights provides a single 'Nash non-cooperative outcome', the sole ownership for the full-rights system and the open access for the no rights system.

The easier solution in terms of management is the open access, but it is wasteful (the aforementioned 'Tragedy of the commons'). On the other hand the sole ownership equilibrium has excellent properties of efficiency, but its implementation is technically complicated due to the constant threat of new participants in the fishery. Game-Theoretic models have been developed as the combination of biological models and Nash's solution concepts in terms of the age of the fish at the time to be harvested and the relationship between parent stock and recruitment (Reed, 1980).

Cooperative and non-cooperative management

John Nash stated the first distinction between cooperative and non-cooperative games (Nash, 1953). His classification consisted in separate the games where agreements are not feasible to be non-cooperative and those where agreements are feasible, cooperative games. In the field of fisheries economics have been used both types of games in order to analyse the optimal use of the resource. The common way is to analyse what happen with the biology and the economics of the fishery in the cases of cooperation and non-cooperation aiming to isolate the negative effects of non-cooperation (Levhari and Mirman, 1980). For the case of cooperative management, a well-known paper (Munro, 1979) merges cooperative game theory with the basic economic model of the fishery. He shows that when the cooperative game is unconstrained the way to reach the optimal joint harvest demand is: the patient player should buy to her impatient partner entirely when the program starts and manage the resource as it were singled owned (Munro, 1991a).

An applied computational game-theoretic model (Sumaila, 1997) is developed showing that the so-named *optimum optimorum* is obtained under cooperation with side payments and no predetermined harvest shares and confirming the previous theoretical results of Munro (1979). A case of bargaining applied to the Southern Blue-fin Tuna was analysed (Krawczyk and Tolwinski, 1993) for three countries exploiting a fishery (Australia, Japan, and New Zealand) using multi-cohort bio-models to find the optimal time-depending quotas. Another solution (Kennedy and Watkins, 1986) model the problem as two-agent with linear dynamics.

Transboundary Stock problems

According to Munro (1996) Transboundary Stock Fish may be classified into groups according to their migratory nature. The first one is the stock of fish that migrates between Exclusive Economic Zones (EEZ) of two or more countries (this is the “transboundary” in strict terms). The second group is composed of highly migratory stocks (e.g. tuna). The third group receipt the name of "straddling-fishing-stock" naming the stocks that are migrating among two national territories and international waters. The management of transboundary resources has been treated more formally by McRae and Munro (1989) and Sumaila (1997). Transboundary problem for most migratory species (tuna and salmon) in the North Atlantic is analysed by Munro (1991b) while Arnason (1991) claims that fisheries models for transboundary stocks must be non-autonomous.

The vessels influence on the transboundary stock management has been also analysed (in the context of a two-player non-cooperative game model) finding the optimal number of vessels to employ in the exploitation of the Arcto-Norwegian cod stock (Sumaila, 1995). A three-agent game theoretic-bioeconomic model of the Pacific sardine transboundary stock (Ishimura et al., 2013) points out that cooperative management is necessary -in the context of climate change scenario- to achieve sustainable fisheries.

International conflicts arise when the straddling-fish-stock is harvested by more than one country. When the transboundary resource is found in two countries EEZs the *shared* management was analysed by (Kaitala and Munro, 1993) showing that the non-cooperative theory developed for the study of transboundary resources also applies to straddling stocks.

Intertemporal and Climate Issues

When different agents can have communication and they are able to take common decisions in order to improve their collective payoff, cooperative decisions can be introduced. In this way Breton and Keoula (2012) present a study on coalition formation and coalition stability in resource economics. Then the application of intertemporal analysis is useful and necessary. Most

economic models assume that agents compete among each other, but game-theoretic models of shared stock treat also cooperation to find joint optimal solutions. It is normal to assume that all agents have the same rate of time preferences but there is no reason to believe that the utility streams are the same for consumers, companies and countries. Hence, the study time-consistent equilibria outcomes in fisheries has shown a wide variety of results (e.g. Ekeland et al. (2015)) shows how a non-constant discounting embedded into an overlapping generations model yield many different equilibria).

The management of fisheries in the long rung requires to take into account the climate shifts. A work on climate change fisheries implications shows how distribution of species may change worldwide (Cheung et al., 2009). Tropical populations may move towards north and northern populations can suffer from the temperature increase, affecting both southern and northern fisheries and producing economic losses. The study of deviations (Brandt and Kronbak, 2010) from the agreed cooperative solutions for the cod stock in the Baltic Sea shows that future payoffs to the fishery can decrease due to the climate change. Then, it would be reasonable to assume that similar conclusions can result for the case of domestic and internationally shared fish stocks. The warmer coastal waters in Canada and the United States have meant that salmon populations have increased in the territories of Canada and decreased in southern waters in California, Washington and Oregon, implying that a cooperative agreement can not be carried out and resulting in non-cooperative behaviours (Miller and Munro, 2004)

Different discounting schemes open new possibilities for the analysis and are an important piece of the debates for the policy design. For example, debates about global warming and stock pollutants are very sensitive to discounting. In general the relation between equilibrium environmental policy and discounting is model-dependent. Contributions to this debate include (Stern, 2007), (Nordhaus, 2007), and (Dasgupta, 2008).

3 The Model

A renewable resource exploitation model is described. This model is often used in fisheries and by policy makers (Clark, 2006). We develop this study whithin a dynamic resource-harvesting framework. The implications of the model's predictions are extendable to other resources.
This section is organised as follows: section 3.1 describes the general intuition followed by the model and section 3.2 shows the basic dynamic optimisation model (Munro, 1979).

3.1 General Intuition

The estimations and predictions of the different possible outcomes in fisheries subject to different conditions can be done by mathematical modelling and the use of dynamic systems. The fish is considered as a destructible renewable stock resource, it means that the first use of the resource implies its permanent lost (the fish itself), but the general resource will grow over the time. Thus, a second characteristic is the self-generation. This regeneration has a natural rate, which depends on the stock and the environmental conditions. The central point of the fishery resources is the stock and the biomass dynamics of a fish stock.
The essential problem of stock in resource economics in general and fishery economics in particular is a problem of *intertemporal allocation* or simply the problem of *the amount of stock that should be extracted today and the amount that should be kept for the future*.

The bioeconomical model includes the biological properties and the economic implications. First the stock dynamic $\dot{x}$, is defined as the natural growth rate minus the harvesting rate. The discounted net cash flow for the country-fisheries (PV) is composed by the discount rate, the *profit* (price and cost difference) and the harvesting rate over the time $h(t)$.
The maximum principle application give us the optimal equilibrium biomass x^* and the optimal harvesting rate $h^*(t)$. Analysing different possible scenarios between two players is possible to obtain the optimal outcomes for both. The results show the optimal behaviours that each player, in our case, each country, should have to maximise the joint revenues.

3.2 Optimization Model

To consider a fishery resource management model a single fishery is initially considered. The basic bioeconomical model emerges from the canonical fishery Shaefer model (Schaefer, 1957). The model comes from the Pearl-Verhulst, or logistic, equation of population dynamics (Clark and Munro, 1975). The stock of the fish is evolving according to the state equation:

$$\frac{dx(t)}{dt} = F(x) - h(t) \tag{1}$$

where $h(t)$ is the harvest rate, $x(t)$[1] is the biomass of the fish and $F(x)$ is the natural growth function, the Shaefer model describes it as: $F(x) = rx(1 - x/K)$, where r is constant and represents the intrinsic growth rate and K is the maximum biomass, i.e. $\lim_{t\to\infty} x(t) = K$. The planner choose the harvest, $h(t) = qE(t)x(t)$, where q is the "catchability" coefficient and $E(t)$ is the rate of fishing effort at time t.

The harvest functions are specific for each country and assumed to be equal to the consumption, hence to keep the focus on the realistic cases, we assume that the harvest is bounded between 0 and a maximum level, $\bar{h} < \infty <$. The demand for the fish and the effort input supply functions are assumed as infinitely elastic.

The basic problem for the society is to establish the optimum consumption/harvest path along the time aiming to maximise the social utility. The flow of payoff is $U(t) = (p - c(x_t))h(t)$, for a constant discount rate δ. The unit cost of harvest is $c(x)$, which is a decreasing convex function, and for a constant price p, the objective of each social planner is to maximise the discounted present value (ref e.g. Munro (1979).)

$$PV = \int_0^T e^{-\delta t}(p - c(x))h(t)dt \tag{2}$$

Subject to

$$\frac{dx(t)}{dt} = F(x) - h(t)$$

[1] In other way $\frac{dx(t)}{dt}$ can be interpreted as the rate of investment. (Clark, 2010)

$$0 \leq h(t) \leq \bar{h} \leq \infty$$

$$0 \leq x(t)$$

The fishing effort cost is denoted by $C(E)$, and we assume that $C(E) = aE$ where a is the constant cost of each unit. Furthermore, following the harvest form $E = h/qx$, the total harvest cost is $C(x,h) = ah/qx$, while the unitary cost is $c(x) = a/qx$. The objective functional is linear in the control variable $h(t)$, then it is a linear optimal control problem. The Hamiltonian of the general problem is:

$$\mathscr{H} = e^{-\delta t}(p - c(x))h(t)) + \lambda(F(x) - h(t)) \tag{3}$$

The co-estate variable is λ, which can be interpreted as the shadow price of the resource discounted to the time zero. The Hamiltonian can also be written as $\mathscr{H} = [e^{-\delta t}(p - c(x)) - \lambda]h(t) + \lambda F(x)$ where the switching function is given by:

$$\sigma(x,t) = e^{-\delta t}(p - c(x)) - \lambda \tag{4}$$

Applying the maximum principle we can obtain the optimal biomass x^* and then the optimal equilibrium harvest rate h^*. The golden rule for the equilibrium can be written as[2]:

$$\delta = F'(x^*) - \frac{c'(x^*)F(x^*)}{p - c(x^*)} \tag{5}$$

where the LHS is the 'own interest rate' of the biomass, which consist of the instantaneous marginal product of the resource $F'(x)$ and the so called marginal stock effect (Clark and Munro, 1975). The golden rule is indicating that the optimal level of biomass is the level at which the social rate of discount is equal to the own rate of interest of the resource. Then the equilibrium harvest is given by: $h^*(t) = F(x^*)$.

The objective functional is linear in the control variable $h(t)$, hence the optimal approach path is given by a Bang-Bang approach:

[2]Complete derivation is developed at A.1

$$\begin{cases} h(t) = \bar{h}, & whenever\, x(t) > x^* \\ h(t) = 0, & whenever\, x(t) < x^* \end{cases}$$

This implies that non-harvesting is optimal whenever $x(t) < x^*$ and the maximum harvest is optimum whenever $x(t) > x^*$. The reason is that when the stock of the resource is greater than the optimal level (x^*) the object would be to harvest until reach the steady state of the stock, and in the other hand, when $x(t) < x^*$ harvest to let the resource recover to the steady state level.

When the x^* level is reached, the fishing should be developed at a sustainable-yield basis, any deviation from this stability will result in an investment or disinvestment and hence the stock will suffer a deviation from the optimal level. When instead of one player, two players are sharing the exploitation of the resource, the result is the same if they have identical effort costs and discount rate (Munro, 1979).

4 Pre-commitment Solutions

Heterogeneous Discounting and Pre-commitment Solutions in a Cooperative Setting

Regarding the case of two countries where the social planners have different conditions to set a discount rate, let us define δ_1 for the country 1 and δ_2 for the country 2. Hence, we can assume $\delta_1 \leq \delta_2 \leq \infty$. Also consider that each country has their fishing costs $c_1(x) \neq c_2(x)$, due to different labour costs, available technology, etc. With respect to the harvesting effort, it is considered unique for each country. That can be argued due to the difference in the fleets, the availability, etc. For the case of the prices one can consider a common market for both countries then they are subject to an international price.

Assumption 1: Lets call efficiency ε. When $c_1(x) \neq c_2(x)$, if $c_i < c_j \Rightarrow c_i \equiv \varepsilon \ \forall i, j \in \{1,2\}$. c_i is more efficient country .

Assumption 2: When $\delta_1 \neq \delta_2$, if $\delta_i > \delta_j \Rightarrow \delta_i \equiv \varepsilon \ \forall i, j \in \{1,2\}$. c_i is more efficient country.

It can be seen that the most conservationist country will be that country that has a lower discount rate. In our case, country 1 is the more conservationist. In this sense both countries are not willing to extract the resource at the same rate. When the biomass of the country 1 is at a level $x^*_{\delta 1}$ and the level for country 2 is $x^*_{\delta 2}$ if $\delta_1 < \delta_2$ we have that $x^*_{\delta 1} \geq x^*_{\delta 2}$. Facing this differences in the resource exploitation it is natural to think that both countries will prefer to sign a binding agreement. The central point into the negotiation of the agreement is the harvesting share. The imposition of a time variant harvest share -$\alpha(t)$- or a time invariant -α- will depend on the exogenous considerations for the modelling.

Since we are working into a dynamic framework, one should argue that shares may be dynamic and vary over the time when we are looking for the optimal time path for the shares. However Munro (1979) shows how some time-variant harvest shares result in extreme cases of optimal policies: a partner can be long time periods without receiving returns from the resource. Hence, we can suggest that enduring agreements are difficult to reach. Moreover, due to political or historical agreements between the two countries the harvesting shares can be fixed for long periods, so the time invariant shares are reasonable. We analyse both cases.

4.1 Exogenous α, the generalised case:

Let's suppose that harvest shares are externally imposed and therefore time invariant. In a two-country scheme we can denote that the share for the Country 1 is $0 < \alpha < 1$ and thus the objective functional for each country is:

Country 1:

$$PV_1 = \int_0^\infty e^{-\delta_1 t}(p - c_1(x))h_1(t)dt \tag{6}$$

Where, $h_1(t) = \alpha h(t)$

Country 2:

$$PV_2 = \int_0^\infty e^{-\delta_2 t}(p - c_2(x))h_2(t)dt \tag{7}$$

Where, $h_2(t) = (1-\alpha)h(t)$

Assuming that the management agreement is fixed and it is not possible to make side payments between the two countries, an equal weight would be given for both countries to the manage-

ment preferences. Then, the joint objective functional may be expressed as:

$$PV = \int_0^\infty e^{-\delta_1 t}(p - c_1(x))h_1(t) + e^{-\delta_2 t}(p - c_2(x))h_2(t)dt \tag{8}$$

The corresponding Hamiltonian of the problem is expressed as:

$$\mathscr{H} = (e^{-\delta_1 t}(p - c_1(x))\alpha + e^{-\delta_2 t}(p - c_2(x))(1-\alpha))h(t) + \lambda(F(x) - h(t)) \tag{9}$$

The application of the maximum principle allows us to determine the optimal equilibrium biomass x^* and also the optimal harvesting rate $h^*(t)$.

Proposition 1: The golden rule for the equilibrium is characterised by [3]:

$$\frac{\delta_1 e^{-\delta_1 t}(p - c_1(x))\alpha + \delta_2 e^{-\delta_2 t}(p - c_2(x))(1-\alpha)}{e^{-\delta_1 t}(p - c_1(x))\alpha + e^{-\delta_2 t}(p - c_2(x))(1-\alpha)} = F'(x) - \frac{(e^{-\delta_1 t}c_1'(x)\alpha + e^{-\delta_2 t}c_2'(x)(1-\alpha))F(x)}{e^{-\delta_1 t}(p - c_1(x))\alpha + e^{-\delta_2 t}(p - c_2(x))(1-\alpha)} \tag{10}$$

The result shows a complex weighted average of the discounts rates at the left hand side, and the right hand side is a complex version of the so.-called marginal stock effect (Clark and Munro, 1975). Because the model is linear, the optimal approach path is the faster one or 'bang-bang'. Then, the payoff would give to the high-discount-rate country (in this case Country 2) a major participation in the time close to the present and give dominant preferences to the Country 1 in the future.

4.2 Endogenous α

Let's have a view about the aforementioned case when α is time-variant, and taking into account that the marginal stock effect is significant and side payment cannot be done. For this case $\alpha(t)$ is a control variable rather than a parameter. Then, the maximization of the Hamiltonian with respect to α is:

$$\frac{\partial \mathscr{H}}{\partial \alpha} = h(t)[e^{-\delta_1 t}(p - c_1(x)) - e^{-\delta_2 t}(p - c_2(x))] \tag{11}$$

[3] The entire derivation of the general case is shown in the appendix section A.5, since other cases has been studied (appendix section A.2,A.3 and A.4) those are more restricted cases with similar outcomes) we choose to develop the most general case

Now one can notice that the result of $\partial\mathscr{H}/\partial\alpha$ will be positive (or negative) depending if the inequality in the following equation holds (or reverse):

$$e^{-\delta_1 t}(p - c_1(x)) - e^{-\delta_2 t}(p - c_2(x)) > 0 \tag{12}$$

In the simplified analysis, when the discount rate is the same for both countries, the maximisation of the Hamiltonian in the case of time-variance of α shows that it is depending on the *marginal rent ratio*. In our case, if $\delta_1 = \delta_2$ and if $c_2(x) > c_1(x)$ then $\alpha = 1$. In general α is oscillating between 0 and 1 depending on what cost function is higher in function of x. If $\delta_1 < \delta_2$ we face two possibilities. First, if $c_2(x) > c_1(x)$ then $e^{-\delta_1 t}(p - c_1(x) > e^{-\delta_2 t}(p - c_2(x) \Longrightarrow \alpha = 1$. In the other hand if $c_2(x) < c_1(x)$ then $0 \leq \alpha \leq 1$ and this case is analized in section 4.3.

4.3 Endogenous h_1, h_2

The case of different harvesting functions presents a wide variety of results. This case is relevant because now we face many different situations where the optimal path can be derived. The objective function for each country is the same, where $h_1(t) \neq h_2(t)$ and are not dependent to each other.

The Hamiltonian is now defined as:

$$\mathscr{H} = (e^{-\delta_1 t}(p - c_1(x)) - \lambda)h_1(t) + (e^{-\delta_2 t}(p - c_2(x)) - \lambda)h_2(t) + \lambda F(x) \tag{13}$$

In addition, the expression 13 can be rewritten isolating h1 and h2:

$$\mathscr{H} = \underbrace{(e^{-\delta_1 t}(p - c_1(x)) - \lambda)}_{\sigma_1(x,t)}h_1(t) + \underbrace{(e^{-\delta_2 t}(p - c_2(x)) - \lambda)}_{\sigma_2(x,t)}h_2(t) + \lambda F(x) \tag{14}$$

Notice that $h1$, $h2$, x and λ are functions of time. It is now clear that there are two control variables $h1$ and $h2$. Those are treated as usual control variables with $0 \leq h_1 \leq \bar{h}$ and $0 \leq h_2 \leq \bar{h}$ where $\bar{h} = qE_{max}x$. Since the switching functions are expressed as:

$$\sigma_1(x,t) = e^{-\delta_1 t}(p - c_1(x)) - \lambda \tag{15}$$

$$\sigma_2(x,t) = e^{-\delta_2 t}(p - c_2(x)) - \lambda \tag{16}$$

The maximization condition says that the optimal control variables ($h1$ and $h2$) maximize the Hamiltonian over $0 \leq h_1 \leq \bar{h}$ and $0 \leq h_2 \leq \bar{h}$. Since the Hamiltonian is linear in control we obtain:

$$h(t) \begin{cases} h_1(t) + h_2(t) = \bar{h} & if\ \sigma_1(x,t) > 0\ and\ \sigma_2(x,t) > 0 \\ h_1(t) + h_2(t) = F(x^*) & if\ \sigma_1(x,t) = 0\ and\ \sigma_2(x,t) = 0 \\ h_1(t) + h_2(t) = 0 & if\ \sigma_1(x,t) < 0\ and\ \sigma_2(x,t) < 0 \end{cases} \tag{17}$$

Assumption 3: For both countries, $h_k(t) \geq h_j(t) + h_j(t) \Rightarrow h_k(t) \leq \bar{h_{i,j}}\ \forall i, j \in \{1,2\}$.

Since cases for $\sigma_1(x,t) > 0$ and $\sigma_2(x,t) > 0$ and for $\sigma_1(x,t) < 0$ and $\sigma_2(x,t) < 0$ are clear, we are going to analyse the rest of the cases.

Definition of the additional cases: Expressions $\sigma_1(x,t)$ and $\sigma_2(x,t)$ have to be maximised to find the most rapid approach to the optimal level. To treat the additional cases we organise the possibilities in function of the costs:

- **Case 1**: $c_1(x)0 = c_2(x) = c$
- **Case 2**: $c_1(x)0 < c_2(x)$
- **Case 3**: $c_1(x)0 > c_2(x)$

From here we have that:

Case 1:

$$\left.\begin{aligned} \sigma_1(x,t) &= e^{-\delta_1 t}(p - c(x)) - \lambda \\ \sigma_2(x,t) &= e^{-\delta_2 t}(p - c(x)) - \lambda \end{aligned}\right\} \pm\sigma_1 \equiv \pm\sigma_2 \tag{18}$$

Then, the last expression is telling us that both equations are positive or negative at the same time, hence we can state that:

$$\begin{cases} if \quad \sigma_1(x,t) > 0 \implies h_1(t) = h_2(t) = \bar{h} \\ if \quad \sigma_1(x,t) < 0 \implies h_1(t) = h_2(t) = 0 \\ if \quad \sigma_1(x,t) = 0 \implies \zeta \end{cases} \tag{19}$$

From 19 ζ refers to the case when both countries are at the stationary state (SS), here we can see

that both are extracting to maintain the optimal level (S.S).

Case 2: $c_1(x) < c_2(x)$

$$\sigma_1(x,t) = e^{-\delta_1 t}(p - c(x)) - \lambda \qquad > \qquad \sigma_2(x,t) = e^{-\delta_2 t}(p - c(x)) - \lambda \tag{20}$$

- First we have that if

$$\sigma_1, \sigma_2 > 0 \Longrightarrow h(t) = \bar{h}$$

  For this case is important t remark that if both countries are extracting, the problem is bounded to: $h_1(t) \leq \bar{h}$ and $h_2(t) \leq \bar{h}$ hence,

$$h_1(t) + h_2(t) = 2\bar{h}$$

- If $\sigma_1(x,t) > 0$, $\sigma_2(x,t) < 0$ the Country 1 extracts $h_1(t) = \bar{h}$, and $h_2(t) = 0$ reducing the stock.

- If $\sigma_1(x,t) > 0$, $\sigma_2(x,t) = 0$ the Country 1 extracts $h_1(t) = \bar{h}$, reducing the stock while $\sigma_2(x,t) = 0$ will be possible during a very short time (instantaneous), then this case is mathematically possible but not relevant.

- Finally, if $\sigma_1(x,t) = 0$, $\sigma_2(x,t) < 0$ the Country 1 extracts $h_1(t) \in (0, \bar{h})$, in order to maintain the SS. Here is important to **remark** that the maximum harvest $\bar{h}$ of one player is enough to reduce the stock, it is:

$$F(x) - \bar{h} < 0$$

Case 3: $c_1(x) > c_2(x)$

For this case we can analyse the evolution of both factor relative to each other, for this case is clear that never can happen that:

$$\sigma_1(x,t) > 0 \qquad \wedge \qquad \sigma_2(x,t) < 0$$

Then it can never be the case in which $h_1(t) = \bar{h}$ and $h_2(t) = 0$.

In the other hand, initially in $t = 0$, since $c_1(x_0) > c_2(x_0) \implies \sigma_1(x_0) < \sigma_2(x_0)$. Therefore, if

$\sigma_1(x_0) > 0$, $h_1(0) = h_2(0) = \bar{h}$, the extraction of the resource is at the maximum rate. But if $\sigma_2(x_0) = 0$, $h_1(0) = h_2(0) = 0$, and any of both is extracting the resource.

In the case that in $t = 0$ we were in the stationary state then Country 2, for sure, is going to extract the resource. But in the long run, if $\sigma_1(x,t) < 0$ to high values of t, $t = T$, then $h_1(T) = 0$ and only Country two is extracting the resource (to reach the SS or maintain it). But in other case that in the long run $\sigma_1(x,T) > 0$, we can see that $\sigma_1(x,t)$ is decreasing slowly (if it is positive) than $\sigma_2(x,t)$, because $\delta_1 < \delta_2$ so in the long run we will have different results than the initial state. In fact, for the SS always $h_1^*(t) > 0$ and it is also possible to say that

$$h_1(t) > 0 \implies \sigma_1(x,t) \geq 0 \implies \sigma_2(x,t) \geq 0 \implies h_1(t) \geq 0$$

Hence, while we are not in the SS, we would have that: $\sigma_1(x) \neq 0$ and $\sigma_2(x) \neq 0$ and $h_1(t) = h_2(t) = \bar{h}$.

Finally it is necessary to mention some observations that emerge from the analysis and may be object of further study:

- He have assumed that Country 1 is always more efficient than Country 2 ($c_i(x) > c_j(x) \forall x$).
- Results may be illustrated easily using constant cost functions, but it would restrict the study. Perhaps results would be too unrealistic.
- A very valuable extension would be to analyse cost decreasing functions, by the form $c_i(x) = \frac{a_i}{x}$, outcomes are not trivial and numerical simulations would be valid.

5 Time-Consistent Solutions

Heterogeneous Discounting and Time-Consistent Solutions in a Cooperative Setting

Heterogeneous discounting problems were treated by Marín-Solano and Patxot (2012) in which different discount factors are analysed along the plan horizon (consumption) and the final function. Afterwards De-Paz et al. (2013) proved the case for N players in a Time-Consistent setting.

The main problem for two countries may be expressed as:

$$J(h(\cdot)) = \lambda_1 \int_t^T e^{-\delta_1(s-t)} U^1(x(s), h(s), s)\, ds + \lambda_2 \int_t^T e^{-\delta_2(s-t)} U^2(x(s), h(s), s)\, ds \tag{21}$$

subject to: $\dot{x} = f(x(s), h(s), s), \qquad x(t) = x_t$

Hence, the Time-Consistent solution is obtained by solving the dynamic programming equation (DPE):

$$\begin{aligned} &\delta_1 W^1(x,t) + \delta_2 W^2(x,t) - \nabla_t W^1(x,t) - \nabla_t W^2(x,t) \\ &= \max_{\{h\}} \Big\{ \lambda_1 U^1(x,h,s) + \lambda_2 U^2(x,h,s) + \nabla_x W^1(x,t) \cdot f(x,h,t) + \nabla_x W^2(x,t) \cdot f(x,h,t) \Big\} \end{aligned} \tag{22}$$

with $W^1(x,T) = 0$ and $W^2(x,T) = 0$ and

$$W^1(x,t) = \lambda_1 \int_t^T e^{-\delta_1(s-t)} U^1(x(s), \phi(x(s),s), s)\, ds \tag{23}$$

$$W^2(x,t) = \lambda_2 \int_t^T e^{-\delta_2(s-t)} U^2(x(s), \phi(x(s),s), s)\, ds \tag{24}$$

It has been shown (Marín-Solano and Patxot, 2012) that the value function $W(x,t)$ is continuously differential function in (x,t) and the Time-Consistent solution is obtained by solving the DPE, and for each pair (x,t) there exists $h^* = \phi(x,t)$ with the corresponding state trajectory s.t. h^* maximizes the RHS of equation 22. The derived decision rule h^* is the Time-Consistent Markov perfect equilibrium, and $W^1(x,t), W^2(x,t)$ are the value functions for Countries 1 and 2 respectively in the cooperative problem. Finally, $U^{1,2}(x,s) = U^{1,2}(x(s), \phi(x(s),s), s)$ and $x(s)$ is the solution[4] to $\dot{x}(s) = f(x, \phi(x,s), s)$ with $x(t) = x$. Hence, in order to determine the Time-Consistent solution we set the folowing form to illustrate the joint payoff:

$$\begin{aligned} J_1 + J_2 = J^c = &\int_t^{t+\tau} e^{-\delta_1(s-t)} (p - c_1(x)) h_1(s)\, ds + \int_t^{t+\tau} e^{-\delta_2(s-t)} (p - c_2(x)) h_2(s)\, ds \\ &+ \int_{t+\tau}^{\infty} e^{-\delta_1(s-t)} (p - c_1(x)) h_1(s)\, ds + \int_{t+\tau}^{\infty} e^{-\delta_2(s-t)} (p - c_2(x)) h_2(s)\, ds \end{aligned} \tag{25}$$

[4] see De-Paz et al. (2013) to have the detailed description of the maximisation problem and the generalised results for the logarithmic problem

subject to: $\dot{x} = F(x) - h_1(t) - h_2(t)$ with value functions W_1 and W_2, then the solution to the DPE can be expressed as:

$$\delta_1 W_1 + \delta_2 W_2 = \max_{h_1(s),h_2(s)} \{(p - c_1(x))h_1(s) + (p - c_2(x))h_2(s) + (W_1' + W_2') \cdot (f(x) - h_1(s) - h_2(s))\} \quad (26)$$

$$= \max_{h_1(s),h_2(s)} \{h_1(s)(p - c_1(x) - [W_1' + W_2']) + h_2(s)(p - c_2(x) - [W_1' + W_2']) + [W_1' + W_2']f(x)\}$$

5.1 Exogenous α

We are going to analyse the case when α is exogenous, so notice that $h_1(s) = \alpha h(s)$ and $h_2(s) = (1-\alpha)h(s)$. Therefore, given the last section, it is straightforward to formulate the solution to the DPE for this scenario:

$$\delta_1 W_1 + \delta_2 W_2 = \max_{h(s)} \left\{ (p - c_1(x))\alpha h(s) + (p - c_2(x))(1-\alpha)h(s) + (W_1' + W_2') \cdot (f(x) - \alpha h(s) - (1-\alpha)h(s)) \right\} \quad (27)$$

Maximising with respect to α we have three possible optimal solutions:

- **Case 1**: $p - \alpha c_1(x) - (1-\alpha)c_2 - W_1' - W_2' > 0 \implies h(t) = \bar{h}$
- **Case 2**: $p - \alpha c_1(x) - (1-\alpha)c_2 - W_1' - W_2' < 0 \implies h(t) = 0$
- **Case 3**: $p - \alpha c_1(x) - (1-\alpha)c_2 - W_1' - W_2' > 0 \implies h(t) = h^*(t)$

6 Predictions

Numerical simulations are essential to illustrate the model outcomes under different circumstances. Here we illustrate the evolution of the stock in four different scenarios. We choose illustrative values for the initial state of the resource $x(0) = 100$ where the country start to catch under $x^*(t)$ and $x(0) = 500$ where the country is initially catching above $x^*(t)$. The harvesting rate is $h = 60$ for the maximum level and $h = 0$ for the minimum. Discount rates are simulated for 0.03%, 0.05% and 0.08%. The K (maximum biomass) is set as 1000 and the growth rate for the population is fixed a 0.02, finally for the costs are given values of: 10, 50 and 100. The next

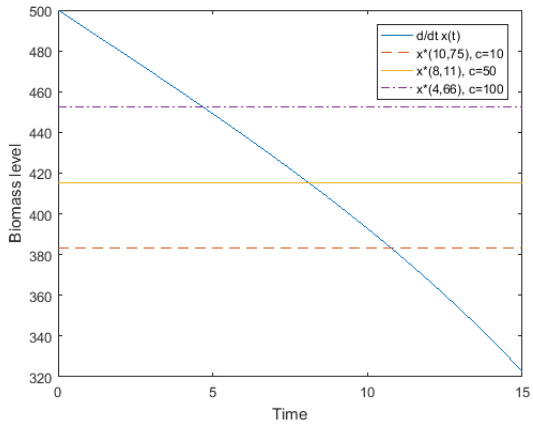

Figure 1: Different cost scenarios

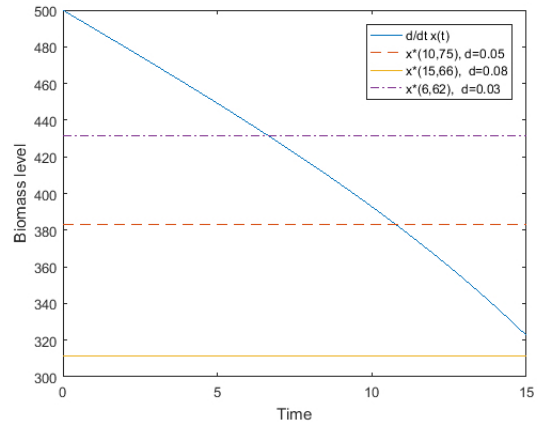

Figure 2: Different discount rate scenarios

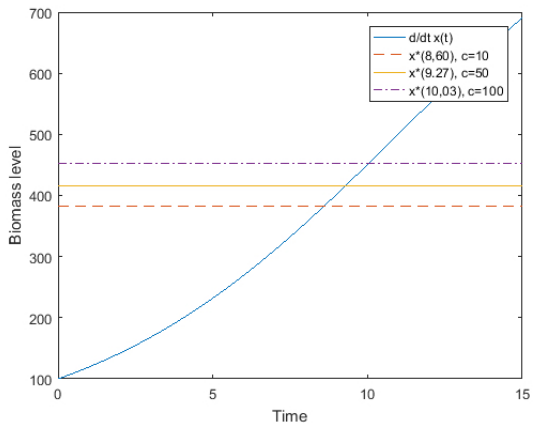

Figure 3: Different cost scenarios

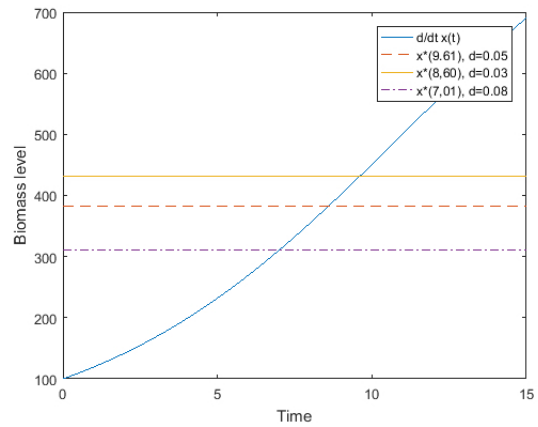

Figure 4: Different discount rate scenarios

figures show the evolution of the resource and the moment when it reaches the optimal level.

Figures 1 and 2 show the case of a country starting above the optimal resource stock. For both cases, the countries start when the stock level is higher that the optimal (500) and the negative evolution (linear in control) is represented. Figure 1 has a fixed $\delta = 0.05$ while figure 2 is for fixed costs $c = 10$. The cost varying simulation show that when a country is facing higher extraction costs, it reaches the optimal level faster than when lower costs. In the other hand, when discounting is heterogeneous we can see that for a higher discount rate, the country reaches the optimal level later than when the discount is smaller. For the opposite case, when the country is facing that the resource is below the optimal level, the desired policy is to harvest at the

minimum level, and let the resource recover to the optimal. In figures 3 and 4 we follow the same parameters for 1 and 2 but this time the initial harvest is below the optimal (100). We can see that the dynamics are the opposite of the previous case. In the different cost scenario, the country with lower costs is reaching first the optimal level, while the lower discount rate has the higher time until the optimal level.

7 Conclusions and Recommendations for Further Work

7.1 Conclusions

This paper shows the equilibrium results derived for the case of two countries sharing a renewable resource, with different levels of efficiency and heterogeneous discount rates. We have contributed to the current literature extending the two-agents existent solutions, in the case of the pre-commitment solution to heterogeneous discounting, dynamic costs and harvest rates. Moreover, using the insights derived by De-Paz et al. (2013) we have proposed the characterising of the Time-Consistent solution.

When two agents are sharing a renewable resource and the optimal strategy is to cooperate we have that the strategy can be treated using the maximum principle. After restricting the problem in some biological parameters (growth rate and maximum biomass) we have obtained the optimal equilibrium biomass (OEB) level and the path to reach it. We find that the most rapid approach to the equilibrium depend on the costs and the discount rate, and depending on the initial conditions it will be reached more rapidly. Also in the long run, for some feasible cases, the final outcome would result in a different stationary state than in the beginning. The most rapid approach is determined by the maximum harvest rate when the biomass level is above the OEB and zero in the opposite case. When both countries are interacting, the optimal policy is to let the most efficient country to harvest alone. The interaction between both countries is determined by the initial decision to cooperate, the extraction paths allow both countries to maximise their benefits taking into account the other country policy.

This paper permits to add more clues about the role of game theory and bio-economic mod-

elling to the renewable resource policy design, we are giving more insights for the cooperation design in order to contribute to the conservation and sustainability of resources and in particular the management of transboundary fishing resources.

7.2 Discussion and Recommendations for Further Work

This paper has shown some results under the light of existing literature, but this results are tied to the accomplishment of some assumptions, thus we have to be careful at the moment to interpret it. Different assumptions may be simplified improving results facing a real policy design. Also we have to pay special attention to the behaviour of the countries when they face opposite starting points and the optimally of the suggested solution of harvest at zero level, while the other country is still harvesting.

Then, further work may address the issue of the optimal solution for the case of opposite initial levels as well as the extension of the Time-consistent solution aiming to determine viable cooperative solution outcomes. In this way we have to remark that there are great opportunities to make use of game theoretic and intertemporal approaches to solve fishery management problems. The use of modern techniques and computational resources may result in notable extraction improvements guaranteeing actual and future generation welfare.

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A Appendix

A.1 Appendix A:

Derivation of the initial case, a single owner, optimal equilibrium biomass x^*:

$$\underset{h(t)}{maximize}\, PV = \int_0^\infty e^{-\delta t}(p - c(x))h(t)dt$$

$$s.t. \dot{x} = F(x) - h(t)$$

$$x(0) = x_0$$

$$x(t) \geq 0$$

$$h(t) = q(t)E(t)x(t)$$

$$0 \leq E(t) \leq E_{max}$$

$$\mathscr{H} = e^{-\delta t}(p - c(x))h(t)) + \lambda(F(x) - h(t))$$

$$\mathscr{H} = [e^{-\delta t}(p - c(x)) - \lambda]h(t) + \lambda F(x)$$

$$\sigma(x,t) = e^{-\delta t}(p - c(x)) - \lambda$$

from $\sigma = 0$:

$$\lambda = e^{-\delta t}(p - c(x))$$

$$\frac{d\lambda(t)}{dt} = e^{-dt}(-\delta)(p - c(x)) + e^{-dt}[-c'(x)\frac{dx}{dt}]$$

$$\frac{d\lambda(t)}{dt} = e^{-\delta t}[-\delta(p - c(x)) - c'(x)\dot{x}]$$

$\dot{\lambda} = e^{-\delta t}[-\delta(p - c(x)) - c'(x)(F(x) - h(t))]$

Over this interval also:

$-\frac{\partial \mathscr{H}}{\partial x} = -[-e^{-\delta t} h(t)c'(x) + \lambda F'(x)]$

$-\frac{\partial \mathscr{H}}{\partial x} = e^{-\delta t} h(t)c'(x) - \lambda F'(x)$

$-\frac{\partial \mathscr{H}}{\partial x} = e^{-\delta t} h(t)c'(x) - e^{-\delta t}(p - c(x))F'(x)$

$-\frac{\partial \mathscr{H}}{\partial x} = \dot{\lambda}$

$\dot{\lambda} = e^{-\delta t}[h(t)c'(x) - (p - c(x))F'(x)]$

equating $\dot{\lambda}$

$e^{-\delta t}[-\delta(p - c(x)) - c'(x)(F(x) - h(t))] = e^{-\delta t}[h(t)c'(x) - (p - c(x))F'(x)]$

$-\delta(p - c(x)) - c'(x)(F(x) - h(t)) = h(t)c'(x) - (p - c(x))F'(x)$

$-\delta(p - c(x)) - c'(x)F(x) + h(t)c'(x) = h(t)c'(x) - (p - c(x))F'(x)$

$-\delta(p - c(x)) - c'(x)F(x) = -(p - c(x))F'(x)$

$-\delta(p - c(x)) = -(p - c(x))F'(x) + c'(x)F(x)$

$-\delta = \frac{-(p-c(x))F'(x)+c'(x)F(x)}{(p-c(x))}$

$\delta = F'(x) - \frac{c'(x)F(x)}{p-c(x)}$

A.2 Appendix B:

Same cost $c_1(x) = c_2(x)$ for different harvesting functions, $h_1(t) \neq h_2(t)$:

$\underset{h_1(t),h_2(t)}{maximize} PV = \int_0^\infty e^{-\delta t}(p - c(x))[h_1(t) + h_2(t)]dt$

$\mathscr{H} = e^{-\delta t}(p - c(x))[h_1(t) + h_2(t)] + \lambda(F(x) - [h_1(t) + h_2(t)])$

$\mathscr{H} = [e^{-\delta t}(p - c(x)) - \lambda][h_1(t) + h_2(t)] + \lambda F(x)$

$\sigma(x,t) = e^{-\delta t}(p - c(x)) - \lambda$

$\lambda = e^{-\delta t}(p - c(x))$

$\frac{d\lambda(t)}{dt} = e^{-dt}(-\delta)(p - c(x)) + e^{-dt}[-c'(x)\frac{dx}{dt}]$

$\frac{d\lambda(t)}{dt} = e^{-\delta t}[-\delta(p - c(x)) - c'(x)\dot{x}]$

$\dot{\lambda} = e^{-\delta t}[-\delta(p - c(x)) - c'(x)(F(x) - [h_1(t) + h_2(t)])]$

Over this interval also:

$-\frac{\partial \mathscr{H}}{\partial x} = -[-e^{-\delta t}[h_1(t) + h_2(t)]c'(x) + \lambda F'(x)]$

$-\frac{\partial \mathscr{H}}{\partial x} = e^{-\delta t}[h_1(t) + h_2(t)]c'(x) - \lambda F'(x)$

$-\frac{\partial \mathscr{H}}{\partial x} = e^{-\delta t}[h_1(t) + h_2(t)]c'(x) - e^{-\delta t}(p - c(x))F'(x)$

$-\frac{\partial \mathscr{H}}{\partial x} = \dot{\lambda}$

$\dot{\lambda} = e^{-\delta t}[[h_1(t) + h_2(t)]c'(x) - (p - c(x))F'(x)]$

equating $\dot{\lambda}$

$$e^{-\delta t}[-\delta(p-c(x))-c'(x)(F(x)-[h_1(t)+h_2(t)])] = e^{-\delta t}[[h_1(t)+h_2(t)]c'(x)-(p-c(x))F'(x)]$$

$$-\delta(p-c(x))-c'(x)(F(x)-[h_1(t)+h_2(t)]) = [h_1(t)+h_2(t)]c'(x)-(p-c(x))F'(x)$$

$$-\delta(p-c(x))-c'(x)F(x)+[h_1(t)+h_2(t)]c'(x) = [h_1(t)+h_2(t)]c'(x)-(p-c(x))F'(x)$$

$$-\delta(p-c(x))-c'(x)F(x) = -(p-c(x))F'(x)$$

$$-\delta(p-c(x)) = -(p-c(x))F'(x)+c'(x)F(x)$$

$$-\delta = \frac{-(p-c(x))F'(x)+c'(x)F(x)}{(p-c(x))}$$

$$\delta = F'(x)-\frac{c'(x)F(x)}{p-c(x)}$$

A.3 Appendix C:

Different costs, $c_1(x) \neq c_2(x)$ and different harvesting functions, $h_1(t) \neq h_2(t)$:

$$\underset{h_1(t),h_2(t)}{maximize}\, PV = \int_0^\infty e^{-\delta t}[(p-c_1(x))h_1(t)+(p-c_2(x))h_2(t)]dt$$

$$s.t.\, \dot{x} = F(x)-[h(t)]$$

where $h(t) = h_1(t)+h_2(t)$ and:

$$h_1(t) = \alpha h(t)$$

$$h_2(t) = (1-\alpha)h(t)$$

$$\mathscr{H} = e^{-\delta t}[(p-c_1(x))\alpha h(t)+(p-c_2(x))(1-\alpha)h(t)]+\lambda(F(x)-h(t))$$

$$\mathscr{H} = e^{-\delta t}[(p - c_1(x))\alpha + (p - c_2(x))(1-\alpha)]h(t) - \lambda h(t) + \lambda F(x)$$

$$\mathscr{H} = (e^{-\delta t}[(p - c_1(x))\alpha + (p - c_2(x))(1-\alpha)] - \lambda)h(t) + \lambda F(x)$$

$$\sigma(x,t) = e^{-\delta t}[(p - c_1(x))\alpha + (p - c_2(x))(1-\alpha)] - \lambda$$

$$\lambda = e^{-\delta t}[(p - c_1(x))\alpha + (p - c_2(x))(1-\alpha)]$$

$$\frac{d\lambda(t)}{dt} = e^{-dt}(-\delta)[(p - c_1(x))\alpha + (p - c_2(x))(1-\alpha)] + e^{-dt}[-c_1'(x)\dot{x}\alpha - c_2'(x)\dot{x}(1-\alpha)]$$

$$\dot{\lambda} = e^{-dt}\{-\delta[(p - c_1(x))\alpha + (p - c_2(x))(1-\alpha)] + [-c_1'(x)\dot{x}\alpha - c_2'(x)\dot{x}(1-\alpha)]\}$$

$$\dot{\lambda} = e^{-dt}\{-\delta[(p - c_1(x))\alpha + (p - c_2(x))(1-\alpha)] + \dot{x}[-c_1'(x)\alpha - c_2'(x)(1-\alpha)]\}$$

$$\dot{\lambda} = e^{-dt}\{-\delta[(p - c_1(x))\alpha + (p - c_2(x))(1-\alpha)] + (F(x) - [h(t)])[-c_1'(x)\alpha - c_2'(x)(1-\alpha)]\}$$

Over this interval also:

$$-\frac{\partial \mathscr{H}}{\partial x} = -[e^{-\delta t}[-c_1'(x)\alpha - c_2'(x)(1-\alpha)]h(t) + \lambda F'(x)]$$

$$\dot{\lambda} = -e^{-\delta t}[-c_1'(x)\alpha - c_2'(x)(1-\alpha)]h(t) - \lambda F'(x)$$

$$\dot{\lambda} = -e^{-\delta t}[-c_1'(x)\alpha - c_2'(x)(1-\alpha)]h(t) - e^{-\delta t}[(p - c_1(x))\alpha + (p - c_2(x))(1-\alpha)]F'(x)$$

$$\dot{\lambda} = -e^{-\delta t}([-c_1'(x)\alpha - c_2'(x)(1-\alpha)]h(t) + [(p - c_1(x))\alpha + (p - c_2(x))(1-\alpha)]F'(x))$$

$$\dot{\lambda} = e^{-\delta t}([c_1'(x)\alpha + c_2'(x)(1-\alpha)]h(t) - [(p - c_1(x))\alpha + (p - c_2(x))(1-\alpha)]F'(x))$$

equating $\dot{\lambda}$

$e^{-dt}\{-\delta[(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)]+(F(x)-[h(t)])[-c_1'(x)\alpha-c_2'(x)(1-\alpha)]\}=e^{-\delta t}([c_1'(x)\alpha+c_2'(x)(1-\alpha)]h(t)-[(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)]F'(x))$

$-\delta[(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)]+(F(x)-[h(t)])[-c_1'(x)\alpha-c_2'(x)(1-\alpha)]=[c_1'(x)\alpha+c_2'(x)(1-\alpha)]h(t)-[(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)]F'(x)$

$-\delta[(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)]+F(x)[-c_1'(x)\alpha-c_2'(x)(1-\alpha)]-[h(t)][-c_1'(x)\alpha-c_2'(x)(1-\alpha)]=[c_1'(x)\alpha+c_2'(x)(1-\alpha)]h(t)-[(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)]F'(x)$

$-\delta[(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)]+F(x)[-c_1'(x)\alpha-c_2'(x)(1-\alpha)]+h(t)[c_1'(x)\alpha+c_2'(x)(1-\alpha)]=[c_1'(x)\alpha+c_2'(x)(1-\alpha)]h(t)-[(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)]F'(x)$

$-\delta[(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)]+F(x)[-c_1'(x)\alpha-c_2'(x)(1-\alpha)]=-[(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)]F'(x)$

$-\delta[(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)]=-F(x)[-c_1'(x)\alpha-c_2'(x)(1-\alpha)]-[(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)]F'(x)$

$\delta[(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)]=-F(x)[c_1'(x)\alpha+c_2'(x)(1-\alpha)]+[(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)]F'(x)$

$\delta=F'(x)-\frac{F(x)[c_1'(x)\alpha+c_2'(x)(1-\alpha)]}{(p-c_1(x))\alpha+(p-c_2(x))(1-\alpha)}$

A.4 Appendix D:

Different, $\delta_1 \neq \delta_2$, and different harvesting functions, $h_1(t) \neq h_2(t)$:

$\underset{h_1(t),h_2(t)}{maximize}\, PV=\int_0^\infty e^{-\delta_1 t}(p-c(x))h_1(t)+e^{-\delta_2 t}(p-c(x))h_2(t)dt$

$s.t.\,\dot{x}=F(x)-[h(t)]$

where $h(t) = h_1(t) + h_2(t)$ and:

$$h_1(t) = \alpha h(t)$$

$$h_2(t) = (1-\alpha)h(t)$$

$$\mathscr{H} = e^{-\delta_1 t}(p - c(x))\alpha h(t) + e^{-\delta_2 t}(p - c(x))(1-\alpha)h(t) + \lambda(F(x) - h(t))$$

$$\mathscr{H} = (e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(p - c(x))h(t) + \lambda(F(x) - h(t))$$

$$\mathscr{H} = (e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(p - c(x))h(t) - \lambda h(t) + \lambda F(x)$$

$$\mathscr{H} = [(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(p - c(x)) - \lambda]h(t) + \lambda F(x)$$

$$\sigma(x,t) = (e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(p - c(x)) - \lambda$$

When $\sigma(x,t) = 0$

$$\lambda = (e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(p - c(x))$$

$$\frac{d\lambda(t)}{dt} = (-\delta_1 e^{-\delta_1 t}\alpha - \delta_2 e^{-\delta_2 t}(1-\alpha))(p - c(x)) + (e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(-c'(x)\dot{x})$$

Over this interval also:

$$-\frac{\partial \mathscr{H}}{\partial x} = -[-c'(x)(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))h(t) + \lambda F'(x)]$$

$$\dot{\lambda} = c'(x)(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))h(t) - \lambda F'(x)$$

equating $\dot{\lambda}$

$$c'(x)(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))h(t) - \lambda F'(x) = (-\delta_1 e^{-\delta_1 t}\alpha - \delta_2 e^{-\delta_2 t}(1-\alpha))(p - c(x)) + (e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(-c'(x)\dot{x})$$

$$c'(x)(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))h(t) - \lambda F'(x) = (-\delta_1 e^{-\delta_1 t}\alpha - \delta_2 e^{-\delta_2 t}(1-\alpha))(p - c(x)) - c'(x)(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(F(x) - h(t))$$

$$c'(x)(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))h(t) - \lambda F'(x) = (-\delta_1 e^{-\delta_1 t}\alpha - \delta_2 e^{-\delta_2 t}(1-\alpha))(p - c(x)) - c'(x)(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))F(x) + c'(x)(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))h(t)$$

$$-\lambda F'(x) = (-\delta_1 e^{-\delta_1 t}\alpha - \delta_2 e^{-\delta_2 t}(1-\alpha))(p - c(x)) - c'(x)(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))F(x)$$

$$-(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(p - c(x))F'(x) = (-\delta_1 e^{-\delta_1 t}\alpha - \delta_2 e^{-\delta_2 t}(1-\alpha))(p - c(x)) - c'(x)(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))F(x)$$

$$-(-\delta_1 e^{-\delta_1 t}\alpha - \delta_2 e^{-\delta_2 t}(1-\alpha))(p - c(x)) = (e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(p - c(x))F'(x) - c'(x)(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))F(x)$$

Dividing both sides by: $(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(p - c(x))$

$$\frac{-(-\delta_1 e^{-\delta_1 t}\alpha - \delta_2 e^{-\delta_2 t}(1-\alpha))(p - c(x))}{(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(p - c(x))} = \frac{(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(p - c(x))F'(x)}{(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(p - c(x))} - \frac{c'(x)(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))F(x)}{(e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha))(p - c(x))}$$

$$\frac{\delta_1 e^{-\delta_1 t}\alpha + \delta_2 e^{-\delta_2 t}(1-\alpha)}{e^{-\delta_1 t}\alpha + e^{-\delta_2 t}(1-\alpha)} = F'(x) - \frac{c'(x)F(x)}{p - c(x)}$$

A.5 Appendix E:

Now we develop the most general case: different discount rates $\delta_1 \neq \delta_2$, different costs $c_1(x) \neq c_2(x)$ and different harvesting functions, $h_1(t) \neq h_2(t)$.

$$\underset{h_1(t),h_2(t)}{maximize}\, PV = \int_0^\infty e^{-\delta_1 t}(p - c_1(x))h_1(t) + e^{-\delta_2 t}(p - c_2(x))h_2(t)\,dt$$

$s.t. \dot{x} = F(x) - h(t)$

where $h(t) = h_1(t) + h_2(t)$ and:

$h_1(t) = \alpha h(t)$

$h_2(t) = (1-\alpha)h(t)$

$\mathscr{H} = e^{-\delta_1 t}(p - c_1(x))\alpha h(t) + e^{-\delta_2 t}(p - c_2(x))(1-\alpha)h(t) + \lambda(F(x) - h_1(t) - h_2(t))$

$\mathscr{H} = (e^{-\delta_1 t}(p - c_1(x))\alpha + e^{-\delta_2 t}(p - c_2(x))(1-\alpha))h(t) + \lambda(F(x) - h(t))$

$\mathscr{H} = [e^{-\delta_1 t}(p - c_1(x))\alpha + e^{-\delta_2 t}(p - c_2(x))(1-\alpha) - \lambda]h(t) + \lambda F(x)$

notice that $\sigma(x,t)$ is multiplying $h(t)$

$\sigma(x,t) = e^{-\delta_1 t}(p - c_1(x))\alpha + e^{-\delta_2 t}(p - c_2(x))(1-\alpha) - \lambda$

When $\sigma = 0$, $\lambda = e^{-\delta_1 t}(p - c_1(x))\alpha + e^{-\delta_2 t}(p - c_2(x))(1-\alpha)$

$\frac{d\lambda(t)}{dt} = -\delta_1 e^{-\delta_1 t}(p - c_1(x))\alpha - e^{-\delta_1 t}c_1'(x)\dot{x}\alpha - \delta_2 e^{-\delta_2 t}(p - c_2(x))(1-\alpha) - e^{-\delta_2 t}c_2'(x)\dot{x}(1-\alpha)$

$-\frac{\partial \mathscr{H}}{\partial x} = \dot{\lambda} = -[(-e^{-\delta_1 t}c_1'(x)\alpha - e^{-\delta_2 t}c_2'(x)(1-\alpha))h(t) + \lambda F'(x)]$

$\dot{\lambda} = (e^{-\delta_1 t}c_1'(x)\alpha + e^{-\delta_2 t}c_2'(x)(1-\alpha))h(t) - \lambda F'(x)$

In this case α is endogenous:

$\frac{\partial \mathscr{H}}{\alpha} = h(t)[e^{-\delta_1 t}(p - c_1(x)) - e^{-\delta_2 t}(p - c_2(x))]$

From $\frac{d\lambda(t)}{dt} = \dot{\lambda}$ and following the same procedures as before we can find the equilibria:

$$\frac{\delta_1 e^{-\delta_1 t}(p - c_1(x))\alpha + \delta_2 e^{-\delta_2 t}(p - c_2(x))(1-\alpha)}{e^{-\delta_1 t}(p - c_1(x))\alpha + e^{-\delta_2 t}(p - c_2(x))(1-\alpha)} = F'(x) - \frac{(e^{-\delta_1 t} c_1'(x)\alpha + e^{-\delta_2 t} c_2'(x)(1-\alpha))F(x)}{e^{-\delta_1 t}(p - c_1(x))\alpha + e^{-\delta_2 t}(p - c_2(x))(1-\alpha)}$$