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ABSTRACT: We study the link between the voters' support for publicly financed education and the retirement system. We show that, when the pension system includes a redistributive component, the support for publicly financed education increases the greater the degree of redistribution provided by the system. Furthermore, support also increases the greater the proportion of income that finances pensions (i.e., the pension system's generosity). This occurs because adults adjust their behaviour in accordance with the returns of the education of the young on their future pensions.

JEL Codes: D7, H31, H42, H44 H52, H55, I22

Keywords: Public education, pension system, voting, OLG

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* We are grateful to M. Bordignon, D. Checchi, H. Cremer, Ph. De Donder, M. Gilli, G. Glomm, T. Krieger, I. Iturbe-Ormaetxe, M. Justman, P. Natale, as well as to seminar participants at the universities of Alicante, Barcelona, Bicocca-Milan, and Toulouse. D.Montolio acknowledges financial support from the Generalitat de Catalunya (grant 2009SGR102). A. Piolatto acknowledges financial support from IVIE, the Spanish Ministry of Science and Innovation (grant ECO2012-37131), and the Generalitat de Catalunya (grant 2009SGR102).
1 Introduction

The recent financial crisis has increased the concerns regarding public debt, with constant calls being made to reduce country deficits and to reform welfare systems. Education and pensions are two publicly financed (or provided) private goods that are often used for inter- and intra-generational redistribution, and, as such, public education and retirement pensions constitute major components of the annual budget: for example, within the European Union, education in 2005 (pre-crises estimates) accounted for as much as 8.3% of GDP (Denmark), with a mean rate of 5.75% for the EU-15 (European Commission 2005), while OECD data for the same year show that mean EU-15 expenditure on retirement pensions was 6.61% of GDP. Today, most western governments are contemplating reforms to both their education and the pension systems. However, voter preferences cannot be ignored by elected politicians: both pensions and education come under very close scrutiny, which limits the policy space of politicians with re-election concerns.

Our paper contributes to the rich literature on the political economy of publicly provided goods, by jointly examining voter concerns for the provision of public education and pensions. We investigate the relation between spending in education and pensions, to determine the extent to which the two are complementary.

To do so, we model the demand for public education when pensions are linked to the earnings of active workers (a pay-as-you-go system), which in turn are dependent on the education the workers had received previously.

We show that agents’ preferences for publicly financed education depend on the pension system and, consequently, we provide a new rationale for voter willingness to have a publicly financed school system. Indeed, on top of the standard “intergenerational altruism” argument (i.e., that parents care about their own children’s education), we observe a link between voters’ utility and the level of education of the whole society. This link emerges when education generates positive externalities (for instance, Gradstein 2000 shows that education helps to reduce social conflicts) or, as in our model, because pensions depend on the workers’ average income, for their redistributive component.

We consider an overlapping generations (OLG) society in which agents care about the education that their children receive. Thus, they choose between a publicly financed or a (costly) private school, and vote on the tax rate to finance public education. In their last period of life they receive a pension. The pay-as-you-go (PAYG) pension system links the

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1For most EU countries, and also for most OECD countries, pension systems are of the pay-as-you-go type albeit with varying degrees of redistribution. Kritzer and Jankowski (2010) provide an overview of the salient characteristics of all European countries’s pension systems.

pension to the population’s average income. Agents anticipate that the average level of education in society has an impact on pensions, the magnitude of which depends in turn on the impact of education on future incomes and on the degree of redistribution of the pension scheme.

We conclude that the agents’ support for publicly funded education is i) decreasing in the contributory component of pensions, also known as the Bismarckian factor, and ii) increasing in the magnitude/generosity of the pension system. In other words, the lower the degree of redistribution characterising the pension system, the less willing agents are to invest in public education, and furthermore agents care more about public education as the stake in their pensions increases.

Indeed, our motivating evidence confirms both results. Figure 1 shows a negative correlation between the Bismarckian factor (the share of the pension that is contributory) and public expenditure in education for OECD countries. In line with our results, the correlation is greater when we only consider the sub-set of countries in which spending on pensions is above the average. Figure 2 supports our second finding, showing that spending on education is positively correlated with the generosity of the pension system, at least when the pension system has a redistributive component. Interestingly, while the correlation is almost null when all the OECD countries are included, when we consider the subset of countries with a Bismarckian factor below the average, we notice a strong positive correlation between expenditure on pensions and expenditure on public education.

Figure 1: Bismarckian factor and total public spending on education (% of GDP) in OECD countries (left-hand graph, correlation: -0.3) and in OECD countries with above-average spending on pensions (right-hand graph, correlation: -0.39).

Authors’ calculations based on World Bank, OECD and Krieger and Traub’s (2009) data.

See appendix A for more details on data sources and definition of variables.
Figure 2: Public spending on pensions and on education (% of GDP) in OECD countries (left-hand graph, correlation: 0.03) and in OECD countries with above-average redistributive pension system (right-hand graph, correlation: 0.52).
Authors’ calculations based on World Bank, OECD and Krieger and Traub’s (2009) data.

A few papers have similarly considered the simultaneous provision of education and pensions, but they disregard the analysis of voter willingness to pay for public education. Kaganovich and Zilcha (1999) focus on the optimal allocation of fixed tax proceeds between education and social security. Pecchenino and Pollard (2002) and Zhang and Zhang (2004) study the impact of social security and education on growth. Soares (2003) studies the preference of agents for public investment in education and how they allocate their time between education and working. Pirttila and Tuomala (2002) use (section IV) the investment in education as a determinant of productivity and study the use of education and pensions to redistribute wealth. Boldrin and Montes (2005) and (2009) undertake normative studies of education and pensions and the optimal intergenerational transfer scheme. All of these studies show that education has positive long run effects on growth, which makes more generous pension systems sustainable. Finally, another strand of the literature considers simultaneously education and pension (e.g., Poterba 1998, Ladd and Murray 2001, Sanz and Velázquez 2007, Cattaneo and Wolter 2009, and Rattsø and Sørensen 2010), but they look at how a change in the composition of society (share of young, adults and elders) affects the provision of pensions and education, given a fixed budget. In our work, instead, we focus on how forward-looking adults change their behaviour and invest in young’s education, to guarantee a sustainable pension system in the future.

The rest of the paper is structured as follows. Section 2 presents the model. In section 3 we study an agent’s optimal behaviour and compute the equilibrium. Section 4 extends the results and examines how retirement concerns can affect voter behaviour. Finally, section 5
concludes.

2 The model

Three cohorts (young, adults and elders) live simultaneously. At the end of each period, young become adults and adults become elders. Young people study, adults work, pay taxes and vote, and in the last stage of their lives, they retire and receive a pension. For the sake of parsimony, we assume no population growth and no discount of the future.\(^4\)

Compulsory education is both publicly and privately provided: the two are mutually exclusive. The quality of education, measured as per student expenditure, is denoted by \(X_P\) for the public sector and \(X_R\) for the private. \(\bar{X}\) is the average quality of education, measured as the cohort’s average spending on instruction. We focus on individual behaviour and, in particular, on the adults’ willingness to pay both for the education of their offspring and to finance public education. We restrict our attention to the agents’ behaviour in periods 0 and 1, and as we only need to focus on education in period 0, we do not need a time index for \(X\). Public education is financed through a universal and proportional tax \(t\) on income \(\omega\) and access to it is free. We assume the quality of education to be homogeneous amongst all public schools. Private schools are costly, and students choose the level of quality they wish to buy.\(^5\) Adults vote on the tax rate for financing public education, and choose the instruction (public or private) that their children receive. In the case of private school pupils, this decision includes the share of budget devoted to education. The residual income is used to consume the numeraire good \(b\). An adult living in period 0 is concerned by their child’s education \(X\), and their own consumption of the numeraire good both in the current and in the next period (respectively \(b_0\) and \(b_1\)), as in the models of “intergenerational altruism”. Their life-time utility function is \(U_0(b_0, b_1, X)\). For the tractability of the model, we assume a quasi-linear specification for \(U_0(b_0, b_1, X)\). Specifically, an adult’s utility function in period 0 is defined as:

\[
U_0(b_0, b_1, X) = b_0 + b_1 + \lambda \omega_0^{\beta} \ln X,
\]

where \(\lambda\) defines the weight of education relative to the consumption of the numeraire good, and \(\beta \geq 0\) determines how income and willingness to pay for education are related. When \(\beta = 0\), all agents care equally about education. When \(\beta > 0\), the utility of education is

\(^4\)Both assumptions have no qualitative impact on the results.
\(^5\)Agents can consider different schools or a single school offering extra services à la carte (i.e., elective classes, gym, library, extra-curricular activities). In equilibrium, agents attend a (costly) private school only if its quality is higher than that of free public schools. This is not an assumption of this model, and out of equilibrium there may be some private schools providing a lower quality of education than that provided by the public schools.
increasing in income. The assumption that the utility of education depends on $\omega_0$ is based on the idea that a positive correlation exists between wealth and the willingness of an agent to invest in the education of their own children.\(^6\)

As for the adults’ income, in period 0 we assume their income to be exogenous and uniformly distributed amongst a unit mass of agents: $\omega_0 \sim U[0, 1]$. However, the income of adults’ living in period 1 is increasing in the education they received in period 0. We assume an adult’s income in period 1 to be

$$\omega_1 = \omega_0 + X^\phi,$$

hence, it is equal to the parental income in the previous period, plus an increasing function of the education received, where we assume that $\phi < 1$. The functional form is not relevant for our results here, although some of the conclusions drawn by the paper do depend on the strictly positive relationship between current investment in education and future income.\(^7\)

The specific functional form, chosen for the sake of the tractability of the model, can be considered a reduced form to account for the fact that earnings are positively correlated with education,\(^8\) and that we observe a high intergenerational persistence of income.\(^9\) The average income in 1 is $\bar{\omega}_1$.

Pensions are the sole source of income for retirees, and are not taxed. Following Casamatta et al. (2000), the pension system is of the PAYG type comprising a contributory and a redistributive component. We denote by $\alpha \in [0, 1]$ the Bismarckian (or contributory) component, i.e., the share of a pension that depends on the agent’s previous contributions. The remaining $(1 - \alpha)$ of the pension, the redistributive component, depends on the average contribution of the current workers. Using $s$ to indicate the income tax that finances pensions, in period 0 each adult pays $s\omega_0$ to the pension system, of which $(1 - \alpha)s\omega_0$ is used to pay current pensions, while $\alpha s\omega_0$ is paid back to the worker in period 1 as a pension. Therefore, a retiree

\(^6\)There are various rationales to explain why the willingness to invest in education should be increasing in income. Our interpretation is that parental income is highly correlated with their level of education and, hence, it is a good proxy for their willingness to invest in their children’s education. Replacing parental education with their income allows us to have just one element of heterogeneity amongst agents. For more on the relation between family income, parental education and their willingness to invest in their offspring’s education, see Griliches and Mason (1972), Steelman and Powell (1991) or Dur et al. (2004).

\(^7\)The idea of education affecting productivity, income and, hence, growth is common in the growth literature. See, for example, Romer (1986) or Gradstein and Justman (1997). Barro (2013) provides a survey of the most relevant contributions.

\(^8\)The idea of income being positively correlated with education is widely supported in the literature. See, for instance, Card (2001) and Lemieux (2006).

\(^9\)A high persistence of income is found, amongst others, by Lee and Solon (2009) for the US. Mazumder (2005) estimates the US Intergenerational Income Elasticity (IIE) to be 0.61. Lefgren, Lindquist, and Sims (2012) estimates for Sweden are slightly lower (between 0.29 and 0.4). The reduced form that we use, combined with the fact that low income agents tend to prefer public education, would suggest a non-linear persistence of income, with a tendency to converge for low incomes. This is precisely reported by Bratsberg et al. (2007) for Denmark, Finland and Norway.
in period 1 receives a pension \( s(\alpha \omega_0 + (1 - \alpha)\bar{\omega}_1) \). We assume both \( \alpha \) and \( s \) to be exogenous.\(^{10}\)

Accounting for both \( t \) and \( s \), the adults’ disposable income in period 0 is \((1-t-s)\omega_0\). Hence, adults’ consumption of the numeraire is \( b_0 = (1-t-s)\omega_0 - X_R \), where \( X_R = 0 \) if a child attends public school. We denote by \( n \) the number of students attending public school. The quality of public school \( X_P \) (corresponding to the per-student expenditure) follows directly from the public school budget constraint \( nX_P \leq (t - s)\omega_0 \), and is:

\[
X_P = \frac{t\omega_0}{n}, \tag{3}
\]

In our model, the only active citizens are adults. Children are not allowed to vote on the tax \( t \) and they have no income; therefore, they cannot make any consumption decisions. Retirees have no interest in the level of education and so decide not to vote, and they use all their income to consume the numeraire good. In the next section we compute the utility maximising behaviour of adults.

### 3 Adults’ utility maximising behaviour

We analyse the optimal behaviour of adults separately, beginning with those that opt for public education, followed by those that opt for the private school system. Furthermore, we analyse the conditions under which an agent prefers public to private education, study the way in which the equilibrium tax to finance public education affects the total expenditure on education, and show that the maximisation problems are well-behaved.

An adult opting for public education should vote for the optimal tax rate \( t \) to finance public education, the maximisation problem would be:

\[
\max_t U(b_0, b_1, X_P) = b_0 + b_1 + \lambda \omega_0^\beta \ln X_P \tag{4}
\]

\[
s.t. \quad b_0 = (1-t-s)\omega_0 \quad b_1 = s(\alpha \omega_0 + (1-\alpha)\bar{\omega}_1) \quad X_P = \frac{t\omega_0}{n}
\]

The first order condition is:

\[
-\omega_0 + (1-\alpha)s \frac{\partial \bar{\omega}_1}{\partial t} + \lambda \omega_0^\beta \left( \frac{1}{t} - \frac{1}{n} \frac{\partial n}{\partial t} \right) = 0. \tag{5}
\]

Equation (5) implicitly defines the optimal tax \( t^*_P \) to finance public education for an adult opting for the public school system. The first element represents the reduction in disposable income as the tax rate increases. The second term accounts for how the agent’s own future

\(^{10}\)See Casamatta et al. (2000) for a model on retirements with vote on \( s \).
income is affected by a change in the average current consumption of education, via the redistributive component of pensions. Finally, the third term accounts for the direct utility change generated by a change in the quality of public education. Specifically, for an increase in \( t \) the first term in the parentheses accounts for the increase in resources invested in public education, while the second one considers the variation of \( X_P \) due to the change in the number of students in public schools.

Alternatively, the maximisation problem of those opting for private education is:

\[
\max_{t, X_R} U(b_0, b_1, X_R) = b_0 + b_1 + \lambda \omega_0^\beta \ln X_R
\]

\[
s.t. \quad b_0 = (1 - t - s)\omega_0 - X_R \\
\quad b_1 = s(\alpha \omega_0 + (1 - \alpha)\omega_1)
\]

The first order conditions are:

\[
-1 + \lambda \omega_0^\beta \frac{1}{X_R} = 0 \quad (7a)
\]

\[
X_R = \lambda \omega_0^\beta \quad (7b)
\]

\[
-\omega_0 + (1 - \alpha)s \frac{\partial \omega_1}{\partial t} = 0 \quad (8a)
\]

\[
\frac{\partial \omega_1}{\partial t} = \frac{\omega_0}{(1 - \alpha)s} \quad (8b)
\]

Equation (7b) defines the preferred consumption of private education \( X_R^* \), while equation (8b) defines the preferred tax rate \( t_R^* \) of an adult opting for private education. For an internal solution we need \( \frac{\partial \omega_1}{\partial t} > 0 \) and \( \frac{\partial^2 \omega_1}{\partial t^2} < 0 \), which occurs if an increase in public expenditure in education induces a concave increase in total expenditure in education.

An adult prefers private over public education if and only if the utility derived is higher. This means that, for an equilibrium tax rate \( t^* \), people attend a private school if and only if \( U(b_0, b_1, X_R^*) \geq U(b_0, b_1, X_P^*) \), that is, if \( -X_R^* + \lambda \omega_0^\beta \ln X_R^* \geq \lambda \omega_0^\beta \ln \frac{t^* \omega_0}{n} \). Then, \( \tilde{\omega} \) is the income for which an adult is indifferent to the two types of schooling:

\[
\tilde{\omega} = \left( \frac{et \omega_0}{\lambda n} \right)^{1/\beta} = \left( \frac{et}{2\lambda n} \right)^{1/\beta},
\]

where \( e \) is the Napier’s constant (or Euler number). By construction, the number of agents attending public school \( (n) \) corresponds to all agents with income below \( \tilde{\omega} \). Hence, the equilibrium value for \( n \) is

\[
n = \int_0^{\tilde{\omega}} f(x) d\omega_0 \quad (10a)
\]
\[ \dot{\omega} = n = \left( \frac{et}{2\lambda} \right)^{\frac{1}{\beta+1}}, \quad (10b) \]

and the quality of public education, given the tax rate, is

\[ X_P = \left( \frac{\lambda t^\beta}{e^{2\lambda^2}} \right)^{\frac{1}{\beta+1}}. \quad (11) \]

Using equation (2), we can now compute the average income in period 1.

\[ \bar{\omega}_1 = \int_0^1 (x + X^\phi) d\omega \quad (12a) \]

\[ \bar{\omega}_1 = \frac{1}{2} + \bar{\omega}X^\phi + \lambda^\phi \int_\omega^1 \omega^\beta \phi d\omega \quad (12b) \]

\[ \bar{\omega}_1 = \frac{1}{2} + \frac{\lambda^\phi}{\beta \phi + 1} + \left( \frac{e}{\lambda} \right)^{\frac{1-\phi}{\beta+1}} \left( 1 - \frac{e^\phi}{\beta \phi + 1} \right) \left( \frac{t}{2} \right)^{\frac{\beta+1}{\beta+1}} \quad (12c) \]

**Proposition 1.** An increase in the tax \( t \) always improves the quality of public school. This attracts more students to the public sector, which mitigates the increase in per capita expenditure, but nevertheless the total impact is positive. However, the total per capita expenditure in private and public education may fall, and this would have a negative impact on the average income in the subsequent period.

**Corollary.** The tax rate preferred by an adult is always higher when opting for the public education system. Furthermore, for adults opting for private education, the preferred tax rate is decreasing and convex in income.

**Proof.** See Appendix B.

Proposition 1 has important policy implications, as it means that an increase in the total expenditure for public education may be offset by a decrease in private expenditure; hence, increasing the tax rate \( t \) may induce a reduction in future total wealth. This is because an increase in the tax to finance public education makes the individual’s budget constraint more binding and, at the same time, public school becomes more attractive. Therefore, fewer people attend a private school (\( \dot{\omega} \) increases). Given that, in equilibrium, the per pupil expenditure in private school is higher than that in the public sector, a shift from the private to the public sector may imply an overall fall in investment in education. This implies that the average income in the subsequent period also falls.

The condition \( \beta > \frac{\phi}{\phi-1} \) guarantees that an increase in the tax rate induces a higher aggregate expenditure on education. It requires that \( \phi \) is sufficiently small compared to \( \beta \). This means either that we need the desire for high quality education to be increasing rapidly in income, or that the returns on education are sufficiently small. Actually, the above-mentioned
condition, together with $\phi \leq 1$, is both necessary and sufficient for the maximisation problem of adults preferring private education to be well behaved, and therefore we assume it to hold.

Using the results from this section, we can now rewrite the first order conditions of both types of agent. For agents opting for public education, equation (5) is rewritten using equations (19a) and (19c) in appendix B:

\[
(1 - \alpha)s \frac{\partial \omega_1}{\partial t} + \lambda \omega_0^\beta \left( \frac{1}{t} - \frac{1}{n} \frac{\partial n}{\partial t} \right) - \omega_0 = 0 \quad (13a)
\]

\[
(1 - \alpha)s \left( \frac{\beta \phi + 1}{\beta + 1} \left( \frac{e}{\lambda} \right)^{\frac{1 - \phi}{\beta + 1}} \left( 1 - \frac{e^{\phi}}{\beta + 1} \right) \left( \frac{1}{2} \right)^{\frac{\beta + 1}{\beta + 1}} t^{-\frac{\beta (1 - \phi)}{\beta + 1}} \right) + \frac{\lambda \omega_0^\beta}{t} \frac{\beta}{\beta + 1} - \omega_0 = 0 \quad (13b)
\]

\[
(1 - \alpha)s \left( \frac{e}{\lambda} \right)^{\frac{1 - \phi}{\beta + 1}} (\beta \phi + 1 - e^{\phi}) \left( \frac{t}{2} \right)^{\frac{\beta + 1}{\beta + 1}} + \beta \lambda \omega_0^\beta - (\beta + 1) \omega_0 t = 0 \quad (13c)
\]

Deriving equation (13b) with respect to $t$, we can see that if a solution to equation (5) exists, then the second derivative is always negative; hence, the stationary point solving equation (5) is a maximum of the problem.

As for agents preferring private education, equation (8b) can be rewritten in closed form as

\[
t = \omega_0 \frac{\beta + 1}{\beta (1 - \phi)^{\frac{1}{\beta + 1}}} \left( \frac{\lambda}{e} \right)^{\frac{1 - \phi}{\beta (1 - \phi)^{\frac{1}{\beta + 1}}} \left( \beta + 1 \right) \left( (1 - \alpha)s (\beta \phi + 1 - e^{\phi}) \right) - \omega_0 \right) \quad (14)
\]

4 The role of pensions

This section analyses how the pension system affects the decision of voters to invest in public education and, therefore, how a change in the pension system may affect the voting equilibrium tax rate $t^*$. 

We rewrite equations (13c) and (8b) respectively as $\Gamma(t^*_P) = 0$ and $\Omega(t^*_R) = 0$. From these equations, we can describe the change in the agent’s optimal tax, when the pension system changes:

\[
\frac{\partial t^*_P}{\partial \alpha} = - \frac{\partial \Gamma(t^*_P)}{\partial \alpha} \frac{\partial \Gamma(t^*_P)}{\partial t^*_P} \quad (15a)
\]

\[
\frac{\partial t^*_R}{\partial \alpha} = - \frac{\partial \Omega(t^*_R)}{\partial \alpha} \frac{\partial \Omega(t^*_R)}{\partial t^*_R} \quad (15b)
\]

\[
\frac{\partial t^*_P}{\partial s} = - \frac{\partial \Gamma(t^*_P)}{\partial s} \frac{\partial \Gamma(t^*_P)}{\partial t^*_P} \quad (16a)
\]

\[
\frac{\partial t^*_R}{\partial s} = - \frac{\partial \Omega(t^*_R)}{\partial s} \frac{\partial \Omega(t^*_R)}{\partial t^*_R} \quad (16b)
\]
Equations (15a) and (15b) describe the change in the preferred tax resulting from a change in the Bismarckian factor \( \alpha \), while equations (16a) and (16b) consider a change in the tax rate \( s \) in order to finance pensions. In both cases, equation (a) refers to the optimal tax of an adult opting for public education, while (b) is for the case of private education.

We are interested in the sign of the previous equations, and we know that all the denominators are negative, therefore we have that:

\[
\text{sign} \left( \frac{\partial t^*_{\ell}}{\partial \alpha} \right) = \text{sign} \left( \frac{\partial \Gamma(t^*_{\ell})}{\partial \alpha} \right) = \text{sign} \left( -s \frac{\partial \bar{\omega}_1}{\partial t} \right) < 0 \quad (17a)
\]

\[
\text{sign} \left( \frac{\partial t^*_{\ell}}{\partial \alpha} \right) = \text{sign} \left( \frac{\partial \Omega(t^*_{\ell})}{\partial \alpha} \right) = \text{sign} \left( -s \frac{\partial \bar{\omega}_1}{\partial t} \right) < 0 \quad (17b)
\]

\[
\text{sign} \left( \frac{\partial t^*_R}{\partial s} \right) = \text{sign} \left( \frac{\partial \Gamma(t^*_R)}{\partial s} \right) = \text{sign} \left( (1 - \alpha) \frac{\partial \bar{\omega}_1}{\partial t} \right) > 0 \quad (18a)
\]

\[
\text{sign} \left( \frac{\partial t^*_R}{\partial s} \right) = \text{sign} \left( \frac{\partial \Omega(t^*_R)}{\partial s} \right) = \text{sign} \left( (1 - \alpha) \frac{\partial \bar{\omega}_1}{\partial t} \right) > 0 \quad (18b)
\]

**Proposition 2.** The equilibrium tax rate \( t \), decided by the majority vote of all adults, is decreasing in the Bismarckian factor \( \alpha \) (the contributory component of the pension system) while it is increasing in the tax rate \( s \) that finances pensions.

**Proof.** From equations (17a), (17b), (18a) and (18b) it can be noted that for all voters the sign of the variation of the preferred tax due to a change in either \( \alpha \) or \( s \) does not depend on income or on the schooling choice. Therefore, we do not need to identify the pivotal voter at the election in order to confirm that the preferred tax rate of the pivotal voter is decreasing in \( \alpha \) and increasing in \( s \).

The intuition behind Proposition 2 is that voters care about their future consumption, which depends on the two components of the pension (the Bismarckian factor and the redistributive component). The redistributive element of the pension depends on the average income of the population, which itself depends on the average rate of investment in education. The more pensions depend on the average level of education (via the redistributive component), the more willing people are to invest in public education, in order to raise the level of education of public school students and, therefore, their own future consumption. The smaller \( \alpha \) is, the greater the degree of redistribution provided by the pension system (i.e., the larger the impact of the average income on the agent’s own pension) and the more pensions depend on the average level of education. Hence, as \( \alpha \) decreases, people care more about public investment in education. By contrast, as \( \alpha \) increases, the private returns on the education of others fall and so agents are unwilling to sacrifice their own current consumption (by paying a higher tax on their income) to increase future average income. In the case of
s, as this increases, the agents’ future consumption increases at the expenses of their current consumption. However, as long as we have a Bismarkian pension system, this simply remains an intertemporal redistribution within the same agent and as such does not affect the voters’ willingness to pay for public education. Nevertheless, given \( \alpha \), a larger \( s \) implies an increase in the level of the wealth that is redistributed through the pension system. Therefore, if we consider public education to be an investment that produces an increase in the future income of others, with the redistributive component of the pension being the returns on the investment, the role of \( s \) is that it increases the returns on the investment and, therefore, a larger \( s \) induces adults to vote for a higher tax rate \( t \).

4.1 The voting equilibrium

The previous results are independent of the voting process and the identity of the decisive voter. We now analyse the voting equilibrium, focusing on the possible types of coalitions that might form when voting on the tax \( t \).

Although the richness of the model limits our possibilities of studying the voting equilibrium and of having a closed-form solution of the model, Proposition 3 does indicate the type of voting equilibrium that the model might generate.

**Proposition 3.** When adults vote on \( t \), two types of equilibrium may prevail. In the first, the median income voter (\( \omega = \frac{1}{2} \)) is decisive, with all agents poorer than the median preferring a higher tax rate. By contrast, voters with income above the median prefer a lower tax rate. In the second, the richest agents form a coalition with some agents with income in the neighbourhood of the median income, and together they seek a reduction in the tax rate. By contrast, the poorest agents are in favour of a high tax rate and form a coalition with those agents with income in the neighbourhood of \( \tilde{\omega} \). The first equilibrium occurs if and only if the median voter's preferred tax is higher than that preferred by the agent with income \( \tilde{\omega} \), i.e.,

\[
t^*_P(\frac{1}{2}) \geq t^*_P(\tilde{\omega}).
\]

**Proof.** See Appendix C.

In our model, agents with income below \( \tilde{\omega} \) attend a (tuition-free) public school, while agents with income \( \omega > \tilde{\omega} \) opt for a (costly) private education and invest more in education than the per pupil expenditure in the public sector. To understand the intuition behind the voters’ behaviour, we must consider that two mechanisms are operating simultaneously: on the one hand, publicly financed education generates redistribution from the wealthier to the poorer agents in society via two channels (public schools and the redistributive component of pensions); on the other hand, richer agents care more for their children’s education, and so are more willing to substitute consumption of the numeraire good for education than are
other agents. The redistribution channel means poorer agents ask for higher taxes, while the other channel means the middle classes (the richest agents attending public schools) ask for higher taxes. If we combine the two effects, the tax preferred by an agent attending public school may either be decreasing in income over all its support, or it may be decreasing up to some income $\omega$, and then increase for $\omega \in [\omega, \tilde{\omega}]$.

When $t^*_P(\frac{1}{2}) > t^*_P(\tilde{\omega})$ (see figure 3 in the appendix), the redistribution effect is so strong that the first half of the population always favours a sufficiently high tax rate, so that we have the very standard result whereby the median voter is pivotal (although preferences are not unimodal). Poor agents are in favour of high taxation, since they profit from wealth redistribution and enjoy public education services, and wealthier agents prefer a lower taxation. This type of equilibrium appears, among others, in Epple and Romano (1996a, 1996b), Cohen-Zada and Justman (2003), and Glomm and Ravikumar (1998).

For the alternative case, when $t^*_P(\frac{1}{2}) < t^*_P(\tilde{\omega})$ (see figure 4 in the appendix), a number of agents in public school are neither sufficiently poor to enjoy significant gains from redistribution, nor rich enough to be sufficiently willing to pay for private education. These agents have an “intermediate” income, i.e., their income is in a neighbourhood of the median income; hence, they are neither the poorest nor the richest in the public school system. This group forms a coalition with the richest agents in the economy, with the aim of lowering the tax rate. This coalition opposes a coalition of the poorest adults (who gain from redistribution) and the middle classes with income in the neighbourhood of $\tilde{\omega}$. This same type of equilibrium may emerge (in a framework without retirement concerns) in the presence of selective vouchers to finance education (see Piolatto 2010).

5 Conclusions

Adults voluntarily use part of their income to finance the education of the young in an act which might be interpreted as intergenerational altruism (Epple and Romano 1996a). However, voters can be expected to want something in return from this investment in education.

We have extended the standard model in which adults vote on the tax to finance public education by adding a second time period during which adults retire and when, if their pensions are redistributive, they might obtain some monetary benefits from the new generation being better educated.

We show that rational voters can increase their future consumption by investing in the education of the young, as the redistributive component of pensions depends on the young’s

\footnote{In Epple and Romano (1996a), this equilibrium occurs under the assumption of single crossing, denoted slope decreasing in income (SDI), while the “ends against the middle” equilibrium prevails under the opposite assumption of slope rising in income (SRI).}
future income. This induces all voters, including those opting for private education, to favour a positive tax to finance public education. Furthermore, by increasing the social returns on education, the preferred tax rate by any agent increases (compared to the case with no retirement concerns). We conclude that, regardless of the identity of the pivotal voter, when retirement concerns are introduced, the preferred tax rate for financing public education increases. This effect would be even greater if we assumed imperfect financial markets or if population growth were higher than the intertemporal discount rate. We can expect an analogous effect to appear as long as education produces other positive externalities on society (i.e., less social conflict, or more technological and scientific progress, possibly leading to better infrastructures, medical treatment and services for the elderly).

The effect of pensions on preferences in education depends on the degree of redistribution provided by the pension system and on its absolute size. As a matter of fact, in a purely Bismarckian (purely contributory) pension system this effect disappears, as there is no link between the education of the young and the agents’ own future pensions. However, under a redistributive pension system, any investment in current education results in an increase in future pensions. The larger the redistributive component of the pension system is, the greater this effect becomes. Hence, we show that voters agree on larger tax rates to finance education when the redistributive component is larger. Moreover, this effect is amplified when pensions represent a larger proportion of total life income. The motivating evidence in the introduction confirms these results, showing that over the last three decades, countries with a larger Bismarckian factor invested less in education, and that there is a positive correlation between expenditure in public pensions and public education.

The model also provides some predictions regarding the type of equilibrium that should emerge. Depending on the value of the parameters, the equilibrium may be of two kinds. In one, the median voter is decisive, with the poorest half of the population in favour of an increase of the equilibrium tax rate and the other half preferring a lower rate. The other equilibrium depends on the preferred tax rate of adults opting for public education being U-shaped: decreasing for low levels of income, and increasing for wealthier agents. This U-shaped form of the preferred tax for adults opting for public education is the result of two opposing forces: redistribution through education (decreasing in income) and the interest for a high quality public school (increasing in income). Some intermediate-income agents may not be rich enough to care for a high quality public education system, but not poor enough to enjoy significant benefits from redistribution. These agents may form a coalition with agents opting for private education, and ask for lower tax rates, while the middle classes, together with the poorest agents in society, asks more public education.

Note that changing the size of the redistributive component of pensions $s(1 - \alpha)$ has a
clear effect on welfare. Investing in education has positive returns, but the amount of current income devoted to education tends to be sub-optimal since adults do not benefit from the increase in wealth that education will produce among the young. With pensions, part of the returns on the investment is enjoyed by adults, which makes them more willing to reduce their current consumption. Still, the share of the return that they enjoy is proportional to $s(1 - \alpha)$. Therefore, the benefit becomes more internalised the larger the redistributive part of the pension becomes. We conclude that aggregate welfare should increase when the redistributive part of the pension is larger.

We leave for future research, the interesting question as to how the political decisions about the tax rate to finance public education affect the voter preferences regarding the pension system. In particular, it would be interesting to allow agents (as in Casamatta et al. 2000) also to vote for the tax $s$ financing pensions and for the Bismarckian factor $\alpha$. 
Appendix

A  Sources used for the motivating evidence

Figures 1 and 2 are constructed using three sources of information: the Bismarckian factor, and public expenditure, as a share of GDP, on education and on pensions. To account for the contributory component of the pension system (the Bismarckian factor), we use Krieger and Traub (2011) estimates, which are computed using microdata (at the household level) taken from the Luxembourg Income Study (2008). Pensions are decomposed into two parts: the redistributive (flat) component, and the contributory (earnings-related) component. The Bismarckian factor, denoted as $\alpha$, is the share of the pension which is contributory, while $1 - \alpha$ is the redistributive share of the pension. Krieger and Traub (2011) results are reproduced in Table 1. The distribution of the Bismarckian factor across countries and years shows a bimodal distribution with one mode around the value 0.08 and the other mode around 0.48 (the Bismarckian factor has a minimum of -0.112 and a maximum of 0.778). This information has been used to determine the cut-off point that defines above- and below-average countries with respect to the redistributive part of the pensions system in Figure 2.

Table 1: Bismarckian factor for selected OECD countries

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Data on public expenditure on education (as a share of GDP) come from the World
Development Indicators (WDI) of the World Bank.\textsuperscript{12} Table 2 reports them for a selection of OECD countries.

Table 2: Public spending on public education as a share of GDP for selected OECD countries

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Finally, data on public expenditure on pensions (old age) as a share of GDP come from OECD.\textsuperscript{13} Table 3 reports them for a selection of OECD countries.

B Proof of Proposition 1

If we derive the previous equations (10b), (11) and (12c) with respect to $t$, we obtain the variation of the number of public school students, of the quality of public school and of the average income in the second period, when the equilibrium tax varies.

$$\frac{\partial n}{\partial t} = \frac{1}{\beta + 1} \left( \frac{e}{2\lambda} \right) \frac{1}{\beta + 1} t^{-\frac{\beta}{\beta + 1}} = \frac{n}{(\beta + 1)t} > 0 \quad (19a)$$

$$\frac{\partial X_P}{\partial t} = \frac{\beta}{\beta + 1} \left( \frac{\lambda}{e^2\beta} \right) \frac{1}{\beta + 1} t^{-\frac{1}{\beta + 1}} > 0 \quad (19b)$$

$$\frac{\partial \bar{\omega}_1}{\partial t} = \frac{\beta \phi + 1 - e^\phi}{\beta + 1} \left( \frac{\epsilon}{\lambda} \right)^{\frac{1-\phi}{\beta + 1}} \left( \frac{1}{2} \right)^{\frac{\beta \phi + 1 - \phi (2(1-\phi))}{\beta + 1}} t^{-\frac{2(1-\phi)}{\beta + 1}} \quad (19c)$$

\textsuperscript{12}http://data.worldbank.org/
\textsuperscript{13}http://www.oecd.org/statistics/
Table 3: Public spending on pensions (old age) as a share of GDP for selected OECD countries

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<td>6.2</td>
<td>6.2</td>
<td>6.3</td>
</tr>
<tr>
<td>UK</td>
<td>3.7</td>
<td>4.0</td>
<td>4.0</td>
<td>4.1</td>
<td>4.0</td>
<td>4.1</td>
</tr>
<tr>
<td>USA</td>
<td>5.0</td>
<td>5.2</td>
<td>5.1</td>
<td>5.3</td>
<td>5.0</td>
<td>5.2</td>
</tr>
</tbody>
</table>

The first two equations are both positive. The sign of equation (19c) is positive if $\beta > \frac{\phi - 1}{\phi}$.

To prove the corollary, notice that if $\beta < \frac{\phi - 1}{\phi}$ or $\phi > 1$, we have a corner solution for the problem of adults opting for private education and their preferred tax is $t = 0$. If $\beta > \frac{\phi - 1}{\phi}$ and $\phi \leq 1$, then both types of agent are in favour of a positive taxation, but then comparing equations (5) and (8b) we can see that in the former we have an extra term $\left(\lambda_0 \omega \frac{1}{\phi} \frac{\partial X}{\partial t}\right)$, which is always positive by equation (19b). It is sufficient to notice that $\frac{\partial^2 \omega_1}{\partial t^2} < 0$, to conclude that for a same income $\omega_0$, the value of $t$ that solves equation (8b) must be lower than that for equation (5). By the same reasoning, if we compare two adults opting for private education, a decrease in $\omega_0$ implies that the optimal value for $\frac{\partial \omega_1}{\partial t}$ is also lower, hence (from $\frac{\partial^2 \omega_1}{\partial t^2}$) it must be that the preferred tax is lower for a wealthier agent. To show that $t^*_R$ is decreasing and convex in income, it is sufficient to derive equation (14) to see that the first derivative is negative and the second is positive.

C Proof of Proposition 3

Proposition 3 characterises the voting equilibrium for the two cases of $t^*_P(\frac{1}{2}) \geq t^*_P(\tilde{\omega})$ and $t^*_P(\frac{1}{2}) < t^*_P(\tilde{\omega})$. The proof is organised as follow: in part A., we derive the optimal tax rate
for adults having opted for public education, and show that it is convex, with a minimum at \( \omega = \bar{\omega} \). In part B., we show that for \( t^*_P(\frac{1}{2}) \geq t^*_P(\bar{\omega}) \), the only possible equilibrium is that the median voter is decisive, with all agents poorer than the median being in favour of a larger tax rate, opposing the remaining agents, who are in favour of a lower tax rate. In part C., we analyse the voting equilibrium when \( t^*_P(\frac{1}{2}) < t^*_P(\bar{\omega}) \), showing that the median voter cannot be decisive, and we show the characteristics of the equilibrium.

A. We rewrite equation (13c) as \( \Gamma(t^*_P) = 0 \). We can describe the change in the agent’s optimal tax, when the income changes as:

\[
\frac{\partial t^*_P}{\partial \omega_0} = -\frac{\partial \Gamma(t^*_P)}{\partial \omega_0} \tag{20}
\]

We know, from the second order condition, that \( \frac{\partial \Gamma}{\partial t^*_P} < 0 \), hence

\[
\text{sign} \left( \frac{\partial t^*_P}{\partial \omega_0} \right) = \text{sign} \left( \frac{\partial \Gamma(t^*_P)}{\partial \omega_0} \right). \tag{21}
\]

Therefore, \( \frac{\partial t^*_P}{\partial \omega_0} > 0 \) if and only if \( \omega_0 > \left( \frac{(1+\beta)\mu}{\beta^2 \lambda} \right)^{\frac{1}{\beta-1}} \equiv \bar{\omega} \). Under the conditions of existence of the model, \( \beta > 1 \), therefore \( \bar{\omega} \) is a minimum and \( t^*_P \) is convex in \( \omega_0 \).

![Figure 3: \( t^*_P(\frac{1}{2}) \geq t^*_P(\bar{\omega}) \)](image)

B. From the corollary of proposition 1, we know that for \( \bar{\omega} \) (as for any level of income), \( t^*_P(\bar{\omega}) > t^*_R(\bar{\omega}) \), furthermore, the \( t^*_R \) is decreasing in income. If \( t^*_P(\frac{1}{2}) \geq t^*_P(\bar{\omega}) \), then there can be no income \( \omega \in [\bar{\omega}, 1] \) such that \( t^*_R(\omega) \geq t^*_P(\frac{1}{2}) \). Finally, if \( t^*_P(\frac{1}{2}) \geq t^*_P(\bar{\omega}) \), then \( \bar{\omega} > \frac{1}{2} \) and for all \( \omega_0 \in \left( \frac{1}{2}, \bar{\omega} \right) \), it must be that \( t^*_P(\frac{1}{2}) \geq t^*_P(\omega_0) \), as represented in Figure 3. Since the preferred tax is decreasing in income for \( \omega_0 < \bar{\omega} \), the preferred one for agents with income \( \omega_0 < \frac{1}{2} \) (who
represent half of the population) is larger than that of the median voter. Meanwhile, for all agents with income above the median the preferred tax is smaller than that of the median voter. We conclude that the median voter is decisive.

\[
\text{Figure 4: } t^*_p(\frac{1}{2}) < t^*_p(\bar{\omega})
\]

C. If \( t^*_p(\frac{1}{2}) < t^*_p(\bar{\omega}) \), then, by continuity, there will be some neighbourhood around \( \frac{1}{2} \) (\( \omega \in [\bar{\omega}, \frac{1}{2}] \) in Figure 4) and around \( \tilde{\omega} \) (\( \omega \in [\tilde{\omega}, \tilde{\omega}] \) in Figure 4) where the preferred tax rate is larger than \( t^*_p(\frac{1}{2}) \).

Define \( \bar{\omega} \equiv \omega : t^*_p(\frac{1}{2}) = t^*_p(\bar{\omega}) \), and \( \tilde{\omega} \equiv \omega : t^*_p(\tilde{\omega}) = t^*_p(\tilde{\omega}) \). By the convexity of \( t \) and \( t^*_p(\frac{1}{2}) < t^*_p(\bar{\omega}) \), it must be that \( \bar{\omega} < \frac{1}{2} \), while \( \tilde{\omega} \) can be larger or smaller than \( \frac{1}{2} \). Since \( \bar{\omega} < \frac{1}{2} \), the group of poorest agents with the highest preferred tax rate (i.e., \( \omega < \bar{\omega} \)) is not large enough to form a winning coalition. We must include some more agents, in order to have a coalition of that half of the population with the highest preferred tax rate. Hence, the coalition is formed by all agents with income \( \omega < \bar{\omega} \), together with some agents with income

\[
\begin{align*}
\omega \in [\tilde{\omega}, \frac{1}{2}] \text{ and } \omega > \tilde{\omega} & \text{ if } \tilde{\omega} > \frac{1}{2} \\
\omega \in [\bar{\omega}, \tilde{\omega}] \text{ and } \omega > \frac{1}{2} & \text{ if } \tilde{\omega} \leq \frac{1}{2}
\end{align*}
\]

so that the coalition’s size is \( \frac{1}{2} \).\(^{14}\)

\(^{14}\)Notice that in Figure 4, we assume that \( t^*_p(\frac{1}{2}) > t^*_p(\bar{\omega}) \), but it could be the other way around. In this case, some voters opting for private education and income sufficiently close to \( \bar{\omega} \) may also join the coalition.
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