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ABSTRACT: This paper derives the optimal pension and tax parameters in a society where individuals differ in two characteristics: rationality and productivity. Rational agents, if not liquidity constrained, smooth consumption over their life-cycle. Myopic agents, by contrast, have ex ante a strong preference for the present and undertake no savings, even though, ex post they regret their decision. Given a paternalistic social objective aiming at maximizing the sum over ex post utilities, this paper shows how both transfer systems interact in their degree of redistribution and generosity. Moreover, it reveals how the optimal policy parameters change if capital markets are imperfect, implying that agents cannot borrow against their retirement benefits. Analytical and numerical results show that in some cases only one transfer system prevails.

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Keywords: Social security, redistributive taxation, myopia, credit constraints

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1 Introduction

For a rational agent, saving is the result of a trade-off between consumption today and consumption tomorrow. The life-cycle model by Ando and Modigliani (1963) assumes that individuals rationally plan their consumption and saving needs in a way that maximizes their expected lifetime utility. This suggests that individuals try to smooth consumption over time, implying accumulation of financial assets while working and dissaving while retired. However, many people experience a significant fall in consumption when retired (see Bernheim, Skinner and Weinberg (2001), Akerlof (2002) and Hamermesh (1984) among many others). Figure 1 illustrates the consumption path predicted by the life-cycle model and the consumption profile that is mainly observed. If this drop in consumption after entering retirement age reflects rational optimization, then the observed consumption profile just reflects true preferences. If, however, individuals are myopic, prone to regret their earlier saving behavior or dynamically inconsistent, this ‘inadequate’ level of savings is an important empirical question.

Bernheim et al. (2001) state that their “findings are difficult to interpret in the context of the life-cycle model” and that “the empirical patterns in this paper are more easily explained if one steps outside the framework of rational, farsighted optimization.” (p.855). A survey of 10,000 employees conducted by Choi, Laibson, Madrian and Metrick (2006) confirms that individuals are far away from being rational farsighted life-cyclers. The authors discover that 68% of employees find their retirement savings rate ‘too low’ relative to their ideal. More than one third intended to increase savings over the next few months. However, only 14% of them did so in the four
months after the survey. Behavioral Economists explain the poor savings for retirement in terms of \textit{bounded self-control} or \textit{lack of willpower} (e.g. Thaler and Shefrin (1981)).

For modeling bounded self-control or lack of willpower to adequately prepare for retirement, the theoretical literature has developed two tools. One is the hyperbolic discounting function which can be traced back to Strotz (1956) and Pollak (1968). A hyperbolic discounter uses a high short-term and a lower long-term discount rate. Since such an agent overvalues the present and undervalues the future, he faces a self-control problem when it comes to retirement savings. The agent overconsumes today and undersaves for the future (see Laibson (1997), Laibson (1998), Diamond and Koszegi (2003) and Schwarz and Sheshinski (2007)). Another tool, which is employed in this paper, is to assume that individuals (partly) ignore future utility when deciding how much to save rather than compute a plan that spans their entire life-cycle utility. In those models, individuals regret their earlier saving decisions and are called myopic (see Feldstein (1985), Cremer, De Donder, Maldonado and Pestieau (2007), Cremer, De Donder, Maldonado and Pestieau (2008)).

The lack to ‘adequately’ save for retirement is cited by many authors as the main justification for the pension scheme (e.g. Diamond (1977), Kotlikoff, Spivak and Summers (1982) and Agulnik (2000)). Feldstein (1985) was among the first who studied theoretically how the presence of myopic agents affects the optimal level of the pension system (see Feldstein and Leibman (2002)). As the optimal level requires balancing the protection of the myopic agents against the cost of distorting resource allocation by the rational agents, he finds that old-age benefits are always positive but may be quite low unless a large fraction of society is completely myopic.

In recent years a vibrant literature has developed which reexamines Feldstein’s canonical analysis. Docquier (2002) shows that the optimal pension contribution rate derived by Feldstein (1985) is Pareto-dominated if the social welfare function not only maximizes the sum of all agents currently alive, but also of all those yet to be born. Imrohoroglu, Imrohoroglu and Joines (2003) study how a society composed of hyperbolic discounters fares in a pay-as-you-go social security program. Their model also accounts for liquidity constraints, unemployment risk and uncertain mortality and income. As they embed social security in a general-equilibrium setting, it does not only provide old-age consumption for the shortsighted individuals, but also distorts labor supply and affects the interest and wage rate. The authors conclude that shortsightedness of individuals must be severe to give scope for social security as a means for improving welfare. Cremer et al. (2007) analyze how the pension parameters of a linear pension scheme should be optimally designed for a society, in which agents differ not only with respect to their rationality but also with respect to productivity. In their framework, the task of retirement benefits is not only to ensure old-age consumption for the myopic agents but also to redistribute from high- to low-income individuals. Their main findings are that the pension system becomes less Beveridgean as the share of rational agents increases. However, if some rational agents are also liquidity constrained this connection is reversed and even targeting towards the poor can be optimal, implying a negative
correlation between past contributions and pension benefits.

So far the economic literature has solely focused on how the pension scheme should be optimally designed in a society that consists not only of life-cyclers but also of myopic agents. However, as can be seen in Figure 1, those two types of agents not only differ in their old-age consumption levels, but also in their consumption while young. Hence, from a welfare perspective it may not only be optimal to have a pension scheme to reduce the variance in consumption while old but also to implement a tax scheme to decrease the gap in consumption while young. Building on the study by Cremer et al. (2007) this paper models the optimal pension parameters in a framework which consists of two redistributive transfer schemes, namely a linear pension scheme and a linear tax system that redistributes among agents during working-age.

The goal of this paper is to answer the following questions. How do the two transfer systems interact and how should they be optimally designed from a paternalistic point of view aiming at maximizing life-cycle utility? How strong is the redistributive concern in each system? Do results change if capital markets are imperfect, implying that individuals cannot borrow against their retirement benefits? And, does society need both a pension and a tax system at all?

To forestall the first result of this paper; with no myopic agents in society the tax and pension scheme are perfectly substitutable. This implies that the government can rely on only one of the two transfer systems to maximize social welfare. But, this result is derived for a society that is solely composed of rational agents who perfectly smooth consumption over their life-cycle. This framework does not incorporate the already mentioned key motivation for a pension scheme, namely to provide old-age benefits for those who have missed to save enough for retirement. As Diamond (2004) states in his Presidential Address “it is inadequate and potentially misleading to study the effects of Social Security in models in which there is no particular reason for Social Security to exist in the first place...[...]...the model of homo economicus, while very useful, is not a fully adequate basis for the design of all policies...” (p.4). By analyzing a mixed society composed of rational and myopic agents this paper shows that there is reason for both transfer schemes to exist; the pension scheme to ensure old-age consumption for the myopic agents and the tax scheme to redistribute among high- and low-income agents.

The rest of the paper proceeds as follows. The next section sets up the Model and derives results analytically. Section 3 gives numerical examples and compares results to those derived in the framework modeled by Cremer et al. (2007) where the government has only pension parameters at hand. A final section concludes, and an Appendix contains most of the proofs.
2 The Model

A mass of agents with unit measure lives for two periods. Individuals are heterogeneous with respect to their exogenous ability \( w_n \) \((w_2 > w_1)\) and with respect to their preferences for the present governed by \( \beta_i \). This gives rise to \( ni \)-agents, where \( \theta_{ni} \) denotes the fraction of type-\( ni \) in society. \( \theta_1 \) and \( \theta_2 \) determine the share of low and high productivity types, whereas \( \pi_n^M \) and \( \pi_n^R \) define the share of myopic and rational agents among each productivity type.\(^1\) Hence, the proportion of the four types in the population amounts to

\[
\begin{align*}
\theta_{1M} &= \theta_1 \pi_1^M, \\
\theta_{2M} &= \theta_2 \pi_2^M, \\
\theta_{1R} &= \theta_1 \pi_1^R, \\
\theta_{2R} &= \theta_2 \pi_2^R.
\end{align*}
\]

If \( \pi_1^M > \pi_2^M \), the share of myopic agents among the low productivity individuals is higher implying a positive correlation between rationality and productivity. The time preference parameter is a binary variable which is one for rational individuals, \( \beta_R = 1 \), and zero for myopic individuals, \( \beta_M = 0 \). Ex post both agents, rational and myopic, have the same intertemporal preferences but only the rational individuals make their decisions in line with these preferences. Myopic individuals do not save for retirement, since ex ante they have a strong preference for the present and consume all their income. They make their consumption decisions according to a discount factor that does not represent their true preferences and when being retired they rue their earlier decisions. It is worth emphasizing that myopic behavior is different from the behavior of rational individuals with high discount rates for future utility. For individuals who have high discount rates it is rational to save little for retirement and enjoy high consumption rates today. For them, utility cannot be increased when they are subject to forced savings. In contrast, forcing myopic individuals to save increases their utility.

2.1 The Transfer Systems

The government’s objective is twofold. On the one hand, it wants to redistribute income from high to low-productivity households. On the other hand, it aims to provide resources to myopic individuals who have missed to save for retirement. The government does not observe \( w_n \), labor supply \( l_{ni} \), savings \( s_{ni} \) and time preferences. However, it knows the joint distribution of productivity and rationality and it observes labor income \( w_n l_{ni} \). Hence, it must rely on distortionary taxes instead of individualized lump-sum transfers. Both the income tax and the pension schedule are assumed to be linear. Net transfers in the tax system amount to

\[
T(w_n l_{ni}) = \tau - tw_n l_{ni},
\]

where \( t \) is the marginal tax rate and \( \tau \) is a uniform lump-sum transfer. For \( T(w_n l_{ni}) \leq 0 \) individual-\( ni \) is a net payer, whereas for \( T(w_n l_{ni}) > 0 \) individual-\( ni \) is a net receiver in the tax system. Individual-\( ni \) has no social security rights.

\(^1\)Due to the unit measure \( \theta_1 = 1 - \theta_2 \) and \( \pi_R^n = 1 - \pi_M^n \) for \( n = 1, 2 \).
uals must also contribute a share $b$ of their pre-tax labor income to the pension scheme. Pension benefits during retirement depend on prior contributions, $bw_nl$, through the formula

$$P(w_nl) = \alpha bw_nl + B,$$

where $\alpha$ is the so-called Bismarckian factor. For $\alpha = 1$, the pension system is purely Bismarckian and each individual’s total contribution is equal to his total pension benefits. For $\alpha = 0$, the pension system is purely Beveridgean and all individuals receive the same pension $B$ irrespective of their prior contributions. The pension system can even be targeted, $-1 < \alpha < 0$, implying that part of pension benefits is decreasing in contributions. As long as $\alpha$ is smaller than 1, there exists redistribution from high-income to low-income individuals.

Both the interest rate and the rate of population growth are assumed to be equal to zero. Hence, it does not matter whether pensions are fully funded or based on the pay-as-you-go principle. The sequence of decision-making is as follows: First, the government sets its policy instruments $p = \{t, b, \alpha, \tau, B\}$. Taking $p$ as given, individuals decide how much labor to supply and how much to save for retirement.

### 2.2 Individual’s Optimization

As in Cremer et al. (2007), life-time utility of individual $ni$ is given by

$$U_{ni} = u(c_{ni} - v(l_{ni})) + \beta_i u(d_{ni}),$$

where $c_{ni}$ and $d_{ni}$ denote first- and second-period consumption, $u(\cdot)$ is utility from consumption and $v(\cdot)$ is the (monetary) disutility of labor supply. This specification of utility is sufficiently general to emphasize the main points at stake, while avoiding additional analytical complexity due to income effects on labor supply. Utility and labor disutility are assumed to be twice continuously differentiable satisfying $u' > 0$, $u'' < 0$, $v' > 0$ and $v'' > 0$. Further, the Inada condition $\lim_{x \to 0} u'(x) = \infty$ is assumed to hold.

Rational individuals maximize (1) with $\beta_i = 1$ subject to first- and second-period consumption determined by

$$c_{nR} = (1 - t - b)w_nl_{nR} - s_{nR} + \tau$$
$$d_{nR} = s_{nR} + \alpha bw_nl_{nR} + B.$$}

Individuals may also be subject to credit market imperfections either because they cannot sell claims on their retirement benefits or because of information asymmetries (see Diamond and Hausman (1984)). For analytical tractability, it is assumed that those credit market imperfections take the form of a non-negativity constraint on savings, $s_{ni} \geq 0$. Hence, the Lagrangean of the rational
where \( \mu_{nR} \), \( \sigma_{nR} \) and \( \gamma_{nR} \) denote the Lagrangean multipliers with respect to first- and second-period consumption and savings. If \( \gamma_{nR} > 0 \), implying \( s_{nR}^* = 0 \), the individual wants to borrow against his future pension benefits but the liquidity constraint prevents him from doing so. Denoting the value of net consumption in period one as \( x_{nR} = c_{nR} - v(l_{nR}) \), the first-order conditions (FOCs) of (4) with respect to \( c_{nR} \), \( d_{nR} \), \( s_{nR} \) and \( l_{nR} \) are given by \(^2\)

\[
\begin{align*}
\frac{\partial L}{\partial c_{nR}} &= u'(x_{nR}^*) - \mu_{nR} = 0, \quad \mu_{nR} \geq 0 \quad (5) \\
\frac{\partial L}{\partial d_{nR}} &= u'(d_{nR}^*) - \sigma_{nR} = 0, \quad \sigma_{nR} \geq 0 \quad (6) \\
\frac{\partial L}{\partial s_{nR}} &= -\mu_{nR} + \sigma_{nR} + \gamma_{nR} = 0, \quad \gamma_{nR} \geq 0, \quad \gamma_{nR}s_{nR}^* = 0 \quad (7) \\
\frac{\partial L}{\partial l_{nR}} &= -u'(x_{nR}^*) + \mu_{nR}(1 - t - b)w_n + \sigma_{nR}b\alpha w_n = 0. \quad (8)
\end{align*}
\]

For rational individuals who are not liquidity constrained, equations (5) to (7) yield \( u'(x_{nR}^*) = u'(d_{nR}^*) \) which implies perfect consumption smoothing \( x_{nR}^* = d_{nR}^* \). The price of period-two consumption relative to period-one consumption is one. In contrast, for liquidity-constrained rational agents, the FOCs amount to \( u'(x_{nR}^*) \neq u'(d_{nR}^*) \). The marginal value of period-one consumption increases, so that the price of consumption in period two relative to period one satisfies \( u'(d_{nR}^*) \leq u'(x_{nR}^*) + \gamma_{nR} \). The optimal saving decision can be summarized as

\[
s_{nR}^* = \begin{cases} 
0.5((1 - (t + (1 + \alpha)b))w_n l_{nR}^* + \tau - v(l_{nR}^*)) & \text{if } \gamma_{nR} = 0 \\
0 & \text{if } \gamma_{nR} > 0,
\end{cases}
\]

where \( \gamma_{nR} = 0 \) indicates either that the capital market is perfect or that optimal savings are non-negative. Note that savings are increasing in ability for a large range of possible policy parameters since

\[
\frac{\partial s_{nR}^*}{\partial w_n} = 0.5(1 - (t + (1 + \alpha)b))l_{nR}^* > 0 \quad \text{for} \quad t + (1 + \alpha)b < 1. \quad (9)
\]

Given the above relationship, the low ability rational individual is more likely liquidity constrained than his high ability counterpart. The optimal labor supply function \( l_{nR}^* \) can be derived by solving (8) for \( v' (\cdot) \) while taking (5) to (7) into account

\[
v'(l_{nR}^*) = \left(1 - t - \left(1 - \frac{u'(d_{nR}^*)}{u'(x_{nR}^*)} \alpha \right) b\right)w_n. \quad (10)
\]

\(^2\)Given the assumptions on utility and labor disutility, the first-order conditions are both necessary and sufficient for a maximum.
It depends on $t$, $b$ and $\alpha$, and also on $\tau$ and $B$ for those who are liquidity constrained. As long as there exists redistribution in the pension scheme, $\alpha < 1$, labor supply of a rational agent is distorted downwards since part of the worker’s contributions will not entitle him to higher retirement benefits. Thus, in terms of deadweight loss from labor supply distortions, pensions are less costly in a Bismarckian than in a Beveridgean system. However, if the individual is liquidity constrained a positive $\alpha$ is less efficiency enhancing as then $u'(d_{nR}) < 1$. 

Turning to the myopic agents. Ex post, the utility function given in (1) with $\beta_i = 1$ is also that of myopic individuals. Ex ante, however, myopic agents disregard the second-period and $\beta_i = 0$. Therefore, their maximization problem amounts to

$$\max_{c_{nM}, l_{nM}} U_{nM} = u(c_{nM} - v(l_{nM}))$$

s.t. $c_{nM} = (1 - t - b)\omega_{n}l_{nM}$.  

Optimization of (11) yields the following optimal labor supply function

$$v'(l_{nM}^*) = (1 - t - b)\omega_{n}.$$ 

In contrast to the optimal labor supply by rational agents, labor supply of the myopic agents is independent of the Bismarckian factor $\alpha$ as they fail to factor in the link between higher earnings and future pension benefits in a (partly) Bismarckian pension system.

Inserting the optimal values $s_{nR}^*$ and $l_{nR}^*$ back into (1) generates the following ex post indirect utility functions for the rational unconstrained ($V_{nR}$), the rational liquidity constrained ($V_{nR}^c$) and the myopic individuals ($V_{nM}$)

$$V_{nR} = u \left( (1 - b - t)\omega_{n}l_{nR} - s_{nR}^* + \tau - v(l_{nR}^*) \right) + u(s_{nR}^* + b\alpha\omega_{n}l_{nR} + B)$$

$$V_{nR}^c = u \left( (1 - b - t)\omega_{n}l_{nR}^c + \tau - v(l_{nR}^c) \right) + u(b\alpha\omega_{n}l_{nR}^c + B)$$

$$V_{nM} = u \left( (1 - b - t)\omega_{n}l_{nM} + \tau - v(l_{nM}^*) \right) + u(b\alpha\omega_{n}l_{nM}^* + B).$$ 

Although savings are zero for the myopic and for the liquidity constrained rational individuals, indirect utility is not the same for both since they differ in their labor supply as long as the pension scheme is not purely Beveridgean. As the rational individuals can always do as good (bad) as their myopic counterpart, namely by saving nothing and ignoring the link between labor income and pension benefits, ex post utilities satisfy: $V_{nM} \leq V_{nR}^c < V_{nR}$ for $n = 1, 2$.

### 2.3 Government’s Optimization

The government aims at maximizing the sum of individual utilities from a paternalistic point of view. The paternalistic policy is selected with the goal of influencing the choices of individuals in

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3 For further discussion of the labor supply effects to the different policy parameters, see Cigno (2008) for rational liquidity constrained agents and Kaplow (2006) for myopic individuals.
a way that will make them better off (see, e.g., Thaler and Sunstein (2003) and O’Donoghue and Rabin (2003)). Therefore, the government maximizes a social welfare function defined over ex post indirect utilities given in (13), that all depend on first- and second-period consumption. The intention is that ex post the myopic individuals in society appreciate having been forced to save for retirement. As a benchmark, the next section derives the first-best allocation before attention is given to second-best solution achieved with the instruments and information available to the government.

### 2.3.1 First-Best

In the first-best solution, the government not only observes productivity but also the degree of myopia. Hence, the government’s optimization problem can be written as

\[
\max_{c_{ni}, d_{ni}, l_{ni}} W = \sum_{n=1}^{R} \sum_{i=1}^{M} \theta_{ni} \left\{ u(c_{ni} - v(l_{ni})) + u(d_{ni}) \right\}
\]

s.t \[
\sum_{n=1}^{R} \sum_{i=1}^{M} \theta_{ni} w_{nl_{ni}} = \sum_{n=1}^{R} \sum_{i=1}^{M} \theta_{ni} \left\{ c_{ni} + d_{ni} \right\}, \quad (14)
\]

where (14) is the resource constraint of the economy. The solution to the above problem yields

\[
x_{ni} = d_{ni} = c \quad \forall ni \quad (15)
\]

and \[
l_{1R} = l_{1M} < l_{2R} = l_{2M}. \quad (16)
\]

In the absence of information asymmetries, the government equalizes (net) consumption levels across individuals and periods. The latter is due to the paternalistic welfare criterion. First-best labor supply is chosen to be the same for rational and myopic agents but to be higher for high productivity types. This first-best solution can be decentralized with individualized lump-sum taxes and transfers among types and periods. In Section 2.3.5 it will be shown that in the special case where agents differ only in rationality, the first-best solution can also be implemented with the policy instruments given in $\mathcal{P}$.

### 2.3.2 Second-Best

To maximize welfare, given the above mentioned information asymmetries, the government has five instruments at hand; the linear tax rate on labor income $t$, the linear pension contribution rate $b$, the lump-sum transfers $\tau$ and $B$ and the Bismarckian factor $\alpha$. The welfare function is given by the sum over ex post utilities determined in (13),

\[
W(t, b, \alpha, \tau, B) = \sum_{n=1}^{R} \sum_{i=1}^{M} \theta_{ni} \left\{ u((1 - t - b)w_{nl_{ni}^*} + \tau - s_{nR}^* - v(l_{ni}^*)) + u(s_{nR}^* + b\alpha w_{nl_{ni}^*} + B) \right\}. \quad (17)
\]
The government taxes labor at a rate \( t \) and \( b \) to finance uniform lump-sum transfers \( \tau \) when young and retirement benefits \( b\alpha w_n l_{ni} + B \) when old. This implies the following budget constraints of the two transfer systems\(^4\)

\[
\begin{align*}
t E w_n l_{ni}^* &= \tau \quad (18) \\
b E w_n l_{ni}^* &= b\alpha E w_n l_{ni}^* + B, \quad (19)
\end{align*}
\]

where \( E w_n l_{ni}^* \) is used for the average income in society given by

\[
E w_n l_{ni}^* = \sum_{n=1}^{2} \sum_{i=M}^R \theta_{ni} w_n l_{ni}^*.
\]

The objective of the government can be written in terms of \( t, b \) and \( \alpha \) only by using the governments’ budget constraints (18) and (19) to eliminate \( \tau \) and \( B \):

\[
\mathcal{W}(t,b,\alpha) = Eu((1-t-b)w_n l_{ni}^* + t E w_n l_{ni}^* - s_{ni}^* - v(l_{ni}^*)) + Eu(s_{ni}^* + b(\alpha w_n l_{ni}^* + (1-\alpha)E w_n l_{ni}^*)). \quad (20)
\]

Differentiating this expression with respect to \( t, b \) and \( \alpha \) and taking the behavioral responses of the individuals into account yields

\[
\frac{\partial \mathcal{W}'}{\partial t} = E \left[u'(x_{ni}^*) \left(-w_n l_{ni}^* + E w_n l_{ni}^* + t E w_n \frac{\partial l_{ni}^*}{\partial t} \right) + (1-\alpha)bu'(d_{ni}^*)E w_n \frac{\partial l_{ni}^*}{\partial t}\right] + \alpha b \sum_{n=1}^{2} \theta_{nM} u'(d_{nM}) w_n \frac{\partial l_{nM}^*}{\partial t} = 0 \quad (21)
\]

\[
\frac{\partial \mathcal{W}'}{\partial b} = E \left[u'(x_{ni}^*) \left(-w_n l_{ni}^* + t E w_n \frac{\partial l_{ni}^*}{\partial b} \right) + \alpha b \sum_{n=1}^{2} \theta_{nM} u'(d_{nM}) w_n \frac{\partial l_{nM}^*}{\partial b} \right] + E \left[u'(d_{ni}^*) \left(\alpha w_n l_{ni}^* + (1-\alpha)E w_n \frac{\partial l_{ni}^*}{\partial l_{ni}^*} \right) \right] = 0 \quad (22)
\]

\[
\frac{\partial \mathcal{W}'}{\partial \alpha} = E \left[uu'(x_{ni}^*)E w_n \frac{\partial l_{ni}^*}{\partial \alpha} \right] + E \left[bu'(d_{ni}^*) \left(w_n l_{ni}^* - E w_n l_{ni}^* + (1-\alpha)E w_n \frac{\partial l_{ni}^*}{\partial l_{ni}^*} \right) \right] = 0. \quad (23)
\]

Equations (21) to (23) reveal that in adjusting labor supply, rational agents take into account the effects on their own life-cycle utility but ignore the effect on tax revenue, which is the difference between what their labor supply produces and the portion they are able to consume after the tax. Myopic agents, by contrast, not only miss to take into account the effect on tax revenue, but also on second period utility when adjusting their labor supply.

\(^4\)As both transfer schemes are linearly conditional on the same tax base \( w_i l_{ni} \) no welfare gains can be achieved by cross-subsidizing the tax system via the pension scheme and vice versa.
An explicit solution cannot be derived from the above system of equations which jointly determines the optimal tax rate $t$, the optimal pension contribution rate $b$ and the optimal Bismarckian factor $\alpha$. However, some interesting results can be obtained by rearranging equation (21) and (22) and solving for $t$ and $b$ respectively. Even though this approach allows to discuss some issues, it must be kept in mind that the three variables are not independent.

**The Optimal Tax Rate**

Rearranging equation (21) and employing the expectation operator, the optimal tax rate for a given pension contribution rate and Bismarckian factor amounts to

$$t = \frac{\text{Cov}(u'(x^*_ni), w_n^*l^*_ni) - (1 - \alpha) b \text{E} u'(d^*_ni) \text{E} w_n \frac{\partial l^*_ni}{\partial t}}{\text{E} u'(x^*_ni) \text{E} w_n \frac{\partial l^*_ni}{\partial t}}. \tag{24}$$

Equation (24) nests the standard result in optimal tax theory (see Atkinson and Stiglitz (1980), p.407). Without any old-age social security scheme, $b = 0$, this formula represents the traditional trade-off between equity and efficiency given by

$$t = \frac{\text{Cov}(u'(x^*_ni), w_n^*l^*_ni)}{\text{E} u'(x^*_ni) \text{E} w_n \frac{\partial l^*_ni}{\partial t}}. \tag{25}$$

The numerator reflects the distributional concern since the covariance between first-period marginal utility and income can be interpreted as a welfare-based measure of inequality. A large negative correlation makes a higher tax rate more desirable. The denominator characterizes the costs of redistribution in terms of the effective elasticity of the tax base. The optimal tax rate should be lower if a higher labor elasticity indicates that the redistributive tax implies a higher deadweight loss. If taxation does not cause distortions ($\partial l^*_ni/\partial t = 0$), redistribution should take place until the correlation between income and first-period marginal utility vanishes. Given both social security systems, the nominator of equation (24) also captures the adverse effects of a positive marginal tax rate on the pension scheme. Lower labor supply due to a positive tax rate likewise cuts the tax base in the pension scheme which in turn reduces the Beveridgean part of the pension scheme. The last term reflects the negative effect on the Bismarckian part of the myopic agents, since compared to rational individuals they fail to take into account the link between old-age benefits and contributions when reducing their labor supply due to taxation.

**The Optimal Pension Contribution Rate**

The optimal pension contribution rate for a given tax rate and Bismarckian factor is given by

$$b = \frac{(1 - \alpha) \text{Cov}(u'(d^*_ni), w_n^*l^*_ni) - \text{E}[w_n^*l^*_ni (u'(d^*_ni) - u'(x^*_ni))]}{(1 - \alpha) \text{E} u'(d^*_ni) \text{E} w_n \frac{\partial l^*_ni}{\partial t} + \alpha \sum_n \theta_n M u'(d^*_nM) w_n \frac{\partial l^*_nM}{\partial b}}. \tag{26}$$

As in equation (24) the denominator reflects the costs of redistribution in terms of the elasticity of labor supply. The first term refers to the Beveridgean part of the pension system whereas
the second term captures the distortive labor supply effects on the contribution related part of the myopic agents. The equity concern is expressed by the first two terms in the numerator of equation (26). Redistributional considerations over the life-cycle are indicated by the covariance term; a negative correlation between first-period’s income and second-period’s marginal utility calls for a lower pension contribution rate and vice versa. The second term reflects the desire to smooth consumption over the life-cycle. For the rational non-liquidity constrained agents this term is nil since savings are chosen so as to perfectly smooth consumption. For the liquidity constrained rational agents for whom \( u'(d^*_{nR}) > u'(x^*_{nR}) \) holds, this term calls for a lower pension contribution rate in order to decrease ‘too high’ old-age benefits. Contrary, for the myopic individuals this term requires a strictly positive pension contribution rate as they miss to save for retirement. The last expression in the numerator captures the adverse effects of a positive pension contribution rate on the tax base in the tax system.

The Optimal Bismarckian Factor

As shown in Appendix A.1, adverse effects of a positive pension contribution rate on myopic labor supply are independent of the optimal Bismarckian factor. However, adverse effects on rational labor supply depend on \( \alpha \) and are largest in a purely Beveridgean system. In order to answer the question as to whether these efficiency losses can be rectified by means of a positive Bismarckian factor, the optimal marginal tax rate \( t^* \) and \( b^* \) as implicitly defined by equations (24) and (26) are substituted into the government’s objective function \( \mathcal{W} \). This yields an optimal value function \( \Omega(\alpha) = \mathcal{W}(t^*, b^*, \alpha) \), which relates a given Bismarckian factor with maximum social welfare if the government implements a linear tax and pension scheme. Evaluating \( \frac{d\Omega}{d\alpha} \) at \( \alpha = 0 \) while taking the envelope theorem into account, yields

\[
\frac{d\Omega}{d\alpha} \bigg|_{\alpha=0} = b^* \text{Cov}(u'(d^*_{n}), w_n l^*_{n}) + (t^* \text{Eu}'(x^*_{n}) + b^* \text{Eu}'(d^*_{n})) \text{Ew}_n \frac{\partial l^*_{n}}{\partial \alpha}.
\]

The first term represents the redistributive impact of \( \alpha \); a positive Bismarckian factor is welfare enhancing if \( \text{Cov}(u'(d^*_{n}), w_n l^*_{n}) > 0 \). Then, consumption inequality over the life-cycle can be smoothed by linking pensions to prior contributions. In contrast, a negative Bismarckian factor has nice redistributinal effects if the covariance is negative. The second term captures the efficiency-enhancing effect on labor supply of the rational individuals, \( \frac{\partial l^*_{nR}}{\partial \alpha} > 0 \) (see Appendix A.1), and calls for a positive Bismarckian factor. Hence, a positive Bismarckian factor is more likely the higher labor supply elasticity of the rational agents and the less negative correlation between first-period income and second-period marginal utility.

To get a better understanding of the optimal design of the two linear social security schemes, the following two sections analyze the extreme settings of \( \pi^*_M = 0 \) (no myopic individuals) and \( \pi^*_R = 0 \) (no rational individuals) for \( n = 1, 2 \).
2.3.3 Only Rational Individuals

In this section the optimal tax and pension contribution rates are derived under the assumption that society consists only of rational individuals. Setting $\pi_n = 0$ and noting that $u'(x_{nR}^*) = u'(d_{nR}^*)$ for rational unrestricted individuals (see Equations (5) to (7)), the FOCs of the government reduce to:

\[
\begin{align*}
t + (1 - \alpha) b &= \frac{\text{Cov}(u'(x_{nR}^*), w_n l_{nR}^*)}{E u'(x_{nR}^*) E w_n \frac{\partial l_{nR}^*}{\partial t}} \\
t + (1 - \alpha) b &= (1 - \alpha) \frac{\text{Cov}(u'(d_{nR}^*), w_n l_{nR}^*)}{E u'(d_{nR}^*) E w_n \frac{\partial l_{nR}^*}{\partial b}} \\
t + (1 - \alpha) b &= -b \frac{\text{Cov}(u'(d_{nR}^*), w_n l_{nR}^*)}{E u'(d_{nR}^*) E w_n \frac{\partial l_{nR}^*}{\partial \alpha}}
\end{align*}
\]

(28)

Given the above system of equations the following result can be derived:

**Proposition 1:** If society consists only of rational individuals who face no binding liquidity constraint, the two transfer systems are perfect substitutable and either the tax or the pension system turns redundant.

The proof can be found in the Appendix A.2. The intuition behind this result is that from the point of view of the rational unconstrained individual only the total tax rate $\xi = t + (1 - \alpha) b$ and total transfers $\tau + B$ matter. This can best be seen by noting that the individual’s life-time budget constraint is determined by

\[
c_{nR} + d_{nR} = B + \tau + (1 - \xi) w_n l_{nR}.
\]

Savings always adjust to perfectly smooth consumption no matter whether redistribution takes place in the tax scheme while working, or in the pension scheme while retired.

There are some more conclusions that can be drawn from equation (37).

1. If the tax rate is chosen to be zero, $t = 0$, the optimal pension parameters are determined by $\xi = (1 - \alpha) b$. For $\alpha < 1$, this implies that the two parameters are perfect substitutes and any optimal contribution-linked pension system can be replicated by a pure Beveridgean system by setting $\alpha = 0$ and $b = \xi$.\(^5\) A pure Bismarckian system, $\alpha = 1$, can only be optimal if there are no redistributational concerns implying $\xi = 0$.

2. Any retirement benefit formula can be optimal as long as the marginal tax rate adjusts accordingly by $t = \xi - (1 - \alpha) b$.\(^6\) For $\alpha < 1$, both the pension and the tax scheme, embody

---

\(^5\)This result is also shown in Bütler (2002) and Cremer et al. (2007).

\(^6\)For this result, see also Kifmann (2008) who studies the optimal age-dependent tax and pension parameters in a society, in which all individuals are rational.
redistributional concerns. However, in a purely Bismarckian pension system $\alpha = 1$, the only effect of pensions on individual behavior is a one-for-one displacement of private savings and all redistributional concerns must be incorporated in the tax system implying $t = \xi$.

The above results hold only for perfect capital markets, implying that savings can be both, positive and negative. When capital markets are imperfect and the individual cannot borrow against his retirement benefits, the following result can be drawn:

**PROPOSITION 2:** If capital markets are imperfect and society consists only of rational agents, the two transfer systems are no longer perfect substitutes. For ‘too strong’ redistributional concerns, the optimal solution can be implemented without a pension system but not without a tax scheme.

The proof can be found in Appendix A.2. When savings are restricted to being positive, it matters from the individual’s point of view, whether redistribution takes place while working or while retired as ‘too large’ pensions may induce a binding liquidity constraint and, hence, lower utility.

In sum, it can be said that the introduction of a linear pension scheme in a society with only rational agents yields no additional welfare gains if the government has already implemented the optimal linear tax scheme. This result is independent on whether capital markets are perfect or not. However, the reverse is not true. This conclusion is in sharp contrast to the next section, where the optimal policy instruments are analyzed for a society consisting only of heterogenous myopic agents.

### 2.3.4 Only Myopic Individuals

In this section the optimal pension and tax scheme are defined under the assumptions that society consists of myopic agents only, implying $\pi^n_M = 0$ for $n = 1, 2$. In section 2.3 it was already shown that no pension scheme can never be optimal in a (partly) myopic society. With respect to the optimal type of the pension scheme the following result can be drawn:

**PROPOSITION 3:** In a society only of myopic individuals, the introduction of a tax scheme does not change the result by Cremer et al. (2007) that the optimal pension scheme is purely Beveridgean.

**Proof:** By noting that $\partial l^*_{nM} / \partial \alpha = 0$ for myopic individuals (see the Appendix A.1), the first-order condition for the Bismarckian factor (equation (23)) simplifies to

$$\frac{\partial \psi}{\partial \alpha} = bE \left[ u'(d^*_{nM}) (w_n l^*_{nM} - E w_n l^*_{nM}) \right] = b \text{Cov}(u'(d^*_{nM}), w_n l^*_{nM})$$
Since $b > 0$ with the made assumptions on utility, the above equation is equal to zero for $\alpha = 0$, implying a flat pension. \textbf{q.e.d.}

In a completely myopic society there is no efficiency reason to link old-age benefits to prior contributions and only the redistributive objective prevails which yields the above result of equal retirement benefits. With $\alpha = 0$, equation (22) and (21) amount to:

$$
t = \frac{\text{Cov}(u'(x_{nM}^*), w_nl^*)}{\text{Eu}'(x_{nM}^*)} - b \frac{\text{Eu}'(d_{nM}^*)}{\text{Eu}'(x_{nM}^*)}.
$$

Without a tax system, equation (30) reduces to equation (9) in Cremer et al. (2007); the optimal pension contribution rate then is larger, the greater the difference between first- and second-period consumption and smaller, the more elastic labor supply. Whether a tax system is welfare enhancing can again best be seen using the envelope theorem. Substituting the optimal pension contribution rate $b^*$ as implicitly defined by equation (30) into the government’s objective function $W$ yields the optimal value function $\Omega(t) = W(b^*, \alpha^* = 0, t)$, which relates a given tax rate with maximum social welfare. Evaluating $\frac{d\Omega}{dt}$ at $t = 0$ while taking the envelope theorem into account, yields

$$
\left. \frac{d\Omega}{dt} \right|_{t=0} = -\text{Eu}'(x_{nM}^*)w_nl^* + \text{Eu}'(x_{nM}^*)Ew_nl^* + b\text{Eu}'(d_{nM}^*)Ew_n \frac{\partial l_{nM}^*}{\partial t}.
$$

With the help of (21) evaluated at $t = 0$ and by noting that $\frac{\partial l_{nM}^*}{\partial t} = \frac{\partial l_{nM}^*}{\partial b}$ (see Appendix A.1), the above equation can be rewritten as

$$
\left. \frac{d\Omega}{dt} \right|_{t=0} = \left(\text{Eu}'(x_{nM}^*) - \text{Eu}'(d_{nM}^*)\right)Ew_nl^*.
$$

Only for $\text{Eu}'(x_{nM}^*) = \text{Eu}'(d_{nM}^*)$ no tax system is optimal. As $\text{Eu}'(d_{nM}^*) = u'(B)$ is independent of productivity and $\text{Eu}'(x_{nM}^*)$ increases with the difference in productivity, a positive tax rate is more likely the larger the variance in productivity. For small differences in ability even a negative tax rate, implying subsidization of labor supply, may be optimal.

\subsection*{2.3.5 No Productivity Differences}

If there are no productivity differences between rational and myopic agents, the following result can be drawn:

\textbf{Proposition 4:} If society consists of myopic and rational individuals who do not differ in productivity, the government is able to entirely offset the ex-post utility loss due to myopia and to implement the first-best solution determined by (15) and (16).
Table 1: Summary of Theoretical Results

<table>
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<th></th>
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<th>imperfect capital markets</th>
</tr>
</thead>
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<td>$t &gt; 0$, $b &gt; 0$, $\alpha &gt; 0$</td>
</tr>
<tr>
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<td>$\xi = t + (1-\alpha)b$</td>
<td>$\xi = t$, $b = 0$</td>
</tr>
<tr>
<td>only myopic agents</td>
<td>$t &gt; 0$, $b &gt; 0$, $\alpha = 0$</td>
<td>no impact</td>
</tr>
<tr>
<td>no productivity differences</td>
<td>$b = -t$, $\alpha = 0$</td>
<td>no impact</td>
</tr>
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</table>

**Proof:** It can be easily verified that $b = -t$, $\alpha = 0$ and $c_{nM}^* = d_{nM}^*$ solve the first-order conditions (21) to (23) for $w_1 = w_2 = w$. $b = -t$ and $\alpha = 0$ imply that labor supply is undistorted and equal across both types $l_{nM}^* = l_{nR}^* = \hat{l}$ which proves (16). As $-tEw_n l_{ni}^* = -tw\hat{l} = -\tau = bEw_n l_{ni}^* = bw\hat{l} = B$ first- and second-period consumption of the myopic can be rewritten as $x_M^* = w\hat{l} - B - v(\hat{l})$ and $d_M^* = B$. Due to $x_M^* = d_M^*$, pensions are given by $B = 0.5(w\hat{l} - v(\hat{l}))$. It remains to show that the solution to the first-order conditions (21) to (23) equalizes consumption levels across types and entirely offsets the ex post utility loss due to myopia which implicates that $x_M^* = d_M^* = \hat{x}_R = \hat{d}_R$, where $\hat{x}_R$ and $\hat{d}_R$ are *laissez-faire* consumption levels by the rational. With no government intervention it is optimal for the rational agent to consume in the first period $\hat{x}_R = w\hat{l} - v(\hat{l}) - \hat{s}_R$ and in the second period $\hat{d}_R = \hat{s}_R = 0.5(w\hat{l} - v(\hat{l}))$. With $b = -t$, $\alpha = 0$ and $B = 0.5(w\hat{l} - v(\hat{l}))$ first- and second-period consumption of the rational are unchanged but total private savings are one for one replaced by compulsory savings as $B = \hat{s}_R$. Hence, $x_M^* = d_M^* = \hat{x}_R = \hat{d}_R$ and the ex post utility loss due to myopia is entirely offset. *q.e.d.*

The pension system redistributes income from the first period to the second to perfectly smooth consumption, while the negative marginal tax rate de facto subsidizes labor supply to exactly offset the labor supply distortions induced by the pension scheme. As long as there are no redistributional concerns, utility losses due to myopia can be entirely offset. Ex post utility is as high as *laissez-faire* utility of a representative rational agent, $V_{nM} = V_{nR}$ for $n = 1, 2$. De facto, each agent pays a lump sum transfer in the first period given by $-\tau$, whereas in the second period each individual receives a lump sum transfer $B$ of the same amount. Whether capital markets are perfect or imperfect turns out irrelevant as private savings of the rational agents are completely replaced by compulsory savings.
2.3.6 Summary of Theoretical Results

Table 1 summarizes the main theoretical results of this section pertaining the optimal policy instruments in a mixed, all rational, all myopic and a society with no productivity differences. As no definite results can be derived concerning the optimal tax rate and the Bismarckian factor in a mixed society, the next section turns to a numerical example. This example not only reveals the sign of those policy instruments but also illustrates how the optimal parameters of the two transfer schemes change with the share of rational agents in different scenarios. Moreover, results are compared with those derived in the framework modeled by Cremer et al. (2007) with only a pension scheme.

3 Numerical Example

This section provides an illustration of the analytical results by means of numerical simulations. In accordance with Cremer et al. (2007) the simulation is based on the following functional form for utility and labor disutility

\[ U_{ni} = \ln \left( c_{ni} - \frac{l_{ni}^2}{2} \right) + \beta_i \ln (d_{ni}) \quad \forall \ ni. \]

The basic parameter values are given by\(^7\)

\[ w_1 \in \{1, 2, 3\}, \quad w_2 = 4 \quad \text{and} \quad \theta_1 = 0.6. \]

To give a comprehensive illustration, computations are executed for different variances in productivity by changing the value of \( w_1 \). Table 2 shows consumption levels, labor supply and utility in the laissez-faire solution. For small values of \( w_1 \) the ordering of net consumption is given by \( \hat{x}_{1R} < \hat{x}_{1M} < \hat{x}_{2R} < \hat{x}_{2M} \) and the difference in productivity is expected to dominate that in time preference, implying redistribution from rich to poor. In contrast, for \( w_1 = 3 \) the ordering switches to \( \hat{x}_{1R} < \hat{x}_{2R} < \hat{x}_{1M} < \hat{x}_{2M} \) and the difference in time preferences should be dominant, pointing to redistribution from myopic to rational agents. In line with the empirical evidence, it is assumed that the share of low productivity agents exceeds the share of high productivity agents.

The following sections analyze the optimal transfer schemes for three different scenarios. Section 3.1 presents the benchmark case in which the optimal policy instruments are derived under the assumption that capital markets are perfect and that productivity and rationality are uncorrelated. No correlation between the two characteristics implies an equal share of rational and myopic

---

\(^7\)Cremer et al. (2007) derive their simulation results under the assumption of a positively skewed Beta (2,4) distribution for wages that vary from 1 to 4. However, here in this more comprehensive framework, the main points at stake can be shown more clearly by employing a discrete distribution in wages.
<table>
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<th>Type</th>
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<th>$\hat{x}_{ni}$</th>
<th>$d_{ni}$</th>
<th>$l_{ni}$</th>
<th>$u(\hat{x}_{ni})$</th>
<th>$\hat{V}_{ni}$</th>
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individuals among both productivity types. Section 3.2 relaxes the assumption of perfect capital markets, whereas Section 3.3 assumes a positive correlation between ability and rationality which requires a higher share of rational agents among the high-productivity types and a higher share of myopic agents among the low-productivity types.

### 3.1 Perfect Capital Markets and no Correlation

First, the case of perfect capital markets and no correlation between productivity and rationality is analyzed, implying $\pi^1_M = \pi^2_M = \pi_M$ and $\pi^1_R = \pi^2_R = \pi_R$. Table 3 presents the optimal values of the government’s instruments for different values of rational individuals, $\pi_R$, and different variances in productivity $w_n$. Variables denoted with a superscript $*$ are obtained under the assumption that both transfer schemes coexist, whereas variables denoted with a superscript $o$ are derived under the assumption that there is only a linear pension system as in Cremer et al. (2007). In calculating these values, any liquidity constraints for the rational individuals in society were assumed away and hence $s^{*o}_{nR} \geq 0$.

**The Pension Contribution Rate**: In a complete rational society, $\pi_R = 1$, one transfer scheme turns redundant (see Proposition 1) which, here, is presumed to be the pension scheme. For $\pi_R < 1$, Table 3 reveals that the generosity of the pension scheme, governed by $b^*$, is positive and stays relatively constant across different shares of myopic agents. Myopic individuals undertake no savings because their immediate ‘self’ induces them to get instant gratification. However, their rational ‘self’ would appreciate a government forcing them to provide for retirement. Consequently, it is optimal for the paternalistic government to introduce a pension system when the share of myopic individuals becomes positive. The three different computations for $w_1$ show that the optimal level of pensions is almost independent of the variance in productivity. By contrast, the pension contribution rate modeled in the framework by Cremer et al. (2007) is higher for large differences
Table 3: Perfect Capital Markets and no Correlation between Productivity and Rationality.

<table>
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<tr>
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<th>$b^*$</th>
<th>$\alpha^*$</th>
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<td>-</td>
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</tr>
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</table>

* As for $\pi_R = 1$ government intervention is solely determined by $\xi = t^* + b^*(1 - \alpha^*)$ (see Proposition 1), the Bismarckian factor and one of the contribution rates, $t^*$ or $b^*$, can be set equal to zero, implying $\xi = t^* = b^0$ if $b^* = 0$ or $\xi = b^* = b^0$ if $t^* = 0$.

in ability as the pension scheme must also account for strong redistributional concerns.

**The Bismarckian Factor**: As the intuition suggests, the optimal Bismarckian factor increases with the share of rational individuals and, as proven in Proposition 3, is zero when all agents are myopic. In a purely myopic society the efficiency enhancing effect of a positive Bismarckian factor is nil. However, in a mixed society a positive $\alpha$ enhances labor supply of the rational individuals. This expands the tax base which in turn provides for every given $t$ and $b$ higher lump sum transfers in the two social security systems. Hence, the less myopic individuals are in society the more the optimal pension system moves away from a purely redistributive Beveridgean system towards a stronger contribution linked Bismarckian system. The optimal Bismarckian factor again turns zero in a society composed only of rational individuals since then all redistribution can be achieved by only taxing people in their working-age. In the framework modeled by Cremer et al. (2007) the Bismarckian factor decreases with the variance in productivity. It may even turn negative for a low disparity in ability, indicating strong redistributive benefits. By contrast, in the framework used in this paper, the Bismarckian factor hardly changes with the variance in productivity. A closer look at the optimal tax rate $t^*$ brings to light why this is the case.

**The Tax Rate**: When the government has the opportunity to utilize the tax system as an
additional instrument to redistribute, the objective behind a negative (positive) $\alpha$, namely to make the system more progressive (regressive), can be achieved more efficiently:

1. For $w_1 = 1$, and hence a large disparity in productivity, welfare maximization calls for redistribution from rich to poor. In the framework by Cremer et al. (2007) this is achieved by a negative Bismarckian factor $\alpha^0$. With both social security systems, however, redistribution from high to low productivity types is obtained via the tax system by setting a positive tax rate $t^*$. The optimal Bismarckian factor continues to be positive to enhance efficiency in both systems.

2. For $w_1 = 3$, implying a low disparity in productivity, even a negative marginal tax rate turns out to be optimal. De facto the optimal tax system subsidizes labor supply. At first sight this result seems counterintuitive, since a negative marginal tax rate turns the tax system regressive with respect to productivity; all individuals pay the same lump sum transfer, $-\tau$, but the high productivity types receive higher absolute labor supply subsidies than the low productivity types, $|tw_1l_1| < |tw_2l_2|$, implying $T_{2i} < T_{1i}$ for $i = R, M$. But, as for a small variance in ability laissez-faire consumption of the myopic is larger than for the rational agents (see Table 2), $\hat{x}_{nM} < \hat{x}_{nR}$ for $n = 1, 2$, the government mainly aims at redistributing to the rational individuals in society. Due to a larger labor supply of the rational agents for $\alpha > 0$, a subsidy on labor together with a positive Bismarckian factor exactly meets this goal. Moreover, subsidizing labor incorporates strong efficiency enhancing effects; a negative marginal tax not only boosts labor supply of the rational agents, as a positive $\alpha$ does, but also encourages labor supply of the myopic agents in society. This in turn expands the tax base and provides for any given $b$ and $\alpha$ higher transfers during retirement.

3. Computations for $w_1 = 2$ uncover that for moderate disparities in productivity a tax rate of zero and, hence, no tax system may also be an optimal solution. In this case, redistributional concerns are to low to offset accompanying efficiency losses by the tax system.

To put it in a nutshell, the role of the government, measured by $t + (1 - \alpha)b$ is larger in a all myopic society since then it pursues two goals (1) providing for old-age and (2) achieving more equality. The extend of the tax system, measured by $|\tau|$, strongly depends on the variance of productivity in society and hardly on the share of myopic and rational agents. The optimal marginal tax rate increases with the variance in productivity starting from being negative for small differences in ability. By contrast, the extent or generosity of the pension system is mainly determined by the pure existence of myopic individuals, their share and the difference in productivity do only play a minor role for the determination of $b^*$. Hence, for a low variance in productivity the redistributional effects of the pension scheme are partly offset by the tax scheme. Compared to the results by Cremer et al. (2007) where the Bismarckian factor plays a strong redistributive role, in this extended model it mainly performs the task of enhancing efficiency in both systems.
Table 4: Optimal Social Security Schemes for Imperfect Capital Markets.

<table>
<thead>
<tr>
<th>π R</th>
<th>t^c</th>
<th>b^c</th>
<th>α^c</th>
<th>W^c</th>
<th>b^o</th>
<th>α^o</th>
<th>W^o</th>
</tr>
</thead>
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<td>0.13</td>
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<td>0.28</td>
<td>0.38</td>
</tr>
<tr>
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<td>0.18</td>
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<td>-0.15</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.32</td>
<td>-</td>
<td>-</td>
<td>0.61</td>
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<td>-1.00</td>
</tr>
<tr>
<td>w_1=2</td>
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<td>0.01</td>
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<td>1.17</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
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<td>0.3</td>
<td>0.03</td>
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</tr>
<tr>
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<td>0.03</td>
<td>0.22</td>
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<td>0.13</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>0.03</td>
<td>0.22</td>
<td>0.21</td>
<td>1.29</td>
<td>0.18</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.20</td>
<td>-</td>
<td>-</td>
<td>1.30</td>
<td>0.18</td>
<td>-0.12</td>
</tr>
<tr>
<td>w_1=3</td>
<td>0</td>
<td>-0.15</td>
<td>0.25</td>
<td>0</td>
<td>2.04</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>-0.12</td>
<td>0.22</td>
<td>0.17</td>
<td>2.05</td>
<td>0.25</td>
<td>0.47</td>
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<td></td>
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<td>-0.12</td>
<td>0.22</td>
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<td>0.62</td>
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<td>-0.12</td>
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<td>2.09</td>
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<tr>
<td></td>
<td>1</td>
<td>0.06</td>
<td>-</td>
<td>-</td>
<td>2.09</td>
<td>0.06</td>
<td>-**</td>
</tr>
</tbody>
</table>

* As stated in Proposition 2, for π R = 1 and imperfect capital markets, it is best to redistribute only via the tax system.
** Here, redistributional concerns are too low to make the liquidity constraint binding which implies perfect substitutability between b^o and α^o.

3.2 Imperfect Capital Markets

This section shows results for the optimal social security structure with imperfect capital markets that may lead to a binding liquidity constraint if the mandatory pension scheme is ‘too large’. The net effects of the presence of liquidity constrained rational agents for whom u'(x_{nR}^*) > u'(d_{nR}^*) holds if γ_{nR} > 0 are very clear-cut. Since the low-ability rational individuals are the first who may suffer from a binding liquidity constraint (see equation (9)) changes in the optimal policy parameters \{t^c, b^c, α^c\} aim at equalizing their marginal utilities. In fact, simulation results reveal that the government can restore intertemporal equality of consumption by using the available policy instruments to ensure that x_{nR}^* = d_{nR}^* (or equivalently γ_{nR} = 0) for n = 1, 2.8

The Pension Contribution Rate: In a completely myopic society, the liquidity constraint

8Note that the government faces two possible regimes. The first regime implies perfect consumption smoothing for the liquidity constrained rational individuals, whereas the second regime implies x_{nR}^* < d_{nR}^* or γ_{nR} > 0. Obviously, the optimum is in the second regime if equating consumptions by the use of the available policy instruments implies higher welfare losses. Computations reveal that this is only the case for w_1 = 1 when π R = 0.3 or 0.6 and when the government has only the pension parameters at hand.
imposed on rational agents turns irrelevant and the optimal tax and pension scheme are the same as in Table 3. As the intuition suggests, in a mixed society where rational agents cannot sell claims against their future retirement benefits, the optimal pension contribution rate is always lower. By contrast, for a society of only rational agents, no old-age benefits via a pension scheme are needed and as stated in Proposition 2, all redistribution should be done via the tax system because the liquidity constraint is never binding and the same level of welfare as with perfect capital markets can be attained. This result is in sharp contrast to the solution in Cremer et al. (2007) where welfare in a complete rational society is lower if the variance in productivity is large. ‘Too strong’ redistributional concerns may induce a binding liquidity constraint and hence lower utility for the constrained agents.

The Bismarckian Factor: Regarding the optimal Bismarckian factor, results are very different to those in Cremer et al. (2007). In their framework, the optimal \( \alpha \) may even turn negative, implying targeting towards the poor. Here, in this more comprehensive model, the optimal pension scheme becomes more Bismarckian compared to the case with perfect capital markets. A stronger link between contributions and pensions de facto reduces old-age benefits of the poor which in turn relaxes their credit constraint; the negative income effect when old makes dissaving less desirable from the poor rational’s point of view.

The Tax Rate: The adjustment in the tax system works in the same direction as in the pension scheme. Here, a relaxation of the poor’s liquidity constraint is obtained via a positive income effect when young by increasing the marginal tax rate which more strongly redistributes from rich to poor in the first-period.

In sum, the existence of credit constrained individuals induces three policy changes compared to Table 3. (1) The pension contribution rate or the generosity of the pension scheme is lowered. (2) The pension scheme turns more Bismarckian, implying higher consumption dispersion in the second period. And (3) the tax system becomes more redistributive, involving lower consumption dispersion in the first period. Hence, higher consumption inequality in old-age seems to be optimal from a redistributive point of view if society is partly composed of myopic agents and if capital markets are imperfect, implying \( s_{r,b}^* \geq 0 \).

3.3 Positive Correlation between Productivity and Rationality

Up to now it was assumed that rationality and productivity are uncorrelated. This section relaxes this assumption and analyzes the case of a positive correlation between the two characteristics \( \pi_M^1 > \pi_M^2 \). \(^9\)

\(^9\)The analysis by Bernheim et al. (2001) gives some evidence that myopia and productivity may be positively correlated as they find that higher wealth is associated with a smaller decline in consumption after retirement even though they have controlled for many life-cycle arguments.
Table 5: Optimal Social Security Schemes for $\pi_1^M > \pi_2^M$.

<table>
<thead>
<tr>
<th>$w_1$</th>
<th>$\pi_R$</th>
<th>$\pi_R^1$</th>
<th>$\pi_R^2$</th>
<th>$t^{pc}$</th>
<th>$b^{pc}$</th>
<th>$\alpha^{pc}$</th>
<th>$\omega^{pc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6</td>
<td>0.4</td>
<td>0.9</td>
<td>0.13</td>
<td>0.20</td>
<td>0.06</td>
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</tr>
<tr>
<td></td>
<td>0.6</td>
<td>0.6</td>
<td></td>
<td>0.12</td>
<td>0.24</td>
<td>0.09</td>
<td>0.54</td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
<td>0.4</td>
<td>0.9</td>
<td>0.00</td>
<td>0.19</td>
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<td>1.29</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>0.6</td>
<td></td>
<td>0.00</td>
<td>0.24</td>
<td>0.11</td>
<td>1.25</td>
</tr>
<tr>
<td>3</td>
<td>0.6</td>
<td>0.4</td>
<td>0.9</td>
<td>-0.16</td>
<td>0.21</td>
<td>-0.11</td>
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</tr>
<tr>
<td></td>
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<td>0.6</td>
<td></td>
<td>-0.15</td>
<td>0.25</td>
<td>0.10</td>
<td>2.07</td>
</tr>
</tbody>
</table>

Table 5 presents the optimal parameters of the two transfer schemes, indicated with a superscript $pc$, for $\pi_1^R = 0.4$ and $\pi_2^R = 0.9$. The chosen values ensure that results can be compared to the above case with no correlation between rationality and productivity since the overall share of low and high productivity types and of rational and myopic agents remains the same. However, there is a shift in probability mass from $2^M$ to $2^R$ and from $1^R$ to $1^M$ agents. Given $\theta_1 = 0.6$ and $\theta_2 = 0.4$ the total share of rational agents amounts to

$$\pi_R = \theta_1 \pi_R^1 + \theta_2 \pi_R^2 = 0.6 \times 0.4 + 0.4 \times 0.9 = 0.6.$$  

From comparing both outcomes the following conclusions can be drawn. (1) When there are more myopic agents among the low-productivity types, the generosity of the pension system can be reduced as myopic agents of low ability require lower old-age benefits to smooth consumption over their life-cycle than their high-ability counterparts. (2) The optimal tax rate hardly changes. And (3) for moderate to small disparities in ability the optimal Bismarckian factor may even turn negative implying targeting towards the poor.

Interestingly, welfare is higher in a society where productivity and rationality are positively correlated than in a society where the two characteristics are uncorrelated. This is due to the fact that the role of the government, measured by $t + (1 - \alpha)b$, is reduced and with it accompanying labor supply distortions.

### 4 Conclusion

Cremer et al. (2007) study the optimal linear pension parameters when society consists of individuals who do not only differ in productivity but also in rationality. Rational agents may also be liquidity constrained. Myopic agents have ex ante a strong preference for the present and undertake no savings, even though, ex post they rue their earlier decision. The social objective is assumed to be paternalistic, aiming at maximizing the sum of ex post utilities. This paper has
extended the model by Cremer et al. (2007) by introducing income taxation. The main results can be summarized as follows:

(i) In a completely rational society where individuals differ only in productivity and where capital markets are perfect, the objective of the government is purely redistributive. Every given degree of redistribution can be achieved with only one of the transfer systems and either the tax or the pension system is redundant. However, if rational individuals are liquidity constrained, the two transfer schemes are no longer perfect substitutes. Welfare can still be maximized having only the tax scheme, but it may be reduced having only the pension scheme as in Cremer et al. (2007).

(ii) If a society is also composed of myopic agents, the pension scheme is needed to ensure old-age consumption. Compared to the Cremer et al. (2007) framework, the generosity of the pension system hardly changes. However, the degree of redistribution in the pension scheme, captured by the Bismarckian factor, may be reversed. When the government has the opportunity to utilize the tax system as an additional instrument to redistribute, the objective behind a negative (positive) $\alpha$ in the Cremer et al. (2007) solution, namely to make the system more progressive (regressive), can be achieved more efficiently with a positive (negative) tax rate. The optimal Bismarckian factor stays positive to enhance efficiency in both systems.

(iii) Numerical simulations show that the generosity of the pension scheme is mainly determined by the pure existence of myopic agents, whereas the marginal tax rate strongly depends on the disparity in productivity. It increases with the difference in productivity starting from being negative for small differences. Both, the tax and the pension contribution rate, hardly change with the share of rational agents. By contrast, the optimal Bismarckian factor increases with the share of rational individuals, but stays relatively constant across different variances in productivity.

(iv) When capital markets are imperfect results change, in this extended framework, only quantitatively; the optimal level of retirement benefits decreases in order to relax an otherwise binding liquidity constraint of the poor rational. Moreover, the degree of redistribution increases in the tax system, whereas it decreases in the pension scheme. In contrast, the degree of redistribution in the Cremer et al. (2007) framework becomes a non-monotonic function of the share of rational agents.

To keep the analysis tractable both transfer systems were assumed to be linear. Cremer et al. (2008) and Tenhunen and Tuomala (2007) relax this assumption and allow for non-linear transfer schemes. However, this approach does not allow them to derive explicit results on how the policy instruments should be optimally designed. Additionally, results strongly depend on the informational assumptions by the government.
A Technical Appendix

A.1 The Effects of $t$, $b$ and $\alpha$ on Labor Supply

This appendix derives the effects of the various policy instruments on labor supply and savings of the myopic and the unconstrained rational individual.

By employing the individual budget constraints to eliminate $c_{ni}$ and $d_{ni}$ from the utility function, the maximization problem is given by

$$U_{ni} = u((1 - t - b)w_n l_{ni} + \tau - \beta_i s_{ni} - v(l_{ni})) + \beta_i u(\beta_i s_{ni} + b\alpha w_n l_{ni} + B) + \gamma \beta_i s_{ni}$$

The first-order conditions for labor supply and savings are

$$-\beta_i u'(x^*_{ni}) + \beta_i u'(d^*_{ni}) + \gamma n_{\beta i} = 0,$$

$$u'(x^*_{ni})(1 - t - b)w_n - u'(x^*_{ni})v'(l^*_{ni}) + \beta_i u'(d^*_{ni})b\alpha w_n = 0.$$

A.1.1 Myopic Agents

For the myopic individual, $\beta_i = 0$, the FOCs given in equation (31) and (32) reduce to $(1 - t - b)w_n - v'(l^*_{nM}) = 0$. Hence, labor supply effects can be simply computed with the implicit function theorem which yields

$$\frac{\partial l^*_{nM}}{\partial b} = \frac{-w_n}{v''(l^*_{nM})} < 0, \quad \frac{\partial l^*_{nM}}{\partial t} = \frac{-w_n}{v''(l^*_{nM})} < 0 \quad \text{and} \quad \frac{\partial l^*_{nM}}{\partial \alpha} = 0.$$

A.1.2 Unconstrained Rational Agents

For the unconstrained rational individual, where $\beta_i = 1$ and $\gamma_{nR} = 0$ which implies $u'(x^*_{nR}) = u'(d^*_{nR})$ and therefore also $u''(x^*_{nR}) = u''(d^*_{nR})$, total differentiation of the FOCs given in equation (31) and (32) yields

$$-2u''(x^*_{nR})ds_{nR} - 2u''(x^*_{nR})b\alpha w_n dl^*_{nR} = \Delta^s$$

$$v''(l^*_{nR})dl^*_{nR} = \Delta^l,$$

with

$$\Delta^s = u''(x^*_{nR})b\alpha w_n dl^*_{nR} + (1 + \alpha)u''(x^*_{nR})w_n l^*_{nR} db + u''(x^*_{nR})w_n l^*_{nR} dt$$

$$+ u''(x^*_{nR})db - u''(x^*_{nR})dt$$

$$\Delta^l = bw_n \alpha - w_n (1 - \alpha) db - w_n dt$$

These equations can be written as the following linear system

$$\begin{bmatrix}
-2u''(x^*_{nR})b\alpha w_n & -2u''(x^*_{nR}) \\
\gamma (l^*_{nR}) & 0
\end{bmatrix}
\begin{bmatrix}
\frac{dl^*_{nR}}{ds_{nR}} \\
\frac{dl^*_{nR}}{dt}
\end{bmatrix}
= \begin{bmatrix}
\Delta^s \\
\Delta^l
\end{bmatrix}.$$
Inverting (33) amounts to

\[
\begin{bmatrix}
\frac{dl^*_nR}{ds^*_nR}
\end{bmatrix} = \frac{1}{D} \begin{bmatrix}
0 & 0 \\
-\nu''(l^*_nR) & -2\nu''(x^*_nR)b\alpha w_n
\end{bmatrix} \begin{bmatrix}
\Delta^s \\
\Delta^l
\end{bmatrix},
\]

where the determinant, given by \(D = 2\nu''(x^*_nR)\nu''(l^*_nR)\), is negative with the assumptions made on utility and labor disutility. From this solution the various labor supply effects for the unconstrained rational agent can be found:

\[
\frac{\partial l^*_nR}{\partial t} = -\frac{w_n}{v''(l^*_nR)} < 0, \quad \frac{\partial l^*_nR}{\partial b} = -\frac{(1 - \alpha)w_n}{v''(l^*_nR)} \leq 0 \quad \text{and} \quad \frac{\partial l^*_nR}{\partial \alpha} = \frac{bw_n}{v''(l^*_nR)} > 0. \tag{34}
\]

### A.2 Proofs

#### A.2.1 Proof of Proposition 1

**Proof:** Making use of (34) and noting that \(\text{Cov}(\nu'(x^*_nR), w_n l^*_nR) = \text{Cov}(\nu'(d^*_nR), w_n l^*_nR)\) for rational unrestricted individuals, each equation in (28) can be written as

\[
t + (1 - \alpha)b = \frac{\text{Cov}(\nu'(x^*_nR), w_n l^*_nR)}{\nu'(l^*_nR) \nu''(l^*_nR)}. \tag{35}
\]

The right hand-side of (35) depends on the expression \(t + (1 - \alpha)b\); equalizing first- and second-period consumption defined in (2) and (3) and solving for savings amounts to

\[
\nu'(x^*_nR) = \nu'(0.5((1 - (1 + (1 - \alpha)b)) w_n l^*_nR - v(l^*_nR) + (t + (1 - \alpha)b) w_n l^*_nR)) \tag{36}
\]

With (36) and by defining \(\xi = t + (1 - \alpha)b\), equation (35) can be rewritten as

\[
\xi = \frac{\text{Cov}(\nu'(0.5((1 - \xi)w_n l^*_nR - v(l^*_nR) + \xi w_n l^*_nR)), w_n l^*_nR)}{\nu'(0.5((1 - \xi)w_n l^*_nR - v(l^*_nR) + \xi w_n l^*_nR)), w_n l^*_nR) \nu'(l^*_nR)} \tag{37}
\]

Since \(l^*_nR = v^{-1}((1 - \xi)w_n)\) the solution to (28) is solely determined by \(\xi\). This implies that the tax and the pension system are perfect substitutes and either \(t \) or \(b\) can be set equal to zero. \textbf{q.e.d.}

#### A.2.2 Proof of Proposition 2

With no tax system, \(t = 0\), the optimal saving decision (equation (39)) reduces to

\[
s^*_nR = \begin{cases} 
0.5((1 + (1 - \alpha)b) w_n l^*_nR - v(l^*_nR) - B) & \text{if } \gamma_{nR} = 0 \\
0 & \text{if } \gamma_{nR} > 0.
\end{cases} \tag{38}
\]
If the pension scheme is ‘too large’ the individual wants to borrow against his retirement benefits, but as the liquidity constraint prevents him from doing so the optimum is in the second row of (38) and $s^*_{nR} = 0$. However, with no pension system, $b = 0$, the optimal savings decision reduces to

$$s^*_{nR} = \begin{cases} 
0.5 \left( (1 - t) w_n l^*_{nR} + \tau - v(l^*_{nR}) \right) & \text{if } \gamma_{nR} = 0 \\
0 & \text{if } \gamma_{nR} > 0,
\end{cases}$$

where the first row can be also written as $s^*_{nR} = 0.5 x^*_{nR}$. Since $x^*_{nR}$ is always chosen to be positive with the made assumptions on utility, savings will never be equal to zero. Therefore, the problem of being credit constrained is only present when the government implements ‘too large’ mandatory saving via a pension scheme. With no pension system the liquidity constraint $s_{nR} \geq 0$ is non-binding and the solution to the individual’s and government’s optimization problem with imperfect capital markets is identical to the one with perfect capital markets for $\xi = t$ and $b = 0$. \textbf{q.e.d.}
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