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Access by Capacity and Peak-Load Pricing [*]

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Abstract

Several European telecommunications regulatory agencies have recently introduced a fixed capacity charge (flat rate) to regulate access to the incumbent's network. The purpose of this paper is to show that the optimal capacity charge and the optimal access-minute charge analysed by Armstrong, Doyle, and Vickers (1996) have a similar structure and imply the same payment for the entrant. I extend the analysis to the case where there is a competitor with market power. In this case, the optimal capacity charge should be modified to avoid that the entrant cream-skims the market, fixing a longer or a shorter peak period than the optimal. Finally, I consider a multiproduct setting, where the effect of the product differentiation is exacerbated.

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1 Introduction

The liberalisation of the electricity, gas and water industries has frequently used vertical desaggregation to promote competition. In the telecommunications sector, however, this policy has not been adopted, except in some countries like the United States, Chile, Bolivia and Brazil. When the main local network operator is also a supplier of long distance telecommunications services, the most valuable instrument to guarantee that all the firms compete at the same playing field level, is to regulate the access to the incumbent firm's network.

The British and the Spanish telecommunications regulatory agencies have recently introduced a system of access by capacity for regulating access to the incumbent's network. This system is based on a fixed charge (flat rate) for the use of some of the incumbent's circuits. Indeed, the principal driver of network costs is the peak-hour capacity cost. Moreover, the proponents of this rule emphasise that a fixed capacity charge would promote an efficient use of the network and the introduction of innovative services. They argue that in contrast to the system of access by time the entrants will have more flexibility to manage their load curve and to fix their end-to-end prices.

The primary objective of this essay is to derive the optimal capacity charge, and to analyse whether the received theory of optimal access pricing may also be applied to the capacity charge problem. I aim to analyse this problem considering a continuous and interdependent demand for telecommunications services and various assumptions for the supply conditions.

The theory of access pricing has been extensively developed by Baumol (1983), Baumol and Sidak (1994), Laffont and Tirole (1994), Armstrong, Doyle, and Vickers (1996), Lewis and Sappington (1999), Carter and Wright (1999) and others.[1] In all these studies the optimal access price is derived from the following assumptions: (a) the demand of the final service does not fluctuate with time; and (b) the access charge is set for one unit of service (e.g. one access-minute).

The purpose of this paper is to provide a theory of how to price capacity when these assumptions are relaxed. The consideration that demand fluctuates with time is relevant, because one of the most important strategies used by the firms that enter the telecommunications sector is to differentiate their service from that of the incumbent by offering different peak and off-peak prices. When this occurs, it is also relevant to consider the interdependencies between the peak and the off-peak demands. On the other hand, in view of the most recent regulatory practice, it is important to modify these models and consider an access system based on a fixed capacity charge rather than on a variable

[1] For a complete analysis of the access pricing problem see Laffont and Tirole (1996), Laffont and Tirole (2000) and Armstrong (2001).

per minute access charge. Finally, for simplicity I consider a model of one-way interconnection. Indeed, after several years of liberalisation the incumbent national operators are still responsible of terminating the main part of the calls. And more importantly for the purpose of this paper, in practice only the incumbent is forced to offer to the market a system of access by capacity.

I begin by considering a market with a regulated incumbent and an unregulated fringe of price-taking entrants. The incumbent operates a network and produces a service which is sold in the final market. The fringe offers the same final service but in order to convey it, it needs to use the monopoly's network. Laffont and Tirole (1994) and Armstrong, Doyle, and Vickers (1996) have characterised the optimal per minute access charge in this setting but, as stated above, they consider a time independent demand. Armstrong, Doyle, and Vickers (1996) derive the optimal access price formulae under various assumptions. These consist of the direct access cost, the incumbent's opportunity cost of providing access and, if the budget constraint of the incumbent firm is an issue, a Ramsey markup.[2] I show that a model of access by time, like the one developed by these authors, and a model of access by capacity, generally imply the same access payment for the entrant.

As in Armstrong, Doyle, and Vickers (1996), the optimal capacity charge consists of the direct capacity costs, the opportunity costs incurred by the incumbent when it provides a unit of capacity to the fringe, and a positive Ramsey term if the incumbent's break-even constraint is an issue. However, with "time-of-use" retail prices, the incumbent's opportunity costs reflect the loss in profit during the peak and the off-peak period.

As an extension to the previous framework, I analyse the capacity charge problem when there is a competitor with market power rather than a competitive fringe. In this case, the length of the pricing periods plays a fundamental role in the establishment of the optimal capacity charge. In particular, I show that the entrant may choose a duration time for the peak period which is longer or shorter than the optimal. By doing so the entrant modifies the peak period price and attracts a part of the customers. However, this affects the optimal length of the incumbent's pricing periods, which in turn affects the optimal prices. In this situation, the optimal policy consists of avoiding the product differentiation by increasing or reducing the capacity charge. If the entrant fixes a lower peak price by enlarging the length of the peak period, an increase of the capacity charge increases the peak price and reduces product differentiation. Subsequently, less capacity is required to satisfy the demand, and so the management of the load improves. If the entrant fixes a higher peak

[2] The authors show how product differentiation, bypass and input substitution possibilities reduce the opportunity cost of access. They demonstrate that these situations reduce the extent to which the incumbent loses final product sales per each additional unit of access supplied.

price but reduces the length of the peak period, a reduction of the capacity charge forces him to extend the duration of the peak price. This would reduce the peak price, and as a result, more capacity is required.

Finally, we study a multiproduct setting where the incumbent and one entrant with market power provide several services. In this case, if the services provided by the incumbent and the entrant differ, the capacity charge should be corrected in order to force the entrant to choose the optimal length of the pricing periods. This may occur even when the entrant has no market power because each service has a particular load curve, and the length of the peak period chosen by the entrant depends on the particular set of services provided. Therefore, as in the single-product case, the optimal capacity charge does not guarantee an efficient use of the network.

The idea of this paper is closely related to the more specific issue of peak-load pricing and access charge. There has been a great deal of research about the properties of peak-load pricing.[3] The research has mostly dealt with the regulation of one utility that faces constant demand within pricing periods, and in the case where the demand in one period is independent to the demand in the other. On the other hand, very few studies consider the problem of the optimal pricing period. Pressman (1970) considers demand interdependencies under conditions of constant demand within exogenously given pricing periods. Craven (1971), Craven (1985), Dansby (1975) and Dansby (1978) allow time varying demand within pricing periods and examine what the optimal length of the pricing periods could be. Crew and Kleindorfer (1986) consider both time-varying and independent demands, but they consider a fixed period length. Burness and Patrick (1991) consider continuous and interdependent demands and determine the optimal pricing period length. These assumptions are specially relevant for the purpose of this paper. The main results emerging from their study are that welfare-optimal prices are set equal to the load-adjusted average, over the respective pricing period, of marginal costs. On the other hand, the pricing period lengths are set so that the optimal value of the welfare function is continuous at the time when the price changes.

Unfortunately, however, the literature on peak-load pricing has scarcely analysed this problem in a multiproduct market with competition. Gersten (1986) examines a model of competition in unregulated markets such as restaurants, theatres, hotels and airlines. Here, firms use peak-load pricing to spread consumers across periods in a profitable way.

Crew and Kleindorfer (1991) examine Ramsey optimal peak-load pricing for postal services and some effects of competitive entry.[4] Competition derives

[3]For survey on peak-load pricing see Crew and Kleindorfer (1986) and Crew, Fernando, and Kleindorfer (1995).

[4]This paper expands Crew, Kleindorfer, and Smith (1990) which consider the problem of peak-load pricing with product differentiation. Product differentiation emerges through

from the capture of a part of postal business by a competitor, e.g. external presorting of mail. They analyse the appropriate discounts per letter to be allowed for external operation, as excessive discounts would encourage inefficient entry into the pre-sort business. They find that when marginal costs for each class of service are constant, the welfare-optimal pre-sort discount (when the break-even constraint of the incumbent is not an issue) is set at exactly the unit cost of pre-sorting. When marginal costs are not constant, the pre-sort discount would depend on the magnitude of the peak problem, with larger discounts given to ameliorate peak loads. Laffont and Tirole (2000) derive optimal Ramsey prices for one unit of peak and off-peak access. In this case the peak-load approach is justified because the network operator has different marginal costs in each pricing period. These two papers have a simple setting in terms of consumer preferences (there are not demand interdependencies and the pricing periods are fixed) and are not suited to analyse the capacity charge problem which, on the contrary, is studied in this paper.

Finally, Escribano and Zaballos (2002) analyse the optimal capacity charge problem. They consider a detailed model of the Spanish market where an incumbent and one entrant provide short and long distance calls to three consumers. The entrants have to face capacity constraints in those points of interconnection through which they are connected to the incumbent network. The authors find that in a model of interconnection by capacity, the entrants' price strategies are detached from those of the incumbent. Therefore, competition is expected to become harder. Although this paper analyses a model of interconnection by capacity, the authors do not provide an explicit formulation of the optimal capacity charge, and do not consider the peak-load pricing strategy that the firms face.

The rest of the paper continues as follows. Section 2 explains the main motivation of the capacity charge policy, and describes some recent applications of this regulation. Section 3 explains the continuous and interdependent demands from consumers. Section 4 describes and solves the model when there is a competitive fringe of price-taking entrants. Section 5 extends the model to consider an entrant with market power. Section 6 analyses a multiproduct setting, given an entrant with market power. Finally, Section 6 concludes the analysis.

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deferred processing of lower priority mail at peak times.

2 The capacity charge in the telecommunications industry

In network industries (telecommunications, electricity, airlines, postal services) demand fluctuates with time.[5] If the price was uniform over time, the amount demanded would rise and fall periodically. Meeting the demand at the peak would require the installation of a capacity that would be under-utilised over the remainder of the cycle. The theory of peak-load pricing developed in the 1940's and 1950's made an important contribution to this problem through the introduction of "time of use" rates. Since then, national authorities regulate utilities using a peak-load pricing policy in order to discourage consumption during the peak periods and encourage off-peak consumption.

In the telecommunications industry, the fast growth of Internet in the mid 1990's has introduced a new problem. As Voice and Internet traffic use the same capacity, an efficient management of the aggregate load curve requires a more careful fixation of retail prices. Demands in each period are controlled by sellers by means of pricing. Reality has shown us that firms can easily move Internet consumption to off-peak periods by increasing its peak price. This is due to the fact that for residential users Internet is more elastic with respect to price than telephony, in spite of the fact that residentials do not have a strong preference for consuming Internet at a concrete time of the day.

In some countries, the use of a excessively low flat rate price for Internet in the off-peak period has caused the so called "evening peak problem" or shifting peak-problem. This implies that consumers stay online during the evening and blow up the incumbent's network. As a result, the evening off-peak consumption may be higher than the morning peak consumption.

With the same goal of ensuring an efficient management of the load curve, and with the additional objective of promoting the entry in the retail market, the "time-of-use" policy is also applied to regulate access to the incumbent's network. Indeed, it is a general practice to fix a peak and an off-peak price for the call-minute access charge.[6] As a consequence, the entrants can establish a peak and an off-peak retail price to emulate the incumbent's tariffs. However, is there any economic sense in applying an access price that depends on call-minutes instead of capacity units?

In the telecommunications sector the principal driver of network costs is the peak-hour capacity cost, which is fixed and does not directly depend on the amount of minutes provided by the network. Therefore, it seems more natural

[5] Koschat, Srinagesh, and Uhler (1995) provide a detailed quantitative study of the optimal peak-load pricing of local telephone calls. Their model incorporates intraperiod variation and uncertainty of demand. Other studies for the telecommunications industry are Park and Mitchell (1987) and Griffin and Mayor (1987).

[6] See Cave (1994) and Mitchell, Neu, Neumann, and Vogelsang (1995).

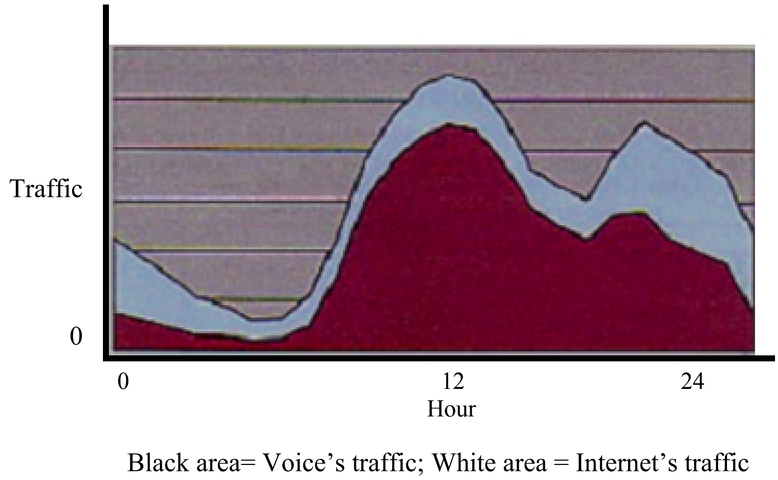

Figure 1: Telecomunications Traffic in Spain in year 2001

and efficient to establish a system of interconnection based on a fixed price for the capacity made available to the entrants (i.e. one circuit of access) rather than on one unit of access (i.e. one access-minute), as it occurs at present.[7]
An access system based on call-minutes access charges generates several distortions in the market.

(1) The incumbent does not have reliable information on the entrant'sneeds for capacity. The incumbent may become overflownwhen the e.ective capacity requested by the entrant exceeds his projected capacity. Whereas, if the capacity requested by the entrant is smaller than the capacity projected, the incumbent has to cover the costs of theoverinvestment of the plant with its own budget.

(2) The entrants have to programme their retail prices taking into account the periods set by the regulator for the access charges. Consequently, they cannot di.erentiate their service from that of the incumbent's by o.ering a di.erent length for each pricing period, or a di.erent number of pricing periods. This problem worsens when the incumbent's retail prices are regulated by means of a price cap, as then, the incumbent can freely modify the pricing periods with the only constraint of meeting the cap.

(3) The entrants do not take advantage of the scale economies that they generate. Indeed, although the capacity cost decreases with the number

[7] Cave and Crowther (1999) describe some advantages of a capacity charge.

of interconnection circuits, the entrants are charged with a constant price per call-minute. Therefore, while the incumbent gets a reduction in costs when it produces more traffic, the entrants do not have any advantage for providing more minutes.

(4) The system does not stimulate an efficient use of the network. An access system based on access-minutes does not provide an incentive to the entrants to use the off-peak excess capacity for providing other services.

The recognition of these problems has led the regulatory authorities of some countries to look for an alternative interconnection system which is not based on access-minutes. In the UK, in 2000, OFTEL compelled BT to commercialise a flat tariff for access to the Internet, called Digital Local Exchange Flat Rate Internet Access Call Origination (FRIACO).[8] This mechanism permits the Internet service providers to emulate the flat tariff offered by BT for the provision of Internet. With the same aim, in February 2001 the German Regulatory Authority introduced a wholesale flat rate additional to its linear pricing scheme.[9]

In Spain, in the summer of 2001, the Telecommunications Regulatory Agency (CMT) developed a system of interconnection by capacity.[10] This system establishes a fixed price per access-circuit and does not take into account the access traffic effectively offered by it.[11] The operators can buy elementary units of 64 Kbit/s of capacity (or multiples of it) that have a predetermined quality. The capacity may be used in all pricing periods, for voice traffic and for the Internet. Moreover, the entrants may resell the contracted capacity that exceeds their needs.[12]

When a system of access by capacity is implemented, the operator who buys capacity chooses the retail tariffs that maximise its profits, using the minimum necessary capacity. The proponents of this system emphasise that it promotes an efficient use of the network because the entrants fill in the unoccupied capacity of the off-peak period with other services. What price

[8] OFTEL (2001).

[9] See Reutter (2001).

[10] CMT (2000).

[11] This tariff structure is not the only one possible. In France and Belgium there is a mixed system in which in addition to a fixed price per circuit there is a variable tariff that depends on the number of minutes that are effectively used.

[12] Aguilar (2002) analyses the case of an entrant in the Spanish market who will prefer to use the system of interconnection by capacity instead of the interconnection by time. He finds that with the actual regulation of the capacity in Spain, the entrant will have to reach a critical mass of consumers before he will be interested in the model of interconnection by capacity. On the other hand, with linear tariffs in the final market, the entrants prefer to be aggressive and choose the system of interconnection by capacity, because in equilibrium everything depends on the market share gained.

should be set for the capacity if these services also compete with those of the incumbent's?

3 Continuous Interdependent Demands

A regulated monopoly operates a network and produces a service, e.g. telephone calls, which can be consumed in $t \in [0, T]$. A rival firm offers the same final service but in order to convey it, it needs to use the monopoly's network. In particular, the rival ($i = 2$) buys capacity from the monopoly ($i = 1$), which can be used to convey the service at any time t. Moreover, a regulator determines the price of the monopoly's final service and the capacity charge in order to maximise social welfare. In the following sections I will analyse how the regulator sets the incumbent's welfare-maximising price and the capacity charge under various conditions. Before that, in this section I analyse the demands of each firm and I define the consumer's surplus.

Each consumer is restricted to buy all the services he consumes from only one firm. Consider a heterogeneous population of consumers denoted by $\theta \in [0, 1]$, where the number of consumers of type θ is given by the distribution function $F(\theta)$. This is assumed to have a continuous density $f(\theta)$.

Let's consider that consumers have continuous and intertemporally dependent quasi-linear preferences. This implies that the utility from consuming q^i at t depends on all values taken by $q^i(t)$ for $t \in [0, T]$. On the other hand, let us assume that there are only two pricing periods, the peak and the off-peak periods. Taking into account this simplification, we have

$$P^i = \begin{cases} P_U^i & \text{for } t \in L_U^i \\ P_L^i & \text{for } t \in L_L^i \end{cases}$$

where $L_U^i = [0, \tau^i]$, $L_L^i = [\tau^i, T]$ and τ^i is the time bound that separates the peak and the off-peak periods. Therefore, the length of the period where firm i sets the peak price, P_U^i, and the off-peak price, P_L^i, are $L_U^i = \tau^i$ and $L_L^i = T - \tau^i = T - L_U^i$, respectively.[13] Burness and Patrick (1991) carefully solve the consumer's problem considering that there are continuous and intertemporally dependent preferences and two pricing periods. Following their

[13] Joskow (1976) and Craven (1971; 1985) analyse the problem of choosing the optimal number of pricing periods.

work, the demand faced by firm i in each pricing period is[14]

$$\begin{aligned} q^i_U(t) &= q(P^i_U, P^i_L, \tau^i, t, \theta), \\ q^i_L(t) &= q(P^i_L, P^i_U, \tau^i, t, \theta). \end{aligned} \tag{1}$$

Taking into account these demands, the consumer's surplus of the type θ during all the interval $[0, T]$ depends on the price in each period $P^i = (P^i_U, P^i_L)$ and on the switch time τ^i. Therefore, we denote the consumer's surplus as $\widetilde{CS}(P^i_U, P^i_L, \tau^i, \theta)$. According to Burness and Patrick (1991) this satisfies the following conditions

$$\frac{\partial \widetilde{CS}(P^i_U, P^i_L, \tau^i, \theta)}{\partial P^i_U} = -\int\limits_0^{\tau^i} q^i_U(t)\, dt; \quad \frac{\partial \widetilde{CS}(P^i_U, P^i_L, \tau^i, \theta)}{\partial P^i_L} = -\int\limits_{\tau^i}^{T} q^i_L(t)\, dt. \tag{2}$$

Moreover, it can be stated that

$$\frac{\partial \widetilde{CS}(P^i_U, P^i_L, \tau^i, \theta)}{\partial \tau^i} = ics_U(P^i_U, P^i_L, \tau^i, \tau^i, \theta) - ics_L(P^i_U, P^i_L, \tau^i, \tau^i, \theta), \tag{3}$$

where $ics_U(P^i_U, P^i_L, \tau^i, \tau^i, \theta)$ is the instantaneous consumer's surplus of type θ in the particular time τ^i, when he is charged with the peak price. In the same sense, $ics_L(P^i_U, P^i_L, \tau^i, \tau^i, \theta)$ is the instantaneous consumer's surplus of type θ in the particular time τ^i, when he is charged with the off-peak price.[15] Equation (3) means that when the length of the peak period is increased, the

[14] The authors consider utility at t as a functional of the function $q^i(t)$, t, and θ. Therefore, utility from consuming q at any t is expressed as $u(\mathbf{q^i}, t, \theta)$, where $\mathbf{q^i} = \{q(t) : t \in [0, 1]\}$. Taking this into account they solve the consumer's problem and obtain the following individual demand

$$q^i_I(t) = q(\mathbf{P^i_I}, \mathbf{P^i_J}, t, \theta), \quad I, J \in \{U, L\}, \quad I \neq J,$$

where $\mathbf{P^i_U} = \{P^i_U(t) : t \in L^i_U\}$ and $\mathbf{P^i_L} = \{P^i_L(t) : t \in L^i_L\}$. For tractability purposes, in this paper we write this demand as

$$q^i_I(t) = q(P^i_I, P^i_J, \tau^i, t, \theta), \quad I, J \in \{U, L\}, \quad I \neq J,$$

where the prices are not a functional.

[15] This result can be obtained from Lemma 2 in Burness and Patrick (1991). This Lemma states that, given optimal consumer behaviour with prices $\mathbf{P_I}$ and $\mathbf{P_J}$, a change in pricing period length has no impact on the type θ consumption within each pricing period. That is, for $I, J \in \{U, L\}, I \neq J$,

$$\int\limits_{t \in I} \nabla_\tau q(\mathbf{P_I}, \mathbf{P_J}, t, \theta) dt = 0,$$

where $\nabla_{\tau^1} q$ is the derivative of q with respect to τ at $t \in [0, T]$.

aggregated consumer's surplus is modified because the peak period lasts longer and the off-peak period lasts less.

Clearly, a type θ buys from the entrant when his consumer's surplus is greater than the consumer's surplus when he buys from the incumbent, i.e., when

$$\widetilde{CS}(P_U^2, P_L^2, \tau^2, \theta) \geq \widetilde{CS}(P_U^1, P_L^1, \tau^1, \theta). \tag{4}$$

This could be the case, for example, when the type θ prefers the lower peak price of the entrant, $P_U^2 < P_U^1$, although the entrant's off peak price is higher, $P_L^2 > P_L^1$ and the peak period lasts longer, $\tau^2 > \tau^1$. The following function reflects the profitability of belonging to firm 2

$$\gamma(P^1, P^2, \theta) = \widetilde{CS}(P_U^2, P_L^2, \tau^2, \theta) - \widetilde{CS}(P_U^1, P_L^1, \tau^1, \theta), \tag{5}$$

where $P^1 = (P_U^1, P_L^1)$ and $P^2 = (P_U^2, P_L^2)$. A series of partitions of $[0,1]$ exists, which separates the consumers who choose the incumbent from those who choose the entrant. These partitions could be characterised by a set of cut-off points θ^*. For analytical convenience, we assume that there is only one cut-off point that separates the consumers.[16] This assumption implies that when consumer θ^* finds profitable to consume from firm 2 at prices P^2 and τ^2, then so does every customer $\theta > \theta^*$. Bearing in mind the previous example, if a consumer prefers firm 2 because it offers a lower peak price than firm 1, although the peak period lasts longer, then any higher consumer type will also prefers firm 2.

Define $\theta^*(P^1, P^2, \tau^1, \tau^2)$ as the market share of the incumbent. Thus, $\theta^* \in (0,1)$ is the largest customer type that consume from the incumbent. From (4) and monotonicity of indirect utility in P^i and in τ^i, it is satisfied that $\frac{\partial \theta^*}{\partial P_U^1} < 0$, $\frac{\partial \theta^*}{\partial P_L^1} < 0$ and $\frac{\partial \theta^*}{\partial \tau^1} < 0$. The opposite sign applies to P_U^2, P_L^2, and τ^2. Therefore, the demand for the incumbent and the entrant could be written as

$$\begin{aligned} q_j^1(P^1, P^2, \tau^1, \tau^2, t) &= \int_0^{\theta^*} q(P_j^1, P_{-j}^1, \tau^1, t, \theta) dF(\theta), \\ q_j^2(P^2, P^1, \tau^2, \tau^1, t) &= \int_{\theta^*}^1 q(P_j^2, P_{-j}^2, \tau^2, t, \theta) dF(\theta), \end{aligned} \tag{6}$$

where $j, -j \in \{U, L\}$. Moreover, we can write the aggregated consumers' surplus in the following way

$$CS(P^1, P^2, \tau^1, \tau^2) = \int_0^{\theta^*} \widetilde{CS}(P_U^1, P_L^1, \tau^1, \theta) dF(\theta) + \int_{\theta^*}^1 \widetilde{CS}(P_U^2, P_L^2, \tau^2, \theta) dF(\theta). \tag{7}$$

[16] An analogous constructions can be pursued considering that there is a finite number of such cut-off points. Moreover, we conjecture that an infinite number of partitions would not modify the conclusions of this paper.

In the rest of the text we often take the convenient assumption that the demand of each firm is additively separable between price and time. This approach is consistent with other works that analyse the peak-load problem. If $q_j^i(P^i, P^{-i}, \tau^i, \tau^{-i}, t) = f(P^i, P^{-i}, \tau^i, \tau^{-i}) + h(t)$, for $j = \{U, L\}$, then at each moment t, a part of the demand depends on the peak and off-peak prices, and the other part of the demand is fixed and does not depend on prices. Note that with this representation, the elasticity of the demand depends only on the prices and is independent of the time at which consumption occurs. Moreover, this formulation allows intraperiod variations of elasticity.

4 A Model of Access by Capacity

We suppose that there is an industry with an incumbent and a large number of entrants, all of whom offer the same service, e.g. telephone calls. When the incumbent and the entrants provide the service to the final users, they incur marginal costs, b_1 and b_2 respectively. These marginal costs reflect the part of the cost of a call that depends on the traffic (backbone switching, information services, billing, etc.). The capacity supply is monopolized by the incumbent. Let β be the incumbent's "stand alone" cost per unit of capacity when it provides the final service to its consumers, or access to the entrants. In contrast to the previous literature, I assume that the marginal cost of providing these activities is negligible, as is generally considered in practice. The capacity charge that the entrants pay to the incumbent for each unit of capacity is denoted by a. The production technology is fixed-coefficient. Each unit of downstream output requires one unit of capacity. Therefore, the firms require as much capacity as the maximum number of units sold during the peak period.

Considering this stylised model of the market, it is possible to analyse how the regulator establishes the welfare-maximising prices of the incumbent's final service and the capacity charge. In this section I consider that the regulator optimally determines the time at which the off-peak price begins. As a consequence of this, the incumbent's switch time, τ^1, is equal to the switch time of all the entrants, τ^2. Therefore, we can write $\tau = \tau^i$ for $i = \{1, 2\}$. If all the entrants offer the consumers the same length for the peak and the off-peak pricing period, they are not able to differentiate their services. As a result, they can be considered as a fringe of price-taking firms.[17] In this situation, it is possible to apply a variant of Armstrong, Doyle, and Vickers (1996) to the capacity charge problem.

Given the fringe prices $P^2 = (P_U^2, P_L^2)$, the switch time τ and the capacity charge a, the fringe will choose to supply the peak period quantity $s_U^2(t)$ and

[17] Alternatively, we can consider a unique entrant that takes as given the prices of the incumbent.

the off-peak period quantity $s_L^2(t)$ in order to maximise their aggregated profit function. This can be formulated as

$$\begin{aligned}\Pi^2(a,\tau) &\equiv max_{s_U^2, s_L^2}: \int_0^\tau (P_U^2 - b_2)s_U^2(t)dt + \int_\tau^T (P_L^2 - b_2)s_L^2(t)dt - aK_2 \\ &= max_{s_U^2, s_L^2}: \sum_j \int_{t\in L_j^2} (P_j^2 - b_2)s_j^2(t)dt - aK_2\end{aligned} \tag{8}$$

subject to the capacity constraint

$$K_2 \geq s_j^{2M},$$

where $s_j^{2M} = max_{t\in L_j^2} s_j^2(t)$ is the maximum level of supply in period $j = \{U, L\}$ and K_2 is the capacity that the fringe contracts from the incumbent. The capacity constraint implies that the quantity supplied by the fringe at any time t can not be higher than the capacity bought from the incumbent. The supply functions satisfy $\frac{\partial s_j^2}{\partial P_j^2} \geq 0$ and $\frac{\partial s_j^2}{\partial a} \leq 0$. If s_U^2 and s_L^2 are the profit-maximising quantities supplied by the entrants, then by the envelope theorem

$$\frac{\partial \Pi^2(a,\tau)}{\partial a} = -s_j^{2M}. \tag{9}$$

Given the incumbent's prices, P^1, and the capacity charge, a, the fringe's equilibrium price, $\hat{P}^2(P^1, a)$, equates the aggregated equilibrium supply of the fringe to the aggregated demand of the fringe in each moment t,

$$\hat{s}_j^2(\hat{P}^2(P^1, a), a, \tau, t) \equiv q_j^2(P^1, \hat{P}^2(P^1, a), \tau, t). \tag{10}$$

In order to simplify the model further we assume that given the incumbent's equilibrium prices, P^1, and the fringe's equilibrium prices, $\hat{P}^2(P^1, a)$, in equilibrium, the quantity sold in the peak period by the fringe is always bigger than the quantity sold in the off-peak period, $s_U^2 > s_L^2$. This assumption is important, because it allows us to elude the shifting-peak case.[18]

Taking this into account, the fringe's equilibrium price, $\hat{P}^2(P^1, a)$, equates the capacity contracted by the fringe to the maximum number of units that the fringe provides in equilibrium in the peak period, $K_2 \equiv \hat{s}_U^{2M}(\hat{P}^2(P^1, a), P^1, \tau)$. Therefore, the equilibrium demand for capacity is

$$\hat{s}_U^{2M}(P^1, a, \tau) \equiv K_2. \tag{11}$$

[18] If this does not occur, the capacity constraint is binding during the off-peak period and the off-peak price should increase in order to avoid an excess demand.

The incumbent's profit over the demand cycle can be represented as

$$\Pi^1(P^1,a,\tau) \equiv \sum_j \int_{t\in L_j^1} (P_j^1 - b_1)\hat{q}_j^1(P^1,a,\tau,t)dt - \beta K_1 + (a-\beta)\hat{s}_U^{2M}(P^1,a,\tau), \tag{12}$$

subject to

$$K_1 \geq \hat{q}_j^{1M},$$

where $q_j^{1M} = max_{t\in L_j^1}\, \hat{q}_j^1(P^1,a,\tau,t)$ is the maximum level of demand in period $j=\{U,L\}$, $\hat{q}_j^1(P^1,a,\tau,t) \equiv q_j^1(P^1,\hat{P}^2(P^1,a),\tau,t)$ is the incumbent's equilibrium demand for the final service at each moment t, and K_1 is the capacity available to the incumbent.

The efficient rule for the capacity charge involves the maximization of social welfare, which we define as the unweighted sum of the consumer's surplus and the firms'profits. The regulator considers the following welfare function

$$W(P^1,a,\tau) \equiv CS(P^1,\hat{P}^2,a,\tau) + \Pi^1(P^1,a,\tau) + \Pi^2(\hat{P}^2,a,\tau). \tag{13}$$

In order to analyse the problem of the regulator, we first characterise the incumbent's welfare maximizing prices and the capacity charge. Further on, I will show the condition that defines the optimal length of the pricing periods. But let us consider the following results

$$\tfrac{\partial(CS+\Pi^2)}{\partial P_j^1} = -\int_{t\in L_j^1} \hat{q}_j^1\, dt, \quad j\in\{U,L\}; \quad \tfrac{\partial(CS+\Pi^2)}{\partial a} = -\hat{s}_U^{2M}.$$

If we write $\mu \geq 0$ as the multiplier associated to the incumbent's capacity restriction, $\lambda \geq 0$ as the multiplier of the break-even constraint $\Pi^1 \geq 0$ and using the Kuhn-Tucker theorem we obtain the following first-order conditions for $P^1 = (P_U^1, P_L^1)$ and a

$$\sum_j \int_{t\in L_j^1} (P_j^1 - b_1)\frac{\partial \hat{q}_j^1}{\partial P_U^1}dt - \sum_j \mu_j \frac{\partial \hat{q}_j^{1M}}{\partial P_U^1} + (a-\beta)\frac{\partial \hat{s}_U^{2M}}{\partial P_U^1} = -\frac{\lambda}{1+\lambda}\int_{t\in L_U^1} \hat{q}_U^1 dt, \tag{14}$$

$$\sum_j \int_{t\in L_j^1} (P_j^1 - b_1)\frac{\partial \hat{q}_j^1}{\partial P_L^1}dt - \sum_j \mu_j \frac{\partial \hat{q}_j^{1M}}{\partial P_L^1} + (a-\beta)\frac{\partial \hat{s}_U^{2M}}{\partial P_L^1} = -\frac{\lambda}{1+\lambda}\int_{t\in L_L^1} \hat{q}_L^1 dt, \tag{15}$$

$$\sum_j \int_{t\in L_j^1} (P_j^1 - b_1)\frac{\partial \hat{q}_j^1}{\partial a}dt - \sum_j \mu_j \frac{\partial \hat{q}_j^{1M}}{\partial a} + (a-\beta)\frac{\partial \hat{s}_U^{2M}}{\partial a} = -\frac{\lambda}{1+\lambda}\hat{s}_U^{2M}, \quad (16)$$

$$\mu_j \geq 0; \quad \mu_j(K_1 - \hat{q}_j^{1M}) = 0, \quad (17)$$

$$\sum_j \mu_j \leq \beta; \quad K_1(\sum_j \mu_j - \beta) = 0. \quad (18)$$

Note that as I discard the shifting-peak case, from (17)-(18) it results that $\mu_U = \beta$ and $\mu_L = 0$. Moreover, given the optimal prices, when the incumbent's break-even constraint does not bind, it should be that $\frac{\lambda}{1+\lambda} = 0$. In this case, from (14)-(16) the socially optimal prices are

$$a = \beta, \quad (19)$$

$$\sum_j \int_{t\in L_j^1} (P_j^1 - b_1)\frac{\partial \hat{q}_j^1}{\partial P_U^1}dt - \beta\frac{\partial \hat{q}_j^{1M}}{\partial P_U^1} = 0, \quad (20)$$

$$\sum_j \int_{t\in L_j^1} (P_j^1 - b_1)\frac{\partial \hat{q}_j^1}{\partial P_L^1}dt = 0. \quad (21)$$

If we assume that demand is additively separable between time and price $\frac{\partial \hat{q}_U^{1M}}{\partial P_U^1} = \frac{\partial \hat{q}_U^1}{\partial P_U^1}$. Moreover, we can use the simplification $L_j^i = \int_{t\in L_j^i} dt$. Therefore,

$$P_U^1 = b_1 + \frac{\beta}{L_U^*}, \quad (22)$$

$$P_L^1 = b_1. \quad (23)$$

where L_U^* is the optimal length of the peak period set by the regulator. Equation (19), (22) and (23) show the first best prices that would guarantee allocative and productive efficiency. Equation (19), in particular, states that the capacity charge is set equal to the direct capacity cost. However, as stated by Armstrong, Doyle, and Vickers (1996), it is not evident that regulators will set the retail prices of the incumbent following the Ramsey principles. On the contrary, it may be more realistic to consider that they will fix P^1 at some level higher than the first best in order to guarantee a certain profit to the incumbent. In this case, from equation (16) the optimal capacity charge is given by the following proposition.

Proposition 1. *When $\frac{\lambda}{1+\lambda} = 0$, $P^1 = (P^1_U, P^1_L)$ are higher than the first best and there are demand interdependencies, the optimal capacity charge is*

$$a = \beta + \sum_j L^*_j(P^1_j - b_1)\sigma_j - \beta\sigma_U, \tag{24}$$

where $\sigma_j = \frac{\frac{\partial \hat{q}^1_j}{\partial a}}{\frac{\partial \hat{s}^{2M}_U}{\partial a}}$.

This proposition states that the optimal capacity charge is higher than the direct costs. The reason for this is that increasing the capacity charge above the direct costs relaxes the need to increase retail prices. Notice that the first term of the right-hand side of equation (24) is the direct cost of the capacity supply. The second term is the incumbent's opportunity cost of providing the marginal unit of capacity to the fringe.

As in Armstrong, Doyle, and Vickers (1996), the opportunity costs can be separated into the product of two factors: the incumbent's marginal profit per unit of final service during all the length of the pricing period, and the displacement ratio defined by σ_j. The displacement ratio is the change in the incumbent's final service sales in period j divided by the change in the incumbent's sales of capacity as the capacity charge is modified. Notice that there is a displacement ratio for each pricing period.

The fringe can use the capacity in all periods. Therefore, when the incumbent supplies the marginal unit of capacity to the entrant, this does not only cause a loss in his peak period profit, but also a reduction in his marginal profit in the off-peak period. Consequently, the incumbent's opportunity costs are the sum of his losses in the peak and off-peak periods. Taking this into account, the optimal pricing rule in (24) could easily be generalised for a different number of pricing periods.

Armstrong, Doyle, and Vickers (1996) show that the optimal per minute access charge is equal to the ECPR.[19] The ECPR states that the access charge should be equal to the direct cost of the access plus the incumbent's opportunity cost of providing access to the fringe. As I have shown, the optimal

[19] This rule was originally proposed by Willig (1979). See Laffont and Tirole (2000) or Armstrong (2001) for a detailed analysis of its properties.

capacity charge should be set in the same way. In this case, however, the term in the ECPR that reflects the opportunity cost of providing capacity should reflect the incumbent's losses in all the pricing periods.

If we now turn to the case where $\frac{\lambda}{1+\lambda} > 0$, the optimal capacity charge should be corrected. The incumbent's break-even constraint binds, for instance, when the incumbent has an increasing return technology for any of the inputs.

Proposition 2. *When $\frac{\lambda}{1+\lambda} > 0$, $P^1 = (P^1_U, P^1_L)$ are higher than the first best, and there are demand interdependencies, the optimal capacity charge is*

$$a = \beta + \sum_j L^*_j (P^1_j - b_1)\sigma_j - \beta\sigma_U - \frac{\lambda}{1+\lambda}\frac{\hat{s}^2_U}{\frac{\partial \hat{s}^{2M}_U}{\partial a}}. \tag{25}$$

This formula states that the optimal capacity charge is the same as in equation (24) minus a negative Ramsey term. The capacity charge is even higher than in (24) to contribute to satisfy the break-even constraint.

It is interesting to note that this pricing rule for the capacity charge has the same structure than the optimal access-minute charge found by Armstrong, Doyle, and Vickers (1996) when there is a fixed coefficient technology, no bypass, and homogeneous products.[20] However, if we modify equation (12) to obtain the optimal time-of-use access prices with our cost structure (i.e. there is a fixed cost of supplying access but the marginal cost of supplying it is zero) we find the following peak and off peak access charges[21]

$$a_U = \frac{\beta}{L_U} + (P^1_U - b_1 - \frac{\beta}{L_U})\sigma_U - \frac{\lambda}{1+\lambda}\frac{\hat{s}^{2M}_U}{\frac{\partial \hat{s}^{2M}_U}{\partial a}\sum_j L_j} \tag{26}$$

$$a_L = (P^1_L - b_1)\sigma_L - \frac{\lambda}{1+\lambda}\frac{\hat{s}^{2M}_U}{\frac{\partial \hat{s}^{2M}_U}{\partial a}\sum_j L_j} \tag{27}$$

[20] In particular, when there is only one pricing period and $T = 1$ our expression (25) becomes

$$a = \beta + (P^1 - b_1 - \beta)\sigma - \frac{\lambda}{1+\lambda}\frac{\hat{s}^2}{\frac{\partial \hat{s}^{2M}}{\partial a}}$$

Using our notation, Armstrong, Doyle, and Vickers (1996) equation (20) for the optimal access charge per minute (when they take into account the budget constraint of the incumbent) is

$$a = c + (P^1 - b_1 - c)\sigma - \frac{\lambda}{1+\lambda}\frac{\hat{s}^2}{\frac{\partial \hat{s}^{2M}}{\partial a}}$$

where c is the incumbent's marginal cost of providing access to the fringe. As the authors demonstrate, their equation (20) can be seen as a variation of the Efficient Component Pricing Rule (ECPR).

[21] Observe that the optimal off-peak access charge will be different when the fringe supply services only in the off-peak period.

These two equations allows us to write the following proposition.

Proposition 3. *When the regulator optimally chooses the switch time τ^i, for $i = \{1, 2\}$, a system of access by capacity and a system of access by time imply the same access payments for the entrant.*

In practice, the system of access by capacity is proposed by some authors because it offers more price flexibility to the entrants. However, it is important to emphasise that in a system of access by capacity the regulator can not use the regulation of the time-of-use access charges to determine the number and the length of the peak and off-peak retail prices. As we will see in the next section, this is a relevant problem when the entrant have market power.

Finally, in order to completely define the optimal prices, we characterise the condition that establishes the optimal length of the pricing periods L_j^*, for $j = \{U, L\}$. To obtain the welfare-maximising L_j^* the regulator maximises the social welfare function in (13) with respect to the time τ at which the peak period ends. Before that, it is useful to consider the following result[22].

$$\int\limits_{t \in L_j^i} \frac{\partial \hat{q}_j^i(P^1, P^2, \tau, t)}{\partial \tau} dt = 0. \tag{28}$$

Taking this into account, the first order condition with regards to the switch time τ is

$$\begin{aligned} &ics_U(P^1, \hat{P}^2, a, \tau^*, \tau^*) + (1+\lambda)(P_U^1 - b_1)\hat{q}_U^1(P^1, a, \tau^*) + (\hat{P}_U^2 - b_2)\hat{s}_U^2(\hat{P}^2, a, \tau^*) = \\ &ics_L(P^1, \hat{P}^2, a, \tau^*, \tau^*) + (1+\lambda)(P_L^1 - b_1)\hat{q}_L^1(P^1, a, \tau^*) + (\hat{P}_L^2 - b_2)\hat{s}_L^2(\hat{P}^2, a, \tau^*). \end{aligned} \tag{29}$$

For notation reduction purposes, we write (29) as

$$V_U(P^1, \hat{P}^2, a, \tau^*) = V_L(P^1, \hat{P}^2, a, \tau^*). \tag{30}$$

where V_j represents the net social welfare in time τ when prices are P_j^1 and $\hat{P}_j^2$. Considering equation (30), we can state the following proposition

Proposition 4. *Given the optimal consumer behaviour with prices P^1 and $\hat{P}^2$, τ^* is chosen so that the optimal value of the welfare function V_j is continuous at the time when the prices are changed.*

This implies that the length of the pricing periods is chosen so that at the optimal switch time τ the value of the net social welfare in time τ with peak

[22] See footnote 13.

prices, V_U, is equal to the value of the net social welfare in time τ with off-peak prices, V_L.[23]

What is important about this result is that V_j does not consider the profit obtained by the incumbent when it provides capacity to the fringe. Indeed, given condition (28), a change in the length of the peak pricing period has no impact on the maximum number of units that the fringe provides in equilibrium during the peak period. On the other hand, the demand interdependencies affect the capacity charge because as they affect the value of the net social welfare V_j, they determine the duration of the pricing periods.

5 Competitor with market power

Competition in network industries is often imperfect. When one firm enters the telecommunications sector it normally uses its market power to attract some specific groups of consumers by offering them a bundle of prices that differ from those of the incumbent. The entrant may choose different pricing periods, beginning and/or ending each pricing period at different moments than the incumbent. To account for this situation, I use the same framework as in the previous section, but instead of considering a competitive fringe, I derive the optimal capacity charge when there is one entrant with market power.[24]

If the competitor has market power and is unregulated, the maximization of social welfare must be carried out under the constraint that the competitor maximises his profits by optimally choosing the price of each period as well as the switch time from the peak period to the off-peak period. Firstly, I derive the entrant's first order profit-maximising conditions. Further on, I will analyse the optimal capacity charge considering that the regulator uses these first order conditions as constraints in his objective function.

Consider that the entrant's profit is the following

$$\Pi^2(P^2, a, \tau) \equiv \sum_j \int_{t \in L_j^2} (P_j^2 - b_2) q_j^2(P^2, P^1, \tau^2, \tau^1, t) dt - aK_2 \tag{31}$$

subject to

$$K_2 \geq q_j^{2M}.$$

[23] This principle that characterises the value of the welfare function when prices are changed is derived and proven by Dansby (1975) and by Burness and Patrick (1991).

[24] Laffont and Tirole (1994) analyse the access charge problem when there is a competitor with market power.

Note that, as in this case the switch time of the entrant is unregulated, τ^2 and τ^1 may be different. Denoting by α, the multiplier associated to the capacity constraint, the first order Khun-Tucker conditions for the problem of the entrant are

$$\int\limits_{t \in L_U^2} q_U^2 dt + \sum_j \int\limits_{t \in L_j^2} (P_j^2 - b_2) \frac{\partial q_j^2}{\partial P_U^2} - \sum_j \alpha_j \frac{\partial q_j^{2M}}{\partial P_U^2} = 0, \tag{32}$$

$$\int\limits_{t \in L_L^2} q_L^2 dt + \sum_j \int\limits_{t \in L_j^2} (P_j^2 - b_2) \frac{\partial q_j^2}{\partial P_L^2} - \sum_j \alpha_j \frac{\partial q_j^{2M}}{\partial P_L^2} = 0, \tag{33}$$

$$\alpha_j \geq 0; \quad \alpha_j (K_2 - q_j^{2M}) = 0, \tag{34}$$

$$\sum_j \alpha_j \leq a; \quad K_2 (\sum_j \alpha_j - a) = 0, \tag{35}$$

$$(P_U^2 - b_2) q_U^2 (P^2, P^1, \tau^2, \tau^1) = (P_L^2 - b_2) q_L^2 (P^2, P^1, \tau^2, \tau^1). \tag{36}$$

Assuming an interior solution ($P_j^2 > 0$, for $j = \{U, L\}$) from (32) we obtain

$$\frac{\sum_j \int\limits_{t \in L_j^2} (P_j^2 - b_2) \frac{\partial q_j^2}{\partial P_U^2} dt}{\int\limits_{t \in L_U^2} q_U^2 dt} - \frac{\sum_j \alpha_j \frac{\partial q_j^{2M}}{\partial P_U^2}}{\int\limits_{t \in L_U^2} q_U^2 dt} = -1, \tag{37}$$

which we rewrite in the form

$$\sum_j \frac{R_j}{R_U} [\int\limits_{t \in L_j^2} \frac{(P_j^2 - b_2) \eta_{jU} dt}{P_j^2} - \alpha_j \frac{\eta_{jU}}{P_j^2}] = -1, \tag{38}$$

where $\eta_{jU} = (\frac{\partial q_j^2}{\partial P_U^2})(\frac{P_U^2}{q_j^2})$ and $R_j = \int_{t \in L_j^2} q_j^2 dt P_j^2$. Notice that when $\frac{\partial q_j^2}{\partial P_U^2} = \frac{\partial q_U^2}{\partial P_j^2}$, we can rewrite the above equation as[25]

[25] see Crew, Fernando, and Kleindorfer (1995).

$$\sum_j [\int_{t^2 \in L_j^2} \frac{(P_j^2 - b_2)\eta_{Uj} dt}{P_j^2} - \alpha_j \frac{\eta_{Uj}}{P_j^2}] = -1, \tag{39}$$

Analogously, repeating the same procedure from equation (33) we obtain

$$\sum_j [\int_{t^2 \in L_j^2} \frac{(P_j^2 - b_2)\eta_{Lj} dt}{P_j^2} - \alpha_j \frac{\eta_{Lj}}{P_j^2}] = -1, \tag{40}$$

Using Cramer's rule and assuming that demand is additively separable we can further simplify equation (39) and (40) to obtain

$$\frac{(P_U^2 - b_2 - \frac{\alpha_U}{L_U^2})}{P_U^2} = -\frac{(\eta_{LL} - \eta_{UL})}{L_U^2(\eta_{UU}\eta_{LL} - \eta_{UL}\eta_{LU})}, \tag{41}$$

$$\frac{(P_L^2 - b_2 - \frac{\alpha_L}{L_L^2})}{P_L^2} = -\frac{(\eta_{UU} - \eta_{LU})}{L_L^2(\eta_{LL}\eta_{UU} - \eta_{LU}\eta_{UL})}, \tag{42}$$

where L_j^2 is obtained by solving equation (36). On the other hand, assuming that $q_U^2 > q_L^2$, from (34) and (35) it is satisfied that $\alpha_U = a$ and $\alpha_L = 0$. Notice that when there are no demand interdependencies (i.e., when $\eta_{UL} = \eta_{LU} = 0$), then (41) and (42) become reduced to the standard Lerner index.

Now, defining $A_U \equiv \frac{(\eta_{LL} - \eta_{UL})}{L_U^2(\eta_{UU}\eta_{LL} - \eta_{UL}\eta_{LU})}$ and $A_L \equiv \frac{(\eta_{UU} - \eta_{LU})}{L_L^2(\eta_{LL}\eta_{UU} - \eta_{LU}\eta_{UL})}$, we can write the entrant's prices as

$$P_U^2 = (\frac{1}{1 + A_U})(b_2 + \frac{a}{L_U^2}), \tag{43}$$

$$P_L^2 = (\frac{1}{1 + A_L})b_2. \tag{44}$$

Given the incumbent's prices, P^1, the switch time, τ^1, and the capacity charge, a, the entrant's equilibrium prices, $\hat{P}^2$, and the profit-maximising switch time, $\hat{\tau}^2$, equate the entrant's equilibrium supply to its demand in each moment t,

$$\hat{s}_j^2(\hat{P}^2(P^1, a), a, \hat{\tau}^2, \tau^1, t) \equiv q_j^2(\hat{P}^2, P^1, \hat{\tau}^2, \tau^1, t). \tag{45}$$

Moreover, the equilibrium demand for capacity is

$$\hat{s}_U^{2M}(P^1, a, \hat{\tau}^2, \tau^1) \equiv K_2. \tag{46}$$

Now, taking into account the entrant's profit-maximising prices we can write the profit function that the regulator considers

$$\Pi^2(\hat{P}^2, a, \hat{\tau}^2) = \sum_j \int_{t \in L_j^2} [(\frac{1}{1+A_j})\frac{\alpha_j}{L_j^2} - \frac{A_j b_2}{1+A_j}]\hat{s}_j^2 dt - a\hat{s}_U^{2M}. \tag{47}$$

The regulator establishes the incumbent's prices, the capacity charge and the incumbent's switch time to maximize

$$W(P^1, a, \tau^1) = CS(P^1, \hat{P}^2, a, \tau^1, \hat{\tau}^2) + \Pi^1(P^1, a, \tau^1) + \Pi^2(\hat{P}^2, a, \hat{\tau}^2), \tag{48}$$

where CS and Π^1 are defined as in the previous section. Next I maximize the welfare function in equation (48) with respect to the capacity charge. Writing $\mu \geq 0$ for the multiplier of the incumbent's capacity constraint and $\lambda \geq 0$ for the multiplier associated to the incumbent's break-even constraint yields

$$\begin{aligned} &\lambda(\hat{s}_U^2 + a\tfrac{\partial \hat{s}_U^{2M}}{\partial a}) + \textstyle\sum_j \int_{t \in L_j^2} [(\tfrac{1}{1+A_j})\tfrac{\alpha_j}{L_j^2} - \tfrac{A_j b_2}{1+A_j}]\tfrac{\partial \hat{s}_j^2}{\partial a} dt \\ &+(1+\lambda)[\textstyle\sum_j \int_{t \in L_j^1} (P_j^1 - b_1)\tfrac{\partial \hat{q}_j^1}{\partial a} dt - \sum_j \mu_j \tfrac{\partial \hat{q}_j^{1M}}{\partial a} - \beta\tfrac{\partial \hat{s}_U^{2M}}{\partial a}] = 0. \end{aligned} \tag{49}$$

where from (17) and (18) we know that $\mu_U = \beta$ and $\mu_L = 0$. Assuming that the demand is additively separable, we simplify equation (49) to obtain the following result.

Proposition 5. *Given a profit-maximising entrant, when* $\frac{\lambda}{1+\lambda} > 0$, $P^1 = (P_U^1, P_L^1)$ *are higher than the first best and there are demand interdependencies, the optimal capacity charge is*

$$\begin{aligned} a = &\varepsilon[\beta + \textstyle\sum_j L_j^*(P_j^1 - b_1 - \mu_j)\sigma_j - (\tfrac{\lambda}{1+\lambda})\tfrac{\hat{s}_U^2}{\frac{\partial \hat{s}_U^{2M}}{\partial a}}] \\ &+ \textstyle\sum_j L_j^*(\tfrac{A_j b_2 L_U^2}{\lambda L_U^2(1+A_j)+L_U^*})\tfrac{\frac{\partial \hat{s}_j^2}{\partial a}}{\frac{\partial \hat{s}_U^{2M}}{\partial a}} \end{aligned} \tag{50}$$

where $\varepsilon = (\frac{L_U^2(1+A_U)(1+\lambda)}{\lambda L_U^2(1+A_U)+L_U^*})$ *and* L_U^* *is the optimal length of the peak period set by the regulator.*

The rule for the optimal capacity charge is complex, but can be given a natural interpretation. First of all, the reader should be convinced about the fact that when the entrant does not have market power, $A = 0$ and $L_U^2 = L_U^*$, the optimal capacity charge in (50) gives precisely the same price as in Proposition 2. When the entrant has market power he finds it profitable to fix a different length of the peak period than what is socially optimal. This is explained in the following Lemma.

Lemma 1. *If prices P^{1*} and a^* are the optimal prices set by the regulator and $\hat{P}^2$ is the optimal price set by the entrant, then a profit maximizing entrant will choose a different length than the socially optimal, $L_U^2 \neq L_U^*$. The concrete value of L_U^2 depends on the consumers' preferences.*

See the proof in the Appendix.

Notice that from (43) an increase of L_U^2 reduces the peak price and a reduction of L_U^2 increases the peak price. Therefore, the entrant can differentiate his product from that of the incumbent's by offering a lower peak price during a longer peak period, or by offering a higher peak price during a shorter peak period.[26] This behaviour allows the entrant to attract the group of consumers with a higher type and to establish a mark-up over its costs in both the peak and off-peak periods.

When some consumers leave the incumbent, the shape of the incumbent's load curve is modified and the optimal lengths of the pricing periods are distorted. In this situation, to maximize social welfare the regulator's objective is to make the entrant choose the optimal retail prices. In particular, the regulator sets a capacity charge that corrects the product differentiation.

In equation (50), the optimal capacity charge of Proposition 2 is modified by the term ε and another negative term is added to this expression. When $L_U^2 > \frac{L_U^*}{1+A_U}$, the time correction term ε is larger than 1. This increases the capacity charge and forces the entrant to reduce the lenght of the peak period and increase the peak price. As a consequence, the differentiation between the bundle of prices offered by the entrant and the incumbent disappears. On the other hand, when $L_U^2 < \frac{L_U^*}{1+A_U}$ it follows that $\varepsilon < 1$. Therefore, the optimal capacity will be lower than the one in Proposition 2. This reduces the profit maximising peak price and the entrant increases the length of the peak period. This also reduces the product differentiation between the entrant and the incumbent. When $P^2 = P^{1*}$ if follows that $L_j^2 > L_j^*$, for $j = \{U, L\}$. Therefore, as the entrant has to reduce the length of the peak period to attract consumers, the entrant's product will not be different from the incumbent's anymore. As a consequence, the entrant will loos its market power ($A_j = 0$) and we are back to equation (25).

[26] The particular strategy that the entrant chooses will depend on the consumers' preferences.

The entrant differentiates its product by enlarging the duration of the peak period. This allows the entrant to fix a mark-up over the two prices. The negative term at the end of equation (50) is a consequence of this behaviour. From (48) it follows that the incumbent's final prices are reduced to lower the monopoly profits of the entrant. To rebalance the consumers' choice between the two firms, the entrant's final prices must also be reduced, and so must be the capacity charge.[27]

Finally, it is important to emphasise that the optimal capacity charge only promotes an efficient use of the capacity when $L_U^2 > \frac{L_U^*}{1+A_U}$. Indeed, only in this case the optimal capacity charge forces the entrant to increase the peak price. As a result, the demand in the peak period is reduced and less capacity is required.

6 The multiproduct industry

In this section I extend the framework of section 4 to consider the case of multiproduct firms. In telecommunications, networks are used to provide different services such as local and long distance telephony, or as dial-up connection to Internet. However, capacity is expensive. Therefore, the prices and pricing periods of each service are chosen to optimise joint use of the network. Our interest here is to analyse whether the optimal pricing rule for the single-product case can be extended to the more realistic multiproduct framework.[28] What is the optimal capacity charge when the incumbent and the entrant provide more than one service? What is the optimal charge if they offer different services?

Consider that an entrant with market power offers the market N final services and that the incumbent provides M services. The services offered by the entrant and the incumbent can be different. As before, and as a matter of convenience, we will only consider two pricing periods, $j = \{U, L\}$, for each service. The consumers only have a provider for each service. This implies that when a consumer buys the service m to the firm i, he is restricted to buy this service to the firm in the peak and the off-peak period. In spite of this, consumers can buy other services from other firms.

Let $P^2 = (P_{1j}^2, ..., P_{Nj}^2)$ be a $[1, N]$x$[U, L]$ matrix of retail prices offered by the entrant and $P^1 = (P_{1j}^1, ..., P_{Mj}^1)$ be a $[1, M]$x$[U, L]$ matrix of retail prices provided by the incumbent. As previously, I consider that there exist intertemporally dependent demands for each service. Moreover, I assume that the demand of each product is independent from the demand of the others.

[27] This effect also appears in Laffont and Tirole (1994), where the authors analyse the optimal access charge when there is a competitor with market power.

[28] Armstrong, Doyle, and Vickers (1996) analyse the access charge problem in a multiproduct setting when the entrants have not market power.

This implies that the aggregated demand of firm i for the product m, $q^i_{mj}(t)$, is only determined by its own prices $P^i_m = (P^i_{mU}, P^i_{mL})$, its own switch time, τ^i, the price of the other firm $P^{-i}_m = (P^{-i}_{mU}, P^{-i}_{mL})$, the switch time of the other firm, τ^{-i}, and the particular point in time.

$$q^i_{mj}(t) = q(P^i_m, P^{-i}_m, \tau^i, \tau^{-i}, t),$$

where $j = \{U, L\}$. Taking this into account, $q^1 = (q^1_{1j}(t), ..., q^1_{mj}(t))$ is the matrix of final products offered by the incumbent and $q^2 = (q^2_{1j}(t), ..., q^2_{mj}(t))$ is the matrix of final products supplied by the entrant.

The entrant has market power and it is unregulated. Therefore, when the regulator sets the capacity charge, she takes into account that the entrant maximises its profits by optimally choosing the price of each service in each pricing period as well as the switch time. The entrant's aggregated profit depends on the prices, P^2, on the switch time, τ^2, and on the capacity charge, a,

$$\Pi^2(P^2, a, \tau^2) \equiv \sum_{n=1}^{N} \sum_{j} \int\limits_{t \in L^2_j} (P^2_{nj} - b_{n2}) q^2_{nj}(P^2, P^1, \tau^2, \tau^1, t) dt - aK_2, \tag{51}$$

subject to

$$K_2 \geq (\sum_{n=1}^{N} q^2_{nj})^M,$$

where $(\sum_{n=1}^{N} q^2_{nj})^M = max_{t \in L^2_j} \sum_{n=1}^{N} q^2_{nj}(P^2, P^1, \tau^2, \tau^1, t)$ is the maximum production level of the n services in the pricing period $j = \{U, L\}$. The capacity constraint in the entrant's problem implies that the pricing periods are the same for all the services. If we denote by $\alpha \geq 0$ the multiplier associated to the capacity constraint, the first order conditions for this problem are

$$\int\limits_{t \in L^2_U} q^2_{nU} dt + \sum_{j} \int\limits_{t \in L^2_j} (P^2_{nj} - b_{n2}) \frac{\partial q^2_{nj}}{\partial P^2_{nU}} - \sum_{j} \alpha_j \frac{\partial q^{2M}_{nj}}{\partial P^2_{nU}} = 0, \tag{52}$$

$$\int\limits_{t \in L^2_L} q^2_{nL} dt + \sum_{j} \int\limits_{t \in L^2_j} (P^2_{nj} - b_{n2}) \frac{\partial q^2_{nj}}{\partial P^2_{nL}} - \sum_{j} \alpha_j \frac{\partial q^{2M}_{nj}}{\partial P^2_{nL}} = 0, \tag{53}$$

$$\mu_j \geq 0; \quad \mu_j(K_2 - (\sum_{n=1}^{N} q_{nj}^2)^M) = 0, \tag{54}$$

$$\sum_j \mu_j \leq \beta; \quad K_2(\sum_j \mu_j - \beta) = 0, \tag{55}$$

$$\sum_{n=1}^{N}(P_{nU}^2 - b_{n2})q_{nU}^2(P^2, P^1, \tau^2, \tau^1) = \sum_{n=1}^{N}(P_{nL}^2 - b_{n2})q_{nL}^2(P^2, P^1, \tau^2, \tau^1). \tag{56}$$

If we simplify these conditions we find that the entrant's profit-maximising prices are

$$P_{nU}^2 = (\frac{1}{1 + A_{nU}})(b_{n2} + \frac{a}{L_U^2}), \tag{57}$$

$$P_{nL}^2 = (\frac{1}{1 + A_{nL}})b_{n2}, \tag{58}$$

where $A_{nj} \equiv \frac{(\eta_{-j-j}^n - \eta_{j-j}^n)}{L_{nj}^2(\eta_{jj}^n \eta_{-j-j}^n - \eta_{j-j}^n \eta_{-jj}^n)}$ and where L_U^2 is obtained by solving equation (56). This structure of prices represents a generalisation of the Ramsey prices when demand interdependencies exist. This is reflected in the following proposition.

Proposition 6. *In a multiproduct setting with demand interdependencies, the smaller (higher) the difference between the own price elasticity (η_{jj}^n) and the cross-price elasticity (η_{j-j}^n) of a service n, the smaller (higher) the peak and off-peak prices.*

The intuition behind this proposition is that the services, for which the price is more important than the moments in which they are consumed, will have a smaller peak and off-peak price and will proportionally contribute less to the firm's profits.

This pricing policy implies a displacement in the consumption of the services that have a higher difference between their own price elasticity and the cross price elasticity, towards the off-peak period. The result of doing so is a better management of the capacity because less capacity is required in the peak period. On the other hand, there is more capacity used during the off-peak period.

Given the incumbent's prices, P^1, the switch time, τ^1, and the capacity charge, a, the entrant's equilibrium prices, $\hat{P}^2(P^1, a)$, equates the entrant's equilibrium supply to its demand in every moment t,

$$\hat{s}^2_{nj}(\hat{P}^2(P^1, a), a, \hat{\tau}^2, \tau^1, t) \equiv q^2_{nj}(\hat{P}^2, P^1, \hat{\tau}^2, \tau^1, t). \tag{59}$$

Moreover, these prices equate the capacity bought by the entrant to the maximum aggregated number of units that the entrant provides in equilibrium during the peak period,

$$(\sum_{n=1}^{N} \hat{s}^2_{nU}(P^1, a, \hat{\tau}^2, \tau^1))^M \equiv K_2. \tag{60}$$

Considering the entrant's profit-maximizing prices, we can rewrite its profit function as

$$\Pi^2(\hat{P}^2, a, \hat{\tau}^2) = \sum_{n=1}^{N} \sum_{j} \int_{t \in L^2_j} [(\frac{1}{1 + A_{nj}}) \frac{\alpha_j}{L^2_j} - \frac{A_{nj} b_2}{1 + A_{nj}}] \hat{s}^2_{nj} dt - a(\sum_{n=1}^{N} \hat{s}^2_{nU})^M. \tag{61}$$

Taking into account the maximum aggregated number of units that the entrant provides in equilibrium, $K_2 \equiv (\sum_{n=1}^{N} \hat{s}^2_{nU})^M$, the incumbent's profit can be written as

$$\Pi^1(P^1, a, \tau^1) = \sum_{m=1}^{M} \sum_{j} \int_{t \in j} (P^1_{mj} - b_{m1}) \hat{q}^1_{mj}(P^1, a, \tau^1, t) dt - \beta K_1 + (a - \beta)(\sum_{n=1}^{N} \hat{s}^2_{nU})^M, \tag{62}$$

subject to

$$K_1 \geq (\sum_{m=1}^{M} \hat{q}^1_{mj})^M.$$

In order to derive the optimal capacity charge, a welfare-maximising regulator considers the following unweighted welfare function

$$W(P^1, a, \tau^1) = CS(P^1, \hat{P}^2, a, \tau^1, \hat{\tau}^2) + \Pi^1(P^1, a, \tau^1) + \Pi^2(\hat{P}^2, a, \hat{\tau}^2), \tag{63}$$

The aggregated consumer's surplus in (63) is defined by

$$CS(P^1, \hat{P}^2, a, \tau^1, \hat{\tau}^2) = \sum_{s=1}^{S} CS_s(P^1, \hat{P}^2, a, \tau^1, \hat{\tau}^2), \tag{64}$$

where $CS_s(P^1, \hat{P}^2, a, \tau^1, \hat{\tau}^2)$ is the aggregated consumer's surplus from consuming service s. On the other hand, S represents all the services provided by the incumbent and the entrant. Therefore, when all firms provide the same services, $S = M = N$. Finally, notice that we assume that all services have the same weight in the welfare function.

The regulator maximizes the total welfare subject to $\Pi^1 \geq 0$ and the incumbent's capacity constraint. Denoting $\lambda \geq 0$ the multiplier of the incumbent's break even constraint and $\mu \geq 0$ the multiplier of its capacity constraint, the first order condition for a is

$$\begin{gathered}\lambda(\sum_{n=1}^{N} \hat{s}_{nU}^2 + a(\sum_{n=1}^{N} \tfrac{\partial \hat{s}_{nU}^2}{\partial a})^M) + \sum_{n=1}^{N} \sum_j \int\limits_{t \in L_j^2} [(\tfrac{1}{1+A_{nj}})\tfrac{\alpha_j}{L_j^2} - \tfrac{A_{nj} b_{n2}}{1+A_{nj}}] \tfrac{\partial \hat{s}_{nj}^2}{\partial a} dt + \\ (1+\lambda)[\sum_{m=1}^{M} \sum_j \int\limits_{t \in L_j^1} (P_{mj}^1 - b_{m1}) \tfrac{\partial \hat{q}_{mj}^1}{\partial a} dt - \sum_j \mu_j (\sum_{m=1}^{M} \tfrac{\partial \hat{q}_{mj}^1}{\partial a})^M - \beta(\sum_{n=1}^{N} \tfrac{\partial \hat{s}_{nU}^2}{\partial a})^M] = 0,\end{gathered} \tag{65}$$

The Khun-Tucker conditions also require that,

$$\mu_j \geq 0; \quad \mu_j(K_1 - (\sum_{m=1}^{M} \hat{q}_{mj}^1)^M) = 0, \tag{66}$$

$$\sum_j \mu_j \leq \beta; \quad K_1(\sum_j \mu_j - \beta) = 0. \tag{67}$$

By simplifying and rearranging equation (65) we obtain the optimal capacity charge rule.

Proposition 7. *In a multiproduct industry where the incumbent provides M services and the entrant provides N services, when $\frac{\lambda}{1+\lambda} > 0$, P^1 are higher than the first best, and there are demand interdependencies,*

$$\begin{gathered} a = \varepsilon[\beta + \sum_{m=1}^{M} \sum_j L_j^*(P_{mj}^1 - b_{m1} - \mu_j)\sigma_{mj} - (\tfrac{\lambda}{1+\lambda}) \frac{\hat{s}_{nU}^2}{(\sum_{n=1}^{N} \frac{\partial \hat{s}_{nU}^2}{\partial a})^M}] \\ + \sum_{n=1}^{N} L_j^*(\tfrac{A_{nj} b_{n2}}{\lambda L_U^2 (1+A_{nj}) + L_U^*}) \frac{\partial \hat{s}_{nj}^2}{(\frac{\sum_{n=1}^{N} \partial \hat{s}_{nU}^2}{\partial a})^M}. \end{gathered} \tag{68}$$

where $\sigma_{mj} = \frac{\frac{\partial \hat{q}^1_{mj}}{\partial a}}{(\frac{\sum_{n=1}^{N} \partial \hat{s}^2_{nU}}{\partial a})^M}$ *and where* $\varepsilon = (\sum_{n=1}^{N} \frac{L^2_U(1+A_{nU})(1+\lambda)}{\lambda L^2_U(1+A_{nU})+L^*_U})$

These formulae can be seen as a generalisation for a multiproduct industry of the optimal capacity charge as seen in the previous section. However, now the interpretation is more complex. The expression inside the brackets has the same interpretation as in the single-product case. The term σ_{mj} is the multiproduct extension of the displacement ratio. It reflects the change in the incumbent's final product m in period j divided by the change in aggregated sales of capacity as the capacity charge is modified. The second term inside the brackets is the incumbent's aggregated opportunity cost when it provides the marginal unit of capacity to the fringe. Remember that in a multiproduct setting, when the incumbent supplies one unit of capacity to the entrant, the entrant can then use the capacity to provide different services. On the other hand, it is interesting to notice that when all the services offered by the incumbent and the entrant are independent, $M \neq N$, the opportunity costs vanish.

$$a = \varepsilon[\beta + (\frac{\lambda}{1+\lambda})(\frac{\hat{s}^2_{nU}}{\sum_{n=1}^{N} \frac{\partial \hat{s}^2_{nU}}{\partial a}})] + \sum_{n=1}^{N} L^*_j(\frac{A_{nj}b_{n2}}{\lambda L^2_U(1+A_{nj})+L^*_U})\frac{\partial \hat{s}^2_{nj}}{\frac{\partial \hat{s}^2_{nU}}{\partial a}}. \tag{69}$$

In this particular case, however, the capacity charge still considers the time correction, ε. Indeed, to maximise social welfare the time correction ε must modify the capacity charge when the length of the peak period chosen by the entrant, L^2_U, is different from the optimal length, L^*_U. This also occurs in the general case defined by the equation (68). L^2_U and L^*_U are different when the entrant has market power. But they are also different when the entrant has not market power but the bundle of services offered by the incumbent and the entrant differ: One of the firms can produce more services than the other, or it may be that each firm specialises in different services. Only when both firms offer the same services and the entrant has not market power, it follows that $L^2_U = L^*_U$. In this case, the time correction term vanishes.

Finally, note that depending on the specific load curve of the services supplied by the incumbent and the entrant, ε increases or decreases the capacity charge.

Proposition 8. *If $\varepsilon > 1$, the time correction increases the capacity charge and induces an efficient use of the capacity. If $\varepsilon < 1$, the time correction decreases the capacity charge and can induce an inefficient use of the capacity.*

When $\varepsilon > 1$ the time correction increases the capacity cost of the entrant. As a consequence, the entrant increases peak prices, which moves a part of its sales to the off-peak period. The higher are the demand interdependencies the more important is the shift in the consumption from the peak to the off-peak period. Moreover, the entrant can fill in the excess capacity of the off-peak period with more sales. Therefore, the time correction not only allows to maximize social welfare, but it also gives incentives for an efficient use of the network. In the opposite sense, when $\varepsilon < 1$ the time correction reduces the entrant's cost of capacity and hence its peak prices. As a result, a part of the sales would shift to the peak period, increasing the requirement for capacity in the peak period. The overall outcome is a higher excess capacity during the off-peak period.

7 Conclusions

In the telecommunications industry, the recent practices in the regulation of the access to the incumbent's network are challenging the traditional system of interconnection by time. One alternative to the call-minute access charge which has been implemented in the United Kingdom and in Spain is the system of interconnection by capacity. While both countries employ this system, only in Spain it is applied for the provision of all telecommunications services.

The justifications explained by the Spanish Regulatory Agency (CMT) at the time of the introduction of the system emphasised that the principal driver of the incumbent's network's cost is the peak-hour capacity cost. Moreover, the nature of the peak-load problem in the telecommunications industry requires an interconnection system that allows the entrants to emulate the incumbent's "time-of-use" policy in the retail market.

An access system based on a fixed price for capacity made available to the entrants (i.e. one circuit of access) rather than for one unit of access (i.e. one access-minute) may serve to alleviate both problems. As stated by CMT (2001), "*it will be the management of the load curve, the efficient use of the interconnection traffic, the opening of new businesses as sales of the excess capacity, the activities that will determine the optimisation of the interconnection capacity and the way to secure lower interconnection costs*". However, considering that each capacity circuit could be used for the provision of different services at different times, which price should be established for capacity?

This paper has shown how the theory on access charge can be extended to the capacity charge problem. As in the case of the access charge analysed by Armstrong, Doyle, and Vickers (1996), the optimal capacity charge consists of the direct cost of capacity plus the incumbent's opportunity cost of access provision to its competitors. Moreover, we have shown that with a time-varying demand, the opportunity cost of the incumbent accounts for the loss of profits during the peak and off-peak periods. Determining the optimal capacity charge is more complex when competitors have market power, or when they are multiproduct firms. In this case, the optimal charge should take into account that the competitors will try to differentiate their services by choosing a duration of the pricing periods that is not the optimal.

The proponents of the system of access by capacity emphasize that this rule gives incentives for an efficient use of the network because the entrants fill in the unoccupied capacity of the off-peak periods with other services. In this paper I have shown that the optimal capacity charge will not forcedly imply a more intensive use of capacity by the entrants. Indeed, under some circumstances the capacity charge can shift a part of the sells of the entrant from the off-peak to the peak time period.

To sum up, a system of access by capacity can improve the efficiency of the market and induce a more aggressive competition. But it is important to be aware that if the entrants have market power they can "cream-skim" the market and leave the lesser profitable users to the incumbent. In a system of access by time the regulator determines the moment in which the peak period begins and ends, and as a consequence, the entrants can not modify the duration of the peak period to cream-skim the market. A system of access by capacity gives more flexibility to the firms to manage their load. However, this does not forcedly imply a maximization of the social welfare.

Appendix

Proof of Lemma 1. The difference between the welfare function with socially maximizing prices P^{1*}, P^{2*} and a^* and the entrant's profits when it fixes the profit maximizing price $\hat{P}^2$ is given by

$$\begin{aligned} &R = W(P^{1*}, a^*, \tau) - \Pi^2(\hat{P}^2, a, \tau) \\ &= CS(P^{1*}, P^{2*}, a^*, \tau) + \Pi^1(P^{1*}, a^*, \tau) + \Pi^2(P^{2*}, a^*, \tau) - \Pi^2(\hat{P}^2, a, \tau). \end{aligned} \tag{70}$$

With socially optimal prices when $\frac{\lambda}{1+\lambda} > 0$, $\Pi^1(P^{1*}, a^*, \tau) = 0$. Moreover, as $\tau^2 = \tau^1$, the entrant cannot differentiate its product from that of the incumbent. As a consequence, $P^{2*} = P^{1*}$ and $\Pi^2(P^{2*}, a^*, \tau) = 0$. Therefore,

$$R = CS(P^{1*}, P^{2*}, a^*, \tau) - \Pi^2(\hat{P}^2, a, \tau). \tag{71}$$

Consequently, in view of equations (3) and (7)

$$\begin{aligned} &\tfrac{\partial R}{\partial \tau} = ics_U(P^{1*}, P^{2*}, a^*, \tau, \tau) - ics_L(P^{1*}, P^{2*}, a^*, \tau, \tau) \\ &-(\hat{P}^2_U - b_2) q^2_U(\hat{P}^2_U, \hat{P}^2_L, P^{1*}, \tau, \tau) + (\hat{P}^2_L - b_2) q^2_L(\hat{P}^2_L, \hat{P}^2_U, P^{1*}, \tau, \tau). \end{aligned} \tag{72}$$

Following Lemma 1 in Burness and Patrick (1991) we can write (72) in the following way

$$\frac{\partial R}{\partial \tau} = +\sum_{i=1}^{2} \int_{P_U^i}^{\infty} q^i(p^i, P_L^i, P^{-i}, \tau, \tau)dp^i - \sum_{i=1}^{2} \int_{P_L^i}^{\infty} q^i(p^i, \infty, P^{-i}, \tau, \tau)dp^i$$
$$-(\hat{P}_U^2 - b_2)q_U^2(\hat{P}_U^2, \hat{P}_L^2, P^{1*}, \tau, \tau) + (\hat{P}_L^2 - b_2)q_L^2(\hat{P}_L^2, \hat{P}_U^2, P^{1*}, \tau, \tau). \tag{73}$$

The proof consists of showing $\frac{\partial R}{\partial \tau} < 0$, so that the sufficient conditions imply that the optimal profit maximising peak length of the entrant, L_U^2, exceeds the optimal welfare maximising peak length that the regulator sets, L_U^*. Considering that there are demand interdependencies, we can further simplify equation (73) as

$$\frac{\partial R}{\partial \tau} = \sum_{i=1}^{2} \int_{P_U^i}^{\infty} (q^i(p^i, P_L^i, P^{-i}, \tau, \tau) - q^i(p^i, \infty, P^{-i}, \tau, \tau))dp^i$$
$$+\sum_{i=1}^{2} \int_{P_U^i}^{P_L^i} q^i(p^i, \infty, P^{-i}, \tau, \tau)dp^i - (\hat{P}_U^2 - b_2)q_U^2(\hat{P}_U^2, \hat{P}_L^2, P^{1*}, \tau, \tau) \tag{74}$$
$$+(\hat{P}_L^2 - b_2)q_L^2(\hat{P}_L^2, \hat{P}_U^2, P^{1*}, \tau, \tau).$$

Note that as $\infty > P_U^i > P_L^i$ the first three terms of this expression are negative. If the entrant's profit during the off-peak period is sufficiently small, equation (74) will be negative. This implies that $L_U^2 > L_U^*$. However, it may also occur that $L_U^2 < L_U^*$.

Finally, we must note that when the entrant's prices and the prices fixed by the regulators are the same, $R = CS(P^1, P^2, a, \tau)$. In this case, it is clear that $\frac{\partial R}{\partial \tau} < 0$. This result is obtained by Burness and Patrick (1991), and implies that for any exogenously given price, a monopoly will establish a longer length of the peak price than the regulator. However, this is not necessarily the case when the entrant and the regulator optimise their objective functions. Indeed, the entrant may decide to fix a higher price by reducing the length of the peak period. ∎

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