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A comment on the cost of capital for investments with non-homogeneous components

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Abstract

In this paper, the expression for the cost of capital is derived when net and replacement investments exhibit differences in their effective prices due to a different fiscal treatment. It is shown that, contrary to previous results in the literature, the cost of capital should be constructed under an opportunity cost criterion rather than a historical one. This result has some important economic consequences, since the optimizing firm will take into account not only the effective price for the new investments but also consider the opportunity cost of replacing them.

Keywords: Cost of capital, Grants on net investments

JEL classification: D92, H32, C61

Resumen

En este trabajo se deriva la expresión para el coste de capital cuando inversión neta e inversión de reposición presentan diferencias en su precio efectivo debido a un tratamiento fiscal diferenciado. Se demuestra, contrariamente a resultados previos en la literatura, que el coste de capital debe construirse desde un criterio de coste de oportunidad en vez de coste histórico. Las consecuencias económicas para la empresa maximizadora de ingresos suponen la consideración del coste de oportunidad para las nuevas inversiones y no del coste efectivo de la inversón.

Palabras clave: Coste de capital, subvenciones para inversión neta

Clasificació JEL : D92, H32, C61

1 Introduction

Since Jorgenson (1963), the cost of capital concept has become a useful tool for the study of the investment demand and its sensitivity to fiscal policies. Although several equivalent approaches can be taken in order to determine the expression for the cost of capital, each one exhibits its own characteristics. For instance, the original setting employed by Jorgenson (1963) allowed for differences between economic and tax depreciations for all the assets, but did not take into consideration different types of assets simultaneously. Later, the cost of capital for new (differentiated) assets was derived in Hall and Jorgenson (1967). In almost all the subsequent studies this has been the approach adopted. It assumes that the present value of tax deductions, together with investment incentives such as investment grants or investment tax credits (ITC), determine the effective price of new capital goods, and therefore their cost of capital. This approach handles assets with different fiscal treatment separately, so that interactions between different types of assets at a given time (or the same asset at different time periods) are not considered.

In all these approaches, the investment rate $I(t)$ at a given time t represents an homogeneous unit of investment. Note that the usual description of the dynamics of the state variable, the capital goods stock $K(t)$, is given by the differential equation

$$\dot{K}(t) = I(t) - \delta K(t),$$

where $I(t)$ is the investment rate, and δ the true economic depreciation rate. From the equation above, it can be seen that $I(t)$ presents characteristics (properties) of net investments, $\dot{K}(t)$, and replacement investments, $\delta K(t)$. Then, for every change affecting $I(t)$ (e.g., a change in its price due to an ITC, a direct investment incentive or inflation in the price of capital goods), the two components of $I(t)$ will be affected simultaneously and homogeneously.

In some cases, economic reality requires us to differentiate these two components. For instance, consider incentives which are exclusively aimed at promoting net growth, while the price for replacement investments remains unaffected. This is the problem studied in Ruane (1982), who derives the expression for the cost of capital under a wide range of assumptions about fiscal and financial policies. Other more usual settings which require a differentiation of these two components can arise when other types of investment incentives are considered from a dynamic perspective. For instance, let us consider aids orientated towards promoting first installation plants, which will disappear after some

time, temporary ITC[1], or incremental ITC[2]. In these cases, once an investment has been undertaken and classified as eligible for a subsidy (grant, ITC...), subsequent investments aimed at its replacement might not be eligible for the incentive, or the fiscal conditions could be modified.

Roughly speaking, in order to construct the expression for the cost of capital, Ruane splits one investment unit into the grant component, ϕ, and the non-grant component of investment, $(1-\phi)$, the investment grant rate being ϕ, with the aim of characterizing each investment component depending on its particular features. She works in the straightforward way of postulating a unit increase in the firm's capital stock in period $t-1$, which, for the capital stock in all other periods to remain unchanged, must be compensated for by a decrease in investment of $(1-\delta)$ in period t. However, this method assumes implicitly that, in period t, the proportion between the grant and non-grant component of the investment remains unaffected. As it will be shown, this proportion is not constant, irrespectively of the investment activity.

In this paper, we take as our starting point the same problem as Ruane (1982), i.e., to determine the cost of capital of assets when only net investments are granted, but we derive it directly as solution of a dynamic problem for the firm. Results obtained working in this way indicate that the optimal behavior for the firm is to consider the cost of capital for marginal investments from an opportunity cost perspective rather than a historical one, in a sense that will be specified later. This result has some important economic consequences, since in cases where some differences exist between the different components of investments undertaken at a given time of the planning horizon, the optimizing firm will take into account not only the effective price for the new investments but also consider the opportunity cost of replacing them. Hence, the analysis of the consequences of fiscal policies directed to promote some investments should take this effect into account.

The paper is organized as follows. Section 2 introduces the model. Although it will be written in order to consider the existence of grants, the same model applies for other types of investment incentives, such as ITC. In Section 3, we solve the model and discuss the expression for the cost of capital under different scenarios, deriving the main results of the paper. The conclusions are briefly commented in Section 4.

[1]In Hassett and Hubbard (2002), it is pointed out that, since 1962, the mean duration of a typical state in which an ITC is in effect has been about three and one-half years, and the same length for periods without ITC.

[2]A survey about the history, scope and types of ITC, can be found in Chirinko (2000).

2 The model

In order to determine the cost of capital, we construct and solve a dynamic model for the firm following Jorgenson (1963)[3], while maintain Ruane's notation in order to facilitate comparisons.

Consider a firm that maximizes its present value, defined as the sum of net receipts, $R(t)$, where r is the interest rate:

$$J = \int_0^\infty R(t)e^{-rt}dt. \tag{1}$$

The firm can decide on investments in capital assets I (from now on, we will omit the argument t if it is not strictly necessary). For simplicity, we restrict the analysis to a production process with a single input K, the capital goods stock. Let $S(K)$ be the earnings function with the usual concavity assumptions ($S(K) > 0$, $dS/dK > 0$, $d^2S/dK^2 < 0$), δ the true economic depreciation rate, τ the profit tax rate, ϕ the investment grant rate, and a proportion γ of interest payments can be offset against tax. We also assume that the price of capital goods is constant and equals unity and that the true depreciation allowances equal the replacement value of physical depreciation. Then, the dynamics of the state variable is given by the following differential equation

$$\dot{K} = I - \delta K, \qquad K(0) = K_0. \tag{2}$$

Note that the state variable accounts both for net and replacement investments. In order to define the amount of grants and allowances against taxes, we need to differentiate which amount of capital assets are net and which are replacement investments, as well as their evolution in time, because of differences in the fiscal treatment. We thus define a new state variable, K_N, which accounts for assets qualified as new investments, while the difference $K - K_N$ will account for replacement assets. For instance, assume that the capital goods stock at the initial period $t = 0$ is of 100 units and qualified entirely as new assets, i.e., $K(0) = K_N(0) = 100$. For an economic (linear) depreciation of 10%,

[3]This approach presents some drawbacks when explaining the adjustment process to the desired capital goods stock level. More realistic settings have been addressed in the literature by introducing adjustment costs and/or financial constraints, e.g., in Kort (1988), both assumptions are jointly treated. However, in order to derive the expression for the cost of capital, defined as the minimum pre-tax rate of return for an investment to be profitable, Jorgenson's approach is the most direct way, and for that reason we follow it here.

after one period, at $t = 1$, we have that $K(1) = K_N(1) = 90$ if no investments during this period, say $I(0)$, are carried out. Assume now that $I(0) = 25$, then 10 units of these will be replacement investments and 15 units new investments, so that $K(1) = 115$, while $K_N(1) = 90 + 15 = 105$, that is, as time goes on the amount of new capital goods and replacement capital goods evolve differently. In the case where investments are restricted to the replacement level, that is, $I(0) = 10$, K would stay at a level of 100 units, $K_N(1) = 90$, since no net growth would be achieved, and all the investments would be qualified as replacement investments. Then, K_N increases with net investments, $\dot{K}(t)$, whereas it depreciates at the same rate as $K(t)$, since once a portion of K_N has depreciated, required investments to its replacement will not be eligible for grants. Hence, the dynamics of K_N is given by

$$\dot{K}_N = \dot{K} - \delta K_N, \qquad K_N(0) = K_0. \tag{3}$$

It is assumed (without loss of generality) that investments carried out before the origin of the planning horizon are new investments. Then, profit taxes[4] are

$$T(K, K_N) = \tau \cdot [S(K) - \gamma r(K - \phi K_N) - \psi\delta(K - \beta\phi K_N)], \tag{4}$$

where ψ is the proportion of investments allowable against tax at the true rate of economic depreciation, and β ($\beta = \{0, 1\}$) determines whether these allowances apply to the total investment ($\beta = 0$), or only to the non-grant component of investment ($\beta = 1$)[5]. This last case corresponds to the case where the tax rule is similar to the accounting rule, when the firm accounts for the grant as a deferred income according to the accrual basis, and income is recognized at the economic depreciation rate.

Finally, net receipts are

$$R = S(K) - T(K, K_N) - I + \phi\dot{K} \tag{5}$$

and, as usual, under stationary market conditions, we also assume that there exists a desired firm size that maximizes (1) K_s being the optimal capital stock:

$$\lim_{t\to\infty} K(t) = K_s. \tag{6}$$

[4]Here, we do not consider allowances for free depreciation in order to simplify expressions. It is also assumed that the firm's profits are large enough to enable all tax allowances.

[5]Note that the interest allowances are calculated on the actual expenditure.

3 Cost of capital

The problem to solve is to maximize (1), where R is given by (5), and T is described by (4), subject to (2), (3) and (6). Solving I in (2), the problem is:

$$\begin{aligned}
&\max\left\{J=\int_0^\infty \left[S(K)-T(K,K_N)-(1-\phi)\dot{K}-\delta K\right]e^{-rt}dt\right\},\\
&T(K,K_N)=\tau\cdot\left[S(K)-\gamma r(K-\phi K_N)-\psi\delta(K-\beta\phi K_N)\right],\\
&\dot{K}_N=\dot{K}-\delta K_N,\\
&K(0)=K_N(0)=K_0,\quad \text{and}\quad \lim_{t\to\infty}K(t)=K_s.
\end{aligned}$$

We will solve the problem from a variational viewpoint. To do this, let us define the extended Lagrangian function

$$\begin{aligned}
F=\big[S(K)-\tau\cdot(S(K)-\gamma r(K-\phi K_N)-\psi\delta(K-\beta\phi K_N))\\
-(1-\phi)\dot{K}-\delta K\big]e^{-rt}+\nu(\dot{K}_N-\dot{K}+\delta K_N),
\end{aligned}$$

where ν is the Lagrange multiplier associated to the differential constraint. The necessary optimality conditions, which are also sufficient for this problem, are given by the Euler equations

$$\frac{\mathrm{d}}{dt}\left(\frac{\partial F}{\partial \dot{K}}\right)=\frac{\partial F}{\partial K}\,,\qquad \frac{\mathrm{d}}{dt}\left(\frac{\partial F}{\partial \dot{K}_N}\right)=\frac{\partial F}{\partial K_N}\,.$$

Defining $\mu=\nu\cdot e^{rt}$, after several calculations and rearranging terms, the differential equations above become, respectively,

$$\begin{aligned}
[S'(K)-\tau\left(S'(K)-\gamma r-\psi\delta\right)-\delta] &= r(1-\phi+\mu)-\dot{\mu}\,, &(7)\\
[\delta\mu-\tau\phi(\gamma r+\psi\delta\beta)] &= \dot{\mu}-r\mu\,. &(8)
\end{aligned}$$

Adding (7) and (8) we can solve μ as

$$\mu=\frac{1}{\delta}\left[\tau\phi(\gamma r+\beta\psi\delta)+r(1-\phi)+S'(K)-\tau(S'(K)-\gamma r-\psi\delta)+\delta\right]\,,$$

and differentiating with respect to t, we get

$$\dot{\mu}=-\frac{1}{\delta}(1-\tau)S''(K)\dot{K}\,.$$

Let t_1 be the moment when the steady state is reached, i.e. $\dot{K}(t)=0$, for every $t\geq t_1$. From the equation above, $\dot{\mu}(t)=0$, for every $t\geq t_1$. Therefore, evaluating (7-8) at t_1, we

get

$$\mu(t_1) = \frac{1}{r}\left[S'(K_s) - \tau\left(S'(K_s) - \gamma r - \psi\delta\right) - \delta - r(1-\phi)\right] , \tag{9}$$

$$\mu(t_1) = \frac{\tau\phi}{r+\delta}\left(\gamma r + \psi\delta\beta\right) , \tag{10}$$

so

$$\frac{1}{r}\left[S'(K_s) - \tau\left(S'(K_s) - \gamma r - \psi\delta\right) - \delta - r(1-\phi)\right] = \frac{\tau\phi}{r+\delta}\left(\gamma r + \psi\delta\beta\right) .$$

Finally, rearranging terms we get that the optimal capital goods stock K_s satisfies

$$S'(K_s) = c = \frac{1}{1-\tau}\left[r(1-\phi) + \delta - \tau\frac{\gamma r(r(1-\phi)+\delta) + \psi\delta(r(1-\beta\phi)+\delta)}{r+\delta}\right] . \tag{11}$$

The right-hand side of (11) shows the expression for the cost of capital, c, defined as the minimum pre-tax rate of return which an investment must earn to be profitable.

When allowances are granted on the non-grant component of the investment, ($\beta = 1$), the expression for the cost of capital reduces to

$$c = \frac{1}{1-\tau}(r(1-\phi)+\delta)\left(1 - \tau\frac{r\gamma + \psi\delta}{r+\delta}\right) , \tag{12}$$

and, in the case where depreciation allowances are granted on the total investment, ($\beta = 0$), we get

$$c = \frac{1}{1-\tau}\left[r(1-\phi) + \delta - \tau\frac{\gamma r(r(1-\phi)+\delta) + \psi\delta(r+\delta)}{r+\delta}\right] . \tag{13}$$

With full interest and depreciation deductibility ($\gamma = 1$ and $\psi = 1$), expressions (12) and (13) are, respectively,

$$c = r(1-\phi) + \delta, \tag{14}$$

$$c = \frac{1}{1-\tau}\left[r(1-\phi) + \delta - \tau\frac{r(r(1-\phi)+\delta) + \delta(r+\delta)}{r+\delta}\right] . \tag{15}$$

Expression (14) illustrates the main result of the paper (a similar reasoning leads to similar consequences in the general case, although we choose (14) for simplicity's sake). Note that while the financing cost takes into account the effective required amount for the acquisition of the asset, i.e., the non-grant component of investment, $(1-\phi)$, the depreciation cost, δ, considers the whole cost of one unit of investment, taking into account that replacement investments are not affected either by the investment grants or restrictions on the amount accounted for allowances against taxes. Therefore, the depreciation cost for the last (new) investment is totally defined taking into account the fiscal treatment that will apply on its future replacement investments (opportunity cost), not the fiscal features that apply on the asset acquired at t in the proportions defined by the grant rate[6], i.e., not in terms

[6] cf. expression (1) in Ruane (1982).

of one replacement investment with the same structure as the replaced asset (historical or acquisition cost). A similar reasoning can be applied to the case when allowances are granted on the total investment. From (15), note that the term inside the brackets

$$\tau \frac{\delta(r+\delta)}{r+\delta}$$

accounts for the present value of future tax deductions due to the depreciation cost, where the allowances for the financing cost are defined entirely from the fiscal treatment for replacement investments, i.e., with no grant at all.

Finally, and in order to show consequences of the construction of the cost of capital expression in both approaches, Ruane's and ours, we compare our results with those obtained by Ruane. In the case of complete deductibility of interest and depreciation, when allowances apply to the non-grant component of investments, ($\psi = 1, \gamma = 1$ and $\beta = 1$), she reported the paradoxical result of a positive dependence between the cost of capital and the investment grant rate when $r(1-\tau) < \delta\tau$. In this case, note that even the cost of capital is higher than the corresponding to non-granted investments, ($\phi = 0$). Analyzing equation (14) here derived, this negative effect will never occur. Differentiating (14) with respect to ϕ we obtain a value of $-r$, i.e., an increase in the investment grant rate leads to a proportional reduction in the financing cost, and has no effect in the depreciation cost. Hence, it is always interesting for the firm to access to direct investment incentives on net investment. Obviously, in this case, benefits will be lower than those with grants on gross investment. Nevertheless, as expression (11) defines a higher cost of capital than that derived from a historical criterion, the corresponding optimal firm size K_S will be lower.

4 Concluding remarks

In this paper we have shown that, for situations where the components of one unit of investment present differences in their effective prices, the cost of capital for a value maximizing firm should be constructed under an opportunity cost criterion, taking into account specificities which apply to each investment components. This result has some important economic consequences, since the optimizing firm should take into account not only the effective price for the new investments but also consider the actual replacement cost of the capital goods. Moreover, a proper analysis of consequences of fiscal policies directed to promote some investments should take this effect into account.

Finally, and comparing our results with previous ones in the economic literature, which, under certain assumptions, account for negative effects of investment grants, it is shown that the final effect when grants apply only on net investment always leads to a reduction in the cost of capital, obviously with lower benefits for the firm compared with those when gross investments are granted.

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