DOCUMENTS DE TREBALL
DE LA FACULTAT DE CIÈNCIES
ECONÒMIQUES I EMPRESARIALS

Col·lecció d'Economia

A comparative Long-memory Analysis between Spanish, Mexican and U.S. interest rates

Fernando Espinosa, Klender Cortez and Romà J. Adillon

Adreça correspondència:
Departament de Matemàtica Econòmica, Financera i Actuarial
Facultat de Ciències Econòmiques i Empresarials
Universitat de Barcelona
Avda. Diagonal 690
08034 Barcelona, Spain
e-mails: espinosa@ub.edu, klender@yahoo.com, adillon@ub.edu

Resumen

Existe bastante evidencia empírica a favor de que muchos fenómenos de la Naturaleza se comportan como objetos Fractales. Aunque los fractales son muy útiles en ese sentido, debemos redefinir el concepto para aplicarlo en Finanzas.
Con ese objeto y debido a la extraordinaria importancia del movimiento Browniano en el ámbito de la Física, Química o Biología, consideraremos la generalización que presupone el movimiento Browniano Fracccionario presentado por Mandelbrot.
El principal objetivo de este trabajo es, por tanto, analizar la existencia de memoria o dependencia a largo plazo en tasas instantáneas del tipo interés de diferentes mercados financieros. Concretamente, realizamos un análisis empírico sobre tasas del mercado interbancario español, mexicano y norteamericano. Trabajamos, por tanto, con tres series temporales de datos diarios correspondientes a operaciones a 1 día y considerando un periodo comprendido entre el 28 de Marzo de 1996 y el 21de Mayo 2002. De todos los test existentes en ese tema aplicamos la metodología propuesta en Taqqu, Teverovsky and Willinger (1995).

Abstract

Evidence exists that many natural facts are described better as a fractal. Although fractals are very useful for describing nature, it is also appropiate to review the concept of random fractal in finance.
Due to the extraordinary importance of Brownian motion in physics, chemistry or biology, we will consider the generalization that supposes fractional Brownian motion introduced by Mandelbrot.
The main goal of this work is to analyse the existence of long range dependence in instantaneous forward rates of different financial markets. Concretelly, we perform an empirical analysis on the Spanish, Mexican and U.S. interbanking interest rate. We work with three time series of daily data corresponding to 1 day operations from 28th March 1996 to 21st May 2002. From among all the existing tests on this matter we apply the methodology proposed in Taqqu, Teverovsky and Willinger (1995).

JEL Classification Numbers: C13, C82, E43

Keywords: Long-memory processes, interest rate analysis, Fractional Brownian Motion.

1 Preliminaries

1.1 Statistical Self-Similarity

Definition 1 *We say that a random process* $X = (X_t)_{t \geqslant 0}$ *with state space* $\boldsymbol{R}^d$ *is self-similar or satisfies the property of (statistica/) self-similarity if for each* $a > 0$ *there exists* $b > 0$ *such that*

$$Law(X_{at}, \quad t \geqslant 0) = Law(bX_t, \quad t \geqslant 0) \tag{1}$$

In other words changes of the time scale ($t \to at$) produce the same results as changes of the phase scale ($x \to bx$).

We can see in Shiryaev(1999) that for (nonzero) strictly stable processes there exists a constant $\mathbf{H}$ such that $b = a^{\mathbf{H}}$. In addition, for strictly α-stable processes we have,

$$\mathbf{H} = \frac{1}{\alpha} \tag{2}$$

In the case of (general) stable processes, in place of (1), we have the property

$$Law(X_{at}, \quad t \geqslant 0) = Law(a^{\mathbf{H}} X_t + tD_a, \quad t \geqslant 0) \tag{3}$$

In this point we introduce the following definition.

Definition 2 *If* $b = a^{\mathbf{H}}$ *in Definition 1 for each* $a > 0$, *then we call* $X = (X_t)_{t \geqslant 0}$ *a self-similar process with Hurst exponent* $\boldsymbol{H}$ *or we say that this process has the property of statistical self-similarity with Hurst exponent* $\boldsymbol{H}$*. The quantity* $\mathbf{D} = \frac{1}{\mathbf{H}}$ *is called the statistical fractal dimension of* X*.*

A classical example of a self-similar process is a Brownian motion $X = (X_t)_{t \geqslant 0}$, wich has the property of statistical self-similarity with Hurst exponent $\mathbf{H} = \frac{1}{2}$.

1.2 Fractional Brownian motion

The *Fractional Brownian motion* was introduced by Mandelbrot(1983), as a generalisation of the classical Brownian motion. In that line, we consider the function

$$A\,(s,t) = |s|^{\mathbf{2H}} + |t|^{\mathbf{2H}} - |t-s|^{\mathbf{2H}} \qquad s,t \in \mathbf{R} \tag{4}$$

For $0 < \mathbf{H} \leqslant 1$ this function is nonnegative definite (see, e.g., Shiryaev (1998)), therefore there exists a Gaussian process on some probability space that has the zero mean and the autocovariance function

$$Cov(X_s, X_t) = \frac{1}{2}A(s,t)$$

i.e., a process such that

$$E[X_s X_t] = \frac{1}{2}\left[|s|^{\mathbf{2H}} + |t|^{\mathbf{2H}} - |t-s|^{\mathbf{2H}}\right] \tag{5}$$

Hence

$$E[X_{as}X_{at}] = a^{\mathbf{2H}}E[X_s X_t] = E[(a^{\mathbf{H}}X_s)(a^{\mathbf{H}}X_t)] \tag{6}$$

so that

$$Law(X_{as}, X_{at}) = Law(a^{\mathbf{H}}X_s, a^{\mathbf{H}}X_t)$$

As in the case of a Brownian motion, By (5),

$$E[X_t - X_s]^2 = |t-s|^{\mathbf{2H}} \tag{7}$$

Definition 3 *We call a continuous Gaussian process $X = (X_t)_{t \geqslant 0}$ with zero mean and the covariance function (5) a (standard) fractional Brownian motion with Hurst self-similarity exponent $0 < \mathbf{H} \leqslant 1$.*

By this definition, a (standard) fractional Brownian motion $X = (X_t)_{t\geqslant 0}$ has the following properties:

1. $X_0 = 0$ and $E[X_t = 0]$ for all $t \geqslant 0$
2. X has homogeneous increments, i.e.,

$$Law(X_{t+s} - X_s) = Law(X_t) \qquad s,t \geqslant 0$$

3. X is a Gaussian process and

$$E[X_t^2] = |t|^{\mathbf{2H}} \qquad t \geqslant 0$$

where $0 < \mathbf{H} \leqslant 1$;

4. X has continuous trajectories.

And it could be represented by,

$$B_{\mathbf{H}}(t) = \frac{1}{\Gamma\left(\mathbf{H}+\frac{1}{2}\right)} \int_0^t (t-s)^{\mathbf{H}-\frac{1}{2}}\, dB_{\mathbf{H}}(s) \tag{8}$$

where Γ is the gamma function, $dB(s)$ the previous increments at time $s < t$ of an ordinary Gaussian random process $B(t)$ with average 0 and variance1.

In Mandelbrot and Wallis(1969), the discrete fractional Brownian noise, $X = \{X_n, n \in \mathbf{N}\}$, is defined as

$$X_n = B_{\mathbf{H}}(n+1) - B_{\mathbf{H}}(n) \qquad with \quad n = 0, 1, 2... \tag{9}$$

By formula (5), see Shiryaev (1998) for more deatils, for the covariance function of a (standard) process $B_{\mathbf{H}}$ we obtain that the covariance function $\rho_{\mathbf{H}}(n) = Cov\left(\beta_k, \beta_{k+n}\right)$ is as follows:

$$\rho_{\mathbf{H}}(n) = \frac{1}{2}\left[|n+1|^{2\mathbf{H}} - 2|n|^{2\mathbf{H}} + |n-1|^{2\mathbf{H}}\right] \tag{10}$$

Hence

$$\rho_{\mathbf{H}}(n) \sim \mathbf{H}(2\mathbf{H}-1)|n|^{2\mathbf{H}-2}$$

as $n \to \infty$.

Thus, if $\mathbf{H} = \frac{1}{2}$, then $\rho_{\mathbf{H}}(n) = 0$ for $n \neq 0$, and $(\beta_n)_{n \geqslant 1}$ is a Gaussian sequence of independent random variables. On the other hand, for $\mathbf{H} \neq \frac{1}{2}$ we have that the correlation function is not equal to zero, independently of t. We see from (10) that the covariance decreases fairly slowly (as $|n|^{-(2-2\mathbf{H})}$) with the increase of n, which is ussually interpreted as a *long memory* or a *strong aftereffect*.

From this point, see that the covariance function is negative if $H < \frac{1}{2}$. In this case, we observe an anti-persistent behavior. On the other hand, if $H > \frac{1}{2}$ the covariance is positive, we would consequently obtain a persistent behavior.

For more details about these preliminaries see Beran(1994), Espinosa(2002), Feder(1988), Feller(1951), Mandelbrot(1983), Mandelbrot and Van Ness(1968), Mandelbrot and Wallis(1969), Shiryaev (1998) and Vervaat(1987).

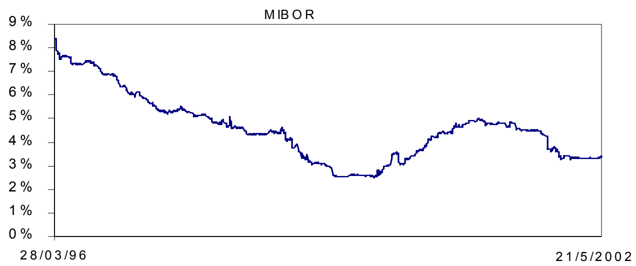
2 A numerical comparative analysis

In this section we carry out an application from the previous sections, improving the estimation methodology. The objective is to find evidence of fractional Brownian motion behavior in the time series under analysis. We also develop a comparative analysis from a random point of view by analyzing the fractability of the time series.

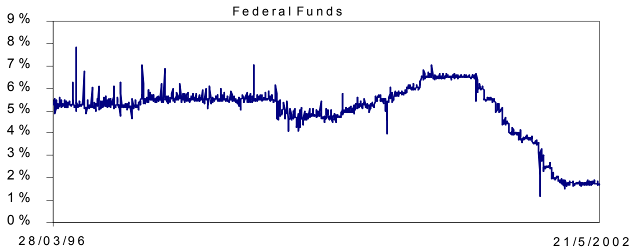
2.1 The data

In this article we perform an empirical analysis on the Madrid Interbank Offered Rate (MIBOR), The Intended U.S. Federal Funds Rate (FF) and the Mexican "Tasa de Interés Interbancaria de Equilibrio" (TIIE).

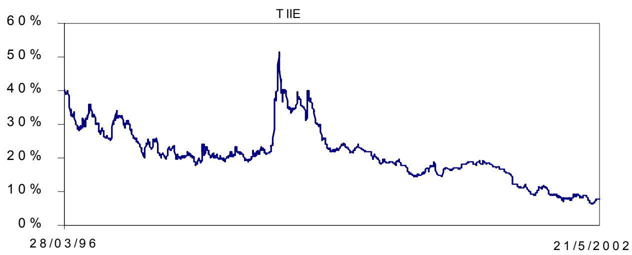
First, we analyze a time series provided by the "Banco de España" from 28th March 1996 to 21st May 2002, with 1492 daily observations of the interest rate for 1 month operations. We can see the behavior in the following figure.



Secondly, we analyze a time series provided by the United States of America Federal Reserve from 28th March 1996 to 21st May 2002, with 2247 daily observations of the overnight interest rate for 1 month operations. The behavior can be seen in the figure below. Since 2001, the weakening in economic activity has become widespread, prompting expectations of further monetary policy easing, and interest rates going down.



Finally we analyze a time series provided by the "Banco de México" from 28th March 1996 to 21st May 2002, with 1540 daily observations of the interest rate for 1 month operations. The highest interest rate in our time interval was on September 15th, 1998. In August of 1998, Russia suffered a severe crises that also affected countries like Mexico. The reaction of "Banco de México" was a restrictive monetary policy that affected the interest rates.



In order to work with the price logarithm (according to the classical time series procedure or Osborne(1959) Theory), we consider that in the interest rate time series, denoted by $\{R_t(\theta), t \geq 0\}$ where θ is the operation maturity and t the time, the loan of one unit of any currency, today (t), will become a quantity $e^{\theta R_t(\theta)}$ at time $t+\theta$, where θ is expressed in years. In other words, the unity return price at time $t+\theta$, is given by $e^{-\theta R_t(\theta)}$. If we take into account that e^x could be approximated by $1+x$, then we obtain the following expression for our processes:

$$R_t(\theta) = -\frac{1}{\theta}\log(P_t(\theta))$$

where $P_t(\theta)$ denotes the price today (t) of a zero coupon bond, which pays one monetary unity at maturity time $t + \theta$.

Therefore, in order to work with the logarithm of prices, it is only necessary to take the first differences of our time series.

Other theoretical interpretations [but not a financial one] of this first difference procedure can be carried out by considering that the time series are transformed into a differenced ($d = 1$) time series, according to a unit root test. For example: the Advanced Dickey-Fuller Test or the The Phillips-Perron Test, whence it is obtained that the time series was integrated once.

For more details about this procedure see Espinosa(2002) and (1997), Hamilton(1994),Box and Jenkins(1970) and Mills(1999).

Taking the first differences of our time series, we can observe the behavior of each one in Appendix 1.

2.2 Detecting Long memory

We estimate the self similarity parameter and/or the intensity of long-range dependence in the time series presented using the methodology described in Taqqu, Teverovsky and Willinger (1995). Summarizing, this estimators are as follows:

- **The R/S Method:** This method implements the algorithm named Rescaled Range Aanalysis which is dicussed for example in detail in B. Mandelbrot andWallis (1969), Mandelbrot (1983). this method was first proposed by Hurst, taken up again by Mandelbrot, and is nowadays applied to finance by Peters (1991) and (1993). This procedure, as it is pointed in the literature about long-memory, produces important mistakes that is why we will propouse a modification, in terms of data analysis.

- **Aggregated Variance Method:** This method computes the Hurst exponent from the variance of an aggregated Fractional Gaussian Noise time series process. The original time series is divided into blocks of size m. Then the sample variance within each block is computed. The slope from the least square fit of the logarithm of the sample variances versus the logarithm of the block sizes provides an estimate for the Hurst exponent **H**.

- **Differenced Aggregated Variance Method:** To distinguish jumps and slowly decaying trends which are two types of non-stationary, from long-range dependence, this method differences the sample variances of successive blocks. The slope from the least square fit of the logarithm of the differenced sample variances versus the logarithm of the block sizes provides an estimate for the Hurst exponent **H**.

- **The Periodogram Method:** This method estimates the Hurst exponent from the periodogram. In the finite variance case, the periodogram is an estimator of the spectral density of the time series. A series with long range dependence will show a spectral density with a lower law behavior in the frequency. Thus, we expect that a log-log plot of the periodogram versus frequency will display a straight line, and the slopw can be computed as $1 - 2\mathbf{H}$. In practice, one uses only the lowest 10% of the frequencies, since the power law behavior holds only for frequencies close to zero. Varying this cut off may provide additional information. Plotting **H** versus the cut off, one should select that cut off where the curve flattens out to estimate **H**. More details can be found in the work of J. Geweke and S. Porter-Hudak (1983) and in Taqqu (1995).

- **The Whittle Estimator:** This method is based on a periodogram analysis. The algorithm is based on the minimization of a likelihood function defined in the frequency domain. For Fractional Gaussian Noise processes the parameter **H** is the unknown parameter which minimizes the function. This approach also allows to compute confidence intervals. For more details about this method see Beran (1994).

The following table summarizes the results obtained.

	R/S	AVM	DAVM	PM	W
MIBOR	0.5739	0.6102	0.6125	0.6103	0.6099
FF	0.4414	0.6208	0.6211	0.625	0.6187
TIIE	0.6013	0.5631	0.5653	0.5701	0.5603

Is important to note that while others estimators give a similar estimation for **H**, the R/S method is given us a wrong lecture. That brings us to the next point.

2.2.1 The modified R/S Analysis

As pointed out in Peters(1991) and (1994), the estimation of **H** under R/S analysis could be biased due to the influence of linear dependency.

To solve this problem, first of all, we recall the methodology first proposed in Hurst (1951). This procedure has the following steps:

We take the interest rate time series, with M observed values, and convert this time series into a new series of first differences of N values,

$$N_i = M_{i+1} - M_i \quad i = 1, 2, 3, \ldots, (M-1) \tag{11}$$

where $N = M - 1$. Following the Box-Jenkins procedure, we will work with stationary time series.

1. We divide N into A continuous sub-periods of range n, performing $A * n = N$. We call each sub-period I_a, where $a = 1, 2, 3, \ldots, A$. Each element inside I_a is called $N_{k,a}$ where $k = 1, 2, 3, \ldots, n$. For every I_a with range n, we calculate the average using the expression,

$$e_a = \left(\frac{1}{n}\right) * \sum_{k=1}^{n} N_{k,a} \tag{12}$$

where e_a = the average of N_i contained in sub-period I_a of range n.

2. We calculate the series $X_{k,a}$ for each subinterval I_a, which will be the accumulated difference series between the observations and the average,

for each subinterval,

$$X_{k,a} = \sum_{i=1}^{k} (N_{i,a} - e_a) \qquad k = 1, 2, 3, \ldots, n \tag{13}$$

3. We define Range as the difference between the maximal value and the minimal value of $X_{k,a}$, for each sub-period I_a,

$$R_{I_a} = \max (X_{k,a}) - \min (X_{k,a}) \qquad \text{where } 1 \leq k \leq n \tag{14}$$

4. We calculate the standard deviation for each sub-period according to the following expression

$$S_{I_a} = \left(\left(\frac{1}{n} \right) * \sum_{k=1}^{n} (N_{k,a} - e_a)^2 \right)^{\frac{1}{2}} \tag{15}$$

5. We take each range R_{I_a} and normalize it by dividing by S_{I_a}. In this way we obtain a rescaled range for each sub-period I_a, R_{I_a}/S_{I_a}. As A are contiguous sub-periods of range n, in order to obtain the R/S measure for the n division, we now calculate the average of all the R/S of each subinterval,

$$(R/S)_n = \left(\frac{1}{A} \right) * \sum_{a=1}^{A} (R_{I_a}/S_{I_a}) \tag{16}$$

6. Now the value of A is increased to the next value. We will use A values such as $2 \leq A \leq$ integer $\left(\frac{M-1}{10}\right)$; or, in other words, n values such as *integer* $\left(\frac{M-1}{2}\right) \geq n \geq 10$, and the steps 1 to 6 are repeated.

Once all the processes for all n have been done, and if we consider that Hurst found that the rescaled range R/S is described for many observations in time by the following empirical relationship, where τ is the full time range considered,

$$R/S = \left(\frac{\tau}{2}\right)^H$$

then, we can rewrite the expression as

$$(R/S)_n = c \cdot n^H \tag{17}$$

$$\log (R/S)_n = \log (c) + H \log (n) \tag{18}$$

were c is a constant value equal to $\left(\frac{1}{2}\right)^H$ and n is equal to τ. Therefore, one can see that the exponent H can be estimated by obtaining the slope in a OLS regression where $\log(n)$ is the independent variable and $\log(R/S)_n$ is the dependent one. We have noted, before, that using this methodology we does not obtain correct estimations for **H.**

First of all we must tackle the problem of the linear dependency. In order to avoid this fact, we have filtered the data using $ARMA$ filters. In other words, using the Box-Jenkins procedure, we have fitted an $ARMA$ model over the differenced time series and applied the R/S analysis over the residuals of the $ARMA$ models. Thus, when the data is filtered with an $ARMA$ model, the short-term linear dependency is eliminated, as indicated in Beran(1994).

After filtering the data, we apply another modification. Not all the data inside the time series has been considered. Only the first 1400 observations have been taken into account, since 1400 is much more mathematically divisible than, for example, 1490 in the case of the MIBOR time series. This has been carried out according to the classical methodology: the n, the number of observations included in each continuos sub-period, has to be a integer value. In the table included in Appendix 2, we calculate the Hurst exponent for different values of number of observations, T (correspond with the number of observations of the differenced time series), considering that not all the range of N values can be used; only N values that produce an integer number of equal partitions can be taken. Note that the results are different depending on the number of observations used to calculate the Hurst exponent.

For example, in the FF time series, if we take a number of observations equal to 1028, we obtain an **H** equal to 0.479... and, taking a number of observations equal to 1398, **H** takes a value equal to 0.784. The question then follows: which one is better? If the second number of observations is a much more divisible number, does it mean that this is the best?. The answer is to modify the traditional methodology in one step; that is, concretely by changing step 2:

2. We divide N into A continuous sub-periods of range n, performing $A * n =$

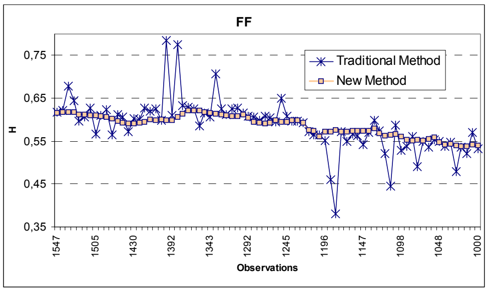
N, where A is an integer value. We begin with $A = \text{integer}\left(\frac{M}{10}\right)$. If n is not an integer value then it has to take the value of the integer of n for all the sub-periods except the last one, which takes the value $n + r$, where $r = N - A * n$. We call each sub-period I_a, where $a = 1, 2, 3, \ldots, A$. Each element inside I_a is called $N_{k,a}$ where $k = 1, 2, 3, \ldots, n$. For every I_a with range n. Then we calculate the average using expression 12. Considering this, we continue with the current methology of the R/S procedure.

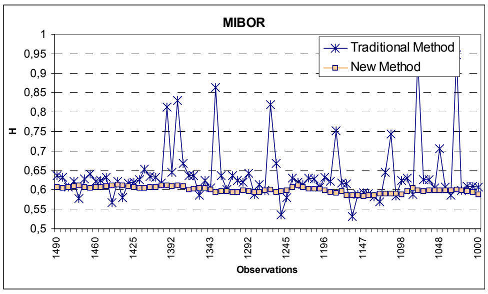
Using all observations of the data in each time series, and the new methodology proposed, we find the following Hurst exponent.

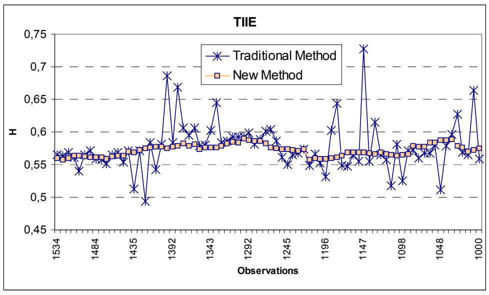
	R/S	R/S modified	AVM	DAVM	PM	W
MIBOR	0.5739	0.6065	0.6102	0.6125	0.6103	0.6099
FF	0.4414	0.6161	0.6208	0.6211	0.625	0.6187
TIIE	0.6013	0.5598	0.5631	0.5653	0.5701	0.5603

Note, that the problem is corrected. Furthermore, taking different values of N, we obtain the table presented in Appendix 3.

As we can see, the Hurst exponent is much more stable than the proposed by Hurst. The following figures show the comparisons of both methodologies.







Finally we can say that, in most cases, **H** is dependent on the number of divisions, applying traditional methodology. We can see that when we work with a different number of divisions, **H** takes different values, and there is no convergence. In the proposed improvement, the value of the parameter **H** is more or less the same independently of the number of divisions taken. Also the results obtained, by the modified methodology, is much more closely to other methods.

3 Conclusions

First of all, concerning the proposed improvements to the traditional methodology, it is worth pointing out that the results obtained are much more reliable than those obtained with the original methodology. However, before this can be unequivocally affirmed further mathematical research is needed.

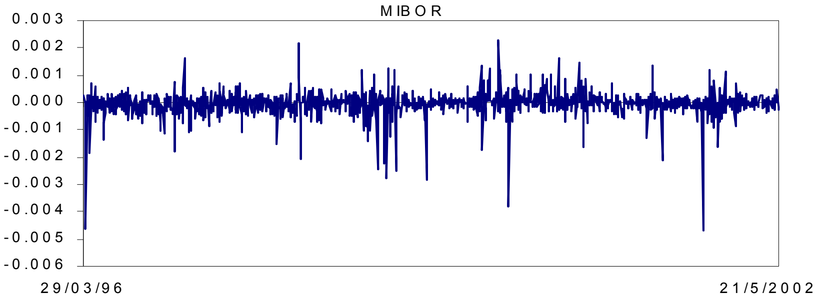
Moreover, The results obtained are not enough far away from 0.5 to justify the existence of a Long-memory behaviour that rules the analised time series.

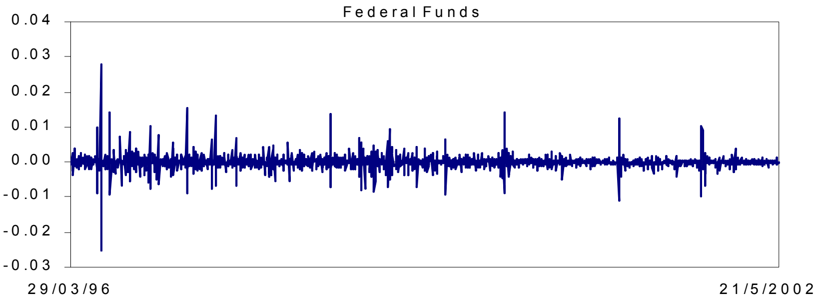
Concretelly, for the MIBOR time series we can say that the **H** will take a value very close to 0.60, for the FF time series 0.61 and for the TIIE time series 0.55. This last case is clarely very close to the Classical Brownian Motion (0.5), but the FF and the MIBOR has a similar behaviour but, also, not so far to conclude the existence of Long-run dependence in the data.

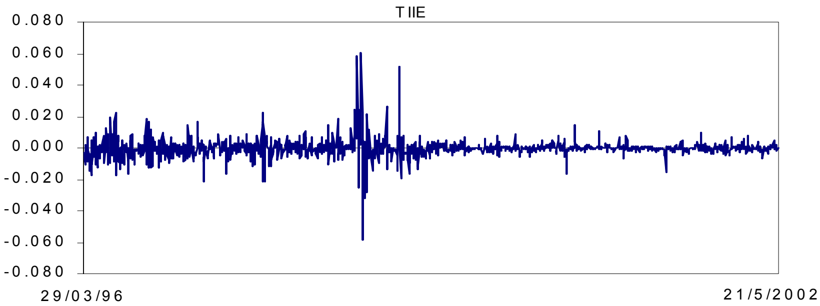
Finally we can state that there is no emprirical evidence of the existence of a long memory process. All the time series analyzed are generated by a random process.

Appendix 1

Differenced time series:







Appendix 2

Time Series: FF			
Observations	H	Observations	H
1547	0,61797951	1266	0,60363478
1545	0,62066567	1258	0,59564059
1544	0,67763902	1251	0,64926782
1534	0,64349166	1245	0,60833399
1525	0,59593992	1236	0,59750012
1521	0,60726487	1230	0,59702135
1510	0,62675184	1224	0,59300897
1505	0,56560191	1216	0,57219874
1500	0,61075932	1210	0,56466364
1490	0,62359758	1200	0,56429343
1475	0,56470446	1196	0,55120216
1460	0,61158008	1189	0,45940154
1450	0,60671136	1182	0,38054485
1445	0,57207934	1175	0,57178613
1430	0,60235351	1168	0,54894181
1420	0,59989371	1161	0,56672712
1414	0,62752747	1152	0,56214295
1410	0,61880858	1147	0,54245739
1406	0,62581012	1140	0,57034069
1400	0,59913418	1133	0,59891775
1398	0,78418299	1125	0,57403267
1392	0,61082656	1118	0,52127772
1384	0,77610711	1112	0,44417394
1378	0,63341259	1105	0,58713889
1370	0,63004878	1098	0,52927131
1364	0,62460011	1090	0,53723669
1357	0,58577421	1083	0,56010687
1350	0,61538274	1076	0,49086504
1343	0,60660723	1070	0,54845832
1336	0,7064432	1062	0,53675164
1328	0,62564779	1056	0,5529559
1320	0,61210655	1048	0,55028807
1314	0,62519072	1040	0,53816839
1308	0,62811875	1035	0,54771091
1300	0,61598569	1028	0,47936282
1292	0,60855953	1020	0,53550195
1287	0,60718231	1014	0,52005957
1280	0,59789697	1007	0,5704099
1273	0,60838425	1000	0,53296544

Time Series: MIBOR			
Observations	H	Observations	H
1490	0,63594356	1266	0,81981859
1488	0,63275042	1258	0,66716992
1485	0,60698768	1251	0,53651102
1480	0,62219858	1245	0,57916027
1475	0,57768027	1236	0,63052875
1470	0,62903169	1230	0,61889491
1464	0,64020376	1224	0,6131119
1460	0,62219419	1216	0,6310198
1455	0,62470841	1210	0,62818859
1450	0,63246308	1200	0,6067645
1445	0,56745703	1196	0,6328322
1440	0,62162093	1189	0,6215299
1435	0,57960074	1182	0,75191829
1430	0,61809937	1175	0,61660347
1425	0,61896546	1168	0,61499917
1420	0,62521989	1161	0,53083093
1414	0,65258335	1152	0,58850098
1410	0,63429254	1147	0,59322559
1404	0,63204011	1140	0,58946859
1400	0,61721749	1133	0,58271404
1398	0,81386719	1125	0,56937789
1392	0,64475077	1118	0,64415576
1384	0,83067704	1112	0,74414794
1378	0,6674031	1105	0,58363692
1370	0,63690197	1098	0,62376364
1364	0,63683607	1090	0,62954153
1357	0,58642598	1083	0,58920016
1350	0,62448536	1076	0,93769911
1343	0,60438809	1070	0,62525243
1336	0,86433902	1062	0,62674466
1328	0,63691778	1056	0,60510956
1320	0,60186384	1048	0,70653681
1314	0,63687569	1040	0,60678017
1308	0,62384175	1035	0,58593135
1300	0,61902963	1028	0,94665338
1292	0,64318108	1020	0,59873653
1287	0,58752032	1014	0,60824696
1280	0,61287665	1007	0,60988009
1273	0,59904919	1000	0,60731756

Time Series: TIIE			
Observations	H	Observations	H
1534	0,56451224	1266	0,60366991
1526	0,56363979	1258	0,58587152
1519	0,56831195	1251	0,56159413
1512	0,56072782	1245	0,55031877
1505	0,53973094	1236	0,56546091
1498	0,56462832	1230	0,56720743
1491	0,57078994	1224	0,57370665
1484	0,55895464	1216	0,54921809
1476	0,55702792	1210	0,56600209
1470	0,55111	1200	0,55191707
1463	0,56480012	1196	0,53172527
1456	0,56819354	1189	0,60255033
1449	0,55357705	1182	0,6434203
1442	0,57087289	1175	0,54872127
1435	0,5129991	1168	0,54706419
1428	0,57062924	1161	0,56561711
1421	0,49364519	1152	0,55511101
1414	0,58416295	1147	0,72793473
1407	0,54285734	1140	0,55557488
1400	0,58144964	1133	0,61534073
1398	0,68622085	1125	0,5648954
1392	0,58370101	1118	0,55800004
1384	0,66837315	1112	0,51803146
1378	0,60623722	1105	0,5817739
1370	0,59471041	1098	0,52503333
1364	0,60563685	1090	0,57154875
1357	0,57920198	1083	0,57200137
1350	0,57960621	1076	0,55961216
1343	0,60202019	1070	0,56696353
1336	0,64525491	1062	0,56812378
1328	0,5842207	1056	0,57970319
1320	0,58617115	1048	0,51187036
1314	0,59210196	1040	0,57930332
1308	0,59263119	1035	0,59673794
1300	0,59244047	1028	0,62784376
1292	0,5987272	1020	0,56902057
1287	0,58170211	1014	0,56524012
1280	0,58875564	1007	0,66313391
1273	0,60071709	1000	0,55827564

Appendix 3

Time Series: FF			
Observations	H	Observations	H
1547	0,61613731	1266	0,59268632
1545	0,6173635	1258	0,59928392
1544	0,61766792	1251	0,59508427
1534	0,6178057	1245	0,59470335
1525	0,61191228	1236	0,59819701
1521	0,61165306	1230	0,59938341
1510	0,61103472	1224	0,59240672
1505	0,60986714	1216	0,57538773
1500	0,60853104	1210	0,57375525
1490	0,60602677	1200	0,56050092
1475	0,6020557	1196	0,57237969
1460	0,59709394	1189	0,57261439
1450	0,59269946	1182	0,57515626
1445	0,5907548	1175	0,57226238
1430	0,59026357	1168	0,57160401
1420	0,59232683	1161	0,57420048
1414	0,59522891	1152	0,57441151
1410	0,59984701	1147	0,57320677
1406	0,59847836	1140	0,57441113
1400	0,59969888	1133	0,57990947
1398	0,59886746	1125	0,56820196
1392	0,59906574	1118	0,56360684
1384	0,60606182	1112	0,56530663
1378	0,61457294	1105	0,56682762
1370	0,62163212	1098	0,56135549
1364	0,62140233	1090	0,55344816
1357	0,62182618	1083	0,55124698
1350	0,61768734	1076	0,55344486
1343	0,61651695	1070	0,55076927
1336	0,61386739	1062	0,55491602
1328	0,6118383	1056	0,55823487
1320	0,6102271	1048	0,54765537
1314	0,60772697	1040	0,54254052
1308	0,60770066	1035	0,54351791
1300	0,61173308	1028	0,54062679
1292	0,60356893	1020	0,537966
1287	0,59531955	1014	0,53834742
1280	0,59392954	1007	0,54100989
1273	0,59151814	1000	0,53913917

Time Series: MIBOR			
Observations	H	Observations	H
1490	0,60658747	1266	0,6005389
1488	0,60605315	1258	0,59503652
1485	0,60753749	1251	0,59604422
1480	0,61020888	1245	0,59854685
1475	0,61111699	1236	0,60752746
1470	0,60801355	1230	0,61061482
1464	0,60599433	1224	0,60636613
1460	0,60805346	1216	0,60373132
1455	0,60766788	1210	0,60274237
1450	0,60892086	1200	0,60366119
1445	0,6111907	1196	0,59971547
1440	0,61312898	1189	0,5945583
1435	0,61030069	1182	0,59327942
1430	0,61017034	1175	0,59706114
1425	0,60758888	1168	0,58632248
1420	0,60569157	1161	0,58518454
1414	0,60528723	1152	0,58643606
1410	0,60621805	1147	0,58334866
1404	0,60695862	1140	0,58511227
1400	0,61167157	1133	0,58660709
1398	0,61115087	1125	0,59134128
1392	0,60994884	1118	0,59097809
1384	0,61162538	1112	0,58948
1378	0,60901086	1105	0,59104934
1370	0,60123992	1098	0,58927041
1364	0,60244222	1090	0,5976797
1357	0,60534376	1083	0,60419733
1350	0,60445299	1076	0,59920093
1343	0,60076043	1070	0,59760905
1336	0,59498686	1062	0,5977493
1328	0,59622082	1056	0,59941249
1320	0,59700782	1048	0,59803382
1314	0,59549911	1040	0,59823292
1308	0,59501815	1035	0,5991202
1300	0,59774858	1028	0,60181222
1292	0,596056	1020	0,59771347
1287	0,59393656	1014	0,59686506
1280	0,59427843	1007	0,59431944
1273	0,59877421	1000	0,58799192

Time Series: TIIE			
Observations	H	Observations	H
1534	0,55981936	1266	0,57663289
1526	0,55733488	1258	0,5749723
1519	0,56008675	1251	0,57416293
1512	0,56434589	1245	0,57375482
1505	0,56327287	1236	0,57289067
1498	0,56286156	1230	0,57077281
1491	0,56109517	1224	0,57409543
1484	0,56152935	1216	0,55703796
1476	0,56076984	1210	0,55939226
1470	0,55830302	1200	0,55838478
1463	0,5623368	1196	0,55846069
1456	0,56428801	1189	0,5600539
1449	0,5640428	1182	0,56093654
1442	0,57014594	1175	0,5643265
1435	0,56838441	1168	0,56933453
1428	0,57247897	1161	0,5690636
1421	0,57446727	1152	0,56917022
1414	0,57571117	1147	0,56925838
1407	0,57754691	1140	0,56692736
1400	0,57711318	1133	0,56623232
1398	0,57506025	1125	0,5689033
1392	0,5770715	1118	0,5665594
1384	0,57915841	1112	0,56444256
1378	0,5821855	1105	0,56313049
1370	0,57817092	1098	0,56443972
1364	0,58068942	1090	0,56660853
1357	0,57370381	1083	0,57819052
1350	0,57594561	1076	0,57693985
1343	0,5764385	1070	0,57801427
1336	0,57599344	1062	0,58430896
1328	0,57875853	1056	0,58352857
1320	0,58244324	1048	0,58764429
1314	0,584845	1040	0,58808264
1308	0,58373357	1035	0,58817336
1300	0,58961646	1028	0,57902348
1292	0,58782031	1020	0,57647005
1287	0,58647116	1014	0,5697108
1280	0,5864931	1007	0,57283518
1273	0,58309105	1000	0,5747546

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