Bjorken-like model for non-central relativistic heavy ion collisions

Author: Joel Gordillo Solana
*Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.**

Advisor: Volodymyr Magas

Abstract: This paper discuses the implementation of a new model to calculate the initial state of quark gluon plasma and the relevance of the vorticity generated on it. Some results based on simulations of $Au + Au$ reactions are also presented.

I. INTRODUCTION

One of the most interesting fields of study is the internal structure of the nuclei, formed by quarks and gluons, the so called partons. From the quantum chromo dynamics (QCD) some features of the behavior of partons are known. On its unperturbed state, the partons are confined forming objects neutral in color so, these partons can't be observed freely on the laboratory. On the other hand, it is expected that in high energetic collisions, were so many new objects are produced, they act like a non-interacting system due to the features of this state, the asymptotic freedom regime. This behavior is known as the quark gluon plasma. Then the partons are observed in this very dense, energetic and hot state.

In the accelerators, nucleus are accelerated by magnetic fields and then collide between each other. After the collision has been produced, the nucleons inside the nuclei undergo a phase transition from hadronic state to quark gluon plasma state[5], and after some time τ_{th} the system thermalizes, which means that is in equilibrium. The evolution of the pre-equilibrium stage of the collision as well as the phase transition are out of the scope of this paper as the main theory used on this work is based on thermodynamics and hydrodynamics that require local equilibrium.

The work will be focused on describing the initial state of quark gluon plasma, when equilibrium is reached, introducing a new model to describe the thermalized intial state, based on the combination of the Bjorken model and some features of the firestreak model. Until now, most of the models disregarded the conservation of the angular momentum of the QGP, but nowadays, hadronic physics have reached a point of high precision experiments that allows to measure observables related to the angular momentum. Recent experiments detected the Lambda particles polarizations and the results indicate that shear and vorticity have to be taken into account to explain the polarizations[2]. This work enhances the study of the vorticity and shear of the initial state, as it has been our main contribution to the improvement done to the model.

Because of the limited space, some calculations will be shortened or omitted, it is assumed that the reader has notions on special relativity as well as on hydrodynamics.

II. QUARK GLUON PLASMA

This new state of matter was discovered at CERN lab in February 2000 in $Pb + Pb$ collisions [9]. This state is formed when two heavy nuclei collide with energies equal or above $10A \cdot GeV$ which are inside the ultra-relativistic heavy ion collision domain. The energies of these kind of process are large enough to consider the heavy ions as classical particles, meaning that classical dynamics can be applied, even the nucleons can be considered as classical for a large extent. Therefore, the hot and dense initial state can be considered as a continuum since the quantum particle effects should be small, thus imply that the QGP can be described using relativistic hydrodynamics. Nowadays, practically all the numerical simulations involves the implementation of relativistic hydrodynamics for intermediate stages of the reaction.

A. Equation of State

In general, finding the equation of state (EOS) is not trivial and it won't be derived in this section. However, on the limit of hight temperatures and densities, the coupling of between quarks and gluons tends to zero, in the asymptotic freedom regime, so the the equation of state can be modelized as a non-interacting gas of N_f (up, down, strange and sometimes charm) quarks that have N_c colors (red, green and blue) and $N_c^2 - 1$ gluons.

$$e(T,\mu) = \frac{\pi^2}{15}(N_c^2 - 1 + \frac{7N_cN_f}{4})T^4 + \frac{N_cN_f}{2}(T^2\mu^2 + \frac{\mu^4}{2\pi^2}) \quad (1)$$

Then, as is considered as an ideal gas, the pressure can be expressed in terms of the energy density with the following expression:

$$P(T,\mu) = \frac{1}{3}e(T,\mu) \quad (2)$$

To take into account the effect of the confinement, which causes the QGP vacuum to be different from our

*Electronic address: jgordiso7@alumnes.ub.edu

normal vacuum, a constant B that bounds the quarks is added to the equations, this is the so called bag model.

$$e_q(T,\mu) = e(T,\mu) + B, \quad P_q(T,\mu) = P(T,\mu) - B \quad (3)$$

B. Simple models

There are two basic and strongly idealized models that try to explain the evolution of the Quark Gluon Plasma. These models has extremely opposite assumptions on the interaction between the partons[10].

Landau Model. This model was thought for lower energy regions (up to a few $A \cdot GeV$) but the relativistic effects on the shape of the nuclei, the Lorentz contraction can't be disregarded, the elongation can be computed as $2R/\gamma$. This model assumes a situation of total stopping of the both nuclei when they collide. Then, the final volume has the shape of a disk, which has the volume of the one of the initial disks and the barionic charge of both of them inside this volume, this implies that the final disk has double baryon density than the previous ones. This enlarged energy density implies more pressure, as the disk is Lorentz contracted, the pressure gradient is greater on the beam axis, so the expansion of the system occurs first in this direction, then, when the system has approximately spherical shape, a expansion on the radial direction occur until the energy density is low enough to allow the freeze-out, the process that allow the particles involved on the QGP to be considered as independent.

Bjorken Model. The Bjorken model was thought for high energy regions, above $10A \cdot GeV$. At these energies, the nuclei become transparent to each other, and the systems pass through each other with its valence quarks almost maintaining their original rapidities.

III. CLASSICAL BJORKEN MODEL

Although the nuclei became transparent to each other, they interact via the interchange of color charge which yields the creation of a chromo-electric field which contains a huge part of the energy of the system.

For e^+e^- collisions it has been observed that the distribution on the rapidities is symmetric on all the interaction zone, that means that the physical magnitudes such as energy, baryon density, or temperature won't depend on rapidities, therefore they only have dependences on the proper time τ of the system defined as:

$$\tau = \sqrt{t^2 - z^2} = t/\gamma \quad (4)$$

because is invariant under Lorentz transformations

1. Basic equations

The Bjorken model respects the conservation laws, which can be written as:

$$T^{\mu\nu}, \nu = 0 \quad (5)$$

Where the energy momentum tensor is $T^{\mu\nu} = (e + P)u^\mu u^\nu - Pg^{\mu\nu}$. Knowing the relations $\frac{\partial u^\mu}{\mu} = \frac{1}{\tau}$ and $u^\mu u_\mu = 1$ and the definition of τ. The above equation leads to:

$$\partial e/\partial \tau = -(e+P)/\tau \quad (6)$$

Assuming the ideal ultra-relativistic EOS 2 and giving $e(\tau_0) = e_0$ as initial condition. The equation is reduced to one that has the analytical solution:

$$e(\tau) = e(\tau_0)\left(\frac{\tau}{\tau_0}\right)^{-4/3} \quad (7)$$

Similar solutions can be derived for other quantities such as baryon density.

The original version of the Bjorken model, can be used for a central symmetric collisions. In addition, this model is assumed to be applied to an infinite system, but in order to evaluate the conservation laws, limits on η must be defined $[-\eta_R < \eta < \eta_R]$.

Finally, nature shows that any of the model assumptions, Landau's or Bjorken's are fully fulfilled. There isn't full transparency or stopping at the energies studied. Transparency appears but the partons are partially stopped by the string tension of the chromoelectric field.

A. Modified Bjorken Model

Most of the collisions produced on the accelerators aren't central, which means that the nuclei doesn't collide by their centers of mass, these feature produces an asymmetry that must be taken into account. In this situation not all the valence quarks of the target and the projectile will interact as they don't cross each other, these are the so called spectators, the ones who interact are known as the participants. Actually, in such asymmetric situation, the resulting system may have a large angular momentum which should be considered. The study of the spectators are left for the moment to future studies.

The angular momentum conservation can be treated as firestreak-like model [3] [7]. This model was thought to be applied to the final state, but it can be used also to describe the initial state [6][4]. The procedure is the following,the transverse plane is divided up into cells of about $1fm^2$ transverse size. The streaks of both, target and projectile corresponding to given transverse coordinates (x_i, y_i) collide and inter-penetrate each other. After a characteristic time, about 1-2 fm/c partons of both, target and projectile form a streak, these streaks will have a finite longitude due to the momenta conservation, the composition of the streaks is hardly asymmetric as some streaks are formed with bore projectile baryon density than others, and vice versa, the only symmetric collision is the one colliding by the center of masses of the system. This asymmetry causes that some of the streaks have forward momentum while other have backwards, at this time, the system presents already angular momentum. The chromo magnetic field generated in the inter-penetration will held the quarks together preventing the system to expand.

Now the Bjorken model can be applied to every one of the streaks, making them to have its own reference system because the model starts to apply at time τ_0 after the initial time t_0 has happened at the touching point in that time z_0. The transformation between t, z and τ, η [8] are:

$$\begin{aligned} t - t_0 &= \tau \cosh \eta, \\ z - z_0 &= \tau \sinh \eta, \\ \tau &= \sqrt{(t-t_0)^2 - (z-z_0)^2}, \\ \eta &= \frac{1}{2} ln \left(\frac{t - t_0 + z - z_0}{t - t_0 + z - z_0} \right) \\ &= Artanh \frac{z - z_0}{t - t_0} \end{aligned} \tag{8}$$

Furthermore:

$$\begin{aligned} dz &= sinh\eta d\tau + \tau cosh\eta d\eta, \\ dt &= cosh\eta d\tau + \tau sinh\eta d\eta, \\ dzdt &= \tau d\eta d\tau \end{aligned} \tag{9}$$

For the Bjorken flow, u^ν is the flow velocity. In Cartesian coordinates, the local flow is ${u_i}^\mu = (cosh\eta, sinh\eta)$. Then the velocity in a point (t,z) is

$$v_z = \frac{z - z_0}{t - t_0} \tag{10}$$

The length of each streak is related to the limits on η. A convenient parametrization is

$$\Delta\eta_i = \eta_{max,i} - \eta_{min,i} \tag{11}$$

Focusing on the central streak is easy to see that $\Delta\eta_c = \eta_{c-max} - \eta_{c-min} = 2\eta_{c-max}$. Knowing this, the initial limits of the central streak can be calculated using (8):

$$z_{c-max} = \tau_0 \sinh \Delta\eta_c, \quad t_{c-max} = \tau_0 \cosh \Delta\eta_c \tag{12}$$

The other side limits are just $z_{c-min} = -z_{c-max}, t_{c-min} = -t_{c-max}$ The peripheral streaks have a distinction between them, the projectile (P) and target(T) streaks are the ones that have forward and backward momentum respectively. In the collider c.m frame the edges will be aligned uniformly that means:

$$z_{P-i-max} = z_{c-max}, \quad z_{T-i-min} = z_{c-min} \tag{13}$$

Due to the boost invariance of the flow, the difference on the rapidities of each peripheral streak is described on the same way as the central streak.

Now, conservations laws of baryon density, energy and moment for each streak are obtained using 8:

$$\begin{aligned} N_i &= N_{Ti} + N_{Pi} = \tau_0 n_i(\tau_0) A \Delta\eta_i \\ E_i &= E_{Ti} + E_{Pi} = 2\tau e_i(\tau_0) Asinh(\frac{\Delta\eta_i}{2}) \cosh(\eta_i) \\ P_i &= P_{Ti} - P_{Pi} = 2\tau e_i(\tau_0) Asinh(\frac{\Delta\eta_i}{2}) \sinh(\eta_i) \end{aligned} \tag{14}$$

Then, the energy density on τ_0 can be calculated,it is assumed that it is the same for both, central and peripheral streaks.

$$e(\tau_0) = e_c(\tau_0) = \frac{E_c}{\tau_0 A 2 \sinh\left(\frac{1}{2}\Delta\eta_c\right)} \tag{15}$$

Where the energy of the streaks,as well as the momentum and the baryon charge, can be inferred from the experimental setup, i.e the beam energy and the impact parameter.

All the calculations were done at the reaction plane, at $y = 0$, now to calculate the length of the streaks at different y, it is again assumed that the energy density at the proper time τ_0 is the same for the peripheral streaks and for the central one

$$e_i(\tau_0) = e_c(\tau_0) = constant \tag{16}$$

As originally $e(\tau_0)$ and $\Delta\eta_c$ are unknown, the difference between the rapidities $\Delta\eta_c$ must be chosen from a dynamical model for the streak expansion, in this work it is a model parameter. In the center of mass frame, the momenta of a given streak and its energy are know from the pre-collision parameters, then, the rapidity associated to it is:

$$\eta_i = Artanh \frac{P_i, z}{E_i} \tag{17}$$

Once η_i is known, $\Delta\eta_i$ can be computed using (15) and taking into account the assumption made before.

$$\frac{1}{2}\Delta\eta_i = Arsinh\left(\frac{E_i}{2\tau_0 e_c(\tau_0) cosh(\eta_i)}\right) \tag{18}$$

Now we can proceed to the initial parameters (x_{i0}, z_{i_0}) of each streak using inverse relations on 8. See [8] for more details.

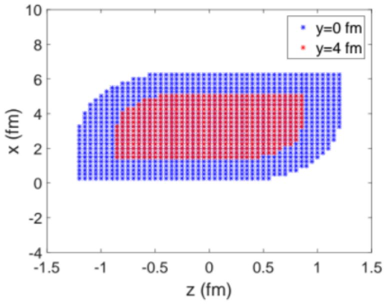

FIG. 1: The initial configuration of the streaks in the reaction plane,on the $[x, \eta, \tau_0 = 1\text{fm/c}]$-hypersurface for $y = 0$ (blue streaks) and for $y = 4\text{fm}$ (red streaks) overlayed. The streak energy density is uniform and it is the same for all streaks. This example is calculated for a Au+Au reaction at 100+100 GeV/nucl. The central streak lenght is 2.36 fm with a uniform energy density of $e_c = 156.31\text{GeV/fm}^3$, and $\tau_0 = 1.0\text{fm}$. The $y = 0$ plane crosses the x-axis at $x = 3.25\text{fm}$. [8]

IV. RESULTS

Some simulations to calculate a possible initial state implementing the model have been done using Au nuclei for both, target and projectile. Gold has an atomic number of 79, and an approximate mass of 197u which is equivalent to 0.938 GeV when it is accelerated at 100 GeV, it has spherical shape with a radius of 6.5 fm approximately. The simulations have been done selecting an impact parameter of the 50% of the sum of the radius of the target and projectile. This energy and projectile corresponds to RHIC(Relativistic Heavy Ion Collider) at BNL(Brookhaven National Laboratory).
To do the calculation, the volume of the nuclei are discretized on a grid. The calculations will be done for each plane zx. Despite the last part of the model have been presented for a fixed τ i.e all the streaks have the same proper time, the calculations are done at a fixed t, so each point has his own τ from (4). Using the pre-collision parameters, $\Delta\eta$ is calculated and then from (8) all the origin points t_0, z_0 are also computed.

Energy density. Once the central energy density is calculated, according to the equation (7) the energy surface can be calculated.
As more peripheral is the streak, smaller is its τ, because of it the edge points have a greater values of energy density.

Velocity. In the given coordinates is easy to compute η from (8). Once these is know is easy to see that the relation between η and v is

$$v = tanh(\eta) \tag{19}$$

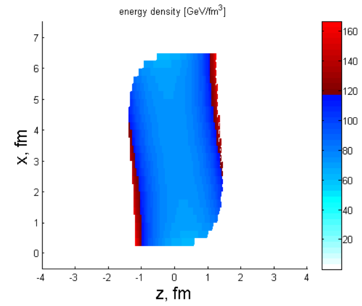

FIG. 2: Representation of the energy profile in the reaction plane at y = 0 propagated to the constant time $t_{is} = 1.78$ fm/c hypersurface.

As expected, the differences on the baryon density leads

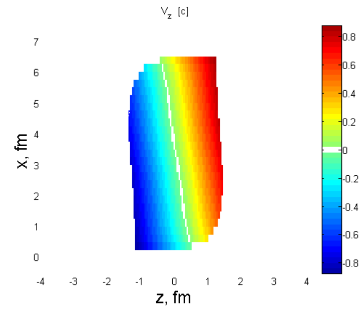

FIG. 3: Representation of the velocity profile in the reaction plane at y = 0 propagated to the constant time $t_{is} = 1.78$ fm/c hypersurface.

to a situation where the projectile side has forward momentum and the target has backwards

Vorticity. Vorticity is a physical magnitude to quantify the rotation of a fluid, which is inferred in each point due to the differential velocity of movement of the surrounding matter of the fluid. Mathematically, the vorticity, is defined as the rotor of the velocity field. In the studied system, only exists y component of the vorticity, as it is confined on the reaction plane [11]

$$\omega_y = -\omega_{zx} = -\frac{1}{2}\nabla \times \vec{v} = -\frac{1}{2}(\partial_x v_z - \partial_z v_x), \tag{20}$$

Once the velocity field is calculated on the points of the grid it is easy to compute numerically the classic vorticity field. The partials derivatives take into account the nearest neighbors of the grid point, so the vorticity of a given plane zx follows the next expression:

$$\omega_y = -\omega_{zx} = \frac{-1}{2}\Big(\frac{v_z^{+0} - v_z^{-0}}{2\Delta x} - \frac{v_x^{+0} - v_x^{-0}}{2\Delta z} + \frac{v_z^{++} - v_z^{--} + v_z^{+-} - v_z^{-+}}{4\Delta x} - \frac{v_x^{++} - v_x^{--} + v_x^{+-} - v_x^{-+}}{4\Delta z}\Big) \tag{21}$$

The system studied has its main movement in the beam direction, which is component z, for this reason the component x of the velocity is negligible, then the expression

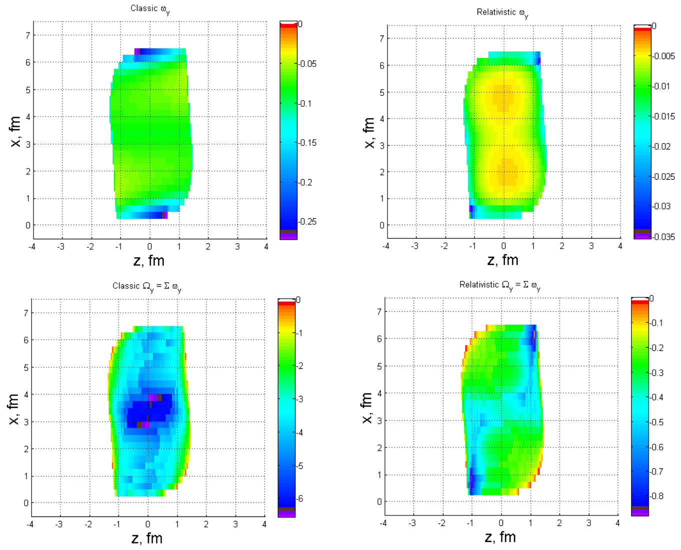

FIG. 4: The reaction plane, y = 0, [x, z] contour plot of the classical vorticity (left upper panel), relativistic vorticity (right upper panel). And figures of the reaction plane, sums of all y planes of classical vorticity (left lower panel) and relativistic vorticity (right lower panel). All of them propagated to the constant time t_{is} = 1.78 fm/c hypersurface.

can be rewritten as:

$$\omega_y = -\omega_{zx} = \frac{-1}{2}\Big(\frac{v_z^{+0} - v_z^{-0}}{2\Delta x} + \frac{v_z^{++} - v_z^{--} + v_z^{+-} - v_z^{-+}}{4\Delta x}\Big) \tag{22}$$

Where all the v_x dependences have been removed. The above expression can be used to calculate all of the inner points of the grid, as all of its neighbor cells are filled with matter, the edges must be treated carefully, imposing the condition that if a cell of the grid isn't filled with matter it hasn't have to be taken into account.

Since the collisions are energetic enough to make the beams be relativistic, the corresponding correction to the velocity has to be applied, having consequences on the vorticity as well.

$$\begin{aligned}\omega_y &= -\omega_{zx} = -\frac{1}{2}\nabla \times \gamma\vec{v} = -\frac{1}{2}(\partial_x \gamma v_z - \partial_z \gamma v_x) \\ &= -\frac{1}{2}\gamma(\partial_x v_z - \partial_z v_x) - \frac{1}{2}(v_z \partial_x \gamma - v_x \partial_z \gamma) \\ &= -\frac{\gamma^{00}}{2}(v_z^{+0} - v_z^{-0})/2\Delta x + \frac{1}{8}[\gamma^{00}(2(v_z^{++} - v_z^{--} + v_z^{+-} \\ &- v_z^{-+} + v_z^{+0} - v_z^{-0})/\Delta x) - {v_z}^{00}(2(\gamma^{++} - \gamma^{--} + \gamma^{+-} \\ &- \gamma^{-+} + \gamma^{+0} - \gamma^{-0})/\Delta x)]\end{aligned} \tag{23}$$

V. SUMMARY. CONCLUSIONS.

As it have been shown, the collision between two nuclei is hardly symmetric, as most of the collision produced are non central the distribution of mass and momentum inside the target and nuclei are different, even if two identical nuclei are considered. The considered collisions are energetic enough to form quark gluon plasma[5], which can be treated as a fluid. Due to the initial asymmetry on the momentum distribution, shear appears between different layers of the system, inducing angular momentum. This angular momentum results on the apparition of vorticity on this fluid. The effect of vorticity has been studied before and neglected.

In this work, the model developed to describe the initial state of the quark gluon plasma shows a state with non-zero vorticity that should be taken into account for the theoretical predictions, as some recent experiment of polarization of lambda particles[2] shows the relevance of the vorticity on the field.

[1] V. K. Magas, L. P. Csernai, and D. D. Strottman, Phys. Rev. C **64**, 014901 (2001); and V. K. Magas, L. P. Csernai, and D. D. Strottman, Nucl. Phys. A **712**, 167 (2002).
[2] M. A. Lisa *et al.* (STAR Collaboration), Invited talk presented at the QCD Chirality Workshop 2016, Feb. 23-26, 2016, Los Angeles, USA.
[3] L. P. Csernai, V. K. Magas, and D. J. Wang, Phys. Rev. C **87**, 034906 (2013).
[4] Long-Gang Pang talk at the XXV International Conference on Ultrarelativistic Nucleus-Nucleus Collisions (Quark Matter 2015), 27 September - 3 October 2015, Kobe, Japan; L. G. Pang, H. Petersen, G. Y. Qin, V. Roy and X. N. Wang, Eur. Phys. J. A **52**, no. 4, 97 (2016).
[5] I. N. Mishustin and J. I. Kapusta, Phys. Rev. Lett. **88**, 112501 (2002); I.N. Mishustin, and K.A. Lyakhov, Phys. Rev. C **76**, 011603 (2007).
[6] I.N. Mishustin and K.A. Lyakhov, Physics of Atomic Nuclei **75**, 371 (2011).
[7] W. T. Deng and X. G. Huang, J. Phys. Conf. Ser. **779**, no. 1, 012070 (2017).
[8] Initial State with Shear in Peripheral Collisions. (2017) V.K. Magas, J. Gordillo, D.D. Strottman, Y.L. Xie and L.P. Csernai
[9] U. Heinz M.Jacob, nucl-th/002042; CERN Press Release of Frebruary 10,2000; http: cern.web.cern.ch CERN/annuncements/2000/NewStateMatter
[10] Introduction to Relativistic Heavy Ion Collisions, Lzlo P. Csernai, 2008
[11] Rotation and Turbulence in Peripheral Heavy Ion Collisions, Djuan Wang, 2014