Automation of the germanium conductivity measurement

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Abstract: The present work, carried out as *Treball Fi de Grau* of the *Degree of Physics* at the *Universitat de Barcelona*, presents the implementation process of improvements to automate the data capture in the experimental realization of the *Laboratory of Modern Physics P13a* practice, that aims to determine the *Germanium Conductivity* (σ), and its inherent forbidden semiconductor *Energy Band Gap* (E_g). By introducing a properly programmed *Arduino M∅ micro-controller* and an ordinary PC (with the appropriate *Python* language program and environment) in the experimental setup, it has been possible to improve the quantity and quality of collected data and also affect the final result, making it more stable and precise. The number of measures taken for the same capture time (12 min approximately) has been increased by a factor of 10, from about 100 to more than 1000, and their precision has also been improved. This has resulted in obtaining a value for $E_g = 0,6615 \pm 0,0002$ (eV), which matches very well the value of the literature $E_g = 0,66$ (eV) considered. The rapid data acquisition and the good results obtained due to its high precision, allow to think that this implementation can be definitive if some improvements are made. In addition, future implementations in other experimental configurations of similar characteristics are possible.

I. Introduction

Semiconductors are well known nowadays due to the large number of studies in the last half of the 20th century, that have lead to the widespread use of this kind of materials in transistors, switches, diodes, photo-voltaic cells, detectors, thermistors and others, massively used as components of integrated circuits in all technology used today.

Studied in depth for the first time by M. Faraday in the early 19th (1839) [1], unlike metals those are mainly characterized by the fact of presenting a negative value for the temperature coefficient of resistance (α). This peculiarity results in a decreasing *Electrical Resistivity* (ρ) and therefore an increase of the *Conductivity* (σ), due to their reciprocal relation, for a rising temperature. At room temperatures they present a wide range of ρ from 10^{-2} to near 10^{14} Ωcm. [2][3]

One of the most studied is *Germanium*, whose chemical symbol is $_{72,630(8)}^{32}Ge$ [4], usually called a diamond-type material due its crystal structure, and categorized as an intrinsic or undoped semiconductor. This is because it is distinguished by not changing its electrical properties due to the possible impurities of its crystalline structure which involves the same concentration of holes ($p = p_i(T)$) than of excited electrons ($n = n_i(T)$) in their respective bands.[2][3]

When heating this kind of materials, the number of carriers in the conduction band increases, not only by the thermally excited electrons in the conduction band but of the holes that they leave behind on the valence band, which also help to increases its electrical conductivity. This increase will strongly depend on the width of the *Energy Band Gap* (E_g) placed between the lowest point E_c and the highest point E_v of the conduction and valence band respectively. The key factor that controls this process is known as *Ratio of the band gap to the temperature*,

$$\frac{E_g}{\kappa_B T} \tag{1}$$

where κ_B is the *Boltzmann constant* [5], so that the higher the temperature the lower this ratio is, which leads to large concentrations of intrinsic carriers and consequently to a greater conductivity.

Starting from the *Fermi-Dirac distribution function* and for temperatures in the *Intrinsic Region* one can approach the energy in each band as it follows (where μ is the *Chemical Potential* or *Fermi level*):

$$E_c - \mu >> \kappa_B T \tag{2}$$
$$\mu - E_v >> \kappa_B T \tag{3}$$

Assuming appropriate values of μ then the *Energy Band Gap*, defined as $E_g = E_c - E_v$, becomes small enough at room temperatures. Knowing the following relations for the intrinsic concentrations of carriers in the conduction band (electrons, e),

$$n_c(T) = N_c(T)e^{-\frac{(E_c-\mu)}{\kappa_B T}} \text{ , where } {}_{N_c(T) = \frac{1}{4}(\frac{2m_c\kappa_B T}{\pi\hbar^2})^{\frac{3}{2}}} \tag{4}$$

and the respective for the valence band (holes, h),

$$p_v(T) = P_v(T)e^{-\frac{(\mu-E_v)}{\kappa_B T}} \text{ , where } {}_{P_v(T) = \frac{1}{4}(\frac{2m_v\kappa_B T}{\pi\hbar^2})^{\frac{3}{2}}} \tag{5}$$

where $\hbar$ is the *reduced Planck constant* [6], if the importance of the impurities as a source of carriers is negligible in the intrinsic case, one can assume that $n_c(T) =$

Crystal	Gap	E_g (eV)		np (cm^{-6})
		0 K	300 K	300 K
Ge	indirect	0,744	0,66	$2,89 \times 10^{26}$

Table I: *Germanium*: type and values for the *Energy Band Gap* when $T \to 0$ and at room temperature (300 K)[2, p. 190]. The concentration of carriers (np) is also shown.[2, p. 206]

$p_v(T) \equiv n_i(T)$ which means:

$$n_i(T) = p_i(T) = 2(\frac{\kappa_B T}{2\pi\hbar^2})^{\frac{3}{2}}(m_e m_h)^{\frac{3}{4}} e^{-\frac{E_g}{2\kappa_B T}} \qquad (6)$$

Since the electrical conductivity is the sum of the electrons and holes contributions:

$$\sigma = ne\mu_e + pe\mu_h \qquad (7)$$

where μ_e and μ_h are their *intrinsic mobilities* ($\mu_i = \frac{e\tau_i}{m_i}$), then

$$\sigma = ne\frac{e\tau_e}{m_e} + pe\frac{e\tau_h}{m_h} \qquad (8)$$

where e is the *elementary charge* [7], ($m_e = m_h$) the mass for each type of carriers [8] and (τ_e, τ_h) their respective *collision time*.[2][3]

In the intrinsic region around room temperature, while the mobilities depends lightly on temperature (following a power law), conductivity is dominated by the decreasing exponential dependency of the carriers concentration:

$$\sigma_i(T) = 2e^2(\frac{\kappa_B T}{2\pi\hbar^2})^{\frac{3}{2}}(m_e m_h)^{\frac{3}{4}}(\frac{\tau_e}{m_e} + \frac{\tau_h}{m_h})e^{-\frac{E_g}{2\kappa_B T}} \qquad (9)$$

for a wide range of temperatures in this region, it can be considered that the multiplicative factors of the exponential are constant (σ_0). Allowing to approximate the conductivity as:

$$\sigma_i(T) = \sigma_0 e^{-\frac{E_g}{2\kappa_B T}} \qquad (10)$$

A usual way to obtain reliable values of the energy band gap E_g comes from the previous theoretical argument resulting in the expression of Eq. 10. Taking the logarithm of both sides, one can reduce it to a linear relation between $\ln\sigma_i(T)$ and $\frac{1}{T}$ as it follows:

$$\ln\sigma_i(T) = \ln\sigma_0 - \frac{E_g}{2\kappa_B T} \qquad (11)$$

Between 0 and 300 K E_g can vary by about 10% , being the dependence linear at room temperature and quadratic at very low temperature (accepted values for *Germanium* are shown in Table I).

This is the starting point for any experiment that aims at determining the temperature dependent conductivity of an intrinsic semiconductor, in order to deduce the value of its corresponding E_g. So that near room temperature, by simply capturing conductivity and temperature data within the intrinsic range, and performing a linear fit by the appropriate numerical method, the value of the forbidden energy band for the semiconductor can be obtained directly from the slope.[2] [3] [9]

One of these experiments is carried out as a fundamental part of the subject *Laboratory of Modern Physics* of the *Degree of Physics* at the *Universitat De Barcelona*: "*Practice 13a - Germanium forbidden band* "[9]. Precisely, the purpose of the present work is the improvement of the data acquisition of this practice, whose characteristics will be presented later. It should be pointed out that, comparing with how this process has been carried out to date, it has been possible to improve the speed, quantity and quality of the collected data and act also on the final result, making it more stable and precise.

This has been achieved by implementing an *Arduino M∅ board* [10] (a powerful, cheap and free-code microcontroller) within the design of the experiment, accordingly programmed to communicate directly with the *Phywe unit* that incorporates the sample of semiconductor to be studied [11]. This new set up allows to read the necessary three internal analogical signals of the module, corresponding to the measures that finally determine the resulting E_g (maximum V_+ and minimum V_- voltage in the ends of the semiconductor board and its temperature T), at every moment, directly from the experimental unit through a built-in port and thanks to a 9-pin *DB9-type cable* configured for this purpose. (Fig. 1)

In addition, a script has been programmed in *Python* language allowing, with the appropriate *Integrated Development Environment (IDE)* and packages, not only to communicate any *PC* with the *Arduino M∅* through its *USB serial* port and to capture the information coming from the board, but also to return a *Comma-Separated Values (CSV)* file that stores all the data, in the desired temperature range, for further analysis by the students.

II. Material and methods

A. Setup and equipment

The experimental setup in use today consists of the following material:

1. A *Germanium* sample ($20 \times 10 \times 1$ mm^3) inserted between the ends of a printed circuit.
2. An inverted U-shaped module with the circuit.
3. Phywe Power supply ($0 - 12$ V DC / 6 and 12 V AC).
4. Tripod base and support rod for sustaining the module where the sample is set up.
5. A pair of red/blue connecting cords that connect both ends of the sample with the digital multimeter to measure the current and voltage across the circuit.

The mentioned module includes a digital display where both the intensity of the current, flowing through the circuit, and the temperature of the sample can be controlled. Also, this setup is provided with a safety system that prevents the sample from reaching temperatures beyond $170 - 175°C$, which could melt circuit contacts.[11]
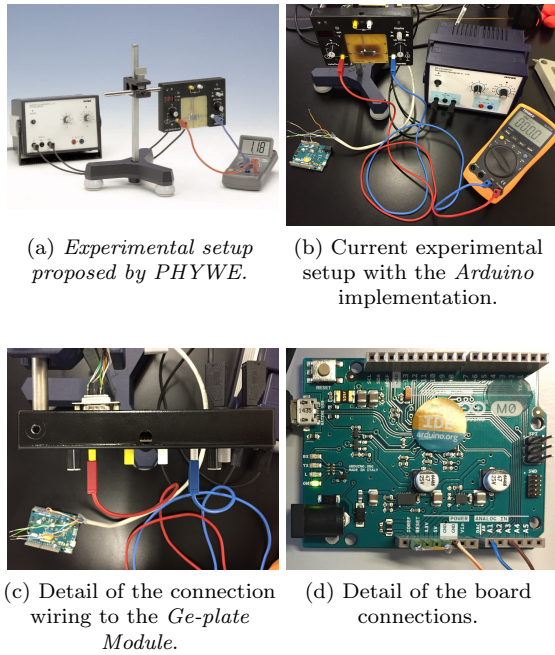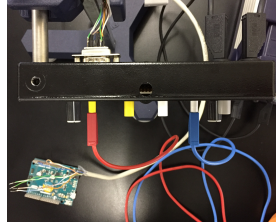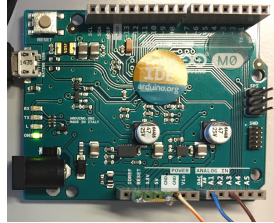
(a) *Experimental setup proposed by PHYWE.*

(b) Current experimental setup with the *Arduino* implementation.

(c) Detail of the connection wiring to the *Ge-plate Module.*

(d) Detail of the board connections.

Figure 1: The first image shows the experimental assembly recommended by the supplying company, which is the one that has been used until the new implementations. The three other images, account for the new implementations made, detailing the appearance of the modified *DB9-type* connection cable and the *Arduino M∅ board* configuration.

B.   Improvements

There have been two basic hardware implementations in the typical configuration: the *Arduino M∅ micro-controller* and the *DB9-type cable* that allows its communication with the module that incorporates the semiconductor. The *Arduino* implementation allows to directly control which signal is to be read from the module at any time and send this value via the *USB serial* port, by programming the board through its own *IDE* in order to interact with the analogical pins (in this case: $A0$, $A2$ and GND). Thankfully, after contacting *PHYWE Systeme GmbH*, they provided the pins of the *Ge-plate Module* rear port, and the control tension corresponding to the internal analogical signal related with T, V_+ and V_-. This has helped to modify the cable in such a way that the desired pins of the board can be connected to the respective signal pins in the module (Fig. 1b, 1c, 1d).

When programming the board, the resolutions of the implemented functions $analogWriteResolution(b = 8$ bits) and $analogReadResolution(b = 12$ bits) have been taken into account, which implies a range of 2^b levels $(0 - 255$ and $0 - 4095)$ to control the value to read and to determine the values sent via the *USB* for each variable of interest, respectively. A reference voltage for the micro-controller can be set by bridging both pins, that

of the $5V$ internal voltage with the *IOREF* corresponding to the *In/Out* tension of that board (as shown in Fig. 1d). Thus, one can establish the maximum voltage for the highest bit level available (255), and also ensure their stability. This 5 V internal voltage was checked with a multimeter in order to ensure its real value. On the one hand, that leads to a direct relation between the control tension voltage and its equivalent bit level (Table IIa) while, on the other hand, it gives the conversion factor in *volts* (V) for the signals captured (Table IIb).

After the first tests, an offset appeared between the value of temperature read through the micro-controller and the one shown in the module screen. By implementing a linear adjustment for the tension values of both temperatures, maximal and minimal, this reading becomes correct for each read (Table IIc). Each measurement order requires a certain amount of time to stabilize the subsequently read value. This has been accounted for, after each measure, by introducing a stabilization time before the next one. In addition, in order to get an statistically representative value, it has been decided to determine the final value for V_+, V_- and T as an average value of $N = 100$ measures.

By looking at the behavior of the received values, to improve the stability and resolution of the readings, an exponential moving average (which converges rapidly) has been applied to the average value of V_-, since it presented a lower resolution and stability (1%) than obtained for V_+, and T $(0, 03\%)$. For the final calculations, all temperature values have been converted from *Celsius degrees* (°C) to *Kelvin* (K) and the voltages transformed into *millivolts* (mV).

It is also necessary to comment on the existence of an offset between the value of intensity I (mA) shown on the display (this difference is also influenced by the lack of precision in the value of the screen, which had only one decimal) and that calculated by the processor. Probably due to the fact that the value of the internal resistance of the module ($R_i = 10$ Ω), which allows to give this value according to the value of V_- $(I = \frac{V_-}{R_i})$, is not the same as the provided by the manufacture company. In spite of this, and after several experiments, it is stated that this offset does not affect the final result since it represents a constant value that simply affects the ordinate at the origin of the linear adjustment to be made.

As for the *Python* programming mentioned before, the final version has the output restricted to a single *CSV* file containing the data collection (T, $\Delta V = V_+ - V_-$, I), since the student is required to process data on his own and make the necessary calculations with the consequent propagation of errors to finally obtain E_g. In order to be able to compare the methods described below, those *Python* scripts have been used with all its capacities in the present work. Thus, all calculations, graphs and adjustments are performed completely automatically, and instantly after completing the capture, when the required cooling process reaches 30°C.

Tension input (V)	Read Value	Byte level
$0,5 - 1,5 \Rightarrow 1$	T	51
$1,5 - 2,5 \Rightarrow 2$	V_-	102
$2,5 - 3,5 \Rightarrow 3$	V_{Hall}	153
$3,5 - 4,5 \Rightarrow 4$	V_+	204

(a) Control voltages for reading the variables to be measured and their equivalence in bytes.

Function	Resolution	Levels	Tension V	Tension mV
Read	12	4095	5	5000
Write	8	255	5	5000

(b) Conversion factors in *volts* (V) and *millivolts* (mV).

Values	V (V)	T (°C)
Minimal	$0,56$	$23,5$
Maximal	$3,46$	$172,5$

(c) Linear adjustment of the temperature analogical signal.

Table II: Considerations to be taken into account in the micro-controller programming, both to determine which signal is to be read at any time and to manage the received signals and the subsequent interpretation of their values. The linear adjustment of the temperature analogical signal is: $T = B + AV$ ($A = 51,38 \frac{°C}{V}$, $B = -5,27°C$)

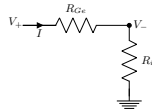Figure 2: $I = \frac{V_-}{R_i}$; $R_{Ge} = \frac{V_+ - V_-}{I}$ $\implies R_{Ge} = (\frac{V_+}{V_-} - 1)R_i$

C. Methods

With the present configuration, while an electric current (at practically constant intensity) coming from the power supply flows between the ends of the printed circuit, creating a potential difference, the *Germanium* sample is heated by the resistance R_{Ge} to the maximum temperature (auto-controlled by the device to avoid the fusion of the circuit contacts) that will not overcome the 173°C (see the diagram of the internal circuit in Fig. 2). After this, the system is let to cool down on its own. During this heating and cooling processes, values of both voltage (on the multimeter) and temperature (on the module display) are collected in order to have a collection of pairs of data for later analysis. With the set of data pairs (T(°C), ΔV(V)) obtained, the respective *Conductivity (σ)* values for the final calculation can be determined using:

$$\sigma = \frac{lI}{A\Delta V} \tag{12}$$

where $l \equiv$ Ge-plate length = 20 mm; $A \equiv$ Ge-plate cross section = 10 mm^2; $I \equiv$ current intensity (in mA); $\Delta V \equiv$ potential difference (in mV). In this situation, the units of the conductivity are $[\sigma] \equiv (\Omega\text{mm})^{-1}$.

Based on how students have collected the data so far, two very similar methods can be identified.

1. Method I

While the assembly is cooled down to 30°C (just over room temperature), the data are visually collected by reading the module display every 2°C. Likewise, as quick as it can be, the potential difference measured between the ends of the printed circuit is read in the multimeter. At the same time, the data has to be registered by hand or introduced in the PC.

This method carries two remarkable problems. First, the cooling process is very fast at the beginning (espe-

cially in the range between the maximum temperature and $\approx 75°C$) and the variation in the display is faster than one can read, implying imprecision in the measurements. Also, even in the subsequent slowly cooling down region, due to the lack of decimals in this temperature value the number of data is scarce.

2. Method II

In recent years, although the whole process is carried out in the same way as in Method I, students have taken advantage of the new capabilities of their portable devices (phones, tablets, etc.) to record the evolution of both screens, multimeter and Phywe module, while the cooling process is taking place. In this way, it has been possible to fine tune the readings since the subsequent play of the scene can be performed as slowly as desired. But is still limited, because the temperature is only shown with a resolution of $\pm1°C$ and, additionally, the processing of the data has to be done later.

3. Method III

With the implementation of the present work, the experimental equipment has been setup as it has been previously described, and this has changed the method of data acquisition. By connecting the properly programmed *Arduino M0 board* (*TFG.io*) to the *Ge-plate Module* and the working PC, and launching the created *Python* scripts (*TFG.py* and *TFGFuncions.py*), while the sample is heated and during its subsequent cooling, the data from the serial port are stored. The micro-controller is able to provide via the *USB serial*, average values of 100 readings of V_+, V_- and T and their respective standard errors, as well as the correspondingly calculated values T (K), $\frac{1}{T}$ ($\frac{1}{K}$), ΔV (mV), I (mA), σ and $\ln \sigma$ ($\frac{1}{\Omega\text{mm}}$) with their propagated errors.

III. Results

The corresponding $E_g \pm \delta E_g$ values are obtained after performing an *Ordinary Least Squares (OLS)* linear fitting for the data pairs of each method (see graphs in Fig. 3), with the appropriate error propagation (See Table III).

| Adjustment and Results	Method I	Method II	Method III
N (pairs)	72	144	1339
r^2	0,9997	0,9998	0,9999
$a \pm \delta a$ ($\frac{\text{K}}{\Omega\text{mm}}$)	-3893 ± 10	-3901 ± 6	-3838 ± 1
$b \pm \delta b$ ($\frac{1}{\Omega\text{mm}}$)	$7,29 \pm 0,03$	$7,31 \pm 0,02$	$7.312 \pm 0,004$
$E_g \pm \delta E_g$ (eV)	$0,671 \pm 0,002$	$0,672 \pm 0,001$	$0,6615 \pm 0,0002$

Table III: The data capture of the three methods were performed at a time with a total elapsed time of 12 minutes. The *OLS* linear fitting performed is:

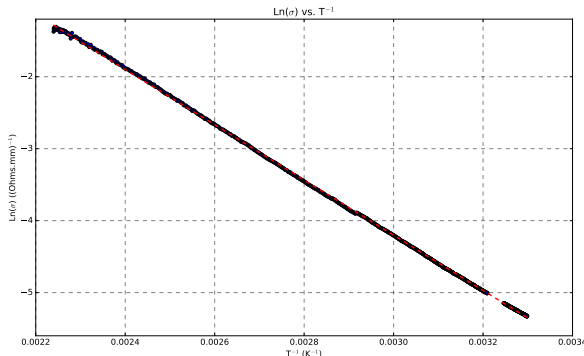$$y = b + ax \implies \ln \sigma_i(T) = \ln \sigma_0 - \frac{E_g}{2\kappa_B}\frac{1}{T}$$

With what is finally obtained:

$$E_g = -2\kappa_B a \text{ and } \delta E_g = E_g \sqrt{(\frac{\delta a}{a})^2}$$
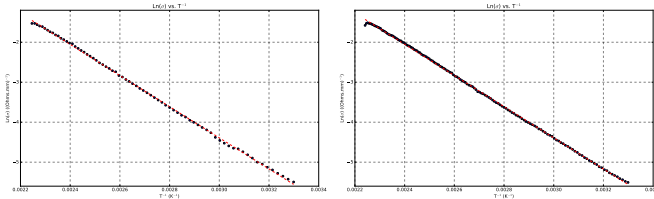
(a) Method I

(b) Method II

(c) Method III

Figure 3: Linear dependence between $\ln \sigma(T)$ and $\frac{1}{T}$ is evident in all three graphs, but the larger number of data obtained with *Method III* turns the statistic greatly improved, and the subsequent adjustment is more accurate.

IV. Conclusions

From the results presented before, the main conclusions of this work are the following:

1. The micro-controller implementation enhances the number of data taken for the same acquisition time (12 min approximately): they have been increased by a factor of 10, from about 100 to more than 1000.
2. The data precision has been also considerably improved, and this has provided better statistical analysis.
3. The rapid data capture and its high precision have produced a value for $E_g = 0,6615 \pm 0,0002$ (eV) for the *Method III*, which matches very well the typical value assumed as relevant in the literature ($E_g = 0.66$ eV)[2, p. 190].
4. This result is highlighted by the values obtained for *Method I and II*, which do not match the value just mentioned.

A possible improvement of this work is the inclusion of a small graphical environment that allows the student more interaction with the data acquisition process.

Regarding the results, it is logical to think that this implementation can be extrapolated to other experiments, those that require an automated data capture and that present similar configuration characteristics (for instance, the experimental determination of the *Hall effect* in a silver sample) in the *Laboratory of Modern Physics*.

Acknowledgments

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[6] $\hbar = 6,582119514(40) \times 10^{-16}$ eV.s, Ibid.

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