The EPR controversy

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Abstract: Under the title "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" Einstein, Podolsky and Rosen questioned the orthodox quantum-mechanical description of physical phenomena. This work aims to summarize and explain this argument and the consequent response by Bohr, provides the critical analysis of contemporary theorists to the referred debate and briefly exposes the author's opinion on the controversy.

I. INTRODUCTION

The aim of this essay is to expose the EPR argument and analyze Bohr's version and arguments on it, as it became one of the most popular responses. It is often assumed by contemporary theorists that Bohr triumphed over Einstein on their dialogue over EPR, but many voices have claimed that the approach made by Bohr was not as satisfactory as popularly known. This essay analyzes the EPR paradox (section III) and Bohr's reply (section IV) using the original papers that generated the debate (references [1] and [2] respectively). Further analysis to these papers written by Mara Beller (reference [4]) and Albert Fine (references [5] and [6]) are also used as secondary bibliographic material in order to see a contemporary theoretical approach to the paradox. During the conclusions, as a compendium of the analysis of the exposed Einstein-Bohr debate, my own opinion on the controversy is briefly exposed (section V).

This essay is halfway between Physics and History of Science, and is intended to cover the physical approach to the issue using the historical-philosophic arguments as a complement to it.

II. HISTORICAL CONTEXT

In 1925 Heisenberg published a series of papers where he formulates Quantum Mechanics (collaborating with Born and Jordan). Later on, in 1926, Schrödinger published his version of Wave Mechanics. These mentioned versions initially seemed to collide but later they realized that both meant the same. The discussions were accompanied by new results (probabilistic interpretation of the wave function, uncertainty principle, etc.) that allowed to establish quickly an orthodox interpretation. The 5^{th} Congress of Solvay, held on 1927, is considered to be the starting point for many important debates, and it is when actually the Einstein-Bohr debate started. The period comprised between 1927 and 1932 is often taken as the Golden Era for Quantum Mechanics development, during which Nuclear Physics and the relativist version of Quantum Mechanics were developed.

When "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" written by Einstein-Podolsky-Rosen was published in 1935 it was a very unexpected attack, because it meant a strong challenge to the orthodox philosophy of quantum physics. Even Schrödinger confessed that he was discontent with the orthodoxy, later known as "Interpretation of Copenhagen". Five months after the previously mentioned paper came out, Bohr's reply was published also in *Physical Review* under the same title. It became very popular because this response challenged the Criterion of Reality proposed by EPR. A debate between Einstein and Bohr followed, Einstein gave his own later versions of the EPR argument (it is said that the EPR paper was originally written by Podolsky) and after Einstein's death, experiments analogous to the described in EPR have been realized in order to determine if Bell's inequalities are violated (due to Bell's analysis of EPR).

III. THE EPR PARADOX

The EPR argument starts pointing out when a physical theory is complete and, if so, to what physical reality it does correspond to. Two main premises are presented.

The first premise, referred as Condition of Completeness, states that: "every element of the physical reality must have a counterpart in the physical theory".

The second premise, referred as Criterion of Reality, considers: "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity".

These previous definitions describe physical quantities as elements of the physical reality and recall that these conform a physical theory. The aim of the authors was to prove that Quantum Mechanics was incomplete, so once they have established these criteria they proceed to present two excluding possibilities:

(A) The quantum-mechanical description of reality given by the wave function is not complete.

(B) When the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality.

One of the previous assumptions must hold. If quantum mechanics is complete (being (A) FALSE) the second option must hold (being (B) TRUE) and vice versa. Instead, if quantum mechanics is incomplete (so (A) is TRUE) then the uncertainty principle is disrupted (so (B) is FALSE).

From this point, the information contained by a wave function is considered complete (premise (A) is FALSE) and corresponds to what can be measured without perturbing the system, referring to the Criterion of Reality.

Let us consider two systems, I and II, that interact during a time $t = T$. At t=0 the states of the systems were known. At $t > T$ the equation of state of I+II can be known and the wave function Ψ is designed for this combined system. After this time has passed, they no longer interact. Further measurements are made upon the first system by reduction of the wave packet.

If a_1, a_2, a_3,... are the eigenvalues of quantity A pertaining to system I, and $u_1(x_1), u_2(x_1), u_3(x_1)$,... the corresponding eigenfunctions, then Ψ, as function of x_1 and x_2 (variables describing the first and the second system respectively) is

$$\Psi(x_1, x_2) = \sum_{n=1}^{\infty} \psi_n(x_2) u_n(x_1). \qquad (1)$$

Then quantity A is measured and is found to have value a_k, collapsing the wave packet to the single term $\psi_k(x_2)u_k(x_1)$, where system I is left in state $u_k(x_1)$ and system II is left in $\psi_k(x_2)$. If instead, quantity B is measured in system I, being b_1, b_2, b_3,... the eigenvalues and $v_1(x_1), v_2(x_1), v_3(x_1)$ the eigenfunctions, we have a new expansion with new coefficients

$$\Psi(x_1, x_2) = \sum_{s=1}^{\infty} \varphi_s(x_2) v_s(x_1), \qquad (2)$$

the measured value is b_r, collapsing the state to $\varphi_r(x_2)v_r(x_1)$, stating $v_r(x_1)$ and $\varphi_r(x_2)$ for how first and second system are left respectively after the measurement.

According to the authors, during both measurements, because the systems no longer interact, no change is done to the second system by measuring the first one. Also, in the situation presented by EPR, both the relative position and the total linear momentum along the same axis (that is zero) are conserved. Anyway, from previous development, we see that two different wave functions ψ_k and φ_r can be assigned to the same reality of the second system after the measurements on the first one.

It can also happen that these two wave functions are eigenfunctions of the operators corresponding to P and Q, being those non-commuting operators, with corresponding eigenvalues p_k and q_r. The authors propose to proceed to do measurements of quantities as pointed out before, where A is the momentum and B the position.

Let us now suppose that the two systems are two particles, and the wave function for this combined system is:

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} e^{(2\pi i/h)(x_1 - x_2 + x_0)p} dp. \qquad (3)$$

From this equation, if we measure the momentum of the first particle, the corresponding eigenfunction (with eigenvalue p) will be:

$$u_p(x_1) = e^{(2\pi i/h)px_1}, \qquad (4)$$

And so, $-p$ will be the eigenvalue for the second particle, and will correspond to the eigenfunction:

$$\psi_p(x_2) = e^{(2\pi i/h)(x_2 - x_0)p}. \qquad (5)$$

If instead of the momentum, we measure the position of the first particle obtainable from Eq. (3) we get:

$$v_x(x_1) = \delta(x_1 - x), \qquad (6)$$

being its corresponding eigenvalue x. The eigenvalue obtained then for the second particle will be $x + x_0$ and it will correspond to the eigenfunction:

$$\varphi_x(x_2) = \int_{-\infty}^{\infty} e^{(2\pi i/h)(x - x_2 + x_0)p} dp = h\delta(x - x_2 + x_0), \qquad (7)$$

where the inverse Fourier transform was applied in order to express it by means of Dirac delta-function. If ψ_p states as eigenfunction of P, and φ_x states for Q, since

$$PQ - QP = h/2\pi i, \qquad (8)$$

we have obtained, by measuring A and B, eigenfunctions of non-commuting operators that correspond to physical quantities belonging to the same reality (system II). By this result, the premise (B) is FALSE (we obtained operators corresponding to two physical quantities that don't commute but simultaneous realities have been assigned). This result was obtained by assuming a complete description of the wave function, i.e., considering (A) to be FALSE. Consequently, negating (A) lead to negating (B), and it was shown at the beginning of the argument that if one premise was false, the other one had to hold. By these means, we cannot deny (A) and so, the quantum-mechanical description of reality given by the wave function is not complete.

IV. BOHR'S RESPONSE

A. Bohr's argument

Two main parts of Bohr's response are commented in this text: his mathematical representation of EPR and his experimental representation of it. For his experimental representation, several arrangements are presented in order to finally expose the EPR arrangement in a clear way. This section resumes most of Bohr's response but does not cover the whole content of it, for an obvious lack of space.

Although it is part of a footnote, Bohr recreates the EPR situation through a mathematical approach. He states that for a mechanical system consisting of two subsystems I (q_1p_1) and II (q_2p_2), which satisfy the usual commutation rules, it is possible to replace the two respective pairs of canonically conjugate variables by two pairs of new conjugate variables (Q_1P_1) and (Q_2P_2) by an orthogonal transformation corresponding to a rotation of θ in the planes (q_1q_2), (p_1p_2). Stating:

$$\begin{aligned} q_1 &= Q_1 cos\theta - Q_2 sin\theta \qquad & p_1 &= P_1 cos\theta - P_2 sin\theta \\ q_2 &= Q_1 sin\theta + Q_2 cos\theta \qquad & p_2 &= P_1 sin\theta + P_2 cos\theta. \end{aligned}$$

These variables satisfy:

$$[Q_1P_1] = ih/2\pi, \qquad\qquad [Q_1P_2] = 0,$$

and from here it can be seen that we can clearly assign definite numerical values to both Q_1 and P_2. From writing these variables in terms of (q_1p_1) and (q_2p_2)

$$Q_1 = q_1 cos\theta + q_2 sin\theta, \qquad P_2 = -p_1 sin\theta + p_2 cos\theta,$$

a subsequent measurement of either p_2 or q_2 would allow us to predict p_1 or q_1 respectively. Once this deduction has been introduced, the EPR recreation that Bohr introduces corresponds to the particular situation where the two particles have an angle $\theta = -\pi/4$ for their positional coordinates and components of momentum. The wave function of the composite system of EPR, from formula (3), corresponds to the special choice of two infinitely narrow slits and $P_2 = 0$.

Being EPR introduced, it is interesting to see Bohr's opinion of it. The main idea arising from Bohr's analysis is that he considers that no distinction can be made between what we are measuring and the measuring arrangement. This statement leads him to the following argument: if a measurement is done with an experimental setup and another measurement of another physical quantity has to be done, the setup (analogous to the frame of reference) needs to be changed and therefore, the system is disturbed. That consideration makes him find ambiguous the meaning of the expression "without in any way disturbing a system", considering that the Criterion of Reality proposed by EPR is not accurate enough. To defend his reasoning, he gets into the discussion of some examples.

Bohr introduces several experimental arrangements using diaphragms. In order to differentiate the setups (and refer to them in a clear way afterwards), a notation is introduced to name the different diaphragms, but it does not appear on the original text (it has been introduced using the notation from reference [3]). On the following figures two setups are shown, obtained from reference [4], in order to illustrate Bohr's idea of diaphragm, because he does not introduce figures during his argumentation, but none of them should be considered as the exact representation of the concerned setups introduced later.

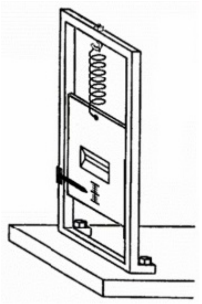

FIG. 1: Apparatus suspended by weak springs for momentum measurement ([4] p.148).

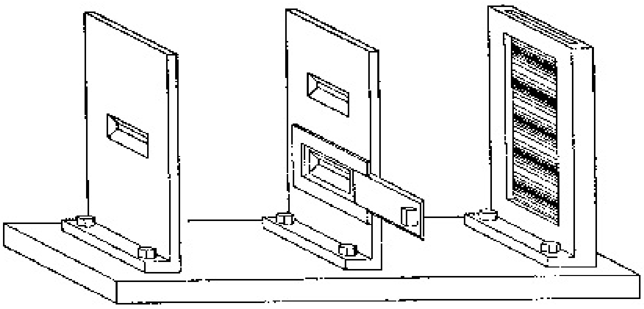

FIG. 2: Apparatus rigidly bolted for position measurement ([4] p.148).

Let us now introduce the arrangements, consisting of diaphragms. The width of the slits of these diaphragms is assumed large compared to the wave-length of the incident particles, so the width can be taken as the uncertainty of the position of the particle relative to the setup, in a perpendicular direction to the diaphragm. The uncertainty associated to the momentum is related to the possible exchange of momentum between the diaphragm and the particle.

Firstly, the description of a diaphragm with a single slit in it is introduced, let us name it D_1, and it "may form part of some more or less complicated experimental arrangement". Those "more complicated" arrangements are then commented, considering equal the support of the setup and the frame of reference. The two basic arrangements, A_1 and A_2, are now introduced. Presenting the foundations of the behaviour of these arrangements allows Bohr to introduce the last, and more complex arrangement, A_3: the experimental representation of the EPR situation.

In the first arrangement, A_1, D_1 is rigidly fixed to the support, and there is no possibility of knowing the momentum exchanged between the particle and the diaphragm. The momentum is then indeterminate, and the position component relative to the support can be measured.

In the second arrangement, A_2, D_1 is not rigidly fixed but remains still connected to the common support by a system of springs that allows the recoil of D_1 when a particle passes through the slit. This second arrangement lets a free choice after the particle has passed the slit: the first option proposes adding a second rigid fixed diaphragm and measuring position; the second option adds a moving diaphragm, not fixed, for the measurement of momentum. If the mass of this diaphragm is considered much bigger than the particle, we can approximate the situation back to the first option and have a setup for position measurement.

In order to represent the EPR thought experiment, a last arrangement, A_3, is introduced: a two-slit diaphragm, let us name it D_2, with two free particles with a given initial momentum passing simultaneously through each slit. The diaphragm is not rigidly fixed, suspended by weak springs. The momentum of this diaphragm can be measured before and after the passing of the particles, so $P_1 + P_2$ can be known, as well as $Q_1 - Q_2$. The widths of the slits have to be small compared to their difference of position, so the uncertainties associated to each one is small compared to the distance between them and we can, as said, know the difference of position just after the passage. The purpose of Bohr introducing this specific arrangement is to obtain, by means of an ideal experimental setup, the same wave function introduced by EPR.

By measuring the momentum of the particles before and after them passing the diaphragm, we get $P_1 + P_2$ and $Q_1 - Q_2$, being the conjugate quantities completely unknown. In order to determine either momentum or position, a subsequent single measurement has to be done. There is a free choice of what we want to measure, as it happens in the EPR mental experiment, by adding the suited setup for what we want to measure (either A_1, for position; either A_2 for momentum). If we measure the position of the first particle by this added diaphragm, we exclude the possibility of measuring the momentum (we can predict Q_2, but we cut ourselves from the possibility of measuring P_1, and so from predicting P_2). Analogously, if we measure the momentum, we can measure P_1 and predict P_2, and no measurement can be done of Q_1, so we cannot predict Q_2. There is no possibility of simultaneous prediction, contrary to the EPR, because a simultaneous reality can only be given to simultaneously measured variables. As he points out, "such measurements demand mutually exclusive experimental arrangements". There is a "finite and uncontrollable interaction" between the object and the measuring arrangement in quantum theory, so the condition of what measuring arrangement we are using (and so what prediction can be done regarding the future behaviour of the system) is in itself an influence which perturbs the system, although not being a mechanical one. Through these reasonings Bohr concludes that the Criterion of reality of EPR leads to a situation where this influence, inherent part of the measured element, is obviated, and we are facing an ambiguity as regards what is "physical reality".

B. About Bohr's argument

Mara Beller was a historian of science who, in collaboration with the philosopher Arthur Fine and after many discussions with him, published some studies on quantum physics analysis from a philosophical point of view. On her book *Quantum Dialogue*, she analyses Bohr's response to EPR and offers a very skeptical vision to his arguments. She distinguishes two lines of argument in Bohr's response (named as the "two voices"), which are: the operational voice and the philosophical voice. That second voice is also argued by Fine on his own studies.

Let us first discuss the operational voice. Going back to the arrangements introduced by Bohr, Beller reasons how the arrangement for position measurement violates the EPR case. If we get back to A_3 for the position measurement we can see that $Q_1 - Q_2$ has only a definite value at the instant when the two particles pass the two slits, and becomes indefinite at any other point. If a second diaphragm spatially separated is used as proposed in order to measure Q_1, because we no longer know $Q_1 - Q_2$, we cannot predict Q_2. Only under the very specific condition where these two diaphragms were merged (in order to measure Q_1 at the same moment that the two particles pass the two-slit diaphragm) this measurement could be done.

In addition, both $Q_1 - Q_2$ and $P_1 + P_2$ can be simultaneously determined with either measuring Q_1 or P_1 and predicting Q_2 or P_2 in the EPR setup. In Bohr's case, only $Q_1 - Q_2$ or $P_1 + P_2$ can be determined according to the measured variable: the change in the mechanical arrangement in order to pursue either a measurement of position or momentum implies an indirect mechanical disturbance, whereas EPR clearly state that no real change takes part in the second system. Beller considers that the EPR is not challenged by Bohr because he does not replicate the EPR conditions.

Both Beller and Fine are concerned about the philosophical voice in Bohr's argument. The reading of Bohr's explanation of EPR, state the authors, is often taken as the original EPR argument, but the issues he points out are not the main point of the original argument. According to Beller and Fine, EPR is about the interpretation of wave functions and its aim is to challenge how adequate is the state description given by the wave function. The Criterion of Reality is only introduced to check, after an eigenstate is assigned to the unmeasured system by wave-reduction, that the associated eigenvalue is an element of reality. Bohr states it is not clear why EPR can predict Q_2 and P_2 at the same time if the measurements on the first particle are not pursued simultaneously and focuses on the Criterion of Reality for that, but simultaneity during the measurements is not the main concern of EPR and the Criterion is only used as a secondary tool.

As a last hint, it is interesting to get in the wording used by Bohr, according to the authors. During their argument, EPR talk about "elements of physical reality", while Bohr talks about the meaning of the notion of reality on his argument. During their discussion, EPR talk about the elements of reality as physical variables, while the wording reformulation of Bohr converts EPR into a metaphysical discussion of what physicists mean when they talk about reality. Beller and Fine consider that this has a strong rhetorical effect and the reader enters Bohr's frame of mind and it is, in this wording ground that he creates, that Bohr "defeats" the EPR argument.

V. CONCLUSIONS

Beller is categorical considering Bohr's response as unsatisfactory. Although being under the influence of my readings of Beller and Fine, and considering too that his answer is not a victory over EPR, I would not affirm that there is a clear resolution to who won the Einstein-Bohr debate. Going back to the critique of A_3 for the position measurement, the need of merging the diaphragms in order to measure the position of the first particle and maintain $Q_1 - Q_2$ known is indeed a difficulty, but it is not fair to hold Bohr responsible for that: Bohr just showed the consequences of preparing a system with the wave function proposed by EPR. It should be remembered that Bohr's experimental arrangements are thought experiments involving idealized apparatus (but this does not affect the theoretical argument because, as he points out in a footnote, "the procedures in question are equivalent with atomic processes, where a corresponding application of the conservation theorem of momentum is well established"). In fact, the first experimentally feasible measure of EPR was proposed by Bohm in 1951, using two conserved spins. Moreover, Beller considers Bohr's approach as non-mathematical (being this a mistake for her since EPR focuses on the wave function description), but she ignores Bohr's mathematical recreation of the EPR because of the irrelevant presence of a footnote in front of the long and tough argument line during the whole response. Moreover, I find Bohr's response to the paradox very interesting because it involves a significant change on the meaning of "disturbance", where an influence such as a non-mechanical disturbance is defined for the first time.

As Beller and Fine remark, Bohr focused on the Criterion of Reality during his argumentation. I find interesting to point out that on his later versions of the EPR argument, Einstein himself drops the use of that Criterion (which was introduced by Podolsky during his writing of the paper). In my opinion, the EPR argument is structured in a way that masks the main idea of it (completeness or not of the information contained in a wave function) because of the use of this Criterion and the exclusive disjunctions.

Although mentioning that the two systems are correlated after the interaction (referring to a composed system I+II), the entanglement concept, which is in fact the main issue derived from the EPR paradox, has not been used during the development of this essay because of the intention to cover in detail the primary sources of study and stick to the point with its own wordings as much as possible (Schrödinger introduced the concept of entanglement after the presented debate). Being this said, referring to entanglement leads to, as a final remark, point out that even if EPR intended to question the completeness of quantum mechanics, their paradox (which challenged the theory on its rising period) supposed a constructive critic to the building of the theory and many were the positive consequences derived: that referred concept of "entanglement" was later introduced and discussed and further studies were and are still pursued on that issue.

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