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Energy Sustainability

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ABSTRACT: We address the problem of decision making under "deep uncertainty," introducing an approach we call Robust Portfolio Decision Analysis. We introduce the idea of Belief Dominance as a prescriptive operationalization of a concept that has appeared in the literature under a number of names. We use this concept to derive a set of non-dominated portfolios; and then identify robust individual alternatives from the non-dominated portfolios. The Belief Dominance concept allows us to synthesize multiple conflicting sources of information by uncovering the range of alternatives that are intelligent responses to the range of beliefs. This goes beyond solutions that are optimal for any specific set of beliefs to uncover defensible solutions that may not otherwise be revealed. We illustrate our approach using a problem in the climate change and energy policy context: choosing among clean energy technology R&D portfolios. We demonstrate how the Belief Dominance concept can uncover portfolios that would otherwise remain hidden and identify robust individual investments.

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I. Introduction

In this paper, we present a prescriptive approach to decision making under “deep uncertainty”. This problem refers to a situation in which there is significant disagreement about probability distributions over relevant outcomes (McInerney, Lempert, and Keller 2012). Deep uncertainty is pervasive in instances of collective decision making, and has particular relevance today in the context of climate change. In fact, an important reason for why governments have been slow to address climate change is the uncertainty that surrounds it. (Oreskes and Conway 2011) argue that some groups have emphasized this uncertainty in order to prevent regulations aimed at addressing climate change. In addition, both ends of the political spectrum have spent considerable time and resources arguing against specific solutions, with, for example, some on the right opposing solar and wind energy, and some on the left opposing nuclear and carbon capture, dueling over uncertainties surrounding costs and other implications. These arguments have led to a conservative approach, with few solutions moving forward at a speed that is needed to avoid serious climate damages (Edenhofer et al. 2014).

The broad question we tackle is how to approach deep uncertainty in the development of public policy strategies, where deep uncertainty is defined as a situation in which experts or models generate conflicting beliefs over future states of the world. Our approach, which we call Robust Portfolio Decision Analysis, has three key characteristics. First, we focus on portfolios of individual alternatives, rather than one-dimensional alternatives. Second, we identify non-dominated portfolios, rather than narrowing in on “optimal” portfolios. Third, we use the set of non-dominated portfolios to identify robust individual alternatives, thus finding common ground. This approach serves the purpose of helping to overcome stalemates, while avoiding bad solutions in cases where there is disagreement over beliefs.

The question of how to make decisions under deep uncertainty may be restated as a question of how to synthesize multiple conflicting beliefs. This question has been approached in the literature in a number of different ways. The most traditional approach is to aggregate beliefs to produce a single, portable probability distribution (see (Clemen and Winkler 1999) (Cooke and Goossens 2008) (Hora et al. 2013) (Lichtendahl, Grushka-Cockayne, and Winkler 2013) for discussions of aggregation methods). The resulting distribution can then be used in a Subjective Expected Utility (SEU) framework (See for example (Baker and Solak 2014; Kelly and Kolstad 1999; Keller, Bolker, and Bradford 2004)). The SEU framework satisfies a set of axioms laid out by (Von Neumann and Morgenstern 2007; Savage 1954), and has long been held up as an example of rationality.

A second set of approaches has been gaining interest in recent years, arguing that SEU is not “externally consistent” (Heal and Millner 2014; Gilboa, Postlewaite, and Schmeidler 2009). These approaches allow for ambiguity aversion and apply non-traditional decision rules to the set of beliefs, thus synthesizing them in the context of a decision. This set of approaches includes Maxmin (Gilboa and Schmeidler 1989; Ribas, Hamacher, and Street 2010), α-Maxmin, Minimax Regret, KMM Ambiguity Aversion (Klibanoff, Marinacci, and Mukerji 2005) and Soft Robustness (Ben-Tal, Bertsimas, and Brown 2010). (Stoye 2011) has shown that the first three of these can be derived by relaxing some axioms required for SEU, while adding others.

In this paper, we build on a concept from the literature that relaxes the axiom of completeness, thus producing a dominance concept. We translate this concept from the descriptive and normative literature into an operational prescriptive concept that we call Belief Dominance. Versions of this dominance concept have been discussed and axiomatized in the economics literature, and referred to, variously, as: admissibility, Knightian decision making, objectively rational, or strong Pareto (Anscombe and Aumann 1963; Bewley 2002; Gilboa et al. 2010; Stoye 2011; Galaabaatar and Karni 2013). These papers discuss the concept of Belief Dominance from a descriptive point of view, aiming to describe and rationalize individuals’ behavior in the face of deep uncertainty, and from a normative point of view, by focusing on the set of axioms that allow a preference relation to be represented mathematically. Our concept is closest to (Gilboa et al. 2010)’s “objectively rational” decision making.

(Danan et al. 2016) move this concept, which they call Unambiguous Preferences, farther toward a normative application, applying it to define robust social decisions. They focus on a social decision maker that has to synthetize multiple preferences and multiple beliefs. Most relevant to our work is the second part of their paper, which focuses specifically on the case where stakeholders have common preferences. They prove that a social preference satisfies a kind of Pareto principle if and only if it can be represented as the concept we call Belief Dominance over the union of all stakeholders’ beliefs.

Some versions of the same concept have appeared in the prescriptive literature, but only in limited form and very specific contexts. (Liesiö, Mild, and Salo 2007, 2008) use the concept in additive weighting models where incomplete information is modeled as intervals; (Grushka-Cockayne, Reyck, and Degraeve 2008) applies the model in (Liesiö, Mild, and Salo 2007, 2008) to an air traffic management problem. (Iancu and Trichakis 2013) apply the concept to robust optimization for linear optimization problems (although they call it Pareto Efficiency, which we will argue is not precise).

In (Danan et al. 2016), following much of the economics literature, individuals have direct preferences over probability distributions, rather than over outcomes and risk. Thus, one of our contributions is to re-state the concept of Belief Dominance in a general framework with clear distinctions between alternatives, beliefs, and preferences (Howard 1988), so that it can be operationalized as a prescriptive decision rule (Bell, Raiffa, and Tversky 1988).

Another common theme from the literature is the relationship between Belief Dominance and other well-known decision rules. Axiomatizations vary from author to author under slightly different definitions of the dominance rule; as well as between papers by the same author, as some combinations of the axioms imply other combinations. (Stoye 2012) has shown that "admissibility", a version of Belief Dominance that does not limit the set of distributions, in some sense encompasses many other decision rules. It can be derived from a set of axioms that, relaxed one at a time and replaced by completeness, lead to Subjective Expected Utility, Minimax, α-Maxmin, or Minimax Regret, respectively. Similarly, (Danan et al. 2016) show that if a preference relation is a completion of Belief Dominance, then it can be represented by a general decision rule they call a variable caution rule. This decision rule can be parameterized to represent SEU or Minimax. They note that the converse is true (any decision rule that can be represented as a variable caution rule is a completion of Belief Dominance) under the condition that the variable caution rule is limited to satisfy Belief Dominance.

To sum up, our contribution relates to making this concept useful to decision makers. We show clearly that the belief-non-dominated set encompasses the choices that result from applying a range of common decision rules. In addition, we extend the set of decision rules commonly considered in this literature to include KMM ambiguity aversion. In fact, we illustrate that the belief non-dominated set goes beyond those solutions that are optimal for any specific set of beliefs or any specific decision rule, to uncover other defensible solutions that may not otherwise be revealed. Thus, the use of this dominance concept is more powerful than sensitivity analysis in that it will always include the optimal solution for any relevant belief, combination of beliefs, or decision rule (see (Wallace 2000) for a discussion of sensitivity analysis).

This general idea – providing and evaluating multiple alternatives rather than a single best decision – has been applied in a set of bottom-up exploratory approaches including Robust Decision Making (RDM) (Rosenhead, Elton, and Gupta 1972; Lempert and Collins 2007), Decision Scaling (Brown et al. 2012), and Info Gap (Ben-Haim 2004); see (Kalra et al. 2014) for a discussion of how these types of models can help lead to agreement over decisions. These methods typically analyze a small set of pre-defined alternatives for robustness and then suggest possible new alternatives based on the analysis (see (Herman et al. 2015) for a review). These approaches synthesize the range of beliefs and models within a decision context by visually communicating the range of possible outcomes implied by the range of beliefs. Our approach complements these approaches in that we use available probabilistic information to derive a good set of alternatives that can then be analyzed with the above methods.

An approach somewhat parallel to ours is Many Objective Robust Decision Making, MORDM (Kasprzyk et al. 2013; Hadka et al. 2015) which, like our method, uses optimization techniques to identify a set of good alternatives for subsequent analysis; in their case, the subsequent analysis uses RDM. MORDM, however, as indicated by its name, focuses on cases with multiple objectives, using Pareto Satisficing as its criteria for identifying a set of good alternatives for subsequent analysis. In future work, our method could be combined with MORDM to produce a set of alternatives that are non-dominated in terms of both objectives and beliefs.

Another key contribution is to extend this framework to portfolios of alternatives; and synthesize these two pieces to provide insights about individual alternatives. Most other robustness approaches, and all other normative applications of this dominance concept, are agnostic about the structure of alternatives, but in practice tend to focus on one-dimensional alternatives. The focus on portfolios has a significant advantage in situations where there are multiple stakeholders, such as in public policy. Our method can highlight individual alternatives that are robust in the sense that they are part of the portfolio regardless of individual beliefs. This allows stakeholders with conflicting sets of beliefs to find some common ground, which is well known to improve the outcomes of negotiation and deliberation (Mansbridge and Martin 2013). When applied to climate change, this portfolio approach, which might include a varied set of mitigation and adaptation strategies, may open up the dialog to a wider group of constituencies, laying hope for a societal solution to this global challenge (CRED 2014; Baker, Bosetti, and Anadon 2015). Indeed, scholars in the field of public engagement have suggested that discussion focused on a broad selection of solutions may appeal to, and mobilize, a wider range of stakeholders than a sole focus on the consequences of climate change (Roser-Renouf, Maibach, and Leiserowitz 2014).

While the method we present is general, we illustrate the practical applicability by grounding it in a climate-related proof-of-concept: energy technology R&D portfolios in response to climate change. There are multiple beliefs over the future performance of key energy technologies, which, in turn, can be mapped into beliefs over the overall cost of addressing climate change or implementing clean energy policies. Multiple studies report different distributions over the future costs of solar photovoltaics (PV), nuclear, biofuels, and other technologies, often conditional on specific policy interventions (Baker, Bosetti, Anadon, Henrion, and Reis 2015). The problem decision-makers face is to use these multiple views –which are often in disagreement–to define a portfolio for pursuing energy-related research and development. In principle, policy makers would want to identify the composition of the energy innovation portfolio to meet their objective, be it reducing energy imports or Greenhouse gas emissions from the energy sector.

In Section II, we define the theoretical framework, draw parallels with stochastic dominance and multi-criteria decision making, and provide a set of theoretical results showing that Belief Dominance encompasses other robustness concepts. Section III introduces a specific application of the methodology to the case of energy R&D portfolio selection. Section IV discusses the flexibility and extensions of the RPDA approach, and Section V concludes.

II. Robust Portfolio Decision Analysis – theoretical framework

There are two pieces to the theoretical framework that we introduce here. The first is the concept of Belief Dominance, defined so that a portfolio *A* dominates *B* if *A* is preferred to *B* for all probability distributions that represent plausible beliefs concerning the outcomes of these portfolios. In our application example, a portfolio of R&D investments in energy technologies would dominate another if it is preferred across the full set of experts' beliefs concerning the impact of R&D on the cost and performance of energy technologies. From this information, we build the set of non-dominated portfolios. The second piece of the framework allows us to move from the set of non-dominated portfolios to derive implications about individual alternatives composing the portfolio. Again, to use our specific example, this represents shifting the focus to R&D investments into specific technologies to find, for example, those that are present in all non-dominated portfolios or those that are never present.

II.1 Belief Dominance

Here we present the concept of Belief Dominance, following the modeling paradigm used in, for example (Rothschild and Stiglitz 1971; Epstein 1980; Athey 2002; Baker 2006) and consistent with (Hadar and Russell 1969; Bertsimas, Brown, and Caramanis 2011).[1] Following Howard(Howard 1988), we focus on the three key elements of a decision problem: preferences, alternatives, and beliefs. Consider the following generic decision model:

$$\max_{\mathbf{x}} V\left(\mathbf{x}, f\right) \tag{1}$$

where *V* represents Expected Utility:

$$V\left(\mathbf{x}, f\right) \equiv \int U\left(\mathbf{x}, \mathbf{z}\right) f\left(\mathbf{z}; \mathbf{x}\right) d\mathbf{z} \tag{2}$$

where $\mathbf{x} \in X \subseteq \mathbb{R}^n$ is a compact *n*-dimensional vector of decision variables, with each vector $\mathbf{x}$ representing an *alternative*; we assume that the set of alternatives, *X*, is finite. $\mathbf{z} \in \mathrm{Z} \subseteq \mathbb{R}^m$ is a realization of the *m*-dimensional random variable *Z* with probability distribution *f*, where *f* represents (possibly endogenous) *beliefs*; and *U* is an objective function, representing *preferences*.[2] Belief dominance compares alternatives over sets of beliefs.

Here we must clarify what we mean by *beliefs*, and endogenous versus exogenous uncertainty. In our framework a belief is a function *f* with two arguments:

$$f : Z \times X \to \mathbb{R}$$

[1] We note there is another strand of literature with a different, but wholly consistent, modeling paradigm, including (Klibanoff, Marinacci, and Mukerji 2005); in which the central concept is that of an "act".

[2] We note that this objective function may contain calculations (how decisions and the outcome of random variables combine into outcomes of interest); it may contain what is sometimes called a value function, providing weightings over different criteria; or it may contain what is called a utility function, representing preferences over risk.

For any given $\mathbf{x} \in X$, *f(**z**; **x**)* is a probability distribution over outcomes **z**, in the usual sense. The uncertainty is exogenous if *f(**z**; **x**) = f(**z**; **x'**)* for all $\mathbf{x}, \mathbf{x}' \in X$; it is endogenous otherwise. To be a *belief*, a function *f*, must be associated with a particular expert (or a particular aggregated expert). Different experts will have different beliefs, represented by f_1, f_2 , for example. See Figure 1 for an illustration.

		$\mathbf{x}_1$	$\mathbf{x}_2$	…	$\mathbf{x}_n$
	f_1	$f_1(z,x_1)$	$f_1(z,x_2)$	…	$f_1(z,x_n)$
Beliefs	…	…	…	…	…
	f_m	$f_n(z,x_1)$	$f_2(z,x_2)$	…	$f_2(z,x_n)$

Figure 1 presents an illustration of the concept of endogenous beliefs. Each row is associated with an expert or an aggregated expert and represents a belief. The probability distributions may vary depending on the alternative. If all distributions in a row are the same, then the belief is exogenous.

Define the set Φ as a compact set of beliefs. We define *Belief Dominance* as follows: an alternative ***x*** *belief dominates* alternative ***x'*** over a set Φ of beliefs if and only if

$$V\left(\mathbf{x}, f\right) \geq V\left(\mathbf{x}', f\right) \ \forall f \in \Phi \tag{3}$$

and the inequality is strict for at least one *f*. We write this as $\mathbf{x} \succ \mathbf{x}'$. Note, this definition is specific to the decision problem as defined by U, which represents the mapping of the primitives (decision variables and random variables) to metrics of interest (such as costs or benefits) and includes the decision maker's preferences, such as weightings over different attributes and attitudes towards risk.

A version of this concept was introduced most prominently in Bewley (2002), although that paper refers back to (Aumann 1962). Bewely required strict dominance under every prior. (Stoye 2012) investigates the concept under the largest possible set of priors. (Gilboa et al. 2010) introduced the concept we adopt, requiring the priors to belong to a closed convex set. This concept is appropriate since our intension is to operationalize a decision rule, rather than to try to discern preferences from choices. We are interested in the particular case where there are multiple beliefs, derived from multiple experts or models. Thus, a set of priors will exist, and as shown below in Section II.1.b. the relevant set will necessarily be closed and convex.

II.1.a Comparison with other dominance concepts

For intuition, we restate Belief Dominance and put it in context with other common dominance concepts, explicitly stating what is fixed, what is compared, and what is defined by a set. For ease of comparison we consider the case of exogenous uncertainty, and slightly abuse the notation defined above by referring to the beliefs as a function of only the random variable[3].

- *Belief*: Fix U; alternative x dominates alternative x' if

$$\int U(\mathbf{x}, z) f(z) dz \geq \int U(\mathbf{x}', z) f(z) dz \;\; \forall f \in \Phi \tag{4}$$

- *Stochastic*: Fix x; distribution f dominates distribution g if

$$\int U(\mathbf{x}, z) f(z) dz \geq \int U(\mathbf{x}, z) g(z) dz \;\; \forall \mathrm{U} \in \Upsilon_S \tag{5}$$

- *Pareto*: Fix f; alternative x dominates alternative x' if

$$\int U(\mathbf{x}, z) f(z) dz \geq \int U(\mathbf{x}', z) f(z) dz \;\; \forall \mathrm{U} \in \Upsilon_P \tag{6}$$

Note that traditionally the sets of utility functions considered in stochastic dominance, Υ_S specify the moments of the utility function with respect to the random variable Z (Levy,Haim 2015). For example, First Order Stochastic Dominance specifies that the set Υ_S contains all functions increasing in z. In contrast, the sets of utility functions traditionally considered in Pareto dominance, Υ_P, specify different weightings over different objectives (Ngatchou, Zarei, and El-Sharkawi 2005). Often these objectives are discrete criteria, such as “cost”, “safety”, “reliability”; such problems are often referred to as Multi-Criteria Decision Making or MCDM. We note that in the traditional statement of Stochastic dominance, the decision “x” is typically not identified or noted; similarly, the traditional statement of Pareto dominance is in a deterministic setting,

Each dominance concept holds constant one element of a decision problem: Belief dominance holds preferences constant; stochastic dominance holds alternatives constant; Pareto dominance holds beliefs constant. The concepts rank different elements of decision problems: Belief dominance and Pareto dominance rank alternatives; stochastic dominance ranks beliefs or probability distributions. The

[3] Note that in each case there is an additional requirement that the inequality be strict in at least one instance.

concepts are robust to different elements of decision problems: Belief dominance is robust to the set of beliefs; stochastic dominance and Pareto dominance are robust to different types of preferences.

Our concept differs from the others in that the disagreement or uncertainty is not over *preferences*, but over *beliefs* about the outcomes of alternatives, represented by sets of probability distributions. When there are multiple different beliefs (for instance, when they are stated by experts who do not agree), we suggest that analysis should yield a set of non-dominated alternatives just as in the cases of stochastic and Pareto dominance; and furthermore, a broader disagreement over beliefs should lead to a larger set of non-dominated alternatives.

Finally, we note that Belief Dominance and Pareto dominance are computationally equivalent: methods appropriate for identifying the non-dominated set in MCDM can be extended to find the belief non-dominated sets. They are not, however, conceptually equivalent, as beliefs and preferences are two distinct elements of a decision problem.

II.1.b Non-Dominated Sets

Again parallel to the other dominance concepts, we are interested in applying Belief Dominance to determine the set of alternatives ***x*** which are non-dominated. An alternative ***x*** is non-dominated if there is no other alternative ***x***′ that belief-dominates it. Define X_{ND} to be the set of non-dominated solutions. Figure 2 provides a visual illustration of this concept.

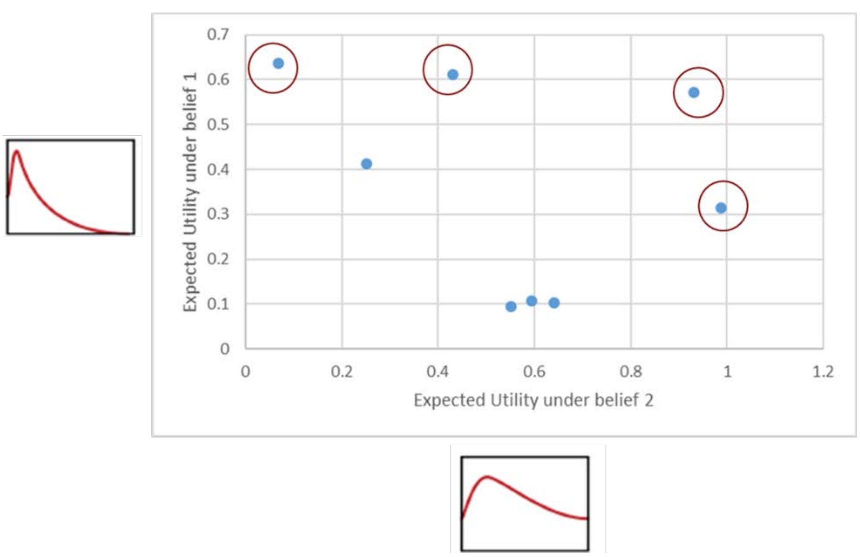

Figure 2 Blue points represent the expected values of alternatives under each of the two beliefs, shown as probability densities near the relevant axes. Belief-non-dominated set includes circled points.

Note that by the linearity of the integral, if an alternative is dominated over a finite set of beliefs, it is also dominated over the convex combination of these beliefs. This implies that if the presence of dominance is established for all individual beliefs, then dominance also holds for all *combinations* of such beliefs. Conversely, if dominance between $\boldsymbol{x}$ and $\boldsymbol{x}'$ does not hold in the convexification of the belief set, there must exist at least two distinct beliefs which rank the alternatives $\boldsymbol{x}$ and $\boldsymbol{x}'$ differently. Thus, from now on, we assume that the set Φ is the convexification of all the relevant beliefs.

This simple result illustrates the power of this method with respect to traditional parametric sensitivity analysis. It has long been understood that sensitivity analysis –in this case finding the optimal solution under a number of candidate probability distributions – is not guaranteed to reveal the actual optimal solution (see (Wallace 2000) for seminal paper). That is, the optimal solution is not guaranteed to be contained in the space spanned by the deterministic solutions (or the solutions of individual probability distributions). The non-dominated set does not have this problem: the optimal solution for any convex combination of the candidate distributions is guaranteed to be in the non-dominated set. Any solution that is optimal for any probability distribution that is a convex combination of the candidate distributions will be part of the non-dominated set.

II.1.c Comparison with Decision Rules

Stoye (2012) argues that the concept of admissibility (Belief Dominance less the requirement for a closed set of priors) "exhausts the overlap between many reasonable decision rules in a precise axiomatic sense." Specifically, Stoye (2012) shows that Expected Utility, Maxmin Utility, and MiniMax Regret are each characterized by different subsets of the axioms defining admissibility. In this paper, as the focus is shifted to the prescriptive usage of these rules, we focus specifically on how the sets of optimal alternatives that result from different decision rules relate to the belief-non-dominated set; and we expand beyond the decision rules considered in Stoye (2012).

We consider the set of decision rules that allow for *probability distributions over outcomes*, but consider *multiple possible beliefs*. For example, we consider the concept of Maxmin Expected Utility (Gilboa and Schmeidler 1989), which chooses alternatives under the worst belief, rather than the simple Maxmin, which considers the worst possible outcome. For conciseness of exposition, we will drop the reference to Expected Utility in each of the robustness concepts. We define each concept precisely below or in

Appendix AI. We explore the concepts of Maxmin Expected Utility (Gilboa and Schmeidler 1989); Maximax Expected Utility; α-Maxmin Expected Utility (Ghirardato, Maccheroni, and Marinacci 2002), where the decision-maker considers the weighted average of the worst expected payoff and the best expected payoff; Minmax Regret with multiple priors (Hayashi 2008); and the (Klibanoff, Marinacci, and Mukerji 2005) Ambiguity Aversion framework (KMM from now on), which is parallel to Expected Utility, incorporating an ambiguity aversion function in a similar role as a risk aversion function. We note that Subjective Expected Utility using averaged probabilities (SEUa from now on, (Cerreia-Vioglio et al. 2013)) can be regarded as a special case of KMM.

In the remainder of this section we show that the belief-dominance concept encompasses all of these other robustness concepts, in the sense that at least one optimal solution under each of these other concepts is in the belief-non-dominated set. We point out that any optimal solution to a robustness concept which is not in the belief-non-dominated set is (1) no better than those optimal solutions that are in the belief-non-dominated set under the robustness concept; and (2) strictly worse than the solutions in the belief-non-dominated set under at least one plausible probability distribution.

The Robustness concepts fall into two classes. Concepts in the first class, which includes Maxmin, Maximax, α-Maxmin, and Minmax Regret, choose the optimal solutions based on a subset of the beliefs in Φ. This implies that there may be multiple optimal solutions, some of which may be dominated across the full range of distributions. We note that (Iancu and Trichakis 2013) were the first to point out this characteristic in the special case of Maxmin and a linear problem. They define and characterize the set of optimal solutions under Maxmin which are also belief-non-dominated, which they refer to as Pareto Robustly Optimal solutions.

The second class includes KMM and SEUa. Under the condition that all distributions in Φ have a strictly positive weight or second order probability, all optimal solutions to these Robustness concepts are belief-non-dominated.

Let us define some terminology. Let C be a robustness concept, where:

Definition $C \in \{\text{maxmin, maximax}, \alpha\text{-maxmin, minmax regret, KMM, SEUA}\}$

A solution $x^C \in X^C \subseteq X$ is optimal under C ("C-optimal") if it is optimal under the robustness concept C. Here we provide two examples.

Example 1: Define $\underline{V}(x) \equiv \min_{f\in\Phi} V(x, f)$.

The Maxmin decision problem can be written

$$\max_{x\in X} \min_{f\in\Phi} \int U(x,z) f(\mathbf{z};x) d\mathbf{z} = \max_{x\in X} \underline{V}(x) \tag{7}$$

The set of solutions which are optimal under Maxmin is defined as follows:

$$X^{Mm} = \left\{ x^0 \in X \mid \underline{V}\left(x^0\right) \geq \max_{x\in X} \underline{V}(x) \right\} \tag{8}$$

Example 2: KMM and SEUa require a weighting or Second Order Probability (SOP) distribution over the First Order beliefs in Φ. Let π represent the second order cumulative probability distribution over the possible distributions in Φ. KMM also requires the definition of an ambiguity aversion function Ψ, similar to a utility function, but its concavity represents ambiguity aversion rather than risk aversion. The set of KMM-optimal solutions is as follows:

$$X^{KMM} = \left\{ x^0 \in X \mid \int_{\Phi} \psi\left(V\left(x^0, f\right)\right) d\pi \geq \max_{x\in X} \int_{\Phi} \psi\left(V(x, f)\right) d\pi \right\} \tag{9}$$

See Appendix AI for formal definitions of the C-optimal sets under the other concepts.

First, we show that belief non-dominance is transitive.

Lemma 1: Belief non-dominance satisfies the transitive property: for alternatives A, B, C

$A \succ B \text{ and } B \succ C \Rightarrow A \succ C$.

See Appendix AII for proof.

We now present the key to our central result: a Lemma showing that if a solution belief dominates a C-optimal solution, then that solution itself must be C-optimal.

Lemma 2: If $x \in X^C \text{ and } x' \succ x \text{ then } x' \in X^C$

Here we present the proof for C=Maxmin. The proof for the remainder of the Robustness concepts is similar and is presented in Appendix AIII.

Proof: (Maxmin) Since Φ is compact, we can choose belief f' such that $V(x', f)$ is minimized. We then have:

$\min_{f\in\Phi} V(x', f) = V(x', f') \geq V(x, f')$ by definition of Belief Dominance

$V(x, f') \geq \min_{f\in\Phi} V(x, f)$ by definition of the minimum

$\min_{f\in\Phi} V(x, f) \geq \max_{\hat{x}\in X} \min_{f\in\Phi} V(\hat{x}, f)$ by definition of Maxmin. Thus, these together imply that $x' \in X^{Mm}$

QED

We now show that at least one optimal solution to robustness concept C is in the belief-non-dominated set.

Theorem 1: If robustness concept C satisfies Lemma 2 then $X^C \cap X_{ND} \neq \varnothing$

Proof: Define the set X_{ND}^C as the C-optimal solutions which are non-dominated by any other C-optimal solution $x \in X^C$:

$$X_{ND}^C = \left\{ x \in X^C \mid \text{there does not exist } x' \in X^C \text{ such that } x' \succ x \right\}$$

Note that X^C is non-empty due to the compactness of the set of alternatives. The set X_{ND}^C can be built by examining the elements of X^C one by one and removing those x that are dominated by some other $x' \in X^C$. Since Belief Dominance is transitive, this set is not empty. Furthermore, the elements of X^C are non-dominated in the entire set X, by Lemma 2. **QED**

Theorem 2: Under the assumption that all beliefs in Φ have a strictly positive weight or SOP, all optimal solutions under SEUa and KMM are in the belief-non-dominated set:

$X^C \subseteq X_{ND}$ for $C = [KMM, SEUa]$

Proof: Let C = KMM. Assume the converse: that there is an optimal solution x* which is not in the set of non-dominated alternatives, i.e., it is dominated. That implies there exists a solution x** such that $V(x^{**}, f) \geq V(x^*, f) \forall f \in \Phi$. Because Ψ is strictly increasing, and the integral is linear, this implies:

$$\begin{aligned} &\Psi\left(V(x^{**}, f)\right) \geq \Psi\left(V(x^*, f)\right) \forall f \in \Phi \Rightarrow \\ &\int_\Phi \Psi\left(V(x^{**}, f)\right) d\pi \geq \int_\Phi \Psi\left(V(x^*, f)\right) d\pi \end{aligned} \tag{10}$$

By Belief Dominance and the assumption that all beliefs have a positive weight in the distribution π over the set of Φ , it follows that the last inequality is a strict one. This in turn implies that x* cannot be optimal under KMM, a contradiction.

To prove this for SEUa, note that by Corollary 2 in (Klibanoff, Marinacci, and Mukerji 2005), KMM reduces to SEUa if Ψ is linear. **QED**

We note that Theorem 2 does not hold for robustness concepts Maxmin, Maximax , alpha-Maxmin and Minmax regret. Each of these concepts uses only a subset of the beliefs in Φ; therefore some of the C-optimal solutions may be dominated by other C-optimal solutions. This is discussed in the case of Maxmin in (Iancu and Trichakis 2013).

Moreover, we note that there may be solutions in the non-dominated set which are not solutions to any of the robustness concepts. This brings us to a key difference between belief-non-dominance and the other robustness concepts. All other concepts present the decision maker with fully ordered sets of solutions, causing decision makers to narrow their consideration based on the choice of robustness concept. As can be seen by the profusion of robustness concepts in the literature, there is no agreement in the literature on which concept is best. Therefore, the non-dominated set gives decision makers the option to choose a solution to a particular robustness concept, but also to go beyond these concepts, perhaps incorporating qualitative concerns that may be quite difficult to model.

II.2 Deriving recommendation for individual alternatives from portfolio-level analyses

Our approach builds on the ideas of Robust Portfolio Modeling (Liesiö, Mild, and Salo 2007, 2008) which supports the selection of a portfolio of alternatives (such as R&D projects) from a large set of candidates. Specifically, the extension of Robust Portfolio Modeling (RPM) to scenario analysis (Liesiö and Salo 2012) employs set inclusion to capture uncertainties about the decision maker's risk preferences and beliefs by accommodating (1) sets of feasible utility functions over outcomes and (2) sets of feasible probability distributions over distinct scenarios. Results are obtained by determining which portfolios are non-dominated, in the sense that there does not exist any other portfolio that would be at least as good for all feasible combinations of utility functions and probabilities, and strictly better for some such combination.

The conceptual breakthrough in RPM is to analyze the set of non-dominated portfolios to inform choices among individual alternatives by dividing these alternatives into three categories. First, those individual alternatives that are contained in all non-dominated portfolios belong to the *core*. Second, individual alternatives that are not contained in any non-dominated portfolios are *exterior*. Finally, the *borderline* consists of individual alternatives that are included in some, but not all, non-dominated portfolios. To define this mathematically, let **x** be a vector including projects indexed by $i = 1..I$, and define $x_i = 1$ if project *i* is invested in and 0 otherwise. Recall that X_{ND} is the set of non-dominated portfolios. We define the three sets as follows:

$$\begin{aligned} core &\equiv \{i \mid x_i = 1 \ \forall \mathbf{x} \in X_{ND}\} \\ ext &\equiv \{i \mid x_i = 0 \ \forall \mathbf{x} \in X_{ND}\} \\ bord &\equiv \{i \mid i \notin core \text{ and i} \notin ext\} \end{aligned} \tag{11}$$

Individual Projects:	a	b	c	d	e	f
Portfolios						
1	1	0	0	1	1	0
2	1	0	1	1	1	0
3	1	0	0	1	1	0
4	0	0	1	1	0	1
5	0	0	0	1	0	1
6	0	0	1	1	0	1
		↑ exterior		↑ core		

Table 1: illustration of the core and exterior projects among the six non-dominated portfolios composed of individual projects a, b,..,f.

Table 1 provides an illustrative example, in which the 6 rows represent the 6 non-dominated portfolios; and the projects *a-f* can be invested in or not. In this case project *b* is in the exterior, project *d* is in the core; all other projects are in the borderline.

An important theoretical result is that when uncertainties are reduced–in the sense that the set of feasible probability distributions becomes smaller–all core and exterior individual alternatives stay in their respective sets (see Theorem 2 in (Liesiö and Salo 2012)). As a result, recommendations concerning the selection of core individual alternatives and the rejection of exterior individual alternatives are robust to learning, because these recommendations stay valid as additional information is obtained. For example, an individual technology investment that is in the core over a finite set of probability distributions will remain in the core for combinations of feasible probability distributions, including any subset of these distributions. Thus, research aimed at deriving recommendations that are more conclusive should be focused on the borderline individual alternatives: for instance, it is possible to analyze if these borderline individual alternatives can be enhanced to make them equally attractive as some core individual alternatives (Gregory and Keeney 1994); or if gathering more information about the borderline individual alternatives allows them to be moved into the core or the exterior.

On the other hand, this result implies that when additional perspectives are added, making the feasible set of probability distributions larger, the core and exterior sets may become smaller. In an extreme case with many beliefs, all individual alternatives might belong to the borderline, providing little useful information.

The core and exterior sets are extreme, in that they represent individual alternatives that are present or missing in every non-dominated portfolio. Another approach is to consider measures of individual alternatives that are on a continuum rather than black or white. One such measure is the Core Index (CI), which is defined as the ratio between the number of portfolios which contain an individual alternative versus the total number of non-dominated portfolios (Liesiö, Mild, and Salo 2007). The resulting CI values can then be employed to obtain tentative guidance as to which individual alternatives are most important to analyze further. For example, referring to the illustrative example presented in Table 1, project *a* has a CI of 0.5.

III. Application: Energy Technology R&D Portfolio in Response to Climate Change.

As an illustration, we apply our framework to the question of how to allocate research funds across a wide variety of energy technologies with varying potential for improvement and differing impacts on the economy and environment. This is a complex research question, which has been approached through different avenues, including (i) the development of a broad range of integrated assessment models (IAMs)[4] and (ii) multiple studies of expert judgments on the potential for technological change (Anadón et al. 2012; Anadón, Chan, and Lee 2014; Baker and Keisler 2011a; Baker, Chon, and Keisler 2009c, 2009b, 2008; V. Bosetti et al. 2012; M. V. E. Catenacci, Bosetti, and Fiorese 2013; Chan et al. 2011; Fiorese et al. 2013, 2014). The IAMs have been useful for developing insights on the relative importance of technologies and the speed of their adoption (see Clarke et al. 2014 for a complete review). Nevertheless, there are considerable challenges from the viewpoint of decision and policymaking, including the large number of assumptions that are required and the significant uncertainties associated with these assumptions. Studies of expert judgments, on the other hand, have provided explicit probability distributions over the potential for technological change; but there are a number of independent and disparate studies, and thus incorporating them into the already computationally-complex IAMs becomes a challenge. In this setting, we explore how these two individual approaches can be combined in an integrative framework to derive robust model-based conclusions while recognizing the uncertainties that have been expressed by multiple stakeholders (see Figure 3 for an influence diagram of the decision process).

[4] http://www.globalchange.umd.edu/iamc/

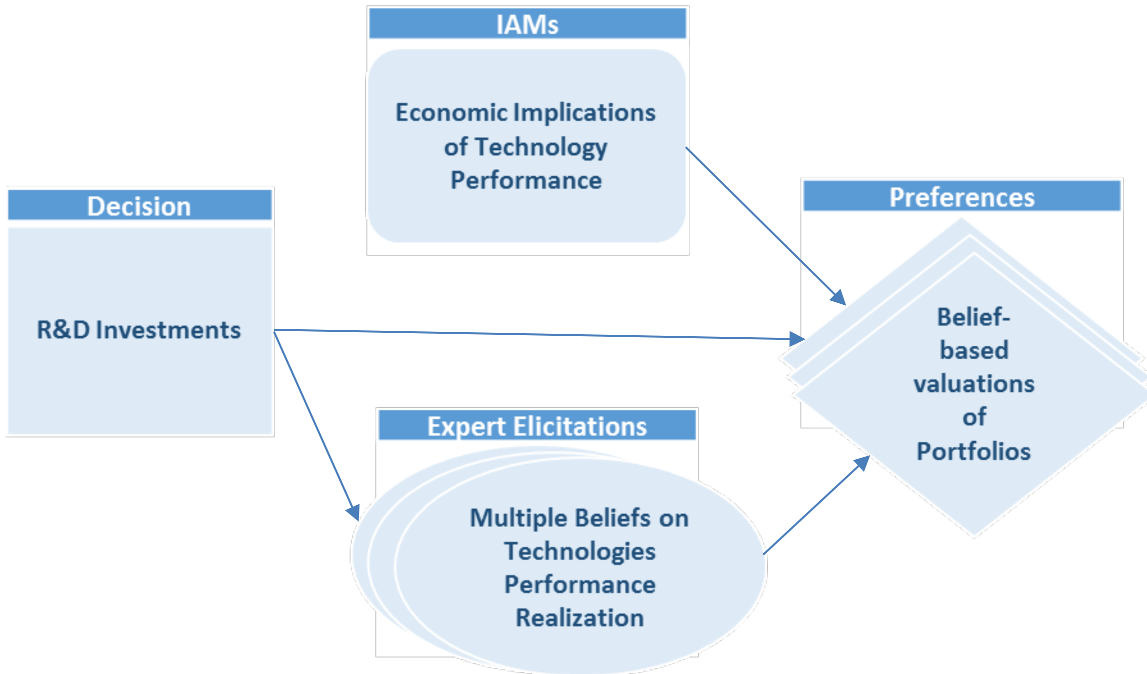

Figure 3: An influence diagram of the decision problem. Square nodes represent decisions; oval nodes uncertainties; rounded squares model calculations; and diamond nodes the objective value. Arrows represent influence.

We use data on the economic implications of future technologies' performance as estimated by a specific IAM, GCAM (e.g. (Kim et al. 2006)). GCAM has been extensively used to explore the potential role of emerging energy supply technologies and the greenhouse gas consequences of specific policy measures or energy technology adoption. It provides insights into the interactions of energy technologies with each other and with the wider economy and the environment.

We integrate the GCAM model with data derived from three large expert elicitation studies of energy technologies (summarized in (Baker, Bosetti, Anadon, Henrion, and Aleluia Reis 2015)). These data allow us to model multiple beliefs about key energy technologies' performances, conditional on the level of R&D investments. Because beliefs over technological performances conditional on R&D investments differ across experts, and it is not known which expert(s) may be right, the problem involves deep uncertainty. We incorporate this deep uncertainty over technological prospects by applying our concept of Belief Dominance in deriving sets of core, exterior, and borderline investments. We illustrate how these sets can be used to inform further research into the individual alternatives and provide insights to decision makers on near term R&D actions.

III.1 Energy technology portfolio model

For this proof of concept, the problem investigated is that of allocating R&D funds across various energy technologies in the specific context of climate change. The decision problem is to choose a portfolio $\boldsymbol{x}$ of investments that minimizes the expected total abatement costs (C) plus R&D investment costs. Bolded characters represent vectors; $\boldsymbol{x}$ is a vector of investments in the different technologies.

Using our terminology from above:

$$\min_{\mathbf{x}} \int U(\mathbf{x},\mathbf{z}) f(\mathbf{z};\mathbf{x}) d\mathbf{z} \tag{12}$$

where $\mathbf{z}$ is the vector of technologies' performances. The objective is:

$$U(\mathbf{x},\mathbf{z}) = C(\mathbf{z}) + \kappa B(\mathbf{x}) \tag{13}$$

$C(\mathbf{z})$ is the total net present value of abatement costs (in trillions of dollars, using a discount rate of 3%), given the realization of the technological performance, $\mathbf{z}$, where abatement is defined as a reduction in emissions below a Business-as-usual baseline. Note that while the objective function in (13) is separable in **x** and **z**, the overall objective in (12) connects them, since the beliefs are endogenous and depend explicitly on **x**. This cost *C* is calculated through the integrated assessment model GCAM; and is in comparison to a Business-as-Usual baseline. We concentrate on a specific climate policy aiming at stabilizing global average temperature at roughly 2°C by the end of the century[5], which is implemented as a constraint on emissions that is compatible with a given climate stabilization scenario.

$B(\mathbf{x})$ is the budget required for the R&D portfolio **x**. To calculate the total social cost of investing in a specific R&D portfolio we multiply the amount of the R&D budget by κ, an opportunity cost multiplier.[6]

An index, τ, accounts for the multiple expert surveys describing the future probabilistic evolution of technological performance $\boldsymbol{z}$ as a function of R&D investment decisions; f is indexed by τ, $f_\tau(\boldsymbol{z};\boldsymbol{x})$.

[5] This is implemented by means of a constraint on CO2-equivalent concentration in the atmosphere set at 450 ppm equivalent (i.e. including other greenhouse gasses using the global warming potential concept) which translates into a likely probability of maintaining the temperature below the 2°C target.

[6] Theory suggests that the cost to society of R&D investment may be higher than the actual dollars spent. We use a value of κ=4. See (Nordhaus 2002; Popp 2006) for details. Previous work ((Baker and Solak 2014) and (Baker, Olaleye, and Aleluia Reis 2015)) has not shown strong sensitivity to this assumption

For this paper, we consider three sets of probability distributions derived from three large multi-technology expert elicitation projects carried out independently by researchers at three institutions: UMass Amherst (Baker and Keisler 2011b; Baker, Chon, and Keisler 2009a, 2009b, 2008), Harvard (Anadón et al. 2012; Anadón, Chan, and Lee 2014; Chan et al. 2011), and FEEM (Valentina Bosetti et al. 2012; M. Catenacci et al. 2013; Fiorese et al. 2013, 2014).[7] We also consider an aggregation of these three as a separate belief (referred to as Combined – see (Baker, Bosetti, Anadon, Henrion, and Aleluia Reis 2015)). The Combined distribution was derived using Laplacean mixing and then smoothed using a fitted piecewise cubic distribution; therefore, it is not a simple convex combination of the other three studies. This results in four probability distributions over the outcomes of technological change $\boldsymbol{z}$, i.e. $\tau = 1,2,3,4$. See Figure A1 in Appendix AIV for a visualization of the multiple distributions used in this analysis.

The portfolios, **x**, consist of investments into five key energy technologies: solar PhotoVoltaics (PV), nuclear fission, Carbon Capture and Storage (CCS), electricity from biomass ("bio-electricity"), and liquid biofuels. The cost of investment $B(\boldsymbol{x})$ is the sum of the cost of investment for each individual project. The cost of investment for each individual project is the net present value of the annual cost over 20 years using a discount rate of 3%. Table 2, based on (Baker, Olaleye, and Aleluia Reis 2015) reports data on R&D cost assumptions for different levels of investments. We use an opportunity cost multiplier of κ=4.

Solar			Nuclear			Biofuels			Bio-electricity			CCS		
Low	Mid	High	Low	Mid	High	Low	Mid	High	Low	Mid	High	Low	Mid	High
1.7	4.0	33.0	6.2	19.2	178.3	1.4	3.7	20.3	1.4	3.0	16.9	5.3	17.1	168.1

Table 2: Annual R&D expenditures, in millions of dollars, assumed constant over a 20 year period.

There exist several ways of implementing the general problem presented in equation (12). Given the specific data we are working with, we take portfolios x to be vectors of binary variables, with $\mathrm{x_i} = 1$ if project i is invested in, and 0 otherwise. Each of the 5 technologies can be invested in at a low, medium, or high level; so each technology is associated with three mutually exclusive binary variables: exactly one decision variable associated with each technology will be equal to 1. The portfolio, given the three levels

[7] For this proof of concept we consider each team as a separate belief rather than each individual expert. We did this because the individual elicitations were gathered in different ways by the different teams making the individual beliefs quite difficult to standardize as compared to the aggregated beliefs.

of investments into the five technologies, is a 3×5 vector of binary variables; three, mutually exclusive, levels of investment by five technologies result in 3^5= 243 possible portfolios.

The vector of realizations $\boldsymbol{z}$ contains eight components including a cost for each of the five technologies and an efficiency for CCS, biofuels, and bio-electricity[8],[9]. In order to make the set of simulations with GCAM computationally feasible, we use the technique of importance sampling in a new way. Using an average of the low, mid, and high Combined distribution, we randomly draw 1000 points of the random vector $\boldsymbol{z}$; each outcome is represented by the 8-dimensional vector $\mathbf{z}_l$, where *l={1,2,…,1000}.* Each of these vectors is evaluated using GCAM, resulting in 1000 values of $C(\mathbf{z}_l)$. We then apply importance sampling to re-calculate the probability of each point depending on the investment portfolio. (Baker, Olaleye, and Aleluia Reis 2015) used a set of diagnostics based on (Owen 2015) and found that the samples performed in the acceptable range, with the possible exception of the biofuels and CCS efficiency parameters for the UMass and Combined distribution. See (Baker, Olaleye, and Aleluia Reis 2015) for more details.

Thus, we have a set of technology values, $\mathbf{z}_l$, $l = 1,..,1000$; and the (discrete) probability of a particular technology value realization, $f_\tau(\mathbf{z}_l;\boldsymbol{x})$, which depends on the elicitation study, τ, and on the portfolio, $\boldsymbol{x}$. We define $H(x;\tau)$, the discrete version of the objective function in equation (12), given a specific set of beliefs, τ, as follows:

$$H(\mathbf{x};\tau) \equiv \sum_{l=1}^{1000} f_\tau(\mathbf{z}_l;\mathbf{x})\{C(\mathbf{z}_l)+\kappa B(\mathbf{x})\} \quad (14)$$

We say that a portfolio $\boldsymbol{x}$ *belief dominates* $\boldsymbol{x}'$ if $H(\boldsymbol{x};\tau) \geq H(\boldsymbol{x}';\tau)$ for all τ, with a strict inequality for at least one of the beliefs. A portfolio **x** is non-dominated if there is no portfolio that dominates it and it is strictly better than at least one portfolio.

[8] The costs for these three technologies are capital costs; efficiencies are used to estimate operating costs.
[9] A complication is that the Harvard probability distributions do not distinguish between biofuels and electricity from biomass. For this initial proof of concept we assume that the investment is evenly divided between the two technologies.

As the number of portfolios is small, we first calculate the expected cost for each of the 243 portfolios, using equation (14), then identify non-dominated sets using the simple cull algorithm introduced by (Yukish 2004).

III.2. Results

III.2. a. Applying Belief Dominance to Portfolios

Out of the 243 possible portfolios, thirteen are non-dominated across the four probability distributions. Table 3 shows the non-dominated portfolios. They are listed in ascending order of the expected cost for the Combined distribution. The first five columns provide the definition of the portfolios by showing the investment level in each technology. The last four columns show the objective value under the four different probability distributions. The objective values are color coded, with the highest cost in each column the darkest red.

Portfolio	Technologies					R&D	Objectives ENPV(Cost in billions of $2005)			
	Solar	Nuc	BF	BE	CCS	($millions)	Combined	Harvard	FEEM	UMass
1	Low	High	High	High	Mid	234	20736	21770	24327	15509
2	Low	Mid	High	High	Mid	75	20768	21654	24188	15720
3	Low	High	Mid	High	Mid	218	20838	21929	24525	15301
4	Mid	High	High	High	Mid	237	20889	21588	24345	15813
5	Low	Mid	Mid	High	Mid	59	20912	21806	24434	15213
6	Mid	Mid	High	High	Mid	78	20922	21513	24163	16162
7	Mid	High	Mid	High	Mid	220	21084	21741	24548	15509
8	Low	High	Low	High	Mid	215	21135	21417	24307	20029
9	High	Mid	Low	High	High	239	21136	21325	22747	20003
10	Mid	Mid	Mid	High	Mid	61	21144	21659	24379	15528
11	High	High	Low	High	High	398	21320	21581	22901	19324
12	Low	High	Mid	High	Low	206	21334	22744	25468	15153
13	Low	Mid	Mid	High	Low	47	21491	22671	25442	15142

Table 3 Non-dominated portfolios. Columns 2-6 report the R&D investment level for each technology, Low, Mid or High. Column 5 is annual R&D investment. The last 4 columns report the Expected NPV of total abatement costs plus investment cost associated with each of the portfolios under the four sets of beliefs. Higher costs are emphasized by darker red colors.

Portfolio 1 is optimal under *Combined* distribution; Portfolio 9 is optimal under both Harvard and FEEM distributions, and is also is the Maxmin solution; Portfolio 13 is optimal under the UMass distribution and is also the solution to Maximax; Portfolio 6 is the MiniMax Regret solution. Letting α vary between 0 and 1, we find the α-Maxmin optimal portfolios as 9, 11, 2, 5, 13, progressively increasing the ambiguity aversion. If we simply give equal weight to the four elicitation beliefs, the optimal portfolio is portfolio 2; if we give equal weight to FEEM, Harvard, and UMass, the optimal portfolio is 5. We performed a KMM analysis, using an exponential ambiguity aversion function, with an ambiguity tolerance parameter similar to a risk tolerance parameter in an exponential utility function.[10] Only two portfolios are optimal across the range of values for this parameter: for ambiguity tolerance below 5,023 billion the optimal is the Maxmin portfolio, i.e. Portfolio 9; for ambiguity tolerance above 5,024 billion, the optimal is Portfolio 2.

Table 4 summarizes these results. Note that even considering a wide range of Robustness concepts and variations within those, the non-dominated set contains a number of portfolios which were not uncovered by these other methods, namely, 3, 4, 7, 8, and 12.

Table 4 List of non-dominated portfolios highlighting which are solutions to robustness concepts. KMM uses an exponential ambiguity aversion function, maximizing the cost subtracted from 26,000; cut off for high ambiguity tolerance is 5024 billion. Shaded rows are not solutions to any of the robustness concepts considered.

Portfolios	Robustness Concepts			
	SEUa	***a-Maxmin***	***Minmax Regret***	***KMM (equal weights)***
1	*Combined* distribution			
2	Equal weight	α = 0.7		Higher Ambiguity Tolerance
3				
4				
5	Equal weight on *Harvard*, *FEEM*, *UMass*	α = 0.1 … 0.6		
6			Minmax Regret	
7				
8				
9	*FEEM*, *Harvard*	α = 0.9, 1 (Maxmin)		Lower Ambiguity Tolerance
10				
11		α= 0.8		
12				
13	*UMass*	α = 0 (Maximax)		

[10] We maximized (26,000 – Cost in table 3).

III.3.b Insights into Individual Alternatives

We can use the results on belief-non-dominated portfolios to derive some robust results among the individual technologies. In Table 3, we see two technologies with robust results. Bio-electricity has a high investment in every non-dominated portfolio, so Bio-Electricity-High is in the core. This technology appears to be good regardless of what probability distribution is used to evaluate it. Nuclear has either a Mid or High investment in every non-dominated portfolio, so Nuclear-Low is excluded. In this proof of concept, given the use of the GCAM model and the choice of the 2°C climate target, it is robust to invest in nuclear at least at the mid- level, regardless of the probability distribution used.

Given these insights, decision makers could incorporate other concerns to identify an overall portfolio investment. For example, if budgets were tight, Portfolio 13, with the lowest budget, could be chosen. If nuclear is controversial, it could be funded at mid levels.

Although providing multiple solutions is one of the advantages of this method, it may still be the case that a decision maker calls for one individual recommendation, rather than a set of non-dominated portfolios. A possibility would be to use the Core Index in order to identify the portfolio made up of individual technology investments that have the highest level of agreement. Recall that the Core Index is the ratio of the number of non-dominated portfolios that contain a project with the total number of non-dominated portfolios. For example, among all non-dominated portfolios in Table 3, Solar Low and Nuclear High each have a Core Index of 7/13=0.54. There is the least agreement among Biofuels, with the most common investment being Biofuels Mid with a CI of 0.46; and, after bio-electricity, the most agreement among CCS, with CCS Mid having a CI of 0.69. Also of note is that both the high and low CCS investments are associated with very high costs for at least one of the teams.

Among all belief non-dominated portfolio, the investments with the highest CIs are solar-Low, Nuclear-High; Biofuels-Mid; Bioelectricity-High; CCS-Mid. This corresponds to portfolio 3. Interestingly, this is one of the portfolios that is not a solution to any of the individual robustness concepts.

These results are conditional on a specific choice of climate stabilization goal and on the model used to represent technology implications for society. For example, these results depend on the model used to assess the societal costs of reducing emissions (in this case, GCAM); ideally an analysis would include multiple models to account for this additional dimension of uncertainty and obtain a more complete set of belief-non-dominated portfolios. Some technological patterns may be common to most models; for example, the key role for Bio-Electricity with CCS has been widely documented as reported in (Clarke et al. 2014). Other results may depend on patterns specific to individual models; for example GCAM tends to be more favorable toward nuclear power than most other IAMs. In order to derive a more robust assessment of the future socio-economic value of technological improvements, it is critical to perform a full analysis using multiple integrated assessment models and multiple climate targets.

As Table 2 shows, the R&D investment amounts vary considerably from technology to technology. For example, the "high" investment amounts for bio-electricity and biofuels are similar to the "mid" amounts for nuclear and CCS. For context, the lowest total investment among the 243 portfolios (with a low investment in each) is \$16 million per year; the highest total investment is \$417 million per year. This compares to a range between \$47-398 million per year among the non-dominated portfolios. The key driver of the size of the budgets among the non-dominated portfolios is the difference between a medium and a high investment in nuclear. The smallest non-dominated portfolio has a very low investment in solar and biofuels, while the largest non-dominated portfolio implies disproportionally larger investments in CCS and Nuclear.

IV. Flexibility of the Framework

Section III presented a proof of concept of a new method to aid decision processes in the face of deep uncertainty and conflicting beliefs. Here we illustrate the flexibility of this framework.

We note that there are many different types and sources of deep uncertainty in general, and specifically in the climate change world. In this paper, we have addressed multiple beliefs about one specific type of uncertainty: uncertainty over well-defined parameters (such as technology costs) represented by probability distributions. Another type of uncertainty is sometimes called "model uncertainty," and refers to the uncertainty that is derived from the representation of processes in models. For example, in our analysis we have employed a single specific IAM, the GCAM model, to translate technology parameters into societal costs and benefits. There exist a variety of IAMs that could be employed to provide the same analysis; this would likely result in different rankings over portfolios.

Our framework of Belief Dominance is flexible enough to allow for considering different models as a source of different beliefs. Reconsider our model presented in equation (12). Let m represent a particular model. The objective function becomes:

$$\mathrm{H}(\mathbf{x};\tau,m)=\int U(\mathbf{x},\mathbf{z},m)f_{\tau}(\mathbf{z};\mathbf{x})\,dz$$

where

$$U(\mathbf{x},\mathbf{z},m)=C(\mathbf{z},m)+\kappa B(\mathbf{x})$$

Individual values of the abatement cost, C, depend on the outcomes of technological change, $\boldsymbol{z}$**,** and on the model used to estimate them, m . Thus, the combination of a particular distribution over $\boldsymbol{z}$ and a particular IAM produces a particular distribution over total abatement costs.

A portfolio $\boldsymbol{x}$ *belief dominates* $\boldsymbol{x}'$ if

$$\mathrm{H}(\mathbf{x};\tau,m)\geq \mathrm{H}(\mathbf{x}';\tau,m)\ \ \forall\,\tau,m \tag{15}$$

Moreover, our framework is not limited to traditional portfolio problems, such as technology R&D. It can be applied more broadly to a wide range of applications, including a broader interpretation of climate change policy. Individual alternatives can include not only investments into energy technologies, but other technology policies, such as standards or subsidies, as well as other climate change policies, such as carbon taxes, carbon caps, international trade agreements, or near-term adaptation decisions. Uncertainties can include not only technological progress, but damage uncertainty, socio-economic uncertainties, and model uncertainty. Finally, this framework can be applied to other domains besides climate change, such as combatting terrorism or designing a healthcare system.

V. Conclusions

We present Robust Portfolio Decision Analysis as a promising approach to deal with problems of decision making in the face of deep uncertainty, situations characterized by conflicting sources of information. The two key aspects of our approach are that (1) it allows us to define non-dominated portfolios of strategies or decisions, in the face of multiple, conflicting beliefs over relevant outcomes; and (2) it allows us to derive insights and implications about individual strategies by looking at the portfolio-level results. We show that our method encompasses and generalizes many existing robustness concepts.

We demonstrate our approach on the specific case of designing a portfolio of publicly-funded research and development investments in future energy technologies. Applying our method, we uncover multiple portfolios which are not solutions to any of the commonly used robustness concepts. This has value in avoiding the use of a somewhat arbitrary rule to mathematically resolve disagreement and in providing the decision maker with flexibility to explore trade-offs which are difficult to model. Moreover, it provides information about individual strategies which are found in all of the portfolios. In our example, we find some common ground among the divergent expert beliefs, namely that a high investment in bioelectricity, and at least a mid-investment in nuclear, are robust, given the specific climate goal and integrated assessment model used for the analysis. Policy negotiators could build on this common ground, incorporate non-quantifiable criteria, and perhaps commission more information where it is most likely to impact decisions, such as into biofuels.

In this paper we have focused on the situation in which there is deep uncertainty, but objectives are well-defined (e.g. total costs in our example). In reality, many of the problems involving deep uncertainty (including climate change) also involve multiple stakeholders with conflicting objectives; this group of problems is often called "Wicked" (Churchman 1967). In order to address both of these aspects of wicked problems, our framework would need to be extended to include analysis of multiple objectives. The concepts we introduce here may inform the MORDM framework, allowing for the visualization of trade-offs in both objectives and beliefs. Alternatively, the Robust Portfolio Decision Analysis framework could be extended to include methods from MORDM to identify Pareto optimal or Pareto Satisficing alternatives.

This method presents innovative and useful elements that can generate important steps forward in the decision-making approach to several societal problems that are affected by deep uncertainty. It does not ignore knowledge, nor does it ignore uncertainty and disagreement. It has promise to provide analytically rigorous support to decision making under deep uncertainty while preserving flexibility for decision makers. The combination of finding common ground and preserving flexibility may help to catalyze difficult dialog.

Appendix

A.I. Definition of robustness concepts

Example 3 (maximax, MM): Define

$$\bar{V}(x) \equiv \max_{f \in \Phi} V(x, f) \qquad \text{(XVI)}$$

The Maximax decision problem can be written

$$\max_{x \in X} \max_{f \in \Phi} \int U(x, z) f(\mathbf{z}; x) d\mathbf{z} = \max_{x \in X} \bar{V}(x) \qquad \text{(XVII)}$$

The set of solutions which are optimal under Maximax is defined as follows:

$$X^{MM} = \left\{ x^0 \in X \mid \bar{V}\left(x^0\right) \geq \max_{x \in X} \bar{V}(x) \right\} \qquad \text{(XVIII)}$$

Example 4 (⍰-Maxmin, ⍰Mm). The ⍰-Maxmin decision problem can be written as

$\max_{x \in X} \left[\alpha \underline{V}(x) + (1-\alpha) \bar{V}(x) \right]$ where $\alpha \in [0,1]$ is a fixed value representing the level of ambiguity aversion. The set of solutions optimal under ⍰-Maxmin is defined as follows:

$$X^{\alpha Mm} = \left\{ x^0 \in X \mid \alpha \underline{V}\left(x^0\right) + (1-\alpha) \bar{V}\left(x^0\right) \geq \max_{x \in X} \left[\alpha \underline{V}(x) + (1-\alpha) \bar{V}(x) \right] \right\} \qquad \text{(XIX)}$$

Example 5 (Minmax Regret, mMR); Define regret $R(x, f) \equiv \left[\max_{\hat{x} \in X} V(\hat{x}, f) - V(x, f) \right]$.

The Minmax Regret problem can be written as

$$\min_{x} \max_{f} R(x, f)$$

The set of optimal solutions is defined:

$$X^{mMR} = \left\{ x^0 \in X \mid \max_{f} R\left(x^0, f\right) \leq \min_{x} \max_{f} R(x, f) \right\} \qquad \text{(XX)}$$

A.II. Proof of Lemma 1

Lemma 1: Belief non-dominance satisfies the transitive property: $x^A \succ x^B \text{ and } x^B \succ x^C \Rightarrow x^A \succ x^C$.

Proof: Assume x^A belief dominates x^B and x^B belief dominates x^C. This implies:

$$\int U\left(x^{A};\mathbf{z}\right)f\left(\mathbf{z};x^{A}\right)d\mathbf{z}\geq\int U\left(x^{B};\mathbf{z}\right)f\left(\mathbf{z};x^{B}\right)d\mathbf{z}\ \forall f\in\Phi$$
$$\exists f^{A}st\int U\left(x^{A};\mathbf{z}\right)f^{A}\left(\mathbf{z};x^{A}\right)d\mathbf{z}>\int U\left(x^{B};\mathbf{z}\right)f^{A}\left(\mathbf{z};x^{B}\right)\mathrm{d}\mathbf{z}\text{ by definition of belief dominance}$$
$$\int U\left(x^{B};\mathbf{z}\right)f\left(\mathbf{z};x^{B}\right)d\mathbf{z}\geq\int U\left(x^{C};\mathbf{z}\right)f\left(\mathbf{z};x^{C}\right)d\mathbf{z}\ \forall f\in\Phi$$
$$\Rightarrow\int U\left(x^{A};\mathbf{z}\right)f\left(\mathbf{z};x^{A}\right)d\mathbf{z}\geq\int U\left(x^{C};\mathbf{z}\right)f\left(\mathbf{z};x^{C}\right)d\mathbf{z}\forall f\in\Phi\text{ (combining line 1\&3 by transitivity of inequality) and}$$
$$\int U\left(x^{A};\mathbf{z}\right)f^{A}\left(\mathbf{z};x^{A}\right)d\mathbf{z}>\int U\left(x^{C};\mathbf{z}\right)f^{A}\left(\mathbf{z};x^{C}\right)\mathrm{d}\mathbf{z}\text{ (combining line 2 \& line 3)}$$
(XXI)

Therefore, x^A belief dominates x^C: the concept is transitive. **QED**

A.III. Proof of Lemma 2

Proof: (Maximax) Since Φ is compact, we can choose $\hat{f}$ such that $V(x, f)$ is maximized. We then have:

$V\left(x', \hat{f}\right)\geq V\left(x, \hat{f}\right)$ by definition of Belief Dominance

$V\left(x, \hat{f}\right)=\max_{f\in\Phi}V\left(x, f\right)\geq\max_{\hat{x}\in X}\max_{f\in\Phi}V\left(\hat{x}, f\right)$ by definition of maximax. Thus, these together imply that $x'\in X^{MM}$ **QED**

Proof: **:** (?-Maxmin) Since Φ is compact, we can choose belief f' such that $V(x', f)$ is minimized. We then have:

$$\underline{V}\left(x'\right)=\min_{f\in\Phi}V\left(x', f\right)=V\left(x', f'\right)\geq V\left(x, f'\right)\text{ by definition of belief dominance} \quad \text{(XXII)}$$

$$V\left(x, f'\right)\geq\min_{f\in\Phi}V\left(x, f\right)=\underline{V}\left(x\right)\text{ by definition of the minimum} \quad \text{(XXIII)}$$

Similarly, we can choose belief $\hat{f}$ such that $V(x, f)$ is maximized. We then have

$$\overline{V}\left(x\right)=V\left(x, \hat{f}\right)\leq V\left(x', \hat{f}\right)\text{ by definition of belief dominance} \quad \text{(XXIV)}$$

$$V\left(x', \hat{f}\right)\leq\max_{f\in\Phi}V\left(x', f\right)=\overline{V}\left(x'\right)\text{ by definition of the maximum} \quad \text{(XXV)}$$

We can multiply inequalities (X)-(XI) through by α and inequalities (XII)-(XIII) by $(1-\alpha)$ and add, resulting in

$$\alpha \underline{V}(x') + (1-\alpha)\bar{V}(x') \geq \alpha \underline{V}(x) + (1-\alpha)\bar{V}(x) \qquad \text{(XXVI)}$$

Thus, $x' \in X^{\alpha Mm}$ **QED**

Proof: (Minmax Regret: mMR) : Since Φ is compact, we can choose f' that maximizes regret for x', i.e. such that $R(x', f)$ is maximized.

$$\begin{aligned}
&\max_f R(x', f) = \max_f \left[\max_{\hat{x} \in X} V(\hat{x}, f) - V(x', f) \right] \\
&= \max_{\hat{x} \in X} V(\hat{x}, f') - V(x', f') \ \{\text{by defintion of } f'\} \\
&\leq \max_{\hat{x} \in X} V(\hat{x}, f') - V(x, f') \ \{\text{since } V(x', f') \geq V(x, f') \text{ by belief dominance}\} \\
&\leq \max_f \left[\max_{\hat{x} \in X} V(\hat{x}, f) - V(x, f) \right] \ \{\text{by definition of the maximum}\} \\
&= \max_f R(x, f)
\end{aligned} \qquad \text{(XXVII)}$$

Thus, $x' \in X^{mMR}$

Proof (KMM): $x' \succ x \Rightarrow V(x', f') \geq V(x, f') \Rightarrow \Psi(V(x', f')) \geq \Psi(V(x, f'))$ for any increasing function Ψ $\Rightarrow \int_\Phi \Psi(V(x', f'))d\pi \geq \int_\Phi \Psi(V(x, f'))d\pi$ since the integral is linear and probabilities p are positive. Therefore

$$x' \in X^{KMM} = \left\{ \hat{x} \in X \mid \int_\Phi \Psi(V(\hat{x}, f))d\pi \geq \max_{x \in X} \int_\Phi \Psi(V(x, f))d\pi \right\}$$

(SEUa):By corollary 2 in KMM, SEUa reduces to KMM when Ψ is linear.

A.IV. Figure A1: representation of probability distributions

Here we reprint a figure from (Baker, Bosetti, Anadon, Henrion, and Aleluia Reis 2015), illustrating the standardized data set of four sets of beliefs over eight technology parameters.

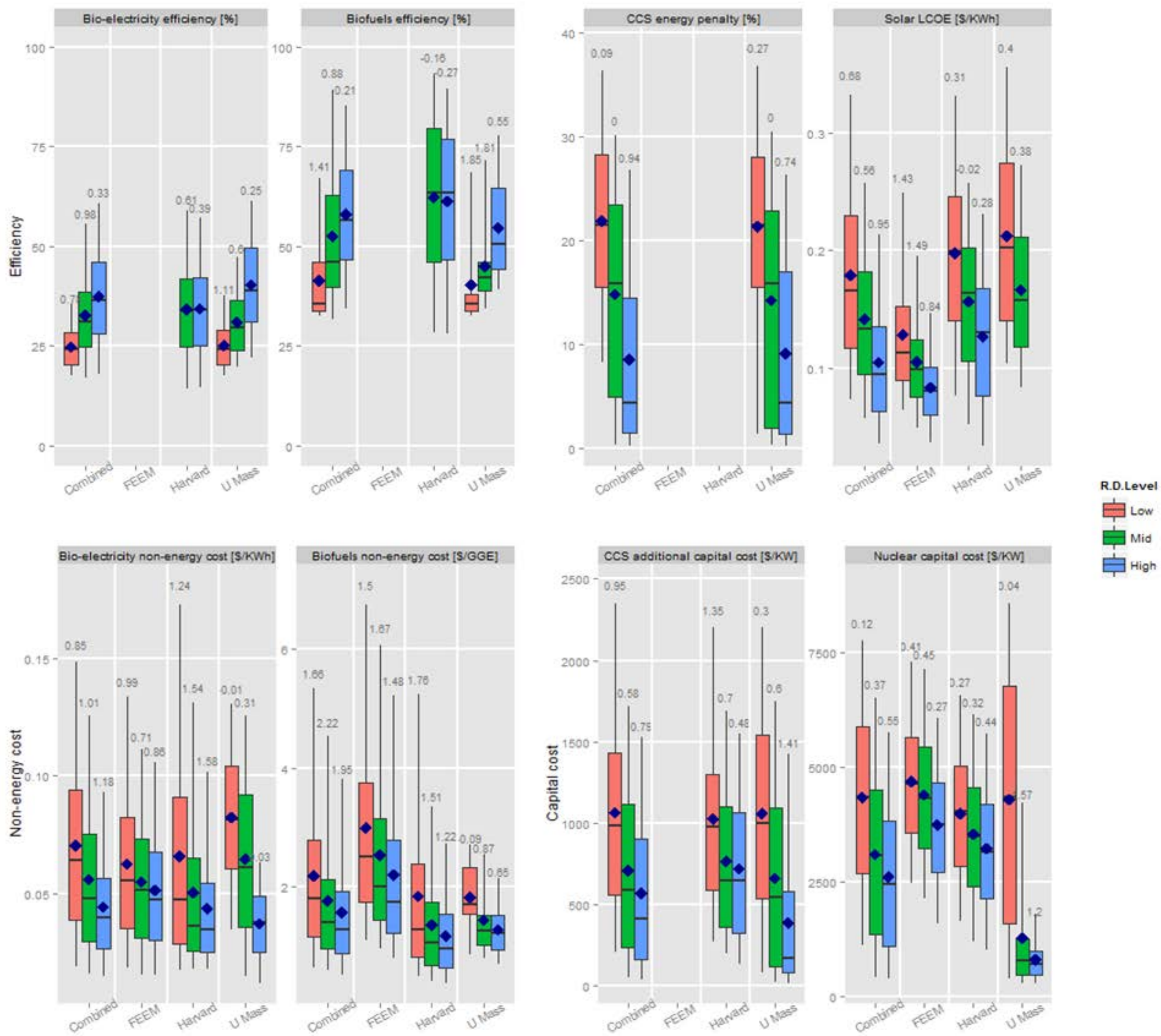

Figure A1: Reprinted from [Baker et al 2015] (need permission). **:** 2030 costs and efficiency elicitation results across studies and R&D levels. We show the Combined distribution of the three studies using equal weights ("Combined"), the FEEM aggregate, the Harvard aggregate, and the UMass aggregate and technologies by R&D level (Low, Mid, and High). The box plots show the 5th, 25th, 50th, 75th, and 95th percentiles for each of the distributions, the diamond the mean value, and the black number the skewness of the distribution.

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