Measurement of the B$^+$ meson lifetime

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Abstract: This research aims to measure the mean lifetime for the B$^+$ meson, a composition of a u and a $\bar{b}$ quark, by using experimental data obtained from LHCb. Making use of Monte Carlo (MC) generated data and simulated data, which includes the effects of a detector, we can study the effects over the data in order to obtain the physical parameters underlying the process. In order to find the mean lifetime τ of the B$^+$ meson we will proceed by two different ways, and we will make use of the simulated Monte Carlo data to prove the validity of both methodologies.

I. INTRODUCTION

An unstable particle against a given interaction has a certain probability of decaying into other less-massive particles per unit of time. This is the case of the B$^+$ meson, composed by an *up* quark (u) and a *bottom* anti-quark ($\bar{b}$), and as it is an unstable particle it can therefore decay as follows:

$$B^+ \longrightarrow J/\psi + K^+$$

after such decay, J/ψ will also decay in two muons, $J/\psi \to \mu^+ + \mu^-$ and the detector will be able to detect both the muons and K$^+$. From those reconstructed muons the J/ψ vertex is found, which due to its decay under the strong interaction, its creation and decay take place at a same point, which is exactly the vertex of decay of the B$^+$ meson. Combining the measured J/ψ and the K$^+$ we can derive the physical quantities of the B$^+$ such as momentum or, combining it with the beams vertex collision, its lifetime. By considering a great number of events we will end up with a distribution for such quantities.

A decay can also be described by the decay width Γ, which is related to the width of the cross-section curve as a function of the energy. It is also closely related to the mean lifetime as $\Gamma \propto \frac{1}{\tau}$, and is a measure of the probability that a specific decay process occurs in an interval of time. From here we can also define τ as a measure of the time that the particle takes to decay, i.e. $\frac{1\text{ps}}{\tau[\text{ps}]}$ gives us the probability that the particle has decayed in any given picosecond. Therefore, the decay time for unstable particles follows an exponential distribution, Eq. (1).

$$f(t) = A\exp(-t/\tau) \qquad (1)$$

LHCb is one of the detectors that can be found in the LHC at CERN, and it focuses its research on the b quark. By means of proton collisions, $b - \bar{b}$ quark pairs are produced and will compose other particles, as the B$^+$ meson that we will study. In the detector, once the collision between two protons has taken place, the products of such collision fly in any direction. As we want to detect as most events as possible, the design of the LCHb is made in order that most of the mesons fall in the covered region. In any case, there is a loss of some of the particles created. Such effect - that the detector does not encompass all the space around the collision - is represented by the acceptance $\mathcal{A}$. It does also include the efficiency of the detector to reconstruct the identified events. Another relevant effect is the resolution, $\mathcal{R}$, which is the effect that tells us about the precision of the detector. We will study both effects in depth in the corresponding sections. Because of all these effects, the physical distribution of the studied quantities behind the process is modified and what we can see from the results after the detector does not correspond with an exponential decay. Such effects modify the exponential distribution as:

$$\mathcal{P}(t) = [\text{Physics}(t') \times \mathcal{A}(t')] \otimes \mathcal{R}(t, t') \qquad (2)$$

where "Physics" means the exponential decay, Eq. (1). It can be seen in Eq. (2) how resolution applies both on the physics as well as on the acceptance; through the development of the project, we will see which is the effect of the resolution on acceptance and if it is important to keep this order, or if we could take the acceptance out of the convolution.

II. VERIFICATION WITH MONTE CARLO GENERATED DATA

First of all, we begin with generated MC data, which has to reflect what we expect from theory and it only contains the physical information from the p-p collision and decay of the created B$^+$ particles, i.e. an exponential distribution for the mean lifetime of different B$^+$ mesons.

This data does not contain any effect due to the detector, so it will help us derive afterwards, by comparing to the one passing through a simulated detector, one of the effects that may change our measurements and must be corrected in order to get the physical values behind our observations: the acceptance $\mathcal{A}$.
Now, from the MC data shown in Fig. 1 we can fit an exponential function as Eq. (1) and get its corresponding value for the mean lifetime, τ.

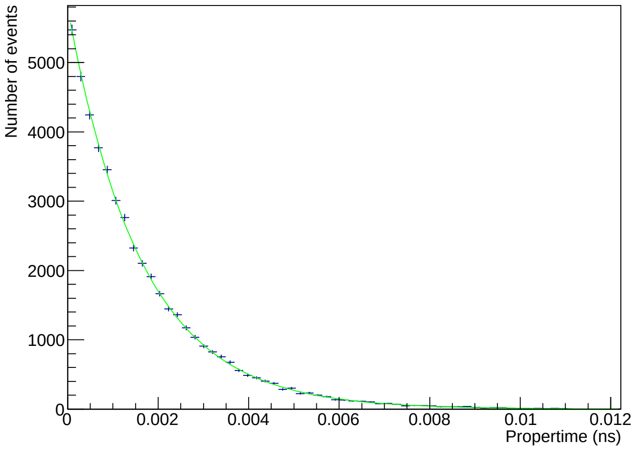

FIG. 1: Lifetime distribution for the Monte Carlo generated data. It has not been shown the tail of the exponential in order to see clearer the fit of the data.

To generate such data, the value introduced for the mean lifetime is the experimental one [2], which should be reobtained when fitting the data. And the result obtained by means of the fit is: $\tau = 1.636 \pm 0.007$ ps. Comparing this result with the experimental world average - the one used to generate that data - from [2], $\tau_{exp} = 1.638 \pm 0.004$ ps, we can see they compare well.

III. ACCEPTANCE

The proper time acceptance $\mathcal{A}$ is one of the most relevant effects of the detector on our measurements. It reflects the number of events falling into the region covered by the detector, so they can be measured, and it also considers the efficiency when reconstructing and selecting the events. As the particle decays, the resulting particles can travel in any direction, and due to mechanical limitations and high cost of materials, the detector can only cover a given solid angle of the whole space, leading to a loss of the detection of some of the particles. In addition, the events should be reconstructed and separated from possible background. Both effects will reduce the B$^+$ signal sample, and this lost is represented by the acceptance that can be computed as the ratio between the proper time distribution of the reconstructed and selected events (t^{true}) and the Monte Carlo generation proper time (t^{MC}). This implies its calculation as follows, [1]:

$$\mathcal{A} = \frac{N(t^{true})}{N(t^{MC})} \tag{3}$$

that is, it is the ratio between the generated candidates and the reconstructed ones as a function of the lifetime. In Figure 2 it can be seen the result of such ratio and also the fitted function we use to describe it.

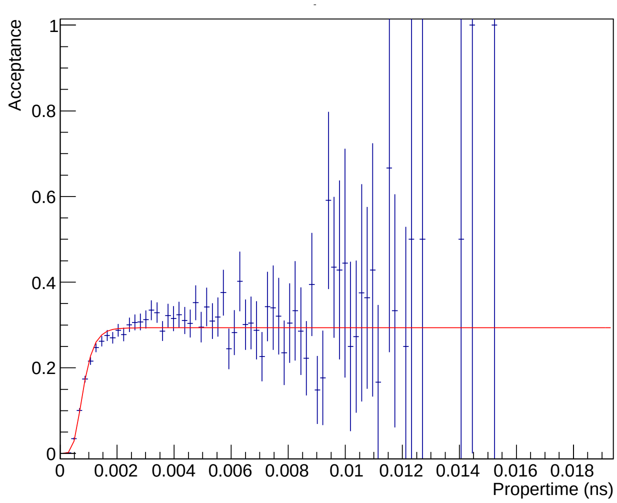

FIG. 2: Proper time acceptance, with the best fitted function achieved. It can be seen how it presents an increase of the function's value at low proper times, while it remains constant for larger values of time.

The $\mathcal{A}$ seems to behave as a sigmoid function, so we use a similar function to parametrize its dependence on τ, Eq. (4).

$$\mathcal{A}(t) = p_0 \exp\left(-p_1 \exp(-p_2 t)\right) \tag{4}$$

with $p_0 > 0$, so it is a positive defined function for any t. The results for such parameters are shown in Table I.

Parameters	B$^+ \to$ J/ψ + K$^+$
p_0	0.294 ± 0.004
p_1	14.4 ± 1.0
p_2	3780 ± 140 ns^{-1}

TABLE I: Fitted parameters for the acceptance function.

Even though being this the best fitting function we could find, by looking at Fig. 2 it can be seen that there is a small region between $t \in [1.5, 4.0]$ ps where the fit is not so accurate; this may lead afterwards to a bias when using the function to extract the life-time from real data. It can also be seen from Fig. 2 how for very large times the acceptance has been taken as a constant value (around 0.29).

Due to the difficulty when finding a good fitting function, we will see how the fit of the acceptance is the most relevant source for systematic errors, and that chosing a function or another one can lead to different values for τ, which will be studied in depth in following sections.

IV. RESOLUTION

The other relevant effect is the proper time resolution $\mathcal{R}$. While acceptance has to do with the geometrics and space covered by the detector as well as its effectiveness to identify the events and reconstruct them, the resolution gives information about the precision of the measurements, which is limitted. To get such information, the proper time resolution can be computed as Eq. (5), which involves comparing the real value and the one that we measured, in order to see how our measurements differ from the real ones, [1] and [3].

$$\Delta t = t^{rec} - t^{true} \tag{5}$$

and so, we expect that the difference between the generated and the reconstructed data is not so high, and that it is close to zero.

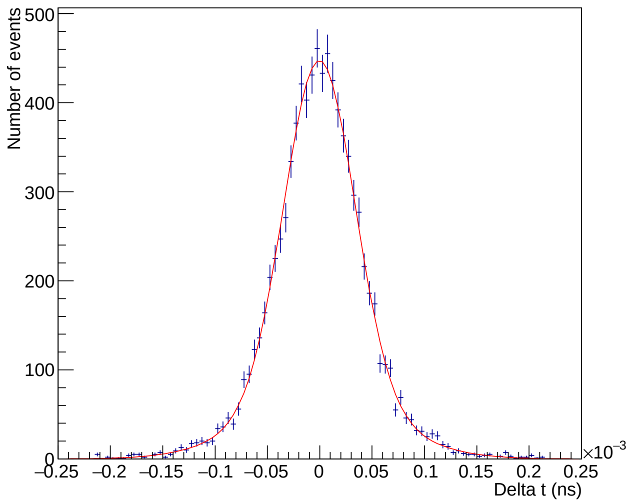

FIG. 3: Proper time resolution, with its fitting function.

From Fig. 3 it can be seen that it behaves as a guassian centered close to zero. However, a simple gaussian does not fit the function very well, so the solution is to consider a sum of two gaussian with the same mean value but different widths, so our resolution function has an expression as Eq. (6).

$$\mathcal{R}(t) = p_0 \exp\left(-\frac{(t-p_1)^2}{2p_2^2}\right) + p_3 \exp\left(-\frac{(t-p_1)^2}{2p_4^2}\right) \tag{6}$$

The results for the parameters are gathered in Table II.

Parameters	B$^+ \rightarrow$ J/ψ + K$^+$
p_0	64 $\pm$ 14
p_1	(-3 $\pm$ 4)$\cdot10^{-7}$ ns
p_2	(6.8 $\pm$ 0.4)$\cdot10^{-5}$ ns
p_3	384 $\pm$ 13
p_4	(3.36 $\pm$ 0.10)$\cdot10^{-5}$ ns

TABLE II: Fitted parameters for the resolution function.

A. Dependence of the parameters with τ

We can also study if the resolution has some dependency on the proper time, i.e. if the parameters of the resolution change sharply for different intervals of the proper time t^{true} or if they present a similar value through all the range. To do so, we study the proper time resolution distribution for ranges of t^{true} and fit a function as Eq. (6), from here we get the mean value μ and the lowest width σ, which describes well the central width - while the largest value is needed to fit the lowest part of the distribution - and we compare it with the values found for the entire range.

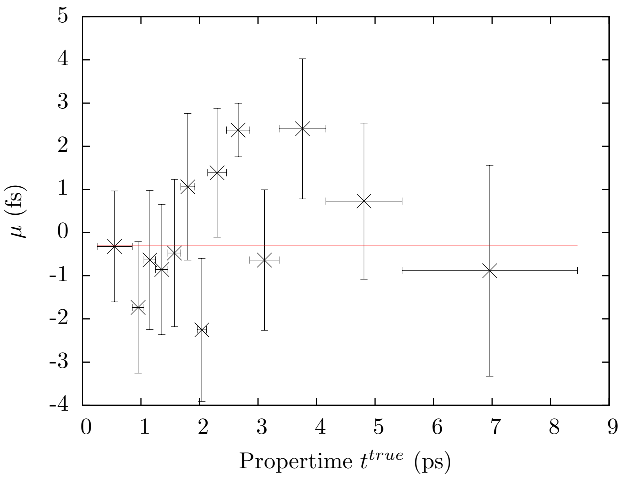

FIG. 4: Variation of the mean value μ of the resolution as a function of the true proper time for the process. In red it can be seen the value for the whole interval.

We can see in Fig. 4 that the mean value of the gaussians for different intervals of t^{true} is not so different from the one considering the whole interval, with all values oscillating around zero. Due to the results shown above, we make the decision not to consider any of this dependence, as there is not a great change between the results in the whole interval, and the dependence on different ones.

For what concerns the width of the resolution, we can see in Fig. 5 that it increases with proper time. By simplicity and given that the obeserved dependence is restrained to a not so wide range ($\sigma \in [28, 43]$ fs) we will neglect such variation when making the corrections. However, and as a larger width makes the exponentials of Eq. (6) smaller, this may imply that the resolution has a stronger effect at lower proper times.

V. FITTING METHODS

Once the functions of acceptance and resolution are found, we proceed with the correction of both effects to our data. In order to do it, the data has to be fitted with a function as Eq. (2) where all the parameters should be

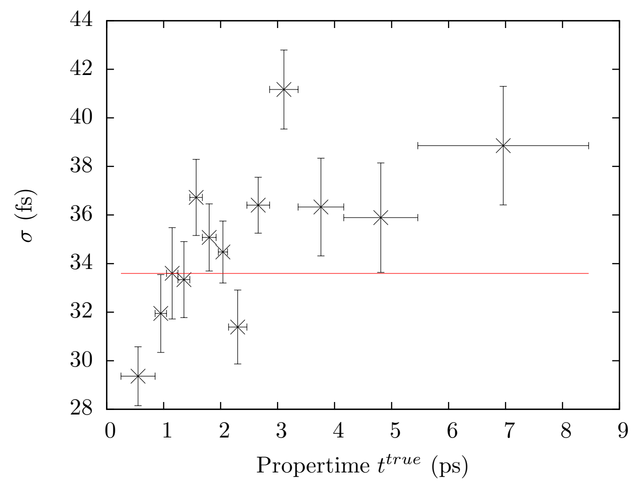

FIG. 5: Dependence of the σ width of the resolution as a function of t^{true}. In red there is specified the value when considering the entire interval for proper times.

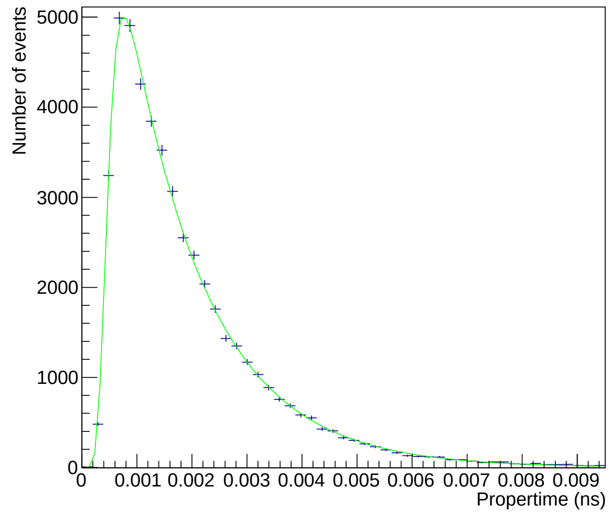

FIG. 6: Real data distribution for the decay time, with the fitted function in green. It is shown the interval from $t \in [0, 9.5]$ ps to see how the fit coincides with the experimental points. The rest of the interval is the flat tail of an exponential.

fixed with the ones found, except the parameters of the exponential decay, which are left free in order to find τ. To find such value, we will proceed by two different ways:

A. Correction of all effects simultaneously: in this approach the real data without modifications will be taken and fitted by means of Eq. (2).

B. Previous correction of acceptance: here, acceptance will be substracted from real data and then the resulting data will be fitted by a function of an exponential convoluted with the resolution. This approach will help us to know whether the acceptance and resolution are related or not, and how.

A. Simultaneous correction of acceptance and resolution on real data

Fixing the parameters of both the acceptance and the resolution when proceeding with the convolution, we obtain a function that is impossible to give a convergent fit to our data. This may be due to the lack of correspondence between the acceptance found by means of Monte Carlo data and the simulations with the one of the real data, so it leads us to think that the acceptance in the real data may have a different behaviour, i.e. that the Monte Carlo acceptance does not reproduce the data acceptance. To try to use Eq. (2) to fit our data, we decide to set free the parameters of the acceptance and proceed with the fit. From it we obtain the results shown in Fig. 6, where we can see how the fit converges and fits correctly the experimental points.

From the fit shown in Fig. 6 we obtain $\tau = 1.449 \pm 0.008$ ps, where this error only covers the statistical error. This great difference may be caused because of the non-correspondence between the acceptances of different sets of measurements. We set the parameters of the acceptance free; thus, not only the parameters of the exponential are modified in order to get the best fit, but also the ones of the acceptance, leading to a combination of all of them to get the best fit. Another probable reason of such error is that the function used is not the correct one; we will deal with this possibility in Sec. **VI**.

B. Correction of resolution to acceptance-corrected data

The other method we can take to get τ is the following: we have the real data and we make the correction of the acceptance effect by dividing the decay-time distribution from the data by the acceptance distribution bin per bin and propagating the corresponding errors. From here, we find that the lower proper time region presents very high values, and this is due to the fact that, in that interval, the acceptance has a close-to-zero value. As there is no way to know information from such low values, we will not be able to recostruct the physical information there, and so we will not take into account this region when fitting the distribution to find τ. To study such parameter we will restrain to the interval $t \in [1.1, 8.0]$ ps.

If we fit the resulting histogram of such division in the interval we said above with an exponential distribution as Eq. (1), we find: $\tau = 1.32 \pm 0.02$ ps, where this error only covers the statistical error. One must think that here we have only corrected the acceptance, but it stills contains the effects of the resolution. We proceed now with its correction by taking a modified expression of Eq. (2), and considering Eq. (7).

$$\mathcal{P}'(t) = \text{Physics}(t') \otimes \mathcal{R}(t, t') \tag{7}$$

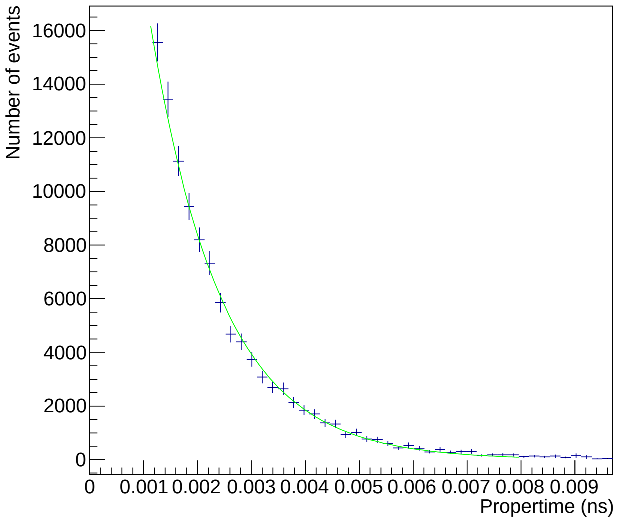

FIG. 7: Real data distribution for the decay time with the effect of the acceptance previously corrected, and the fitted function plotted in green.

If we do now fit the histogram of the division said above on the same interval with a function as Eq. (7) and letting the parameters of the exponential free, we will obtain the fit shown in green in Fig. 7. Here we obtain $\tau = 1.32 \pm 0.02$ ps, which is the same as we obtained without considering the effect of the resolution. From here we can infere that the resolution has not affected the result and that it is only effective to the results when acting on the acceptance, i.e. we cannot take $\mathcal{A}$ out of the convolution of Eq. (2), because we see here that the acceptance and resolution are related. Thus, we can conclude that this method is not the right one, and that we should proceed with the one explained in Sec. **V.A**.

VI. SYSTEMATIC ERRORS

Given the discrepance of our result in Sec. **V.A** when comparing with [2], we may think that the acceptance fitting function might not be the proper one. We can therefore repeat the same procedure as in Sec. **V.A**, but considering another function − also a sigmoid-like function − that fitted well the $\mathcal{A}$, Eq. (8). From here we obtain $\tau^* = 1.466 \pm 0.007$ ps, whose error is the statistical one. We see both results are quite similar, so maybe there is another source of systematic error. We can estimate such systematic error considering different ranges for the fit appearing in Fig. 6. When proceeding with the fit for different ranges, we get different values in a wide range for τ, and by considering the further values from the one found using the whole range when fitting the function, we get that the systematic error is of 0.15 ps, being it the main source of error.

$$\mathcal{A}^* = \frac{1}{1 + p_0 \exp\left(-p_1 t\right)} + p_2 \tag{8}$$

VII. CONCLUSIONS

In this work we have defined and studied the effects modifiying the measurements of the mean lifetime in a detector: the acceptance and the resolution. Using data from LHCb of events involving B$^+$ mesons, we are able to extract the functional forms of both $\mathcal{A}$ and $\mathcal{R}$ by using MC and simulated data. Concerning the resolution - which has been fitted by the sum of two gaussian - we study the dependence of its parameters with the lifetime and conclude that the differences are neglegible, and so we consider the parameters in the whole range. We see also that there is no correspondence between the acceptance found by MC and the one for the real data, which leads us to a non satisfying result: $\tau = 1.449 \pm 0.008$ ps and to the need to find possible systematic errors. Considering a different function for the acceptance, we find that it has not a strong effect on the result, so we try to estimate such error by other means: considering different ranges for the fit, and we find that the systematic error takes a value of 0.15 ps. Thus, the final result is: $\tau = 1.45$ ps $\pm$ 0.01 ps (stat) $\pm$ 0.15 ps (sys), which we find to be compatible with the average value, [2].

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