Holographic Description of 2-Dimensional Quantum Black Holes

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Abstract: In this work we obtain and analyse a black hole solution on the 1-brane of the Randall-Sundrum II braneworld in the 3-dimensional BTZ black hole bulk. Interpreting the results in the light of the conjectured duality between d-dimensional quantum-corrected black hole solutions and black holes on the brane in the AdS_{d+1} braneworld, we have found that there exists a lower bound for the size of a 2-dimensional quantum black hole. At the end, we suggest that this lower bound could be an *informon*, i.e. a remnant of the quantum black hole the existence of which has been proposed in the literature as a possible solution of the black hole information paradox.

I. INTRODUCTION

In the framework of quantum field theory in curved spacetime (QFTCS), there is a natural way of taking into account, at least to a certain order, the effect of quantum fields on the spacetime. More specifically, what one expects is that the back-reaction effects of quantum fields on the gravitational field are governed by the semi-classical Einstein's equations [1]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa_d^2 \langle\psi| T_{\mu\nu} |\psi\rangle, \quad (1)$$

where $|\psi\rangle$ is a state corresponding to a certain configuration of the present quantum fields. The constant κ_d^2 is the fundamental mass scale in d dimensions and it has units of $[\kappa_d^2] = E^{2-d}$ [1]. In the two dimensional case the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ vanishes identically and κ_d^2 becomes dimensionless so we can set it equal to one. In this case, (1) becomes

$$\Lambda g_{\mu\nu} = \langle\psi| T_{\mu\nu} |\psi\rangle \quad (2)$$

and the dynamics of the metric is of purely quantum origin. This fact will be crucial for our work. A black hole solution of (2) with a conformal field theory (CFT) was found in [2]. In the Schwarzschild gauge it reads

$$ds^2 = -(\lambda^2 x^2 + 2\mu|x| - 1)dt^2 + \frac{dx^2}{\lambda^2 x^2 + 2\mu|x| - 1}, \quad (3)$$

where $\lambda^2, \mu > 0$ depend on parameters of the CFT, such as the number of fields. However, in this paper we do not study (3) directly in the framework of QFTCS. Instead, we will take an holographic point of view. Specifically, our contribution consists of studying the properties of a black hole solution on the brane of the AdS_3 braneworld and map the results obtained into the quantum black hole (3), assuming the validity of the holographic conjecture stated in [3]. Explicitly, the conjecture states that

"The black hole solutions localised in the brane in the AdS_{d+1} braneworld which are found by solving the classical bulk equations in AdS_{d+1} with the brane boundary conditions, correspond to quantum-corrected black holes in d dimensions, rather than classical ones."

A braneworld in AdS_{d+1} is a model in which the observable universe, i.e. the universe where we live in, is a d-dimensional slice (the brane) embedded in an asymptotically AdS_{d+1} spacetime (the bulk). While quantum fields are trapped on the brane, gravity can access the bulk. In our particular case, we work on the Randall-Sundrum type II (RSII) [4] braneworld in the 3-dimensional BTZ black hole bulk [5].

II. THE BLACK HOLE ON A 1-BRANE IN BTZ

In this section we obtain a black hole solution on the 1-brane of the RSII braneworld. To do so, we follow a similar procedure to that performed in [6].

A. The BTZ Black Hole

The only asymptotically AdS black hole solution known in 2+1 dimensions is the BTZ black hole, and it plays the role of the bulk in our braneworld. It is a (2+1)-dimensional solution of the Einstein-Maxwell equations, but in this work we consider the non-rotating, neutral version given by

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 d\theta^2 \quad (4)$$

where $F(r) = \frac{r^2}{L^2} - m$ ($r > 0$), L is the AdS_3 length, $\mathcal{M} := m/\kappa_3^2$ is the mass of the black hole being $1/\kappa_3^2$ the 3-dimensional Planck mass and $\theta \sim \theta + 2\pi$. In order to understand some parts of the following subsection, it is important to notice that the singularity at $r = 0$ of (4), hidden by the horizon at $r = \sqrt{m}L$, is not a singularity in the curvature of the spacetime, but a singularity in its causal structure. Indeed, the BTZ black hole is a quotient of AdS_3 by the identification of points by means of a discrete subgroup of its isometry group,

$\{e^{2\pi n\vec{\xi}}\}_{n\in\mathbb{Z}}$, generated in turn by an specific Killing vector, $\vec{\xi}$ [7]. In order to preserve causality after performing the identification, one has to restrict the physical spacetime to be the region of $AdS_3/(P \sim e^{2\pi n\vec{\xi}}P)$ in which $\vec{\xi}$ is spacelike, i.e. $g(\vec{\xi},\vec{\xi}) > 0$. In conclusion, $BTZ = \{AdS_3/(P \sim e^{2\pi n\vec{\xi}}P) | g(\vec{\xi},\vec{\xi}) > 0\}$, and there exists a gauge in which the induced metric on BTZ is (4), $\vec{\xi} = \partial_\theta$, and $r > 0$ correspond to the region $g(\vec{\xi},\vec{\xi}) > 0$. Analytical prolongation beyond $r = 0$ would lead to closed timelike curves because of the identification, and for this reason $r = 0$, that lies at an affine-finite distance, is a physical singularity. It is important to have it clear for the following subsection. The RSII set up in the BTZ bulk is constructed by splitting up the bulk into two separated parts, gluing two copies of one of the parts along a 1-brane and performing a $\mathbb{Z}_2$-symmetry on the metric w.r.t. the brane (a mirror), that is, if $x \sim_{\mathbb{Z}_2} y$ then $g(x) = g(y)$. In the following subsection we write the action of the theory, solve the equations of motion and interpret the solution for the brane.

B. Solution for the Bulk and the Brane

Let us take the convention that greek indices run from zero to two, while latin indices run from zero to one. We shall assume the existence of two charts $\{x^{\pm\alpha}\}$, each at one side of the brane Σ, a chart $\{y^a\}$ on Σ and the first junction condition $[h_{ab}] := h^+_{ab}(\Sigma) - h^-_{ab}(\Sigma) = 0$ in order to have a distributionally well defined curvature, where h_{ab} is the induced metric on Σ. We define the normal vectors $n^{\pm\alpha}$ to point away from Σ into the adjacent space. Notice that under these assumptions the extrinsic curvatures satisfy $K^+_{\alpha\beta}(\Sigma) = K^-_{\alpha\beta}(\Sigma) := K_{\alpha\beta}(\Sigma)$ [8]. Our bulk is present at both sides of the boundary brane Σ, so two Gibbons-Hawking-York boundary terms [9] together with the bulk Einstein-Hilbert action are required for the gravitational part of the action. The 1-brane action is proportional to its world-volume (which is an area, since a 1-brane is a string actually), being the proportionality constant its tension σ. Hence, the action of the theory reads

$$\mathcal{S}[g_{\alpha\beta}, h_{\alpha\beta}] = \frac{1}{2\kappa_3^2}\int d^3x\sqrt{-g}(R + \frac{2}{L^2}) + \\ + \frac{1}{\kappa_3^2}\int_\Sigma d^2y\sqrt{-h}(K^+ + K^-) - \frac{2\sigma}{\kappa_3^2}\int_\Sigma d^2y\sqrt{-h} \quad (5)$$

where $1/\kappa_3^2$ is the 3-dimensional Planck mass. The action variation w.r.t. $g^{\alpha\beta}$ and $h^{\alpha\beta}$ give (note we are using the projector $h_{\alpha\beta}$ rather than the induced metric h_{ab})

$$\delta\mathcal{S} = \frac{1}{2\kappa_3^2}\int d^3x\sqrt{-g}(R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} - \frac{1}{L^2}g_{\alpha\beta})\delta g^{\alpha\beta} + \\ + \frac{1}{\kappa_3^2}\int_\Sigma d^2y\sqrt{-h}(\langle K_{\alpha\beta} - Kh_{\alpha\beta}\rangle + \sigma h_{\alpha\beta})\delta h^{\alpha\beta} \quad (6)$$

where $\langle A\rangle := (1/2)(A^+(\Sigma) + A^-(\Sigma))$. The bulk equations of motion are nothing but Einstein's equations and are solved by (4). Let us now rewrite the boundary equation in the form

$$K_{\alpha\beta} = \sigma h_{\alpha\beta}. \quad (7)$$

To solve (7) for the 1-brane we begin with the static ansatz

$$\Sigma:\ 0 = \Theta(r,\theta) := \theta - \Psi(r), \quad \Psi \sim \Psi + 2\pi \quad (8)$$

which leads to the normal vector $n_\alpha = \frac{\pm\partial_\mu\Theta}{\sqrt{|\partial_\mu\Theta\partial^\mu\Theta|}} = \pm A(0, -\Psi'(r), 1)$, where $\Psi'(r) := \frac{d}{dr}\Psi(r)$ and $A := \frac{r}{\sqrt{F(r)(r\Psi'(r))^2+1}}$. Now we prolongate n_α to the rest of the spacetime and compute the extrinsic curvature. Plugging the result into (7), the solution for $\Psi(r)$ is immediately found, giving two branes that are symmetric w.r.t. the x-axis and that wrap around an infinite number of times (see FIG.1),

$$\Psi_\pm(r) = \pm\frac{\log\left(\frac{2\sigma^2L^4m + 2\sigma L^2\sqrt{m}\sqrt{r^2(1-\sigma^2L^2)+\sigma^2L^4m}}{Lr}\right)}{\sqrt{m}}. \quad (9)$$

Therefore, in order to have a brane defined at $r \to \infty$ one has to impose $\sigma^2L^2 < 1$. Choosing $(y^0 = t, y^1 = r)$

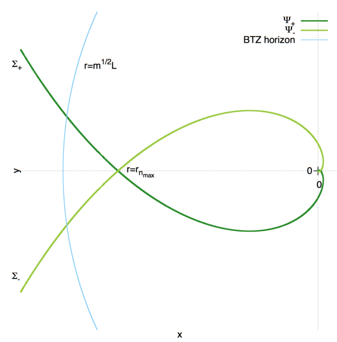

FIG. 1: $\Psi_\pm$ solutions, where $x := r\cos(\theta)$, and $y := r\sin(\theta)$.

on Ψ_+ and Ψ_-, the pull-back of the metric reads

$$ds^2 = -\left(\frac{r^2}{L^2} - m\right)dt^2 + \frac{\phi(r)}{\frac{r^2}{L^2} - m}dr^2 \quad (10)$$

being $\phi(r) = \frac{2\alpha r^2}{L(\beta r^2 + \alpha^2)}$, $\alpha = 2\sigma^2L^3m$ and $\beta = 4\sigma^2L^2m(1-\sigma^2L^2)$. Nevertheless, (10) is not a black hole yet. To see why, let us recall that the BTZ solution has a causal singularity at $r = 0$ instead of a curvature one. In (10) the curvature is also well behaved at $r = 0$, so if analytic prolongation beyond $r = 0$ does not lead to closed timelike curves, then there are no physical reasons to prohibit such an extension and $r = 0$ would not be a singularity. Indeed, moving to the Schwarzschild gauge through

$$\rho(r)=\frac{\sqrt{(1-\sigma^2L^2)r^2+\sigma^2L^4m}}{1-\sigma^2L^2} \tag{11}$$

the metric reads

$$ds^2=-\tilde{F}(\rho)dt^2+\frac{d\rho^2}{\tilde{F}(\rho)}, \tag{12}$$

with $\tilde{F}(\rho)=\frac{1-\sigma^2L^2}{L^2}\rho^2-\frac{m}{(1-\sigma^2L^2)}$. However, this spacetime is nothing but AdS_2 with cosmological constant [10]

$$\Lambda_2=-\frac{1-\sigma^2L^2}{L^2} \tag{13}$$

and of course it does not contain nor singularities neither horizons.

C. Black Hole on the Brane

In order to have a black hole, we shall construct the 1-brane using parts of both branches Ψ_+ and Ψ_-. The radius at which Ψ_+ and Ψ_- intersect are given by

$$r_n=L\frac{4\sigma^2L^2e^{n\pi\sqrt{m}}m}{e^{2n\pi\sqrt{m}}-4\sigma^2L^2(1-\sigma^2L^2)m} \tag{14}$$

with $n\in\mathbb{Z}$ and $\Psi_\pm$ approach infinity with asymptotic angle

$$\theta_\infty(\sigma^2)=\pm\frac{1}{2\sqrt{m}}\log\big(4\sigma^2L^2(1-\sigma^2L^2)m\big). \tag{15}$$

Indeed, when $\theta_\infty(\sigma^2)/\pi\in\mathbb{Z}$ the branches become parallel for large r, the denominator in (14) vanishes for $n=\theta_\infty(\sigma^2)/\pi$ and therefore the last intersection point lies at infinity. It is easy to check that $\frac{dr_n}{dn}<0$ $(\forall\sigma,m,n)$, so when $\theta_\infty(\sigma^2)/\pi\notin\mathbb{Z}$ there exists a last intersecting point given by $r_{n_{max}}$, being

$$n_{max}(\sigma^2)=\left[\frac{1}{\pi}\theta_\infty(\sigma^2)\right]_\rightarrow \tag{16}$$

where the operator $[\cdot]_\rightarrow$ takes the first integer coming after the $\mathbb{R}$-number it contains. For $r>r_{n_{max}}$ the branches do not intersect and approach infinity with asymptotic angle given by (15). The 1-brane of our braneworld, Σ, can be constructed cutting out the parts of Ψ_+ and Ψ_- at which $r<r_{n_{max}}$, and gluing the remaining branches, that we will refer to as Σ_+ and Σ_- respectively, in the last intersection point $r=r_{n_{max}}$ (see FIG.1). Let us first study the branch Σ_+. It is convenient to consider the Schwarzschild gauge (12) here. In Σ, $\rho>\rho(r_{n_{max}})$, so introducing the coordinate $x:=\rho-\rho(r_{n_{max}})$, $0<x<\infty$, the metric on Σ_+ reads

$$ds^2_{\Sigma_+}=-(\lambda^2x^2+2Mx-N)dt^2+\frac{dx^2}{\lambda^2x^2+2Mx-N} \tag{17}$$

where $\lambda^2:=-\Lambda_2$, $M:=\frac{1}{L}\sqrt{\frac{r^2_{n_{max}}}{L^2}(1-\sigma^2L^2)+\sigma^2L^2m}$ and $N:=(m-\frac{r^2_{n_{max}}}{L^2})$. Performing the same procedure on Σ_- but defining $x:=-(\rho-\rho(r_{n_{max}}))$, $-\infty<x<0$ instead, allows us to write down the metric for Σ in the compact form

$$ds^2_\Sigma=-f(x)dt^2+\frac{dx^2}{f(x)}, \tag{18}$$

where $f(x):=\lambda^2x^2+2M|x|-N$. The horizon of (18) lies at

$$|x_h|=\frac{-M+\sqrt{M^2+\lambda^2N}}{\lambda^2}, \tag{19}$$

and it will only exist if $N>0$, i.e. if the last intersecting point is inside the BTZ horizon, $r_{n_{\max}}<\sqrt{m}L$. Provided that $N>0$, we can rescale the coordinates as $\tilde{x}:=\frac{x}{\sqrt{N}}$, $\tilde{t}:=\sqrt{N}t$, and the metric becomes

$$ds^2_\Sigma=-\tilde{f}(\tilde{x})d\tilde{t}^2+\frac{d\tilde{x}^2}{\tilde{f}(\tilde{x})} \tag{20}$$

where $\tilde{f}(\tilde{x}):=\lambda^2\tilde{x}^2+2\mu|\tilde{x}|-1$, $\mu:=\frac{M}{\sqrt{N}}$ (from now on we remove the tildes). Using standard conformal compactification techniques to study the causal structure of (20), we obtained the Kruskal and Penrose diagrams shown in FIG.2. Inspection of these diagrams leads one to conclude that (20) contains a black (and white) hole region. This is precisely the black hole studied in the following sections of this paper. The curves $x=0$ in the Penrose diagram are not straight lines because we have cut the brane precisely at $x=0$ and have not allowed it to reach $r=0$. Actually, $x=0$ is a geometrical geodesic of *spacelike* character. This means that a free falling particle can be 'at rest' (instantaneous) at $x=0$. Moreover, the extrinsic curvature of $\{(t,x=0)\}\in\Sigma$ is precisely $K_{\partial\Sigma}=\mu$. For these reasons, and other interesting ones discussed in [6] we shall interpret our black hole (20) as a singularity caused by the presence of a particle with mass μ sitting at $x=0$. In the following sections, we go beyond the work done in [6] by studying the properties of the black hole on the brane and mapping the results into the corresponding quantum black hole, assuming the validity of the holographic conjecture.

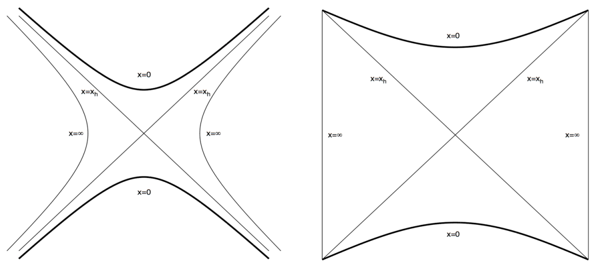

FIG. 2: Kruskal diagram (left) and Penrose diagram (right).

III. ANALYSIS OF THE BLACK HOLE SOLUTION ON Σ

Here we prove that the black hole (20) on Σ is not allowed for some values of the tension of the brane, σ. Since the physical magnitudes in (20) are σ and μ, let us set $L = 1$ and regard m (note that it is dimensionless) as a fixed parameter. For convenience, we shall work with σ^2 rather than σ. Notice that now $\sigma^2 \in (0, 1)$. The key point to see that the black hole (20) is not allowed for some σ^2 is to realise that by variating the tension new intersecting points (IP) between Ψ_+ and Ψ_- can appear from the infinity. Indeed, (16) is a step function of σ^2 and its unit jumps at certain values of σ^2 are translated to jumps of the last IP $r_{n_{max}}(\sigma^2)$ from a finite value to infinity (see discussion below (15)). That is, by variating σ^2 we can make appear new intersecting points from infinity. However, as discussed in Section II C the last IP has to be inside the BTZ horizon, $r_{n_{max}} < \sqrt{m}$, in order to have a black hole on Σ. If we keep variating σ^2 once a new IP has appeared from infinity, it could eventually end up crossing the BTZ horizon and hence we would have a black hole on Σ again, but the tensions at which $r_{n_{max}} > \sqrt{m}$ do not allow such a black hole.

Let us now find the values of σ^2 at which the black hole on Σ is not allowed. First, we have to obtain the tensions σ_n^2 at which new IPs appear from infinity solving $n = \frac{1}{\pi}\theta_\infty(\sigma_n^2)$. Before doing it, though, we shall perform a brief analysis of the function $\frac{1}{\pi}\theta_\infty(\sigma^2)$. It has a maximum at $\sigma^2 = 1/2$ and satisfies $\frac{1}{\pi}\theta_\infty(\sigma^2) < 1$ $(\forall m > 0, \sigma \in (0, 1))$, which means that $n_{max} \leq 1$ $(\forall m > 0, \sigma \in (0, 1))$. The function exhibits three different behaviours depending on m: for $m < 1$ it is always negative while for $m > 1$ it vanishes twice and widens its positive range as m increases. In the critical case $m = 1$ it vanishes only once at its maximum (see FIG.3). For reasons that will become clear in the next section, we can restrict ourselves to study the case $m > 1$ only. In this situation, there exist two solutions for each $n < 1$, given by

$$\sigma_{n\pm}^2 = \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{\exp\left(2n\pi\sqrt{m}\right)}{m}}\right). \tag{21}$$

For the case $r_{n=1}(\sigma^2)$, we see that $r_1(\sigma^2) < \sqrt{m}$ $\forall\sigma^2 \in (0, 1)$ and furthermore it is the last IP in $\sigma^2 \in (\sigma_{0-}^2, \sigma_{0+}^2)$. Hence, the first result is that the tensions $(\sigma_{0-}^2, \sigma_{0+}^2)$ allow a black hole on Σ. Studying $r_0(\sigma^2)$ in $[\sigma_{0+}^2, 1]$, it is easy to see that it is monotonically decreasing and $> \sqrt{m}$, so the second result is that in $[\sigma_{0+}^2, 1]$ the black hole on Σ is forbidden. A similar (although a bit more tedious) analysis performed in the lower range $[0, \sigma_{0-}^2]$ leads one to conclude that the forbidden intervals of tension are

$$\left(\cup_{n=0}^{-\infty}[\sigma_{nH}^2, \sigma_{n-}^2]\right) \cup [\sigma_{0+}^2, 1] \tag{22}$$

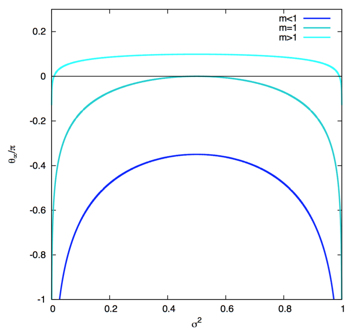

FIG. 3: Behaviour of $\frac{1}{\pi}\theta_\infty$ vs σ^2 for different bulk masses.

where

$$\sigma_{nH}^2 = \frac{1}{2}\left(1 + \frac{\exp\left(n\pi\sqrt{m}\right)}{\sqrt{m}} - \sqrt{\left(1 + \frac{\exp\left(n\pi\sqrt{m}\right)}{\sqrt{m}}\right)^2 - \frac{\exp\left(2n\pi\sqrt{m}\right)}{m}}\right) \tag{23}$$

is the tension at which the IP $r_n(\sigma^2)$ passes through the BTZ horizon, $r_n(\sigma_{nH}^2) = \sqrt{m}$. In the following section we conclude our contribution assuming the validity of the holographic conjecture in order to regard (20) as a quantum black hole, and then study and interpret the implications of the result (22).

A. Consequences and Results on the Quantum Black Hole

One necessary condition to apply the holographic conjecture is that the bulk must be classical. In our case, this means $m \gg 1$. Hence, before making any reference to the conjecture, we shall rewrite the result in (22) as an expansion to a certain order in a parameter ϵ, which in turn has to be a power of $1/m$. Soon we will see that in order to obtain an analytical expansion in ϵ the appropriate parameter is $\epsilon = 1/\sqrt{m}$. Furthermore, choosing this parameter yields to a better physical interpretation of the expansion, since the BTZ black hole temperature is $T_{BTZ} \sim \sqrt{m}$ [5]. Then, an expansion in ϵ is actually an expansion in the inverse of the temperature of the black hole living in the bulk. As a first study, we shall restrict our expansion to the first order in ϵ or the leading order in its inverse, and leave to future work the analysis of higher order corrections. Before performing the expansion of (22), let us remark that since n ranges from 0 to $-\infty$ we are not allowed to truncate the expansion of the exponential and we have to retain it. Expanding the other factors to leading order, the forbidden intervals (22) read

$$\left(\cup_{n=0}^{-\infty}\left[\epsilon^2\frac{\exp\left(2n\pi\epsilon\right)}{4}, \epsilon^2\frac{\exp\left(2n\pi\epsilon\right)}{4}\right]\right) \cup \left[1 - \frac{\epsilon^2}{2}, 1\right]. \tag{24}$$

That is, to leading order in ϵ the forbidden intervals squeeze to become a discrete spectrum of forbidden tensions, which constitutes the first result of this work. However, it is clearly a second order correction and we leave its study to future work. Staying in the first order in ϵ, we see that all the forbidden tensions lie at the unphysical points $\sigma^2 = 0$ and $\sigma^2 = 1$ and, therefore, do not lead to any physical consequence. This is equivalent to say that the allowed range $(\sigma_{0-}^2, \sigma_{0+}^2)$ covers the whole interval $\sigma^2 \in (0,1)$ in a first order expansion in ϵ. Let us now study the consequences of this fact on the mass μ, that we shall make dimensionless defining $\tilde{\mu} := \frac{\mu}{\lambda}$ (recall $1/\lambda$ is the AdS_2 length). For $\sigma^2 \in (\sigma_{0-}^2, \sigma_{0+}^2)$, μ is given by

$$\mu(\sigma^2) = \sqrt{\frac{(r_1(\sigma^2))^2}{m - (r_1(\sigma^2))^2} + \sigma^2} \tag{25}$$

and it is a monotonically increasing function of σ^2. Evaluating $\tilde{\mu}_\pm := \tilde{\mu}(\sigma^2 \to (\sigma_{0\pm}^2)^\mp)$ and performing the expansion to the first order in ϵ and the leading order in its inverse, we have

$$\tilde{\mu}_+ = \frac{2}{\epsilon}, \qquad \tilde{\mu}_- = \frac{\epsilon}{2}. \tag{26}$$

The existence of an upper and a lower bound in the mass $\tilde{\mu}$, and the fact that one is the inverse of the other, is the second result we have obtained and it is rather surprising. Studying a possible invariance or duality $\tilde{\mu} \to 1/\tilde{\mu}$ is left for future work. In order to understand the bounds $\tilde{\mu}_\pm$ we shall evaluate the size of the black hole horizon in both cases, again to first order in ϵ. At $\tilde{\mu}_-$ we obtain

$$|x_h|(\tilde{\mu}_-) = \frac{1}{\lambda}. \tag{27}$$

Now we are in conditions to assume the holographic conjecture and we shall interpret the last result in its light. What (27) is telling us is that the black hole on Σ can not be larger than the AdS_2 length $\frac{1}{\lambda}$. At the same time, we know that the horizon of classical black holes in AdS is larger than the AdS length. Therefore, if we take the holographic point of view and regard (20) as a quantum black hole, our result in (27) is a natural recovery of the separation line between the quantum and classical regimes. Evaluating the horizon at $\tilde{\mu}_+$ gives

$$|x_h|(\tilde{\mu}_+) = \frac{1}{\lambda}\frac{\epsilon}{4}. \tag{28}$$

This is even more interesting because from the holographic point of view we are already in the fully quantum regime and there is still a lower bound $|x_h|(\tilde{\mu}_+)$ for the size of the quantum black hole. Motivated by the results on the black hole information loss problem done in [11, 12], we suggest that the lower bound for the size of the horizon could be interpreted as the existence of an informon, i.e. a remnant of the 2-dimensional quantum black hole proposed in already existent works as a possible solution of the black hole information paradox. However, this is out of the scope of this paper and we leave the study of the black hole in these lines to future work.

IV. CONCLUSIONS

We have obtained a black hole solution on the 1-brane of the RSII braneworld and studied its causal structure. Analysing the solution we have found that the black hole on Σ is forbidden for a spectrum of tensions. Studying the consequences of this in the large m limit, we have obtained the intriguing result of a lower bound for the size of the black hole. Interpreting the results in the light of the holographic conjecture, we suggest that the lower bound could be understood as an informon.

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