Consumption composition and macroeconomic dynamics*

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July 2, 2014

Abstract

We analyze the transitional dynamics of a model with heterogeneous consumption goods where convergence is driven by two different forces: the typical diminishing returns to capital and the dynamic adjustment in consumption expenditure induced by the variation in relative prices. We show that this second force affects the growth rate if the consumption goods are produced with technologies exhibiting different capital intensities and if the intertemporal elasticity of substitution is not equal to one. Because the aforementioned growth effect of relative prices arises only under heterogeneous consumption goods, the transitional dynamics of this model exhibits striking differences with the growth model with a single consumption good. We also show that these differences in the transitional dynamics can give raise to large discrepancies in the welfare cost of shocks.

JEL classification codes: O41, O47.

Keywords: multi-sector growth models, transitional dynamics, consumption growth.

*Financial support from the Government of Spain through grants ECO2012-32392, ECO2012-34046 and ECO2011-23959; PR2009-0162 and SM2009-0001; the Generalitat of Catalonia through grants SGR2009-00350 and SGR2009-1051; and the Xunta de Galicia through grant 10PXIB300177PR is gratefully acknowledged. Alonso-Carrera also thanks the Research School of Economics (Australian National University) for its hospitality. Caballé also thanks the financial support from the ICREA Academia program. The paper has benefited from comments by participants in the World Congres of the Econometric Society (Shanghai), DEGIT (Los Angeles), ESEM (Milan), SAEe (Granada), ASSET (Padova), Australasian Workshop in Macroeconomic Dynamics, PET13 (Lisbon), Conference on Structural Change, Dynamics, and Economic Growth (Pisa) and seminars in UPV (Bilbao), IAE-CSIC (Barcelona), Australian National University, University of Melbourne, Monash University, Macquarie University, University of Wollongong, National University of Ireland (Maynooth), Geary Institute (UCD), GREQAM (Marseille), Universidad de Murcia, Universidad de las Islas Baleares, Universitat Rovira i Virgili and Universidade do Minho.

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1. Introduction

There is a recent growing interest in accounting for patterns of economic growth and business cycles using growth models with heterogeneous consumption goods.[1] This literature basically focuses on studying how the dynamic adjustment of the sectoral structure affects the aggregate outcomes. An important result of this analysis is that the growth rate of consumption expenditure in these multi-sector growth models directly depends on the variation of the relative price of consumption goods as this variation alters the sectoral composition of consumption demand (see, for instance, Rebelo, 2001; or Ngai and Pissarides, 2007). However, the dynamic behavior of these models are not well understood as the existing studies focus on the balanced growth path (BGP, henceforth) or impose some restrictive assumptions that prevent the relative price of consumption goods from displaying the aforementioned growth effects. Some authors assume that the intertemporal elasticity of substitution (IES, henceforth) is equal to one (see, e.g., Echevarria, 1997; Laitner, 2000; Ngai and Pissarides, 2007; or Perez and Guillo, 2010), or they consider that the consumption goods are produced by means of technologies with identical capital intensities, which makes relative prices to be constant in absence of biased technological change (Kongsamut et al., 2001; or Steger, 2006). Under these assumptions, the dynamics of the aggregate variables are identical to those predicted by the model with a single consumption good: the growth rate of consumption expenditure only depends on the interest rate for consumption-denominated loans. According to this result, the process of convergence would be only determined by the return to capital with independence of the number of consumption goods.

Our main purpose in this paper is to analyze how the adjustment in expenditure induced by the dynamic variation of the relative price modifies the dynamic behavior of the economy and affects the shock propagation mechanism. We relax the aforementioned restricted assumptions on the IES and on the sectoral capital intensities. In this case, the process of convergence is thus driven by two forces: the return to capital and the dynamic adjustment in relative prices. To the best of our knowledge, the present paper is the first in analyzing the transitional dynamics of a growth model with heterogeneous consumption goods when these two forces driving the transition are operative. Let us emphasize that we analyze the role that the change in consumption composition along the transitional dynamics plays in the propagation mechanism of shocks. Therefore, our purpose is neither to analyze the origin of the observed sectoral change nor to study how to reconcile the observed balanced growth of the aggregate variables with the permanent change of the sectoral structure. These are the objectives of the papers of Kongsamut et al. (2001), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), Foellmi and Zweimüller (2008) and Boppart (2011), among others. By the contrary, we simply extend the contribution of a vast literature that has focused on studying the transitional dynamics of growth models with a unique consumption good (see, among many others, Caballé and Santos,1993; or Mulligan and Sala-i-Martin, 1993)

We first prove that the growth effect of the variation of the relative price

[1] Examples include, among many others, Echevarria (1997), Kongsamut et al. (2001), Ngai and Pissarides (2007), or Perez and Guillo (2010).

on expenditure is jointly determined by the IES and the sectoral composition of consumption expenditure. A variation in the relative price of consumption goods will alter the future cost of the consumption basket. Consumers respond to this change in expenditure by adjusting their intratemporal and intertemporal decisions on consumption. On the one hand, they modify the sectoral composition of their consumption baskets to accommodate the impact of price variation on expenditure. This response clearly depends on the degree of substitutability between goods on preferences. In addition, the change in the future consumption expenditure triggers both a substitution and an income effect that result in opposite impacts on savings. Which of the later two effects dominates depends on the magnitude of the IES. Therefore, the existence of several heterogeneous consumption goods is relevant to the equilibrium dynamics.

In order to study the transitional dynamics when the variation of prices displays the aforementioned growth effects, we extend the endogenous growth model with physical and human capital accumulation by Uzawa (1965) and Lucas (1988). More precisely, we analyze a three sector growth model with a homothetic utility function whose argument is a composite good combining two different consumption goods. These goods are produced by means of constant returns to scale technologies that use physical and human capital as inputs. Furthermore, technologies exhibit different capital intensities across sectors, which make the relative prices between consumption goods and between both types of capitals endogenous. Finally, we consider non-logarithmic preferences so that the dynamic adjustment of the relative price of consumption goods alters the growth rate of consumption expenditure.

As occurs in multi-sector growth models with two types of capital, the transitional dynamics will be governed by the imbalances between the two stocks of capital. As we mentioned before, those imbalances in the proposed environment give rise to changes in the relative price of consumption goods, which interact with the diminishing returns to capital to determine the intertemporal allocation of consumption expenditure and saving. Therefore, the adjustment in the relative price is a key element for the dynamic mechanism offsetting the initial imbalances. More precisely, the existence of two different forces governing the transition yields two interesting differences with respect to the transitional dynamics obtained in the standard growth model with a unique consumption good. First, in growth models with a unique consumption good, convergence in the expenditure growth rate occurs from below (above) if the initial value of the ratio of physical to human capital is larger (smaller) than its stationary value. We will show that, under a plausible condition, this behavior is reversed when we allow for heterogeneous consumption goods. It should be noticed that, when that condition is satisfied, the initial effect on expenditure growth of a shock in one of the capital stocks will be the opposite of the one obtained in a model with a single consumption good. Moreover, while the growth rate of consumption expenditure exhibits a monotonic behavior when the diminishing returns to capital is the only force governing the transition, it may exhibit instead a non-monotonic behavior in our extended model. In our model the non-monotonic behavior arises when the aforementioned two different forces acting on the transitional dynamics exhibit opposite growth effects.

The two differences we have just mentioned imply that the patterns of growth along the transition crucially depend on the parameters values of our model. More precisely, we show that the capital intensity ranking across sectors, the IES, and the sectoral share of expenditure crucially determine the nature of the transition. We will simulate the economy in order to illustrate the transitional dynamics and the corresponding propagation mechanism. This numerical simulations show that, contrary to the model with a single consumption good, our model displays a much smaller and more plausible growth rate of GDP when the economy departs from its BGP. Furthermore, our model may exhibit a non-monotonic dynamic response of the growth rate of GDP to the imbalances in the capital ratio. In particular, this rate may display a hump-shaped dynamics when the physical to human capital ratio is initially smaller than its stationary value. This dynamic behavior replicates the stages of economic growth introduced by Rostow (1960), and it is consistent with the time series and the cross section evidence on the growth patterns along the development process (see, e.g., Maddison, 1991; Echevarria, 1997; or Fiaschi and Lavezzi, 2004), and on the post-World War II growth experience of Western Europe and Japan (see, e.g., Christiano, 1989; Papageorgiou and Perez-Sebastian, 2006; or Alvarez-Cuadrado, 2008). Literature has accounted for this non-monotonic behavior by means of considering non homothetic preferences (see, e.g., Steger, 2000; or Alvarez-Cuadrado et al., 2004), or introducing barriers to capital accumulation (see, e.g., Hansen and Prescott, 2002; or Ngai, 2004). In contrast, in our model the non-monotonic behavior of the growth rate of GDP is explained by the presence of the aforementioned two different forces acting on the transitional dynamics.

We also compute the corresponding growth and welfare effects of technological shocks. We show that these effects will strongly depend on the sectoral composition of the composite consumption good when these shocks cause large effects on its unitary cost. Therefore, by considering specific models where the force associated with the dynamics of the relative price between goods is not operative, the existing literature may obtain biased results about the effects of technological shocks.

The paper is organized as follows. Section 2 presents the ingredients of the model. Section 3 characterizes the competitive equilibrium, whereas Section 4 analyzes the equilibrium dynamics of the growth rate of expenditure. Section 5 develops the numerical analysis concerning the transitional dynamics and the effects of technological shocks. Section 6 presents some concluding remarks. The Appendix contains the proofs of all the results of the paper.

2. The economy

Let us consider a three-sector growth model in which the output in each sector is obtained from combining amounts of two types of capital, k and h, which we dub physical and human capital, respectively. More precisely, each sector i produces an amount y_i of commodity using the following production function:[2]

$$y_i = A_i \left(s_i k\right)^{\alpha_i} \left(u_i h\right)^{1-\alpha_i} = A_i u_i h z_i^{\alpha_i}, \ i = 1, 2, 3, \tag{2.1}$$

where s_i and u_i are the shares of physical and human capital allocated to sector i, $z_i = s_i k \,/ u_i h$ is the physical to human capital ratio, $A_i > 0$ is the (constant) sectoral

[2] To ease the notation we omit the time argument of all the variables. Moreover, we use dot notation to indicate the derivative of a variable with respect to time.

total factor productivity (TFP), and $\alpha_i \in (0,1)$ measures the intensity of physical capital in sector i.

We interpret the first sector as the one producing manufactures and assume that the commodity y_1 can be either consumed or added to the stock of physical capital. We denote by c_1 the amount of good y_1 devoted to consumption. We consider the second sector as the one producing food and services devoted to consumption, such as cultural or entertainment goods. Thus, the output of this sector can only be devoted to consumption, which we denote by c_2. Finally, we assume that the commodity y_3 is devoted exclusively to increase the stock of human capital and, therefore, we identify the third sector with the education sector.

We take manufactures as a numeraire, and we denote the relative prices of commodities y_2 and y_3 by p and p_h, respectively. Let w be the rental rate of human capital (i.e., the real wage per unit of human capital) and r be the rental rate of physical capital. We assume perfect sectoral mobility so that the equilibrium values of both rental rates are independent of the sector where the units of physical and human capital are allocated. Firms in each sector behave competitively, so that they choose the amounts of physical and human capital that maximize profits by taken p, p_h, w and r as given.

The economy is populated by an infinitely lived representative agent characterized by the instantaneous utility function

$$U(c_1, c_2) = \frac{x^{1-\sigma}}{1-\sigma}, \tag{2.2}$$

where $x = v\left(c_1, c_2\right)$ and $\sigma > 0$ is the (constant) elasticity of the marginal utility of this composite consumption x. We assume that the function $v\left(c_1, c_2\right)$ is increasing in each consumption good, linearly homogeneous and strictly quasiconcave. The representative agent is endowed with k units of physical capital and h units of human capital. Therefore, the budget constraint of the consumer is given by

$$wh + rk = c_1 + pc_2 + I_k + p_h I_h, \tag{2.3}$$

with I_h and I_k being the gross investment in human and physical capital, respectively,

$$I_k = \dot{k} + \delta k, \tag{2.4}$$

and

$$I_h = \dot{h} + \eta h, \tag{2.5}$$

where $\delta \in [0,1]$ and $\eta \in [0,1]$ are the depreciation rates of physical and human capital, respectively. The representative agent thus maximizes

$$\int_0^{\infty} e^{-\rho t} U(c_1, c_2) dt, \tag{2.6}$$

subject to (2.3), (2.4), and (2.5), where $\rho > 0$ is the subjective discount rate.

3. Competitive equilibrium

Given the initial stocks of physical and human capital, k_0 and h_0, a competitive equilibrium in this economy consists of a path of prices $\{p,\ p_h,\ r,\ w\}$, a path of firm allocations $\{s_i,\ u_i\}_{i=1}^{3}$, and a path of consumer allocations $\{c_1,\ c_2,\ I_k,\ I_h\}$ that are consistent with consumer and firm optimization and with market clearing conditions: (i) $\sum_{i=1}^{3} s_i = \sum_{i=1}^{3} u_i = 1$, (ii) $y_2 = c_2$, (iii) $y_1 = c_1 + I_k$, and (iv) $y_3 = I_h$.

The solution to the consumer's problem is given by the following equations derived in Appendix A:[3]

$$p = \frac{v_2(c_1, c_2)}{v_1(c_1, c_2)}, \tag{3.1}$$

$$\frac{\dot{p}_h}{p_h} = r - \frac{w}{p_h} + \eta - \delta, \tag{3.2}$$

$$\frac{\dot{c}_1}{c_1} = \frac{r - \rho - \delta}{\sigma} + \left(\frac{\varepsilon\xi}{\sigma}\right)\left(\frac{\dot{p}}{p}\right), \tag{3.3}$$

$$\frac{\dot{c}_2}{c_2} = \frac{\left(r - \frac{\dot{p}}{p}\right) - \rho - \delta}{\sigma} + \left[\frac{1 + \xi(\varepsilon - \sigma)}{\sigma}\right]\left(\frac{\dot{p}}{p}\right), \tag{3.4}$$

and the transversality conditions

$$\lim_{t\to\infty} e^{-\rho t} v_1(c_1, c_2)\, v(c_1, c_2)^{-\sigma} k = 0, \tag{3.5}$$

and

$$\lim_{t\to\infty} e^{-\rho t} v_1(c_1, c_2)\, v(c_1, c_2)^{-\sigma} h = 0, \tag{3.6}$$

where ξ is the elasticity of substitution between c_1 and c_2 in $v(c_1, c_2)$, which is given by

$$\xi = \frac{v_1(c_1, c_2)\, v_2(c_1, c_2)}{v(c_1, c_2)\, v_{12}(c_1, c_2)}; \tag{3.7}$$

and ε is the Edgeworth elasticity of the consumption good c_1 with respect to the consumption good c_2 (i.e., the elasticity of the marginal utility of c_1 with respect to c_2), which is given by

$$\varepsilon \equiv -c_2\left(\frac{\partial^2 U/\partial c_1 \partial c_2}{\partial U/\partial c_1}\right) = -(1 - \sigma\xi)\left[\frac{c_2 v_{12}(c_1, c_2)}{v_1(c_1, c_2)}\right]. \tag{3.8}$$

Equation (3.1) tells us that the price ratio p is equal to the marginal rate of substitution between the two consumption goods. Equation (3.2) shows that the growth of price p_h is determined by the standard non-arbitrage condition between the investments in physical and human capital. Finally, equations (3.3) and (3.4) characterize the growth rates of the amounts of consumption goods c_1 and c_2, respectively. As usual, these growth rates depend on the interest rate for loans denominated in the corresponding consumption good. However, the variation of the relative price also affects those growth rates beyond the capital gain it generates.

[3] From now on, the sub-index of a function will refer to the position of the argument with respect to which the partial derivative is taken.

Obviously, a rise in the relative price leads c_2 to be relatively more costly. This will alter the demand for both c_1 and c_2 in a proportion that depends on the degree of substitutability between the two consumption goods in terms of utility, which is jointly determined by the elasticity of substitution in the composite good $v(c_1, c_2)$ and the Edgeworth elasticity.

Observe that the growth rate of c_2 depends on the real interest rate for loans denominated in good y_2, which is given by $r - \dot{p}/p$. Hence, changes on the relative price affect the intertemporal allocation of c_2 by altering the market rate of transformation between capital investment and consumption good c_2. This dynamic effect of the relative price is standard in two-sector growth models with a unique consumption good (as, e.g., in Rebelo, 1991): it is a mere consequence of assuming that the investment and consumption good c_2 are not perfect substitutes. However, as can be seen from (3.3) and (3.4), our point is that changes in relative price in a model with heterogeneous consumption goods have an additional effect on the equilibrium dynamics by altering the composition of the consumption basket.

At this point, we can derive the growth rate of total consumption expenditure, which is defined as $c = c_1 + pc_2$. We will denote by e the fraction of total expenditure on consumption good c_2 so that $pc_2 = ec$ and $c_1 = (1-e)\,c$. As shown in the Appendix B, Equation (3.1) implicitly defines the expenditure share e as a function of relative price p, i.e., $e = E(p)$, with

$$E'(p) = \frac{E(p)\,[1 - E(p)]\,(1-\xi)}{p}. \tag{3.9}$$

Observe that the dependence of consumption expenditure on the relative price is determined by the substitution elasticity ξ. In particular, we see that $E'(p) > 0$ if and only if $\xi < 1$. Intuitively, a rise in relative price p has two opposite effects on the relative expenditure on consumption good c_2. First, given a quantity of c_2, this variation of the relative price increases the expenditure on this good. In addition, the rise in the relative price also leads to a fall in the relative demand for c_2. Which of these two effects dominates depends on the magnitude of the elasticity of substitution between consumption goods.

By log-differentiating $c_1 = (1-e)\,c$ with respect to time, and after some algebra shown in Appendix B, we obtain from (3.3) that

$$\frac{\dot{c}}{c} = \frac{r - \rho - \delta}{\sigma} - \left[\frac{(1-\sigma)\,E(p)}{\sigma}\right]\left(\frac{\dot{p}}{p}\right). \tag{3.10}$$

Equation (3.10) tells us that the growth rate of consumption expenditure is driven by both the interest rate and by the change in the relative price of the two consumption goods. The effect of a rise in the interest rate on the rate of growth of c is summarized by the intertemporal elasticity of substitution $IES = 1/\sigma$. The growth effect of a rise in the growth rate of the relative price is jointly determined by the IES and the fraction of total expenditure on consumption good c_2.[4] On the one hand, the sign of

[4] Note that the effect of relative prices on expenditure growth appears because only the good c_1 can be used as physical capital. If the equilibrium mix of the two consumptions goods could be devoted to investment in physical capital, then the relative price would not affect the growth rate of consumption expenditure c (see Acemoglu and Guerrieri, 2008).

the later effect depends on whether σ is larger or smaller than one. An important part of the previous literature on multi-sectoral growth models commonly uses a logarithmic specification for preferences and this explains why it does not obtain the growth effect of the variation in the relative price. On the other hand, the intensity of the growth effect of changes in the relative price depends on the sectoral allocation of consumption expenditure. The larger the fraction of expenditure on c_2, the smaller the difference between the unit cost of consumption basket and relative price p and, therefore, the larger is the growth effect of a variation in this price.

The intuition on the aforementioned growth effect of the dynamic adjustment of the relative price is as follows. Equation (3.10) is the Euler equation equating the market return from investing one unit of the numeraire y_1 and the growth of the marginal utility arising from consuming this commodity. Given an interest rate and a composition of consumption, an anticipated increase on relative price p pushes future consumption expenditure up. Obviously, this effect is directly related with the expenditure shares and depends on to what extend consumers can reduce the impact of price changes on expenditure by altering the composition of consumption basket (see Equations (3.3) and (3.4)). Moreover, this increase in the future expenditure always exhibits two opposite effects on the intertemporal allocation of expenditure and savings. First, a direct effect of a future rise in relative price is to increase savings to maintain the demands for goods. On the other hand, a future rise in relative price leads consumers to shift expenditure from future to present in order to smooth the expenditure path. Which of these two effects dominates depends on the magnitude of IES. When this elasticity is larger than unity, the income effect dominates and as a result, if the relative price rises, consumers save a large proportion of income for any given interest rate. Conversely, if IES is smaller than one, then the expenditure smoothing motive is strong and, hence, a rise in the relative price leads to a decrease in savings.

At this point, we should also mention that the fraction of expenditure on c_2, which is given by $e = E(p)$, depends on relative price p and, furthermore, the sign of this dependence is determined by the elasticity of substitution between consumption goods. Therefore, current changes in the relative price will determine the growth effects of future variations in this price by inducing the sectoral allocation of expenditure in the next period. For instance, consider that $\xi > 1$ and relative price p is growing along the transition dynamics. In this case the fraction of expenditure on c_2 will decrease so that the growth effect of relative price will also decrease along the equilibrium path. The adjustment on the sectoral allocation of consumption expenditure generates then a mechanism for the propagation of structural shocks.

After having presented the equilibrium conditions on the demand side of our economy, we will now move to the supply side and we will characterize the solution to the optimization problem faced by firms. In particular, firms maximize profits in each sector and, thus, the competitive factors payment must satisfy simultaneously the following equations:

$$r = \alpha_1 A_1 z_1^{\alpha_1 - 1}, \tag{3.11}$$

$$r = p\alpha_2 A_2 z_2^{\alpha_2 - 1}, \tag{3.12}$$

$$r = p_h \alpha_3 A_3 z_3^{\alpha_3 - 1}, \tag{3.13}$$

$$w = (1 - \alpha_1) A_1 z_1^{\alpha_1}, \tag{3.14}$$

$$w = p\left(1-\alpha_2\right)A_2 z_2^{\alpha_2}, \tag{3.15}$$

and

$$w = p_h\left(1-\alpha_3\right)A_3 z_3^{\alpha_3}. \tag{3.16}$$

By using this conditions, we will next derive the sectoral structure at the equilibrium.

3.1. Static equilibrium: sectoral structure

We now proceed to obtain the equilibrium conditions determining the intratemporal allocations of resources across sectors as functions of aggregate variables. To this end, we first derive the physical and human capital ratio in each sector. If $\alpha_1 \neq \alpha_2$, we can combine the system of equations (3.11) to (3.16) to obtain

$$z_i = \psi_i p^{\frac{1}{\alpha_1-\alpha_2}}, \quad \text{for } i = 1, 2, 3, \tag{3.17}$$

where

$$\psi_1 = \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{\alpha_2}{\alpha_1-\alpha_2}} \left(\frac{1-\alpha_2}{1-\alpha_1}\right)^{\frac{1-\alpha_2}{\alpha_1-\alpha_2}} \left(\frac{A_2}{A_1}\right)^{\frac{1}{\alpha_1-\alpha_2}},$$

$$\psi_2 = \left(\frac{\alpha_2}{1-\alpha_2}\right)\left(\frac{1-\alpha_1}{\alpha_1}\right)\psi_1, \tag{3.18}$$

and

$$\psi_3 = \left(\frac{\alpha_3}{1-\alpha_3}\right)\left(\frac{1-\alpha_1}{\alpha_1}\right)\psi_1. \tag{3.19}$$

Finally, we obtain the shares of physical and human capital in each sector. Consider the aggregate ratios $z = k/h$ and $q = c/k$. Then, we combine the technology (2.1) of the sector producing y_2 with $pc_2 = E\left(p\right)c$ and $y_2 = c_2$ to get

$$u_2 = E\left(p\right)\left(\frac{qz}{pA_2 z_2^{\alpha_2}}\right), \tag{3.20}$$

and we use the definition of z_2 to obtain

$$s_2 = E\left(p\right)\left(\frac{qz_2^{1-\alpha_2}}{pA_2}\right). \tag{3.21}$$

Next, we combine the definitions of z_1 and z_3 to get

$$u_1 = \frac{\left(1-u_2\right)z_3 - \left(1-s_2\right)z}{z_3 - z_1}, \tag{3.22}$$

and

$$s_1 = \left(\frac{z_1}{z}\right)\left(\frac{\left(1-u_2\right)z_3 - \left(1-s_2\right)z}{z_3 - z_1}\right). \tag{3.23}$$

3.2. Equilibrium dynamics: aggregate variables

In this subsection we derive the system of dynamic equations that fully determines the equilibrium path of aggregate variables. We first characterize how the dynamics of the relative prices take place. To this end, we use equations (3.12), (3.13), and (3.17), to obtain

$$p_h = \varphi p^{\frac{\alpha_1-\alpha_3}{\alpha_1-\alpha_2}}, \tag{3.24}$$

where

$$\varphi = \frac{\alpha_2 A_2 (\psi_2)^{\alpha_2-1}}{\alpha_3 A_3 (\psi_3)^{\alpha_3-1}}.$$

This previous relationship between the relative prices implies that

$$\frac{\dot{p}}{p} = \left(\frac{\alpha_1-\alpha_2}{\alpha_1-\alpha_3}\right)\left(\frac{\dot{p}_h}{p_h}\right). \tag{3.25}$$

Equation (3.25) shows that the relationship between the growth rate of the relative prices p and p_h only depends on the capital intensity ranking among sectors. Therefore, in our economy the dynamics of both prices p and p_h are fully determined by the non-arbitrage condition (3.2) and equation (3.25). In particular, we combine (3.2), (3.11), (3.14), (3.17), (3.24) and (3.25) to obtain

$$\frac{\dot{p}}{p} = \left(\tfrac{\alpha_1-\alpha_2}{\alpha_1-\alpha_3}\right)\left[\alpha_1 A_1 \psi_1^{\alpha_1-1} p^{\frac{\alpha_1-1}{\alpha_1-\alpha_2}} - \frac{(1-\alpha_1)A_1\psi_1^{\alpha_1} p^{\frac{\alpha_3}{\alpha_1-\alpha_2}}}{\varphi^{\frac{\alpha_1-\alpha_3}{\alpha_1-\alpha_2}}} + \eta - \delta\right] \equiv \kappa(p). \tag{3.26}$$

Note that the right hand side of the previous equation can be written as a function $\kappa(\cdot)$ of relative price p.

We now proceed to the characterization of the growth rate of the two capital stocks. For that purpose, we use (2.1), (2.4), (2.5) and the market clearing conditions to obtain

$$\frac{\dot{k}}{k} = \frac{A_1 u_1 z_1^{\alpha_1}}{z} - [1 - E(p)]\, q - \delta, \tag{3.27}$$

and

$$\frac{\dot{h}}{h} = A_3 (1 - u_1 - u_2)\, z_3^{\alpha_3} - \eta. \tag{3.28}$$

Finally, we combine (3.10) with (3.11), (3.17) and (3.25) to obtain

$$\frac{\dot{c}}{c} = \nu(p) - \left[\frac{(1-\sigma)E(p)}{\sigma}\right]\kappa(p) \equiv \gamma(p) \tag{3.29}$$

where

$$\nu(p) \equiv \frac{\alpha_1 A_1 z_1^{\alpha_1-1} - \rho - \delta}{\sigma}. \tag{3.30}$$

Note that the function $\nu(\cdot)$ defined in (3.30) only depends on relative price p as follows from (3.17). Equation (3.29) shows the two forces governing the transition and the parameters measuring the intensity of these two forces. In particular, the net balance between the two forces depends crucially on IES and the expenditure share $E(p)$, which determines in turn the nature of the transitional dynamics of the economy.

Combining (3.27) and (3.28), we get

$$\frac{\dot{z}}{z} = \frac{A_1 u_1 z_1^{\alpha_1}}{z} - [1 - E(p)]\, q + \eta - \delta - A_3 (1 - u_1 - u_2)\, z_3^{\alpha_3}, \tag{3.31}$$

and combining (3.27) and (3.29) we obtain

$$\frac{\dot{q}}{q} = \nu(p) - \left[\frac{(1-\sigma) E(p)}{\sigma}\right] \kappa(p) - \frac{A_1 u_1 z_1^{\alpha_1}}{z} + [1 - E(p)]\, q + \delta. \tag{3.32}$$

The dynamic equilibrium is thus characterized by a set of paths $\{p,\ z,\ q\}$ such that, given the initial value z_0 of the physical to human capital ratio, solves the equations (3.26), (3.31), and (3.32), and satisfies (3.17), (3.20), (3.21), (3.22) together with the transversality conditions (3.5) and (3.6). As in the standard two-sector growth model, there is a unique state variable z and the transition will be governed by the imbalances between the two capital stocks.

At this point, we derive the following well-known result, which has important consequences for the equilibrium dynamics of our economy.

Proposition 3.1. *The relative price p of consumption goods is constant over time for all initial values of the capital ratio $z = k/h$ when at least one of the following conditions holds: (i) $\alpha_1 = \alpha_2$, (ii) $\alpha_1 = \alpha_3$.*

Proof. See Appendix C.

Obviously, under the conditions pointed out by Proposition 4.1, the growth rate of consumption expenditure only depends on the interest rate. If $\alpha_1 = \alpha_2$, which means that the two consumption goods c_1 and c_2 are produced by means of technologies with the same capital intensity, then the transitional dynamics of our model coincides with that of the two-sector growth model with a unique consumption good introduced by Uzawa (1965) and Lucas (1988), and which was analyzed by, among others, Caballé and Santos (1993) or Mulligan and Sala-i-Martin (1993). When $\alpha_1 = \alpha_3$, which means that the two capital goods k and h are produced by means of technologies with the same capital intensity, then the three sectors in our model are in fact using Ak technologies.[5] Therefore, the dynamics in this case coincides with the dynamics in the Ak growth model with several consumption goods (see, e.g., Rebelo, 1991).

We have just established the conditions under which the growth rate of consumption expenditure depends not only on the interest rate, but also on the growth rate of relative price p. This new dependence requires that the IES be not equal to one and consumption goods be produced by means of technologies with different capital intensities. The previous arguments then explain why previous multi-sector growth models do not find a direct effect of the relative price on consumption growth. Some

[5] Note that when $\alpha_1 = \alpha_3$ the technology producing commodity y_i can be rewritten as $y_i = \widehat{A}_i u_i h$, where $\widehat{A}_i = A_i (z_i)^{\alpha_i}$ is constant for all $i = 1, 2, 3$ and $z_1 = z_3$. Since goods y_1 and y_2 are produced with linear technologies, their relative prices are constant and given by $p = \widehat{A}_1 (1 - \alpha_1) \Big/ \widehat{A}_2 (1 - \alpha_2)$.

of these models consider logarithmic preferences, whereas other models assume that consumption goods are produced with technologies that share the same capital intensity. Obviously, in the later case the variation of relative prices could still affect directly the growth rate of consumption expenditure under exogenous and biased technological change, that is, when the sectoral TFPs grow at exogenous growth rates that are different across sectors (see, e.g., Ngai and Pissarides, 2007). However, if technologies exhibit different capital intensities, the relative price between consumption goods appear as an endogenous channel for the propagation of shocks in fundamentals. In the rest of the paper, we will illustrate the consequences of this endogenous mechanism and, hence, we will assume that $\alpha_1 \neq \alpha_3$ and $\alpha_1 \neq \alpha_2$.

Note that relative prices would also affect the growth rate of consumption expenditure when $\alpha_2 = \alpha_3$, that is, when services and human capital are produced with the same technology.[6] Moreover, this growth effect of prices would also hold if we had assumed a unique capital stock. In this latter case, the dynamics of prices would be driven by the accumulation of the capital stock, whereas in our two-capital model they are driven by the relative accumulation of these two capital stocks. By setting $A_3 = \eta = 0$, our model becomes a neoclassical growth model with two consumption goods and constant efficient units of labor. In this case, conditions (3.11), (3.12), (3.14) and (3.15) jointly define relative price p as a function of capital k. This price then changes along the transition and, therefore, the two forces driving the convergence process are still operative in this model with a single capital type.

We define a steady-state or BGP equilibrium as an equilibrium path along which the ratios z and q and relative prices p and p_h remain constant. The following result characterizes the steady-state equilibrium:

Proposition 3.2. *The unique steady-state value p^* of the relative price solves $\kappa(p^*) = 0$ and the two capital stocks and consumption expenditure grow at the same constant growth rate $g^* \equiv \nu(p^*)$. Moreover, the steady-state value z^* of the physical to human capital ratio and the steady-state value q^* of the consumption expenditure to capital ratio are unique.*

Proof. See Appendix C.

Note that neither the steady-state price level p^* nor the growth rate g^* depend on preferences. As in the standard endogenous growth model with a single consumption good, the steady-state values of these two variables only depend on the technology. In contrast, the steady-state value of the ratios z^* and q^* depend on preferences and, more precisely, on the properties of the function $v(c_1, c_2)$ for the composite consumption good x.[7] The analysis of this dependence is out of the scope of this paper.

4. Transitional dynamics analysis

Let us now analyze how the transitional dynamics is affected by the existence of two heterogeneous consumption goods. For that purpose, we will focus on the behavior of

[6] Note that if $\alpha_2 = \alpha_3$ then the consumption good c_2 and human capital are produced by using technologies with the same capital intensity. In this case, the two relative prices satisfy $p = \frac{A_3}{A_2} p_h$.

[7] The exact expressions for z^* and q^* are given in the proof of Proposition 3.2.

the growth rate of consumption expenditure during the transition. To be able to derive an analytical result, we will consider in this section that the composite good is given by the following Cobb-Douglas function:

$$x \equiv v\left(c_1, c_2\right) = c_1^{\theta} c_2^{1-\theta}, \tag{4.1}$$

with $\theta \in [0,1]$. In this case, the elasticity of substitution ξ in (3.7) is equal to one so that the fraction of expenditure on c_2, $E\left(p\right)$, is constant and equal to $1-\theta$. Hence, the dynamic adjustment on the relative demand for c_2 is perfectly offset by the variation on relative price p so that the sectoral allocation of expenditure does not change. However, the adjustment of consumption basket is still important for equilibrium dynamics because it determines to what extend consumers are able to accommodate their expenditure to changes in the relative price. In Section 5, we will consider a CES aggregator for consumption to numerically illustrate how the adjustment on sectoral allocation of expenditure determines the propagation mechanism.

Proposition 4.1. *The steady-state equilibrium is locally saddle-path stable.*

Proof. See Appendix C.

In the proof of Proposition 4.1 it is shown that the equilibrium value p of the relative price of good c_2 is always equal to its steady state value when $\alpha_1 < \alpha_3$ so that it is constant along the transition towards the steady state. This implies that the growth rate of consumption expenditure is constant and equal to $v\left(p^*\right)$ along the transition when $\alpha_1 < \alpha_3$. Therefore, there is no transition in terms of the growth rate of consumption expenditure in this case. Following Perli and Sakellaris (1998), we will impose from now on the standard assumption that the production of consumption good c_1 (or of physical capital k) is more intensive in physical capital than the production of human capital, $\alpha_1 > \alpha_3$, so that the rate of growth of consumption expenditure will exhibit transitional dynamics.[8]

Assumption A. $\alpha_1 > \alpha_3$.

We proceed with the analysis of the two aforementioned forces governing the transition in this economy. It is important to note that this dynamic analysis is global in the sense that the conclusions obtained from this analysis hold even when the equilibrium path is far from the steady state. As shown in equation (3.29), those two forces are summarized by the terms $v\left(p\right)$ and $\kappa\left(p\right)$, which are functions of the relative price of goods. The function $v\left(p\right)$ collects the growth effect of an increase in the interest rate and $\kappa\left(p\right)$ is a measure of the growth effect of a variation in the relative price.[9] As the two forces only depend on the relative price, the properties of the transition will depend on the slope of the stable manifold relating the price p with the state variable z as this manifold determines the dynamic adjustment of the

[8] The role of the factor intensity ranking in the transitional dynamics of multi-sector growth models is extensively discussed in Bond et al. (1996).

[9] In the proof of Proposition 3.2 we have shown that $\kappa\left(p\right)$ is decreasing when $\alpha_1 > \alpha_3$, whereas it is immediate to see from (3.17) that $v\left(p\right)$ is a decreasing (increasing) function when $\alpha_1 > (<)\, \alpha_2$.

relative price along the transition. We proceed to the characterization of this dynamic adjustment. To this end, we denote the stable manifold relating p and z by $p = P(z)$. Note that the function $P(\cdot)$ is defined on the domain $(0, \infty)$.

Lemma 4.2. *If $\alpha_1 > (<)\alpha_2$ then $P'(z) > (<)0$. Moreover, the range of the function $P(\cdot)$ is $(0, \infty)$.*

Proof. See Appendix C.

The intuition behind this lemma is straightforward. Let us assume that $z_0 < z^*$. In this case, h_0 is large in comparison to k_0 and then the relative price of human capital p_h will be lower than its long-run value and, therefore, this price increases along the transition. This implies that the relative cost of producing the good relatively more intensive in physical capital will decrease along the transition. As firms behave competitively, this means that the relative price of consumption goods p dynamically evolves in such a way that $\kappa(p) > (<)\, 0$ when $\alpha_1 > (<)\alpha_2$. Obviously, the converse is true when $z_0 > z^*$. In any case, we finally conclude that the slope of the stable manifold relating relative price p and capital ratio z is strictly positive (negative) if $\alpha_1 > (<)\alpha_2$. In addition, by using identical arguments, we can directly see that the range of equilibrium values of p is the interval $(0, \infty)$. If the value of the physical to human capital ratio z tends to zero, then human capital becomes an abundant resource whose price tends to zero. Symmetrically, if the value of the capital ratio z tends to infinity, then physical capital becomes so abundant that its price tends to zero, that is, relative price p_h of human capital in terms of physical capital tends to infinity.

Proposition 4.3. *The physical to human capital ratio z exhibits a globally monotonic transition.*

Proof. See Appendix C.

The result in Proposition 4.3 allows us to characterize analytically the global transitional dynamics of the growth rate of consumption expenditure $\gamma = \dot{c}/c$. We should first mention that the coexistence of two forces determining the transition implies that the dynamic path of this variable may be non-monotonic when these two forces have opposite growth effects. To show these non-monotonic dynamics, we use (3.29), (3.17) and (3.26) to obtain the following derivative of the rate of growth of consumption expenditure with respect to the capital ratio z:

$$\frac{\partial \gamma}{\partial z} = \left[\frac{(1-\alpha_1)\, A_1 \psi_1^{\alpha_1 - 1} P(z)^{\frac{\alpha_2 - 1}{\alpha_1 - \alpha_2}}}{\alpha_1 - \alpha_2} \right] \Omega(z)\, P'(z), \tag{4.2}$$

where

$$\Omega(z) = \left(\frac{\alpha_3 \chi \psi_1}{\varphi^{\frac{\alpha_1 - \alpha_3}{\alpha_1 - \alpha_2}}} \right) P(z)^{\frac{1 - \alpha_1 + \alpha_3}{\alpha_1 - \alpha_2}} - \alpha_1 \left(\frac{1}{\sigma} - \chi \right), \tag{4.3}$$

and

$$\chi \equiv \left[\frac{(1-\sigma)(1-\theta)}{\sigma} \right] \left(\frac{\alpha_1 - \alpha_2}{\alpha_1 - \alpha_3} \right). \tag{4.4}$$

According to Lemma 4.2, the function $\Omega(\cdot)$ is strictly increasing in z. Note that if $\chi \in (0, 1/\sigma)$ then there exists a unique value $\overline{z}$ of z, such that $\Omega(z) > (<) 0$ when $z > (<) \overline{z}$. The following result uses these arguments together with Proposition 4.3 to provide conditions for the existence of non-monotonic behavior and to characterize the global transition dynamics of the growth rate $\dot{c}/c$ of consumption expenditure:

Proposition 4.4.

(a) *If $\chi \leq 0$, the time path of the growth rate of consumption expenditure is strictly decreasing (increasing) when $z_0 < z^*$ $(z_0 > z^*)$.*

(b) *If $\chi \in (0, 1/\sigma)$ and $\overline{z} < z^*$, the time path of the growth rate of consumption expenditure strictly decreases when $z_0 > z^*$, monotonically increases when $z_0 \in (\overline{z}, z^*)$, and exhibits a non-monotonic behavior when $z_0 < \overline{z}$.*

(c) *If $\chi \in (0, 1/\sigma)$ and $\overline{z} \geq z^*$, the time path of the growth rate of consumption expenditure strictly decreases when $z_0 < z^*$, strictly increases when $z_0 \in (z^*, \overline{z})$, and exhibits a non-monotonic behavior when $z_0 > \overline{z}$.*

(d) *If $\chi > 1/\sigma$, the time path of the growth rate of consumption expenditure is strictly increasing (decreasing) when $z_0 < z^*$ $(z_0 > z^*)$.*

Proof. See Appendix C.

The results in Proposition 4.4 imply that we can distinguish four types of transition in this economy depending on the value of χ, which is jointly determined by the IES, the expenditure share θ, and the capital intensity ranking across sectors. These different types of transition are represented in Figure 1, where the growth rate $\gamma = \dot{c}/c$ of consumption expenditure is displayed as a function of the capital ratio z. In particular, Panel (i) shows the growth rate of consumption expenditure when $\chi = 0$, i.e., when this rate is not affected by the growth of relative price p. In this case, as in the Uzawa-Lucas model, the growth rate of consumption expenditure is a monotonic function that decreases when $z_0 < z^*$ and increases when $z_0 > z^*$ (see Mulligan and Sala-i-Martín (1993) and Caballé and Santos (1993) for a complete analysis of the transitional dynamics of the Uzawa-Lucas model). In fact, the condition $\chi = 0$ holds when the production structure of the economy coincides with the one in the Uzawa-Lucas model ($\alpha_1 = \alpha_2$), there is a unique consumption good ($\theta = 1$), or preferences are logarithmic ($\sigma = 1$). Moreover, the same type of convergence holds when $\chi < 0$. However, when $\chi \in (0, 1/\sigma)$ the two forces governing the transition have opposite growth effects and the patterns of growth are different from the ones in the Uzawa-Lucas model. On the one hand, the growth rate of consumption expenditure exhibits a non-monotonic behavior when the initial value of the capital ratio is sufficiently far from its stationary value. On the other hand, as shown in Panels (ii) and (iii), we must distinguish two types of transition, depending on the relationship between $\overline{z}$ and z^*. Interestingly, if $\overline{z} < z^*$ the convergence is from below when $z_0 < z^*$ and from above otherwise. Therefore, in this case, the conclusions from convergence are reversed due to the effect of the growth of prices. As shown in Panel (iv), this reversed transition also arises when $\chi > 1/\sigma$. To see the implications of this reversed transition, suppose that the economy suffers a decrease in the stock of physical capital so that the ratio z of physical to human capital goes down. This reduction implies an initial increase in the growth rate of consumption expenditure in a model with a single consumption good, whereas it could result in an initial reduction in the growth rate $\dot{c}/c$ in our general model.

[Insert Figure 1]

5. Numerical Analysis

The results in Proposition 4.4 imply that the transition crucially depends on both the value of the parameters and the initial conditions. We next discuss which is the most plausible type of transition, as well as how quantitatively important are the differences in the transitional dynamics across different parametric scenarios. We address these two issues by following some numerical simulations. In order to fit our model with data, we will consider that the commodity y_1 corresponds to manufactures, the consumption good y_2 is composed of primary goods and services, and h is human capital. We also use the Cobb-Douglas specification (4.1) for the composite good x. We choose a period to be a year. A subgroup of parameters are determined exogenously (i.e., they are chosen based on targets that are independent of the equilibrium allocation), whereas the remaining parameters are jointly determined by imposing the BGP to satisfy some standard targets in the literature. Table 1 reports the targets and the implied parameter values.

[Insert Table 1]

The group of parameters determined exogenously was chosen as follows. The constants A_1 and A_2 are arbitrarily set to one because these parameters only affect the unit of measurement of the two commodities y_1 and y_2. According to Herrendorf et al. (2013), the consumption value added expenditure share on manufacturing decreases during the last 40 years from 0.3 to 0.16. We then select the value of 0.2 for the parameter θ. We choose the value of α_1 by using the income share of capital at the manufacturing sector reported by Valentinyi and Herrendorf (2008) for the US economy. We take the average share of physical capital in the final education output estimated by Perli and Sakellaris (1998) to set α_3. Finally, Perli and Sakellaris (1998) pointed out that the estimates of the depreciation rate η vary widely. We decide to set this rate equal to zero.

We choose the parameters A_3, α_2, δ, and ρ so that the BGP reproduces the following features that literature has measured for US economy. For expositional simplicity, we associate each parameter with the target that provided the most intuition for its value, but all the parameters are determined jointly. We choose the productivity parameters A_3 to target a participation of output from the education sector in GDP of 8%, which was calculated from the estimations in Perli and Sakellaris (1998). We set the value of α_2 to replicate an aggregate labor income share of 0.64. Although this variable has continuously decreased from 0.71 along the period 1960-2010, it has taken values very close to 0.64 during the last two decades. We select the value of δ to account for a ratio of physical capital investment to stock of this capital equal to 0.076 (see, e.g., Cooley and Prescott, 1995). We pin down the value of ρ to target a stationary growth rate of 2%, which is in the range used by the literature and corresponds with the growth rate of US GDP per capita between 1960 and 2010.

Finally, we use three different values for the IES : 0.29, 0.22 and 0.21. As the Proposition 4.4 shows, the IES crucially determines the nature of the transition since it governs the relationship between the two dynamic forces for a given capital intensity ranking across sectors and expenditure share θ. We take these values of IES, which lie in the range of plausible values, to illustrate that small variations in this elasticity change the type of transition dynamics. Using the expression of χ in Equation (4.4), we obtain that $\chi = 0.1948$ for $IES = 0.29$, $\chi = 0.2121$ for $IES = 0.22$ and $\chi = 0.2147$ for $IES = 0.21$. Thus, the considered values of IES replicates the cases (b), (c) and (d) in Proposition 4.4, respectively.[10] Since we have considered three possible values of IES, Equation (3.10) requires three alternatives values of ρ to match the target of the stationary growth rate in each case.

Our calibration implies a plausible sectoral structure along the BGP. Even when we do not observe in the data a stationary sectoral configuration, the sectoral variables predicted by the model are consistent with the average values observed recently in the US economy. In particular, our benchmark economies exhibit a share of manufacturing output on GDP of 28.53% that is very close to the value-added share estimated by Buera and Kaboski (2009). Furthermore, the labor share in the manufacturing and education sectors predicted by the model at the BGP are 29.87% and 11.13%, respectively. These values are in line with the estimations of the former share by Kongsamunt et al. (2001) and of the latter by Jones et al. (1993). Finally, note also that our calibration implies that $\alpha_1 < \alpha_2$. We should mention here the long-standing debate about the capital intensity ranking among sectors producing consumption goods. A crucial point in this discussion is whether housing is considered as a service. If this is the case, since the stock of physical capital embeds residential capital, the service sector will be relatively more physical capital intensive than the manufacturing sector. This is the view that we adopt in our numerical analysis.

We next simulate the response of each of the three parameterized economies to imbalances in the capital ratio (i.e., when $z_0 \neq z^*$), as well as to technological shocks. Our objective is to remark the importance of the dynamic adjustment in consumption expenditure induced by the variation of the relative price for macroeconomic dynamics. To this end, we will compare the transition dynamics of the baseline economies with those of two counterfactual economies: (i) an economy with a single consumption good, i.e., with $\theta = 1$; and (ii) an economy with two goods that are produced with technologies with identical capital intensities, i.e., with $\theta = 0.2$ and $\alpha_1 = \alpha_2$. We parameterize the counterfactual economies so that they exhibit the same equilibrium allocation as the benchmark economies along the BGP. This results in different values of α_1, A_1 and A_2 to replicate the same stationary values of the aggregate labor income share and of the two relative prices p and p_h as in the benchmark economies. Table 1 also shows these parameter values.

[10] Macroeconomic literature usually sets the value of IES in the interval $(0.5, 1)$. The transtional dynamics of the growth rate of expenditure for this range of values is the one corresponding with the case (b) in Proposition 4.4.

5.1. Transitional dynamics

The expression of χ in equation (4.4) implies that it takes positive values when $\alpha_1 < \alpha_2$, $\theta < 1$ and $\sigma > 1$. Thus, the value of χ is positive under our empirically plausible values of the fundamental parameters. In this case, the two aforementioned forces governing the transition display opposite growth effects. In our numerical examples, we show that, if the force associated with the variation of prices dominates then the transition will be different from that of models with a single consumption good. Figures 2, 3 and 4 show that this is the case when the value of σ is sufficiently high. These figures show the dynamic response of some relevant variables to imbalances in the capital ratio. Panels (i), (ii), (iii) display, respectively, the growth rate of consumption expenditure, the growth rate of GDP and the relative price of consumption goods as a function of the deviations of the capital ratio with respect to its stationary value. Furthermore, all panels compare the transitional dynamics of the baseline economy with heterogeneous consumption goods (continuous line) with the transition in the counterfactual economy with a unique consumption good, i.e., with $\theta = 1$ (dashed line).[11] We observe that the differences between the two economies under consideration are quite significant in the three parametric scenarios. Hence, the direct effect of the price adjustment on the intertemporal allocation of consumption expenditure also has important quantitative consequences for macroeconomic dynamics.

[Insert Figures 2, 3 and 4]

The first panel of Figures 2, 3 and 4 illustrates numerically the results in Proposition 4.4. We observe that the dynamic adjustment of consumption expenditure is non-monotonic when $IES = 0.29$ and $IES = 0.22$ in the economy with two consumption goods ($\theta = 0.2$). Moreover, when $IES = 0.21$, the introduction of heterogeneous consumption goods reverses the transition. This occurs because IES determines the relative intensity of the two forces driving the dynamics of aggregate expenditure. When IES is low, the growth effect of changes in the interest rate is low in comparison with the growth effects of changes in the growth of the relative price. In this case, even if the initial values of the economy are close to the corresponding steady-state values, the transition is different from the one arising in an economy where the transition is governed only by the diminishing returns to capital.

The significant effects of the price variation on the intertemporal allocation of consumption expenditure and saving have important quantitative implications for the dynamic behavior of the other macroeconomic variables. As an illustration, Figures 2, 3 and 4 show that the path of the rate of growth of GDP also depends on the value of the parameter θ. We observe two key differences between the response of the growth rate of GDP to the imbalances in the capital ratio in the benchmark economies and that in the economy with $\theta = 1$. Note first that the growth rate of GDP in the benchmark economies displays a hump-shaped dynamics for sufficiently small values of IES when the capital ratio is below its stationary rate. This is consistent with the empirical evidence about the growth patterns observed along the development process (see, e.g.,

[11] The transition in the counterfactual economy with $\theta = 0.2$ and $\alpha_1 = \alpha_2$ is similar to that in the economy with $\theta = 1$. Hence, to simplify the exposition, we do not include this transition in this subsection.

Maddison, 1991; or Fiaschi and Lavezzi, 2004), and on Western Europe and Japan after the World War II (see, e.g., Christiano, 1989; or Alvarez-Cuadrado, 2008). In addition, we also observe that the deviation of the growth rate of GDP from its steady-state value is much smaller and more plausible in the benchmark economies for each value of the capital ratio z. As the reduction on physical capital leads consumers to reallocate resources to producing manufactures in detriment of the other two sectors, the more intensive in services is the composite good (i.e., the smaller is θ), the less the consumer is willing to reduce the production of services necessary to accelerate the accumulation of physical capital.

5.2. Disentangling the propagation mechanism

The previous analysis has highlighted the crucial role played by the dynamic adjustment of the consumption composition in the propagation of shocks on fundamentals. In this subsection we will show that this propagation mechanism depends on the value of the parameter θ and on the capital intensity ranking across the sectors producing the consumption goods. For that purpose, we proceed to study the dynamic adjustments and the welfare costs from two different shocks: a sectoral biased and a sectoral unbiased technological shock. We assume that the economy is initially in a BGP and, unexpectedly, one of these shocks is introduced in a permanent basis. We will compare the welfare cost in our benchmark economies with that arising in the two counterfactual economies.[12] We set the size of the shocks in each economy (i.e., benchmark and counterfactual economies), such that the GDP instantaneously reduces by 5%. We only present the results for the case with $IES = 0.29$ because this elasticity has an insignificant effect on the welfare comparison between the considered economies. This is derived from analyzing the dynamic behavior of the composite good $x = c_1^{\theta} c_2^{1-\theta}$, which is the fundamental variable for welfare analysis. By using conditions (3.3) and (3.4), we obtain

$$\frac{\dot{x}}{x} = \left(\frac{1}{\sigma}\right) \left[\alpha_1 A_1 z_1^{\alpha_1 - 1} - \rho - \delta - (1-\theta)\, \kappa\,(p)\right]. \tag{5.1}$$

Obviously, the growth rate of x also depends on the forces driving the intertemporal allocation of consumption expenditure c: the diminishing returns to capital and the growth rate of prices. However, observe that the net effect of these two forces does not depend in this case on the value of σ.

We first analyze the effects of a biased technological shock consisting on a reduction in the TFP A_1 of the manufacturing sector. We illustrate these effects with the help of Figure 5, which summarizes how the economy responds to the shock; and Table 2, which provides the welfare cost of this shock. The first conclusion is that the welfare cost is more than three times larger in the economy with a unique consumption good. This large difference arises from the fact that the intratemporal substitution between goods in the economy with $\theta = 0.2$ reduces the impact of the shock in the level of the composite good. Note that Equations (3.1) and (4.1) imply $c_1/c_2 = \theta p/(1-\theta)$,

[12] As in Lucas (1987), we measure the welfare cost by the percentage of GDP in which the composite good x should be increased to obtain the same discounted sum of utility as in the hypothetical situation where the shocks do not occur.

so that this consumption ratio behaves qualitatively as the relative price p. Therefore, from Panel (ii) we derive that the consumption ratio instantaneously jumps down after the shock and then monotonously increases to a smaller stationary value. In other words, while the economy with $\theta = 1$ accommodates the shock only by altering the accumulation of both capitals, the benchmark economy also dynamically adjusts the sectoral composition of consumption demand.

[Insert Figure 5 and Table 2]

We also compare our benchmark economy with the counterfactual economy where the two consumption goods are produced by means of technologies with the same capital intensity (i.e., $\alpha_1 = \alpha_2$). Previously, we have compared economies with different expenditure share across goods (i.e., different θ). This analysis allows us to illustrate the biased derived by using models with a single consumption good. However, the difference in the welfare costs across economies cannot be entirely attributed to the dynamic adjustment on consumption composition as we compare economies with different composite consumption good. To fix this inconvenient, we investigate if our conclusions about the role played by the adjustment on consumption composition still maintains by comparing economies with the same θ. Observe that the composition mechanism neither operates in the economy with $\alpha_1 = \alpha_2$. In the later economy the relative price p is exogenously given by the ratio between sectoral TFPs so that there is not a dynamic adjustment of this price and, therefore, neither of the sectoral composition of consumption demand, after the shock. However, in the benchmark economy with $\alpha_1 \neq \alpha_2$, both the relative price and the sectoral composition of consumption also endogenously adjust to the shock along the entire transitional dynamics (see Panel (ii) in Figure 5). This dynamic adjustment on the sectoral composition of consumption allows the benchmark economy to compensate the initial negative impact of the shock in the composite consumption x. Hence, the differences in the welfare costs from shocks between the benchmark and counterfactual economies can be exclusively attributed to the dynamic adjustment on consumption composition. Table 2 shows that these differences in welfare costs are important in the case of the sectoral biased shock. More precisely, the welfare cost is an 5% larger in the economy with $\alpha_1 = \alpha_2$, provided that $\theta = 0.2$.

Panels (iii) and (iv) in Figure 5 show that the response of the sectoral structure are identical in the benchmark economy and in the counterfactual economy with $\theta = 0.2$ and $\alpha_1 = \alpha_2$. Both economies only exhibit permanent differences in the level of the sectoral labor shares. However, the dynamic adjustments in these labor shares are qualitatively similar in the two economies. The labor shares in the manufacturing and services sectors instantaneously jump down after the shock and then monotonously increase to the stationary value. As was mentioned before, the differences between the aforementioned economies are in the dynamic adjustment of relative price and in the sectoral composition of consumption demand. Therefore, the response of each economy to the shock at the aggregate level is also different because the aggregate dynamics are driven by the variation of prices in the benchmark economy but not in the counterfactual economy.

Table 2 also displays the welfare costs of an unbiased technological shock consisting on reducing the TFP in each sector by the same proportion. We observe that the welfare cost is very similar in the benchmark and the counterfactual economies. In fact, we should also mention that the differences in the effects of this shock between the considered economies only arise because the depreciation rates of both capital stocks are different, which makes the shock distort the optimal allocation of capital among sectors. If $\delta = \eta$, then the stationary value of p is not affected by the unbiased shock as it can be derived from (3.17) and (3.26). Therefore, in this case there would not be discrepancies between the economies concerning the dynamic response of the rate of growth of expenditure and the level of composite consumption, so that the welfare cost in the three economies would coincide. We can thus conclude that the discrepancy in the welfare cost of shocks between the economies under consideration only arises when these shocks have permanent effects on relative prices and on the sectoral composition of consumption in the economy with two goods.

The previous analysis confirms the importance of the dynamic adjustment in consumption composition for the propagation of shocks. Therefore, the analysis of shocks may be biased if we consider either models with a unique consumption good or models with several consumption goods that are produced with technologies with identical capital intensities.

5.3. A sensitive analysis: the CES case

We finally analyze how the numerical results depend on the elasticity of substitution between goods. In the previous analysis we have considered a Cobb-Douglas functional form for the composite good, which implies a unitary elasticity of substitution and constant expenditure shares. In this sub-section we study the robustness of the results for the case where the composite good is given by a CES functional form as follows

$$x \equiv v\left(c_1, c_2\right) = \left[\theta c_1^{\mu} + \left(1-\theta\right) c_2^{\mu}\right]^{\frac{1}{\mu}},$$

where $\mu \leq 1$. In this case the elasticity of substitution between goods is given by $\xi = 1/\left(1-\mu\right)$ and the expenditure share in consumption c_2 is

$$E\left(p\right) = \frac{\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{\mu-1}} p^{\frac{\mu}{\mu-1}}}{1+\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{\mu-1}} p^{\frac{\mu}{\mu-1}}}.$$

The results on the response to shocks may change because the elasticity of substitution drives the response of consumption demands and aggregate expenditure to the variation of relative price p. The endogeneity of the sectoral expenditure shares may then determine the intensity of the propagation mechanism. We next study how important are these two channels for the effects from shocks.

We compute the welfare cost of biased technological shocks that instantaneously reduce aggregate GDP by 5%. To this end, we consider several values of the elasticity of substitution: $\xi = \{0.25,\ 0.5,\ 0.75,\ 2,\ 4,\ 10\}$. We then set the value of θ such that the stationary expenditure share for manufacturing is 0.2, i.e., $E\left(p^*\right) = 0.8$. The remaining parameters are calibrated by imposing the same facts as for the Cobb-Douglas case analyzed in the previous subsections.

Table 3 provides the results of these numerical simulations. We observe that the results are quite robust to changes in the values of the elasticity of substitution ξ. The welfare cost of shocks in our benchmark economy decreases with this elasticity. The more complementary are the goods, the more difficult is for the consumer to accommodate the shock by altering the consumption basket. We also check that the differences with the results from the counterfactual economies change very little with the elasticity of substitution. The differences with the economy with $\theta = 1$ are obviously decreasing in ξ, since the welfare cost is invariant in that economy. On the contrary, the differences with the economy with two consumption goods but $\alpha_1 = \alpha_2$ slightly decrease with ξ.

[Insert Table 3]

6. Concluding remarks

We have analyzed the transitional dynamics of an endogenous growth model with two consumption goods. We have shown that the growth rate of expenditure not only depends on the interest rate but also on the growth rate of the relative price of consumption goods. Convergence in this case may be determined by two different forces: the diminishing returns to capital and the dynamic adjustment of consumption expenditure induced by variations in relative prices. In particular, the latter force arises when preferences are not logarithmic and the technologies producing the two consumption goods have different capital intensities. The growth effects of the adjustment in relative prices yield interesting differences with respect to the transitional dynamics obtained in the standard growth model with a unique consumption good. We illustrate these differences using a growth model with two capital stocks that we identify with human and physical capital. First, we show that in contrast with the standard growth model, convergence in the growth rate may occur from above if the initial value of the ratio of physical to human capital is larger than its stationary value and may occur from below otherwise. Second, we show that the growth rate of consumption expenditure may exhibit a non-monotonic behavior when the two aforementioned dynamic forces have opposite growth effects. These differences in the transition have other noteworthy implications.

First, economies with the same interest rate may exhibit different growth rates of consumption along the transition. Therefore, our model provides an additional explanation to the cross-country differences in the growth rates. Rebelo (1992) shows that the introduction of a minimum consumption requirement also implies that the growth rates do not equalize. This occurs because the minimum consumption makes preferences non-homothetic so that the IES is no longer constant along the transition. In this framework, convergence is driven by the interest rate and by the time-varying IES. More recently, Steger (2006) shows that, if there are heterogeneous consumption goods and a unique capital stock, then the IES is not constant and the growth rates do not equalize. Obviously, he derives this result when preferences are non-homothetic. In contrast, we show that, when there are heterogeneous consumption goods, the growth rates are different even with a constant IES because of the adjustment of consumption composition along the transition.

Second, the direct growth effect of dynamic price adjustment is an unexplored channel affecting the persistence and propagation of shocks. According to our results, the welfare cost of shocks will also depend on the sectoral composition of consumption and on the physical capital intensities of the sectors producing the consumption goods. Capital intensity ranking across sectors affects the response of relative prices to shocks, whereas the consumption composition determines the effect of the price variation derived from shocks on the cost of the composite consumption good. Therefore, the empirical estimation of the sectoral composition parameters and the sectoral capital intensities should be an important concern for future research on the assessment of the welfare cost of macroeconomic shocks and on the analysis of business cycles.

A natural extension of our paper is to introduce some of the mechanisms used by the literature to explain the observed structural change: no-homothetic preferences or biased technological change. As was mentioned before, the objective of the present paper was to explain neither the structural change nor the unbalanced growth observed in the data. We have only studied how the dynamic adjustment of the sectoral composition of consumption affects the macroeconomic dynamics. Hence, in our paper structural change is only a transitory phenomenon obtained as a mere by-product of the dynamic adjustment derived from imbalances in the capital ratio. An interesting next step would be to analyze the consequences of introducing sources of permanent structural change. This would generate an interesting endogenous feedback between structural change and capital accumulation that can contribute to the literature on structural transformation. Furthermore, our model seems an appropriate environment to analyze the role that the accumulation of human capital may play in explaining the dynamic transformation of the service sector.

References

[1] Acemoglu, D. and Guerrieri, V. (2008). "Capital Deepening and Nonbalanced Economic Growth," Journal of Political Economy 116, 467-498.

[2] Alvarez-Cuadrado, F. (2008). "Growth Outside the Stable Path: Lessons from the European Reconstruction," European Economic Review 52, 568-588.

[3] Alvarez-Cuadrado, F., Monterio, G. and Turnovsky, S. (2004). "Habit Formation, Catching-up with the Joneses, and Economic Growth," Journal of Economic Growth 9, 47-80.

[4] Bond E., Wang, P. and Yip, C. (1996). "A General Two-Sector Model of Endogenous Growth with Human and Physical Capital: Balanced Growth and Transitional Dynamics," Journal of Economic Theory 68, 149-173.

[5] Boppart, T. (2011). "Structural Change and the Kaldor Facts in a Growth Model with Relative Price Effects and Non-Gorman Preferences," Econ-Working papers 002, University of Zurich.

[6] Buera, F.J. and Koboski, J.P. (2009). "Can Traditional Theories of Structural Change Fit the Data?" Journal of European Economic Association 7, 469-477.

[7] Caballé, J. and Santos, M. (1993). "On Endogenous Growth with Physical and Human Capital," Journal of Political Economy 101, 1042-1067.

[8] Cooley, T. and Prescott, E. (1995). "Economic Growth and Business Cycles,", in T. Cooley (ed.), Frontiers of Business Cycle Research, Princeton University Press, 1-38.

[9] Christiano, L.J. (1989). "Understanding Japan's Saving Rate: The Reconstruction Hypothesis," Federal Reserve Bank of Minneapolis Quarterly Review 13 (2), 10-25.

[10] Echevarria, C. (1997). "Changes in Sectoral Composition Associated with Economic Growth," International Economic Review 38, 431-452.

[11] Fiaschi, D. and Lavezzi, A.M. (2004). "Nonlinear Growth in a Long-run Perspective," Applied Economic Letters 11, 101-104.

[12] Foellmi, R. and Zweimüller, J. (2008). "Structural Change, Engel's Consumption Cycles and Kaldor's Facts of Economic Growth," Journal of Monetary Economics 55, 1317-1328.

[13] Hansen, G. and Prescott, E. (2002). "Malthus to Solow," American Economic Review 92, 1205-1217.

[14] Herrendorf, B., Rogerson, R. and Valentinyi, A. (2013). "Two Perspectives on Preferences and Structural Transformation," forthcoming in American Economic Review.

[15] Jones, L.E. Manuelli, R.E. and Rosi, P.E. (1993). "Optimal taxation in Models of Endogenous Growth," Journal of Political Economy 101, 485-517.

[16] Kongsamut, P., Rebelo, S. and Xie, D. (2001). "Beyond Balanced Growth," Review of Economic Studies 68, 869-882.

[17] Lucas, R. E. (1987). Models of Business Cycles, Basil Blackwell.

[18] Lucas, R. (1988). "On the Mechanics of Economic Development," Journal of Monetary Economics 22, 3-42.

[19] Maddison, A. (2001). The World Economy: A Millennia Perspective, OECD, Paris.

[20] Mulligan, C. and Sala-i-Martín, X. (1993). "Transitional Dynamics in Two-Sector Models of Endogenous Growth," Quarterly Journal of Economics 108, 737-773.

[21] Ngai, R. (2004). "Barriers and the Transition to Modern Growth," Journal of Monetary Economics 51, 1353-1383.

[22] Ngai, R. and Pissarides, C. (2007). "Structural Change in a Multi-sector Model of Growth," American Economic Review 97, 429-443.

[23] Papageorgiou, C. and Perez-Sebastian, F. (2006). "Dynamics in a Non-Scale R&D Growth Model: Explaining the Japanese and South Korean Development Experiences," Journal of Economic Dynamics and Control 30, 901-930.

[24] Perez, F. and Guillo, D. (2010). "Reexamining the Role of Land in Economic Growth," Manuscript.

[25] Perli, R. and Sakellaris, P. (1998). "Human Capital Formation and Business Cycle Persistence," Journal of Monetary Economics 42, 67-92.

[26] Rebelo, S. (1991). "Long-run Policy Analysis and Long-run Growth," Journal of Political Economy 99, 500-521.

[27] Rebelo, S. (1992). "Growth in Open Economies," Carnegie-Rochester Conference Series on Public Policy 36, 5-46.

[28] Rostow, W. (1960). The Stages of Economic Growth, Oxford University Press, Oxford.

[29] Steger, T.M. (2000). "Economic Growth with Subsistence Consumption," Journal of Development Economics 62, 343-361.

[30] Steger, T.M. (2006). "Heterogeneous Consumption Goods, Sectoral Change and Economic Growth," Studies in Nonlinear Dynamics and Econometrics 10, No. 1, Article 2.

[31] Uzawa, H. (1965). "Optimum Technical Change in an Aggregative Model of Economic Growth," International Economic Review 60, 12-31.

[32] Valentinyi, A. and Herrendorf, B. (2008). "Measuring factor income shares at the sectoral level," Review of Economic Dynamics 11, 820-835.

Appendix

A. Solution to the consumer's optimization problem.

The Hamiltonian function associated with the maximization of (2.6) subject to (2.3), (2.4) and (2.5) is

$$\begin{aligned} H &= e^{-\rho t} U(c_1, c_2) + \\ &\quad \lambda (wh + rk - c_1 - pc_2 - I_k - p_h I_h) + \mu_1 (I_k - \delta k) + \mu_2 (I_h - \eta h), \end{aligned}$$

where λ, μ_1, and μ_2 are the co-state variables corresponding to the constraints (2.3), (2.4) and (2.5), respectively. The first order conditions are

$$e^{-\rho t} v_1 v^{-\sigma} = \lambda, \tag{A.1}$$

$$e^{-\rho t} v_2 v^{-\sigma} = \lambda p, \tag{A.2}$$

$$\lambda = \mu_1, \tag{A.3}$$

$$p_h \lambda = \mu_2, \tag{A.4}$$

$$\lambda r - \delta \mu_1 = -\dot{\mu}_1, \tag{A.5}$$

$$\lambda w - \eta \mu_2 = -\dot{\mu}_2. \tag{A.6}$$

Combining (A.1) and (A.2), we obtain Equation (3.1) and, from log-differentiating this equation with respect to time, we get

$$\left[\frac{v_{21}}{v_2} - \frac{v_{11}}{v_1} \right] \dot{c}_1 + \left[\frac{v_{22}}{v_2} - \frac{v_{12}}{v_1} \right] \dot{c}_2 = \frac{\dot{p}}{p}. \tag{A.7}$$

Since $v(c_1, c_2)$ is linear homogeneous, we establish the following relations:

$$v = v_1 c_1 + v_2 c_2, \tag{A.8}$$

$$v_{11} c_1 + v_{12} c_2 = v_{21} c_1 + v_{22} c_2 = 0. \tag{A.9}$$

Using Conditions (A.8) and (A.9) in (A.7), and after some algebra, we obtain

$$\frac{\dot{c}_2}{c_2} = \frac{\dot{c}_1}{c_1} - \xi \left(\frac{\dot{p}}{p} \right), \tag{A.10}$$

where ξ is the elasticity of substitution of $v(\cdot)$ given in (3.7). Using (A.3) and (A.4), we obtain

$$p_h \mu_1 = \mu_2,$$

which implies that

$$\frac{\dot{p}_h}{p_h} + \frac{\dot{\mu}_1}{\mu_1} = \frac{\dot{\mu}_2}{\mu_2},$$

and (3.2) follows from using (A.5) and (A.6). Log-differentiating with respect to time (A.1), we obtain

$$-\rho + \left[\frac{v_{11}}{v_1} - \sigma \left(\frac{v_1}{v} \right) \right] \dot{c}_1 + \left[\frac{v_{12}}{v_1} - \sigma \left(\frac{v_2}{v} \right) \right] \dot{c}_2 = \frac{\dot{\lambda}}{\lambda}.$$

By employing the definition of the elasticity of substitution ξ given in (3.7), we derive

$$-\rho+\left[\frac{v_{11}}{v_1}-\sigma\xi\left(\frac{v_{12}}{v_2}\right)\right]\dot{c}_1+\underbrace{\left(\frac{v_{12}}{v_1}\right)(1-\sigma\xi)}_{-\varepsilon/c_2}\dot{c}_2=\frac{\dot{\lambda}}{\lambda}.$$

Combining this equation with (A.3) and (A.5), and using Conditions (A.8) and (A.9), we obtain

$$-r+\delta=-\rho-\left[\frac{v_{12}c_1}{v_2}\right]\left[\frac{v_2c_2}{v_1c_1}+\sigma\xi\right]\left(\frac{\dot{c}_1}{c_1}\right)-\varepsilon\left(\frac{\dot{c}_2}{c_2}\right).$$

Finally, Equations (3.3) and (3.4) follow from combining the previous equation with (A.10) and combining the resulting expression with (A.8) and (A.9).

B. Deriving the Euler equation on expenditure.

First, we express the solution to the consumer's problem in terms of total expenditure and the fraction of expenditure on c_2. To this end, we use the linear homogeneity of function $v(\cdot)$ to define

$$\widetilde{v}(1-e,e/p)\equiv\frac{v(c_1,c_2)}{c},\tag{B.1}$$

and, moreover, we note that

$$v_i(c_1,c_2)=\widetilde{v}_i(1-e,e/p),\tag{B.2}$$

and

$$v_{ij}(c_1,c_2)=c^{-1}\widetilde{v}_{ij}(1-e,e/p),\tag{B.3}$$

for $i=\{1,2\}$ and $j=\{1,2\}$. Using these properties, we rewrite Condition (3.1) as

$$p\widetilde{v}_1=\widetilde{v}_2,\tag{B.4}$$

the elasticity of substitution ξ given in (3.7) as

$$\xi=\frac{\widetilde{v}_2\widetilde{v}_1}{\widetilde{v}_{12}\widetilde{v}},\tag{B.5}$$

and the Edgeworth elasticity ε given in (3.8) as

$$\varepsilon=-(1-\sigma\xi)\left[\frac{e\widetilde{v}_{12}}{p\widetilde{v}_1}\right].\tag{B.6}$$

Since $\widetilde{v}$ is also linearly homogeneous, we get

$$\widetilde{v}=(1-e)\widetilde{v}_1+(e/p)\widetilde{v}_2,\tag{B.7}$$

and

$$(1-e)\widetilde{v}_{11}+(e/p)\widetilde{v}_{12}=(1-e)\widetilde{v}_{21}+(e/p)\widetilde{v}_{22}=0.\tag{B.8}$$

Condition (B.7) together with (B.4) implies that $\widetilde{v} = \widetilde{v}_1$. Therefore, the elasticity of substitution ξ can finally be expressed in equilibrium as

$$\xi = \frac{\widetilde{v}_2}{\widetilde{v}_{12}}. \tag{B.9}$$

Using the strict quasiconcavity of v and $\widetilde{v}$, we get from applying the implicit function theorem to (B.4) that

$$\frac{\partial e}{\partial p} \equiv E'(p) = \frac{\widetilde{v}_1 + \left(e/p^2\right)\widetilde{v}_{22} - (e/p)\,\widetilde{v}_{12}}{-\widetilde{v}_{21} + (1/p)\,\widetilde{v}_{22} + p\widetilde{v}_{11} - \widetilde{v}_{12}}.$$

By using Condition (B.8), and after some algebra, we obtain

$$E'(p) = -\left(\frac{e}{p}\right)(1-e)\left[\frac{p\widetilde{v}_1}{\widetilde{v}_{12}} - 1\right],$$

so that (3.9) follows from using (B.4) and (B.9).

Log-differentiating $c_1 = (1-e)\,c$ with respect to time, and combining the resulting expression with (3.3), we obtain

$$\frac{\dot{c}}{c} = \frac{r-\rho-\delta}{\sigma} + \left[\frac{\varepsilon\xi}{\sigma} + \frac{pE'(p)}{1-e}\right]\left(\frac{\dot{p}}{p}\right).$$

By combining the previous equation with (B.4), (B.6), (B.9) and (3.9), and after some algebra, we obtain (3.10).

C. Proofs of results

Proof of Proposition 3.1. (i) Consider first the condition $\alpha_1 = \alpha_2$. We can first combine equations (3.11), (3.12), (3.14) and (3.15) to get $z_1 = z_2$. Therefore, by combining equations (3.11) and (3.12), it follows that the relative price between the two consumption goods remains constant and equal to $p = A_1/A_2$.

(ii) Let us now consider the condition $\alpha_1 = \alpha_3$. Observe that in this case conditions (3.11), (3.13), (3.14) and (3.16) imply that $z_1 = z_3$ and, thus, the relative price between the two capitals is constant and given by $p_h = A_1/A_3$. Equation (3.2) implies that the ratio w/r remains constant when p_h is constant. Then, from combining (3.11) and (3.14) we immediately see that z_1 is constant when p_h is constant. Therefore, both the rental rate r and z_2 are constant as follows from (3.11) and (3.13). Finally, equation (3.12) shows that in this case relative price p between the two consumption goods remains constant.■

Proof of Proposition 3.2. The uniqueness of p^* follows from the monotonicity of $\kappa(p)$, which can be shown using (3.26),

$$\kappa'(p) = \left[\frac{(1-\alpha_1)\,A_1\psi_1^{\alpha_1-1}p^{\frac{\alpha_2-1}{\alpha_1-\alpha_2}}}{\alpha_3-\alpha_1}\right]\left[\alpha_1 + \left(\frac{\alpha_3\psi_1}{\varphi^{\frac{\alpha_1-\alpha_3}{\alpha_1-\alpha_2}}}\right)p^{\frac{1-\alpha_1+\alpha_3}{\alpha_1-\alpha_2}}\right] > (<)\, 0 \text{ if } \alpha_1 < (>)\, \alpha_3,$$

and the fact that $\lim_{p\to 0}\kappa(p) = -\infty(\infty)$ and $\lim_{p\to\infty}\kappa(p) = \infty(-\infty)$ when $\alpha_1 < (>)\,\alpha_3$.

Combining (3.20), (3.21) and (3.22), we obtain

$$u_1 = \frac{z_3 - z}{z_3 - z_1} + \left[\frac{E(p)}{pA_2 z_2^{\alpha_2}}\right]\left(\frac{z_2 - z_3}{z_3 - z_1}\right) qz \tag{C.1}$$

and

$$1 - u_1 - u_2 = \frac{z - z_1}{z_3 - z_1} + \left[\frac{E(p)}{pA_2 z_2^{\alpha_2}}\right]\left(\frac{z_1 - z_2}{z_3 - z_1}\right) qz. \tag{C.2}$$

Given the value of p^*, the strict quasiconcavity of $v(c_1, c_2)$ guarantees a unique stationary value of the expenditure shares. Denote by $\widetilde{\theta}$ the value of $1 - E(p^*)$. In a steady state, equations (3.28) and (3.27) simplify to

$$1 - u_1^* - u_2^* = \frac{g^* + \eta}{A_3 (z_3^*)^{\alpha_3}},$$

$$\frac{A_1 u_1^* (z_1^*)^{\alpha_1}}{z^*} - \widetilde{\theta} q^* = g^* + \delta.$$

By using (C.1) and (C.2), the previous two equations can be rewritten as the following system of two equations:

$$z^* + \underbrace{\left(\frac{1 - \widetilde{\theta}}{p^* A_2 (z_2^*)^{\alpha_2}}\right)}_{\phi_1} (z_1^* - z_2^*) q^* z^* = \underbrace{\left(\frac{g^* + \eta}{A_3 (z_3^*)^{\alpha_3}}\right) (z_3^* - z_1^*) + z_1^*}_{\phi_2},$$

$$z_3^* + \left[\phi_1^* (z_2^* - z_3^*) - \frac{(z_3^* - z_1^*)}{A_1 (z_1^*)^{\alpha_1}}\right] q^* z^* = \underbrace{\left[(z_3^* - z_1^*)\left(\frac{g^* + \delta}{A_1 (z_1^*)^{\alpha_1}}\right) + 1\right]}_{\phi_3} z^*.$$

The steady state values of $z^*$and q^* are the unique solution of this system of equations and they are equal to

$$z^* = \frac{\phi_1 \phi_2 (z_2^* - z_3^*) + \phi_1 (z_1^* - z_2^*) z_3^* - \frac{\phi_2 (z_3^* - z_1^*)}{A_1 (z_1^*)^{\alpha_1}}}{\phi_1 (z_2^* - z_3^*) + \phi_1 \phi_3 (z_1^* - z_2^*) - \frac{z_3^* - z_1^*}{A_1 (z_1^*)^{\alpha_1}}},$$

and

$$q^* = \frac{\phi_2 \phi_3 - z_3^*}{\left[\phi_1 \phi_2 (z_2^* - z_3^*) - \frac{\phi_2 (z_3^* - z_1^*)}{A_1 (z_1^*)^{\alpha_1}} + \phi_1 (z_1^* - z_2^*) z_3^*\right]},$$

where the steady-state values of z_i, $i = \{1, 2, 3\}$, satisfy $z_i^* = \psi_i (p^*)^{\frac{1}{\alpha_1 - \alpha_3}}$ as follows from (3.17).■

Proof of Proposition 4.1. Let J be the Jacobian matrix evaluated at the steady state of the system of differential equations formed by (3.26), (3.31) and (3.32),[13]

$$J=\begin{pmatrix} \frac{\partial \dot{p}}{\partial p} & \frac{\partial \dot{p}}{\partial z} & \frac{\partial \dot{p}}{\partial q} \\ \\ \frac{\partial \dot{z}}{\partial p} & \frac{\partial \dot{z}}{\partial z} & \frac{\partial \dot{z}}{\partial q} \\ \\ \frac{\partial \dot{q}}{\partial p} & \frac{\partial \dot{q}}{\partial z} & \frac{\partial \dot{q}}{\partial q} \end{pmatrix},$$

where

$$\frac{\partial \dot{p}}{\partial p}=p\kappa'(p),$$

$$\frac{\partial \dot{p}}{\partial z}=0,$$

$$\frac{\partial \dot{p}}{\partial q}=0,$$

$$\frac{\partial \dot{z}}{\partial p}=z\left\{\begin{array}{c} \underbrace{\left(\frac{A_1 z_1^{\alpha_1}}{z}\right)\left(\frac{\partial u_1}{\partial p}\right)+\left(\frac{A_1 u_1 \alpha_1 z_1^{\alpha_1-1}}{z}\right)\left(\frac{\partial z_1}{\partial p}\right)}_{\epsilon_p} \\ -A_3 z_3^{\alpha_3}\left[\frac{\partial(1-u_1-u_2)}{\partial p}\right]-A_3(1-u_1-u_2)\alpha_3 z_3^{\alpha_3-1}\left(\frac{\partial z_3}{\partial p}\right) \end{array}\right\},$$

$$\frac{\partial \dot{z}}{\partial z}=z\left\{\underbrace{-\frac{A_1 u_1 z_1^{\alpha_1}}{z^2}+\left(\frac{A_1 z_1^{\alpha_1}}{z}\right)\left(\frac{\partial u_1}{\partial z}\right)}_{\epsilon_z}-A_3 z_3^{\alpha_3}\left[\frac{\partial(1-u_1-u_2)}{\partial z}\right]\right\},$$

$$\frac{\partial \dot{z}}{\partial q}=z\left\{\underbrace{\left(\frac{A_1 z_1^{\alpha_1}}{z}\right)\left(\frac{\partial u_1}{\partial q}\right)-\theta}_{\epsilon_q}-A_3 z_3^{\alpha_3}\left[\frac{\partial(1-u_1-u_2)}{\partial q}\right]\right\},$$

$$\frac{\partial \dot{q}}{\partial p}=q\left\{\left[\frac{\alpha_1(\alpha_1-1)A_1 z_1^{\alpha_1-2}}{\sigma}\right]\left(\frac{\partial z_1}{\partial p}\right)-\left[\frac{(1-\sigma)(1-\theta)}{\sigma}\right]\kappa'(p)-\epsilon_p\right\},$$

$$\frac{\partial \dot{q}}{\partial z}=-q\epsilon_z,$$

and

$$\frac{\partial \dot{q}}{\partial q}=-q\epsilon_q.$$

The determinant of the Jacobian matrix is

$$Det(J)=\left(\frac{\partial \dot{p}}{\partial p}\right)\left[\left(\frac{\partial \dot{z}}{\partial z}\right)\left(\frac{\partial \dot{q}}{\partial q}\right)-\left(\frac{\partial \dot{z}}{\partial q}\right)\left(\frac{\partial \dot{q}}{\partial z}\right)\right]=zq\kappa'(p)pA_3 z_3^{\alpha_3}M,$$

[13] In this proof all the variables are valued at the BGP equilibrium. To ease the notation, we omit the asterisk denoting the steady-state.

where

$$M = \epsilon_q \left[\frac{\partial \left(1 - u_1 - u_2\right)}{\partial z} \right] - \epsilon_z \left[\frac{\partial \left(1 - u_1 - u_2\right)}{\partial q} \right] =$$

$$= \left\{ \begin{array}{c} \theta \left(\frac{\partial u_1}{\partial z} \right) - \left(\frac{A_1 u_1 z_1^{\alpha_1}}{z^2} \right) \left(\frac{\partial u_1}{\partial q} \right) \\ - \left[\left(\frac{A_1 z_1^{\alpha_1}}{z} \right) \left(\frac{\partial u_1}{\partial q} \right) - \theta \right] \left(\frac{\partial u_2}{\partial z} \right) + \left[- \frac{A_1 u_1 z_1^{\alpha_1}}{z^2} + \left(\frac{A_1 z_1^{\alpha_1}}{z} \right) \left(\frac{\partial u_1}{\partial z} \right) \right] \left(\frac{\partial u_2}{\partial q} \right) \end{array} \right\}.$$

Using (3.20), (C.1) and (C.2), and after some algebra, M simplifies to

$$M = - \left(\frac{\theta}{z_3 - z_1} \right) \left[\underbrace{1 + \phi_1 z_2 \left(g^* + \delta\right) + \phi_1 z_1 \left(A_1 z_1^{\alpha_1 - 1} - g^* - \delta \right)}_{N} \right].$$

Note that $N > 0$ because

$$A_1 z_1^{\alpha_1} - g^* - \delta = \frac{g^* \left(\sigma - \alpha_1\right) + \rho + \delta \left(1 - \alpha_1\right)}{\alpha_1} > 0,$$

where the inequality follows from the transversality condition, which implies that $\rho > \left(1 - \sigma\right) g^*$. Thus, the determinant is given by

$$Det\left(J\right) =$$

$$- \left(\frac{\theta z q \kappa' \left(p\right) p A_3 z_3^{\alpha_3}}{z_3 - z1} \right) \left\{ 1 + \phi_1 z_2 \left(g^* + \delta\right) + \phi_1 z_1 \left[\frac{g^* \left(\sigma - \alpha_1\right) + \rho + \delta \left(1 - \alpha_1\right)}{\alpha_1} \right] \right\}.$$

By using (3.17) and (3.19), we obtain that $z_3 > (<)\, z_1$ when $\alpha_1 < (>) \alpha_3$ and, therefore, we derive from the proof of Proposition 3.2 that $\kappa' \left(p\right) > (<)\, 0$ when $z_3 > (<)\, z_1$. We then conclude that $Det\left(J\right) < 0$. Next, we obtain the value of the trace,

$$Tr\left(J\right) = \frac{\partial \dot{p}_h}{\partial p_h} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{q}}{\partial q} =$$

$$= \left\{ \begin{array}{c} p \kappa' \left(p\right) + A_1 z_1^{\alpha_1} \left(\frac{\partial u_1}{\partial z} \right) - \frac{A_1 u_1 z_1^{\alpha_1}}{z} \\ - \left(\frac{A_3 z_3^{\alpha_3}}{z} \right) \left[\frac{\partial (1 - u_1 - u_2)}{\partial z} \right] - q \left[\left(\frac{A_1 z_1^{\alpha_1}}{z} \right) \left(\frac{\partial u_1}{\partial q} \right) - \theta \right] \end{array} \right\}.$$

Using (C.1) and (C.2), the trace simplifies, after some tedious algebra, to

$$Tr\left(J\right) = \alpha_1 A_1 \psi_1^{\alpha_1 - 1} p^{\frac{\alpha_1 - 1}{\alpha_1 - \alpha_2}} + \frac{\left(1 - \alpha_1\right) A_1 \psi_1^{\alpha_1} p^{\frac{\alpha_3}{\alpha_1 - \alpha_2}}}{\varphi^{\frac{\alpha_1 - \alpha_3}{\alpha_1 - \alpha_2}}} - \left(g^* + \eta\right) - \left(g^* + \delta\right).$$

Making $\kappa\left(p\right) = 0$, we obtain

$$Tr\left(J\right) = 2 \left(\alpha_1 A_1 \psi_1^{\alpha_1 - 1} p^{\frac{\alpha_1 - 1}{\alpha_1 - \alpha_2}} - g^* - \delta \right),$$

and, by using (3.29) at BGP, we derive

$$Tr(J) = 2[(\sigma - 1) g^* + \rho] > 0,$$

as follows from the transversality condition.

Since the trace of J is positive and the determinant is negative, there exists a unique negative root and the equilibrium is saddle-path stable. When $\alpha_1 > \alpha_3$ the adjustment process of relative price p is stable so that the negative root of the Jacobian J is $p\kappa'(p)$. Otherwise, the dynamic process of p is unstable. In this case, relative price p instantaneously jumps to its stationary value, and the negative root of J is one of the roots obtained from the sub-system of differential equations formed by equations (3.31) and (3.32) with $p = p^*$ for all t.[14]■

Proof of Lemma 4.2. Equation (3.17) shows that all the physical to human capital ratios in the three sectors, z_1, z_2 and z_3, depend positively (negatively) on relative price p when $\alpha_1 > (<) \alpha_2$. We can write the aggregate physical to human capital ratio $z = k/h$ as

$$z = \frac{k_1 + k_2 + k_3}{h_1 + h_2 + h_3}, \tag{C.3}$$

where k_i and h_i are the stocks of physical and human capital used in the production of good i, $i = \{1, 2, 3\}$. When all the ratios z_1, z_2 and z_3 vary in the same direction, the aggregate physical to human capital ratio z also varies in this direction. For instance, if all the ratios z_1, z_2 and z_3 rise, then the following relationship between the increments of the sectoral capital stocks must apply: $\Delta k_1 > \Delta h_1$, $\Delta k_2 > \Delta h_2$, and $\Delta k_3 > \Delta h_3$. Therefore,

$$\Delta k_1 + \Delta k_2 + \Delta k_3 > \Delta h_1 + \Delta h_2 + \Delta h_3.$$

Using the previous inequality in (C.3), and the dependence of the ratios z_1, z_2 and z_3 on relative price p, we obtain the monotonically increasing (decreasing) relationship between the aggregate physical to human capital ratio z and relative price p of human capital along the stable manifold when $\alpha_1 > (<) \alpha_2$.

Note that equation (3.17) implies that $\lim_{p\to 0} z_i = 0 (\infty)$ when $\alpha_1 > (<) \alpha_2$, with $z_i = k_i/h_i$, $i = \{1, 2, 3\}$. This means that either $\lim_{p\to 0} k_i = 0 (\infty)$ or $\lim_{p\to 0} h_i = \infty (0)$ when $\alpha_1 > (<) \alpha_2$. In both cases, we will get that $\lim_{p\to 0} z = 0 (\infty)$ if $\alpha_1 > (<) \alpha_2$. However, $\lim_{p\to\infty} z_i = \infty (0)$ when $\alpha_1 > (<) \alpha_2$, with $z_i = k_i/h_i$, $i = \{1, 2, 3\}$, which means that either $\lim_{p\to\infty} k_i = \infty (0)$ or $\lim_{p\to\infty} h_i = 0 (\infty)$ when $\alpha_1 > (<) \alpha_2$. In both cases, we will get that $\lim_{p\to\infty} z = \infty (0)$ if $\alpha_1 > (<) \alpha_2$. Therefore, as the ratio z may take potentially any value in the interval $(0, \infty)$, the range of values of the price p along the stable manifold is also $(0, \infty)$.■

Proof of Proposition 4.3. In the proof of Proposition 4.1, we have shown that $\kappa'(p) < 0$ if $\alpha_1 > \alpha_3$. This means that the relative price exhibit a monotonic transition. In addition, Lemma 4.2 states that the stable manifold relating prices and the ratio of

[14] Note that the dynamic system characterizing the equilibrium maintains the duality between quantities and prices that emerges in the Lucas-Uzawa-type growth models. More precisely, the dynamic adjustment of prices is determined independently of the quantities and is dictated by the capital intensity ranking across sectors.

capitals is strictly monotone. This implies that the ratio z of capitals must also exhibit a monotonic behavior along the entire transition.■

Proof of Proposition 4.4. Given the sign of $P'(z)$ characterized by Lemma 4.2, we conclude from (4.2) that the growth rate of consumption expenditure γ is increasing (decreasing) when $\Omega(z) > (<) 0$. Therefore, the statement of the proposition directly follows from (4.3). Parts (a) and (d) follow since $\Omega(z) < 0$ when $\chi \leq 0$ and $\Omega(z) > 0$ when $\chi > \frac{1}{\sigma}$. For Part (b) note that we get $\Omega(z) > 0$ along the transition when $z_0 > z^*$ and $\Omega(z) < 0$ when $z_0 < \overline{z} < z^*$. In the first case, the rate of growth of consumption is monotonically decreasing, whereas it exhibits a non-monotonic behavior when $z_0 < \overline{z}$. In particular, if $z_0 < \overline{z}$ the growth rate of consumption expenditure initially decreases and ends up being increasing with time as the dynamic equilibrium approaches its steady state. In Part (c), we have that $\Omega(z) > 0$ along the transition when $z_0 < z^*$ and $\Omega(z) < 0$ when $z_0 > \overline{z} \geq z^*$. In the first case, the consumption growth rate is monotonically decreasing, whereas it exhibits a non-monotonic behavior when $z_0 > \overline{z}$. In particular, if $z_0 > \overline{z}$ the growth rate of consumption expenditure initially decreases and becomes eventually increasing as the equilibrium path approaches its steady state.■

Targets	Value	Parameter	Benchmark
Manufacturing TFP	1	A_1	1
Services TFP	1	A_2	1
Capital income share in manufactures	0.33	α_1	0.33
Capital income share in education	0.11	α_3	0.11
Human capital depreciation	0	η	0
Manufacturing share on expenditure	0.2	θ	0.2
Physical capital investment to stock	0.076	δ	0.056
Education output share on GDP	0.08	A_3	0.1952
Aggregate labor income share on GDP	0.64	α_2	0.405
Intertemporal elasticity of substitution	{0.29, 0.22, 0.21}	σ	{3.5, 4.5, 4.7}
Growth rate of GDP	0.02	ρ	{0.09, 0.07, 0.066}

Note. The contrafactual economies exhibit the baseline values of paramenters except for the following ones:

- Contrafactual 1: $\theta = 1$, $\alpha_1 = 0.3817$ and $A_1 = 0.9621$;
- Contrafactual 2: $\alpha_1 = \alpha_2 = 0.3817$, $A_1 = 0.9621$ and $A_3 = 1.0213$.

This ensures that they exhibit the same equilibrium allocation as the benchmark economies along the BGP.

Table 1. Parameter values

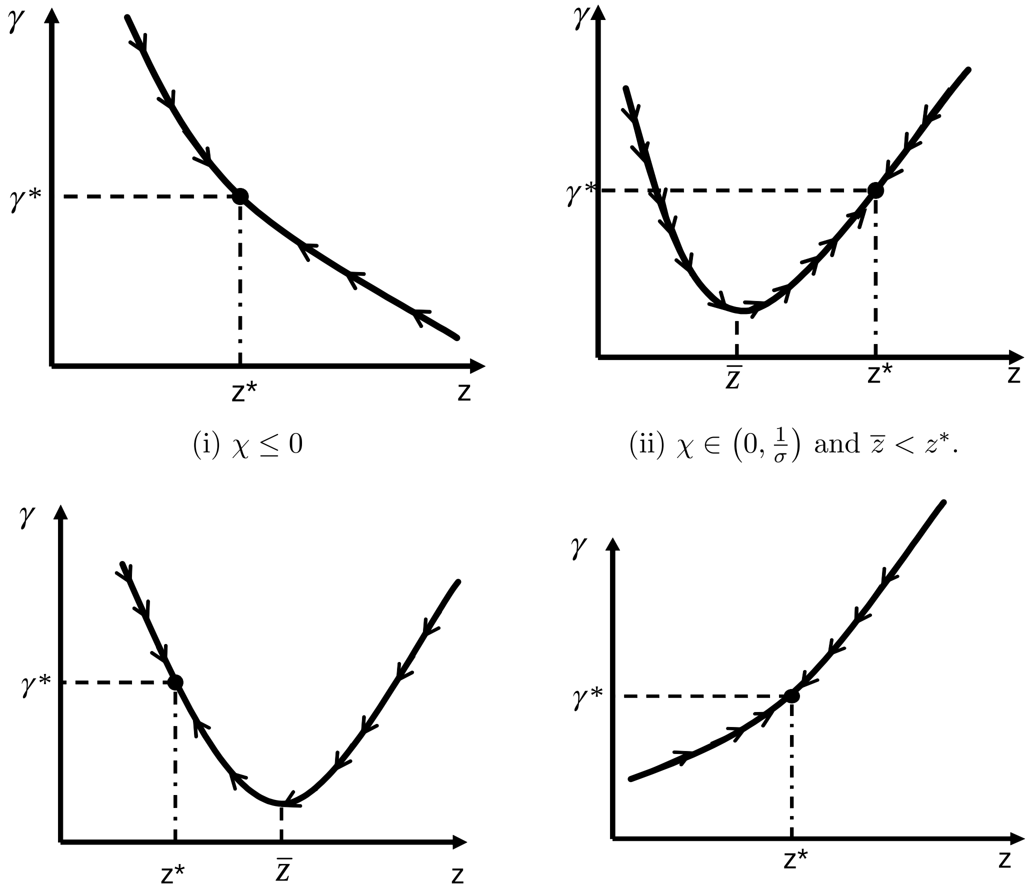
Type of shock (*)		$\theta = 0.2$ (a)	$\theta = 1$ (b)	b/a
Sectoral biased: ∇A_1	$\alpha_1 \neq \alpha_2$ (c)	1.4671%	4.9593%	3.3807
	$\alpha_1 = \alpha_2$ (d)	1.5405%	4.9616%	3.2206
	d/c	1.0500	1.0005	–
Sectoral unbiased: $\dfrac{\nabla A_i}{A_i} = \dfrac{\nabla A_j}{A_j}$	$\alpha_1 \neq \alpha_2$ (c)	5.3358%	5.3136%	0.9958
	$\alpha_1 = \alpha_2$ (d)	5.3445%	5.3140%	0.9943
	d/c	1.0016	1.0001	–

(*) The size of the shocks in all economies is set such that the GDP instantaneously decreases by 5%

Table 2. Welfare cost of technological shocks ($\sigma = 3.5$)

ξ	μ	Benchmark	Economy with $\alpha_1 = \alpha_2$	Economy with $\theta = 1$
0.25	-3	1.4789%	1.5574%	4.9593%
0.5	-1	1.4742%	1.5494%	4.9593%
0.75	-0.33	1.4704%	1.5447%	4.9593%
1	0	1.4671%	1.5405%	4.9593%
2	0.5	1.4545%	1.5233%	4.9593%
4	0.75	1.4274%	1.4888%	4.9593%
10	0.9	1.3543%	1.3941%	4.9593%
The size of the shocks in all cases is set such that the GDP instantaneously decreases by 5%				

Table 3. Welfare cost of a biased technological shock under a CES aggregator for consumption ($\sigma = 3.5$)



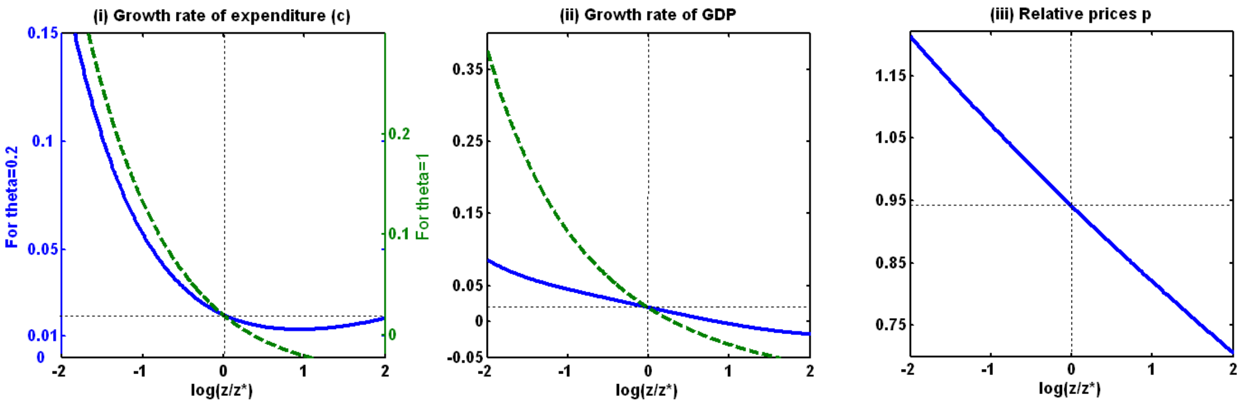
(i) $\chi \leq 0$

(ii) $\chi \in \left(0, \frac{1}{\sigma}\right)$ and $\overline{z} < z^*$.

(iii) $\chi \in \left(0, \frac{1}{\sigma}\right)$ and $\overline{z} > z^*$.

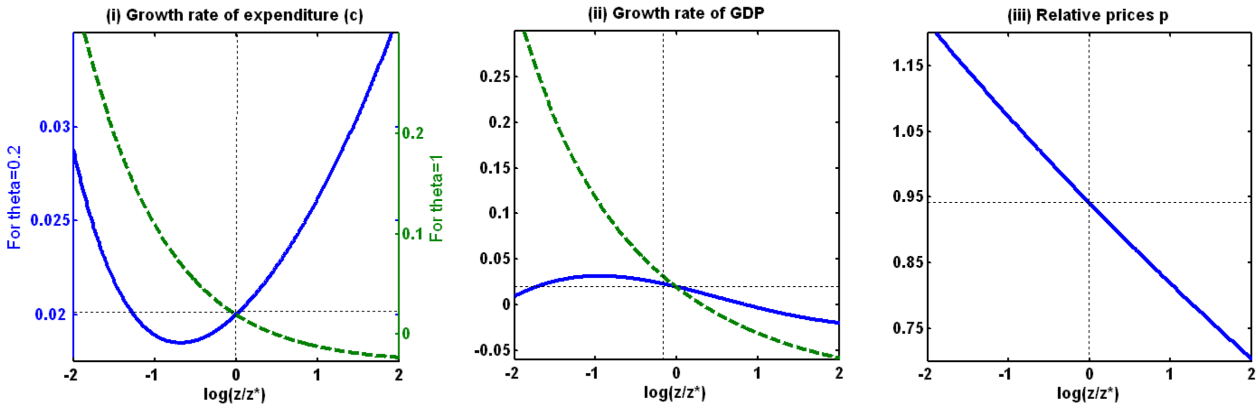
(iv) $\chi > \frac{1}{\sigma}$.

Figure 1. Growth rate of consumption expenditure



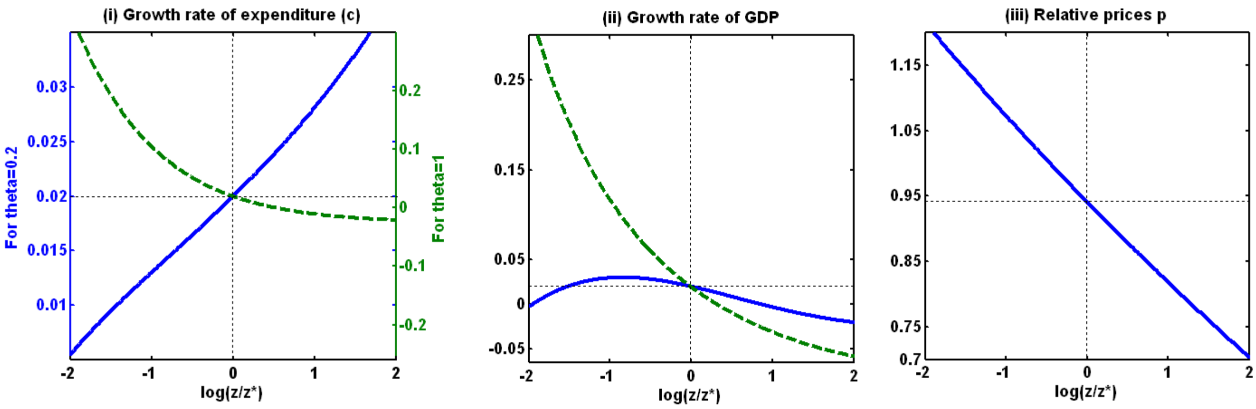
— Economy with $\theta = 0.2$ - - - Economy with $\theta = 1$

Figure 2. Transitional dynamics with $IES = 0.29$.



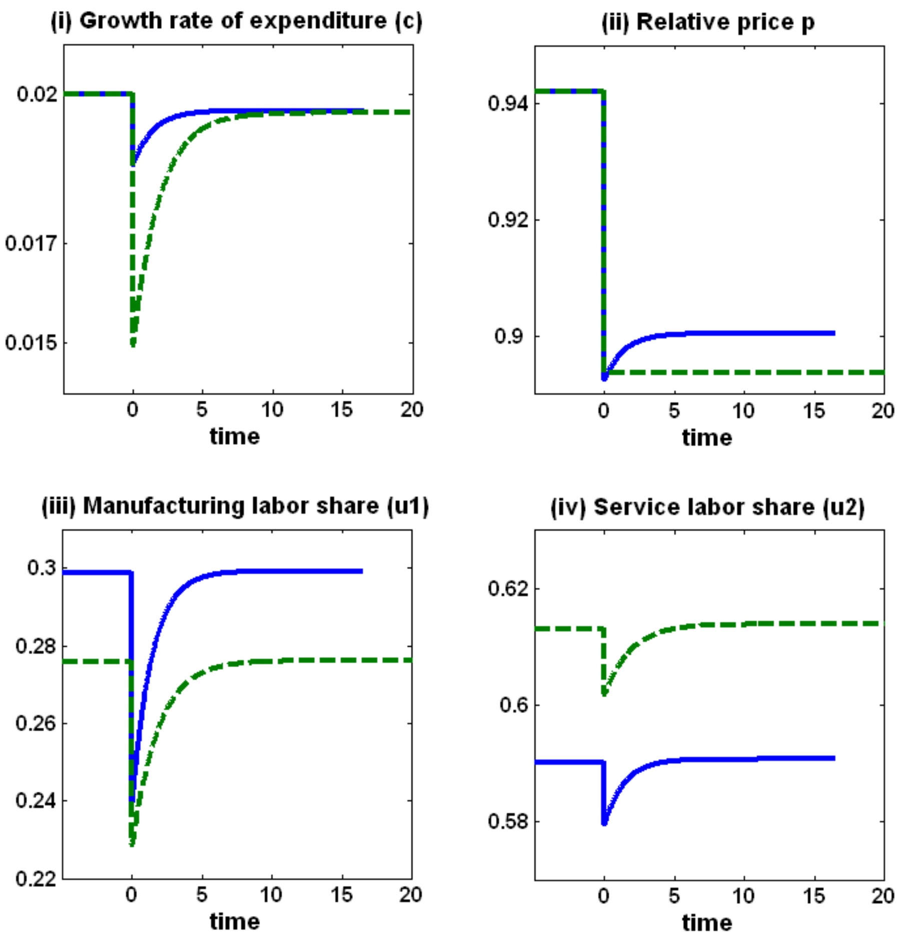
— Economy with $\theta = 0.2$ - - - Economy with $\theta = 1$

Figure 3. Transitional dynamics with $IES = 0.22$.



— Economy with $\theta = 0.2$ - - - Economy with $\theta = 1$

Figure 4. Transitional dynamics $IES = 0.21$.



— Economy with $\theta = 0.2$ and $\alpha_1 \neq \alpha_2$ - - - Economy with $\theta = 0.2$ and $\alpha_1 = \alpha_2$

Figure 5. Dynamic effects of a biased technological shock when $IES = 0.29$.