

Searching for P- and CP-odd effects in heavy ion collisions

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Abstract. In this talk we will summarize the main results from our recent work concerning the possibility that a new metastable phase occurs in some heavy ion collisions (HIC). This phase would be characterized by the breaking of two characteristic symmetries of strong interactions; namely P and CP . We investigate the experimental consequences of parity breaking in such a situation and propose suitable observables to elucidate the presence this phenomenon.

WHY P AND CP MIGHT NOT BE GOOD SYMMETRIES IN HIC

Parity is one of the well established global symmetries of strong interactions. While there are arguments to think that P or CP cannot be broken in the usual QCD vacuum[1], this may not be the case under extreme conditions of temperature and density. Indeed no fundamental principle forbids spontaneous parity breaking for $\mu \neq 0$ or out of equilibrium.

For a long time the possibility that P - and CP -odd condensates may exist as regular phase at finite density has been contemplated, going back all the way to the work of Migdal[2]. The debate was inconclusive by using simple nucleon-meson models and, if anything, the arguments suggested that a condensate of this type was not physically viable. However, more recent –and more complete– effective theory studies indicate that such a phase is a real possibility[3] at ‘moderate’ densities (3 to $8 \times \rho_N$) compared to the normal nuclear density $\rho_N 0.17 fm^{-3}$.

This possibility is surely relevant in astrophysical contexts but it may still be possible to ephemerally produce such a phase in HIC in conditions where the density is high (i.e. large baryonic chemical potential) and temperature relatively low. These conditions may be within reach of future experiments at FAIR and NICA[4]. However this is not the main subject of this presentation.

Rather we would like to consider the possibility that in a violent collision long-lived bubbles with a non-vanishing value of the topological charge, where parity is locally broken, could be produced. The possibility that these topological fluctuations would take place in a finite volume and large T was first proposed by Kharzeev, Pisarski and Tytgat[5] and later considered in a number of works[6, 7]

Large fluctuations in the gauge field could indeed exist in a hot environment and they could generate a local imbalance of topological charge. This picture is supported by lattice simulations[8] although results obtained in Euclidean simulations are hard to connect with the time dependent dynamics that exist in the early stages of a HIC.

The possibility of domains where P is broken is also supported by the *glasma* picture[9]. However domains in this model are typically very small ($\ll 1$ fm) in the initial QGP phase. Of course these domains should expand and grow at later stages. Actually in our work we will be interested exclusively in the hadronic phase that develops later in the collision. We will assume that some domains with a net topological charge and spatial extent > 1 fm exist in this hadronic phase, originally created via the *glasma* mechanism or any other.

Generating an effective chiral chemical potential

If a topological charge T_5

$$T_5 = \frac{1}{8\pi^2} \int_{\text{vol.}} d^3x \epsilon_{jkl} \text{Tr} \left(G^j \partial^k G^l - i \frac{2}{3} G^j G^k G^l \right) \quad (1)$$

arises in a finite volume due to quantum fluctuations of gluon fields and it is conserved for a sufficiently long time in a quasi-equilibrium situation, we can introduce a topological chemical potential μ_θ conjugate to it into the QCD lagrangian via $\Delta \mathcal{L}_{top} = \mu_\theta \Delta T_5$ that plays the role of an effective θ term. The variation of the topological charge is

gauge invariant

$$\Delta T_5 = T_5(t_f) - T_5(t_i) = \frac{1}{8\pi^2} \int_{t_i}^{t_f} dt \int_{\text{vol.}} d^3x \text{Tr} \left(G^{\mu\nu} \tilde{G}_{\mu\nu} \right). \quad (2)$$

We will assume that, temporarily, as a consequence of a topological fluctuation of gluon fields or some other mechanism an effective CP -odd large θ term is generated. We will suppose that this region eventually grows in the hadronic phase to a sufficiently large size. We will also assume that this situation can be treated by equilibrium field theoretical methods.

For *peripheral* heavy ion collisions such a situation may lead to the Chiral Magnetic Effect[10] whereby a large θ term, combined with the large magnetic field due to the colliding nuclei, generates a large electric field and originates charge separation.

For *central* collisions (and light quarks) a large θ term will trigger an ephemeral phase with axial chemical potential $\mu_5 \neq 0$. This comes about because the PCAC equation predicts an induced axial charge to be conserved in the chiral ($m = 0$) limit:

$$\frac{d}{dt} (Q_5^q - 2N_f T_5) \simeq 0, \quad Q_5^q = \int_{\text{vol.}} d^3x \bar{q} \gamma_0 \gamma_5 q = \langle N_L - N_R \rangle, \quad (3)$$

the latter being an average chiral state asymmetry. Neglecting the quark mass is a good approximation for the lightest u and d quarks only. The strange quark is already too heavy and erases the chiral charge in the time scales where this phenomenon could be relevant. In a quasi-equilibrium situation the appearance of a nearly conserved chiral charge can be incorporated with the help of a chiral chemical potential μ_5 .

EFFECTIVE MESON THEORY IN A MEDIUM WITH LOCAL PARITY BREAKING

In principle, from the discussion in the previous section we have two possible isospin structures for μ_5 : (a) An isosinglet pseudoscalar background is expected to be relevant in situations where temperature is the main external driver ($T \gg \mu$) such as in RHIC and LHC, and it would be due to the formation of domains with a non-zero topological charge. (b) An isotriplet pseudoscalar background could be appropriate in situations where $\mu \gg T$, as will be the case in FAIR or NICA, and a true thermodynamic phase forms temporarily. The formation of an isosinglet pseudoscalar condensate cannot be excluded in this case either. Only the situation (a) will be considered in this talk.

Having in mind that although the parity breaking domain may have formed in the early stages of the HIC we are only concerned about its consequences in the later stage hadronic phase we will proceed to build an effective theory for mesons in a P - and CP -odd environment. We will deal in turn with scalars and vector mesons. Note that in both cases a breaking of Lorentz symmetry will occur.

The $J = 0$ sector can be described by using the spurion technique in the Lagrangian

$$D_\mu \implies D_\mu - i\{\mu_5 \delta_{0\mu}, \cdot\} \quad (4)$$

and constructing a generalised sigma model[18] with the light scalar mesons $\sigma, \vec{\pi}, \eta, \eta', \vec{a}_0$. The most relevant operator has dimension $D = 2$. All these hadronic states naturally have a well defined parity in ‘normal’ QCD and partly because of that only the σ and ρ interact strongly with the pion fireball.

The new eigenstates of the hamiltonian do not have a well defined parity. Now due to parity breaking there is mixing in the $\sigma - \eta - \eta'$ and $\vec{\pi} - \vec{a}_0$ channels. The resulting $J = 0$ eigenstates are *all* coupled and are expected to *thermalize* in the HIC fireball.

For vector mesons the most relevant operator has the dimension $D = 3$. P - and CP -odd effects will appear through the Chern-Simons term[7, 11]

$$\Delta \mathcal{L} \simeq \varepsilon^{\mu\nu\rho\sigma} \text{Tr} \left[\hat{\zeta}_\mu V_\nu V_\rho \sigma \right] \quad (5)$$

with $\hat{\zeta}_\mu = \partial_\mu \hat{a}(\vec{x}, t) = \delta_{\mu 0} \partial_0 \hat{a}(t)$ for a spatially homogeneous, time dependent background field $\hat{a}(\vec{x}, t)$. We shall assume here $\partial_0 \hat{a}(t) \sim \hat{\zeta} \propto \mu_5$, as follows from the discussion in the previous section.

Vector mesons will be introduced and treated in the conventional way using the Vector Meson Dominance model[12] and enter the above lagrangian via the corresponding vector current V_μ . Note that for vector mesons there will be no mixing at this order resulting from the previous lagrangian but rather a distortion of the spectrum.

The current V_μ formed by a combination of vector mesons and the photon couples to fermions

$$\mathcal{L}_{\text{int}} = \bar{q} \gamma_\mu \hat{V}^\mu q; \quad \hat{V}_\mu \equiv -e A_\mu Q + \frac{1}{2} g_\omega \omega_\mu \mathbb{I} + \frac{1}{2} g_\rho \rho_\mu^0 \tau_3, \quad (6)$$

where the charge $Q = \frac{\tau_3}{2} + \frac{1}{6}$ and the quark-meson coupling constants $g_\omega \simeq g_\rho \equiv g \simeq 6$. In addition we have to include Maxwell and mass terms

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} (F_{\mu\nu}F^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} + \rho_{\mu\nu}\rho^{\mu\nu}) + \frac{1}{2} V_{\mu,a}(\hat{m}^2)_{a,b}V_b^\mu \quad (7)$$

$$\hat{m}^2 \simeq m_V^2 \begin{pmatrix} \frac{10e^2}{9g^2} & -\frac{e}{3g} & -\frac{e}{g} \\ -\frac{e}{3g} & 1 & 0 \\ -\frac{e}{g} & 0 & 1 \end{pmatrix}, \quad \det(\hat{m}^2) = 0. \quad (8)$$

Particularizing to the present situation we get the following P -odd interaction

$$\mathcal{L}_{P\text{-odd}}(k) = \frac{1}{2} \zeta \varepsilon_{jkl} V_{j,a} N_{ab} \partial_k V_{l,b}, \quad (9)$$

where in the case of an isosinglet pseudoscalar background

$$N_{ab}^\theta \simeq \begin{pmatrix} \frac{10e^2}{9g^2} & -\frac{e}{3g} & -\frac{e}{g} \\ -\frac{e}{3g} & 1 & 0 \\ -\frac{e}{g} & 0 & 1 \end{pmatrix}, \quad \det(N^\theta) = 0. \quad (10)$$

Simultaneous diagonalization of the matrices \hat{m}^2, N leads to

$$N = \text{diag} \left[0, 1, 1 + \frac{10e^2}{9g^2} \right] \simeq \text{diag} [0, 1, 1] \quad \hat{m}^2 = m_V^2 \text{diag} \left[0, 1, 1 + \frac{10e^2}{9g^2} \right] \simeq \text{diag} [0, 1, 1]. \quad (11)$$

Note that after diagonalization the photon itself is unaffected as it remains massless.

Due to (9) different vector meson helicities get a different momentum-dependent correction to its mass[13]. Vector mesons exhibit the following dispersion relation:

$$m_{V,\varepsilon}^2 = m_V^2 - \varepsilon \zeta |\vec{k}|, \quad (12)$$

where $\varepsilon = 0, \pm 1$ is the meson polarization. The position of the poles for \pm polarized mesons is moving with wave vector $|\vec{k}|$. Massive vector mesons split into three polarizations with masses $m_{V,+}^2 < m_{V,L}^2 < m_{V,-}^2$. This splitting unambiguously signifies P breaking. Could the splitting be measured?

POSSIBLE MANIFESTATIONS OF P-ODD EFFECTS IN HIC

When trying to understand the nature of the fireball produced in a HIC it is natural to investigate electromagnetic probes such as photons and leptons. In the previous section we have shown that P breaking has substantial effects on the meson spectrum and this could eventually reflect in their leptonic decay products.

Let us then proceed to investigate possible ‘anomalies’ in dilepton production in various meson decays such as $\rho, \omega \rightarrow e^+e^-$. The total dilepton production also receives potential contribution from the pseudoscalar Dalitz decays $\eta, \eta' \rightarrow \gamma e^+e^-$ and the ω Dalitz decay $\omega \rightarrow \pi^0 e^+e^-$

In fact dilepton production shows a number of anomalies [14, 15, 16]. Perhaps the most obvious one is a surprising enhancement in the low dilepton invariant mass region that has been observed in virtually all experiments for a long time. This excess is to this date unexplained.

We show below data from the two RHIC experiments: PHENIX[14] and STAR[15] at BNL. Other experiments have also seen a similar excess[16]. It is impossible to explain this excess with the standard ‘hadronic cocktail’.

Another peculiarity is a large broadening of the ρ spectrum. This was very clearly observed by the NA60 collaboration[17] several years ago by measuring dimuon production around the combined $\rho - \omega$ peak and carefully subtracting contributions from Dalitz decays. Interpreting these results is not easy and it has remained a big puzzle for a long time. On the other hand, the results seem to give credence to the idea of ‘broadening’ for in-medium ρ mesons as opposed to the ‘shifting mass’ scenario. It is claimed that the large broadening can be understood by ‘conventional’ mechanisms although this involves a certain amount of parameter fitting and it is certainly more of a post-diction than a prediction. It is natural to question whether this issue is really understood.

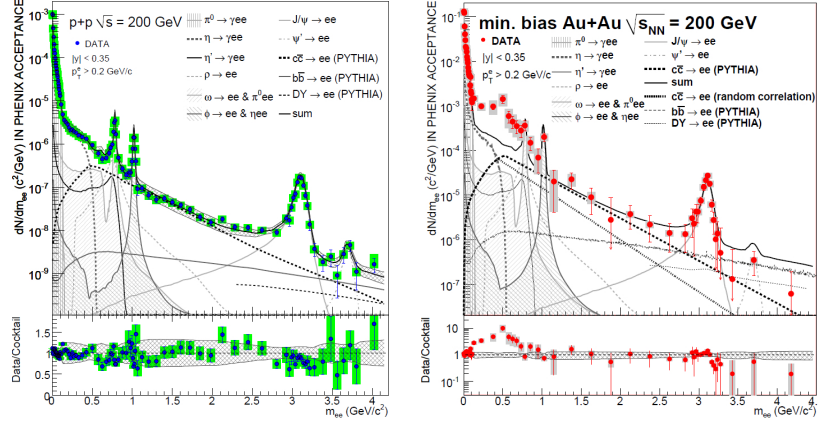


FIGURE 1. Left: the yield of dileptons (e^+e^- pairs) is exceedingly well reproduced in proton-proton collisions by the hadronic cocktail. Right: however for *central* or *quasicentral* collisions the cocktail fails completely to reproduce the data below the ϕ meson resonance. Figures from the PHENIX collaboration.

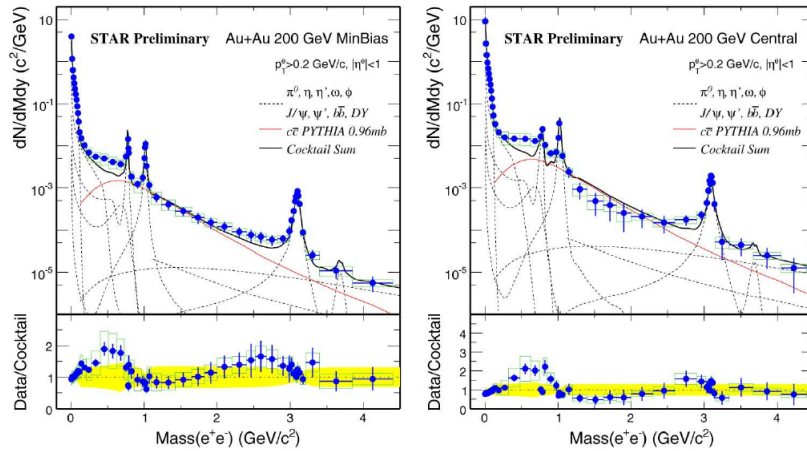


FIGURE 2. Similar results as in PHENIX are obtained by the STAR collaboration. On the left the enhancement is shown for minimum bias events, whereas the right figure only contains central events. This shows that the effect is really present for central or nearly central events. The enhancement is in any case less spectacular than in PHENIX.

Dalitz decays in a P-odd environment

Having seen that there are a number of features in dileptons produced in HIC it is legitimate to ask oneself if local parity breaking could be of some relevance to account for, perhaps in combination with more conventional explanations, some of these effects.

Let us first turn to the modifications induced in the $J = 0$ sector, which contributes to dilepton production via Dalitz decays. In what follows we will always compare to the PHENIX measurements and for that reason we will use the same set of experimental cuts: $|y_{ee}| < 0.35$, $|\vec{p}_t^e| > 200$ MeV and gaussian M_{ee} smearing (width=10 MeV), and the effective temperature $T = 220$ MeV.

Using the effective lagrangian briefly described before for $J = 0$ mesons we determine[18] the spectrum (that as described contains states without definite parity) and, given that they mix among themselves strongly, assume that they thermalize in the fireball that now consists mostly of the lightest of such states. After that we proceed to compute the corresponding Dalitz decays. Many more details can be found in X.Planelis' Ph D thesis[19]. The corresponding predictions for e^+e^- production are shown in Fig. 4

The results represent a net enhancement to dilepton production. Even though they are clearly insufficient to explain the dramatic enhancement in PHENIX in the region 200-500 MeV, they help vis a vis STAR data. However, before

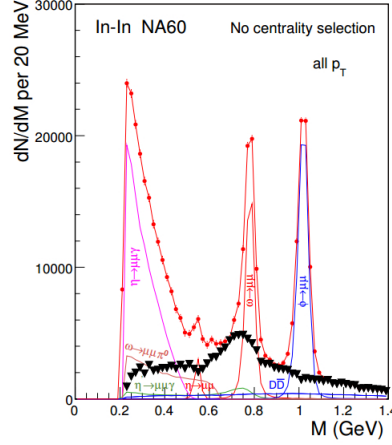


FIGURE 3. A large broadening of the ρ spectral function was measured in a neat way by the NA60 collaboration.

jumping to conclusions the reader should be warned that except for very low values of μ_5 the effective lagrangian so obtained ceases to make sense very soon, a fact that may reflect the need to introduce more resonances. Thus this approach is of limited validity and we do not exclude that a more accurate treatment may actually provide a higher dilepton yield and be part of the solution of the dilepton puzzle in the 200-500 MeV region. We do regard this as a totally open issue at present because unfortunately we have little phenomenological intuition of the properties of ‘broken parity’ QCD.

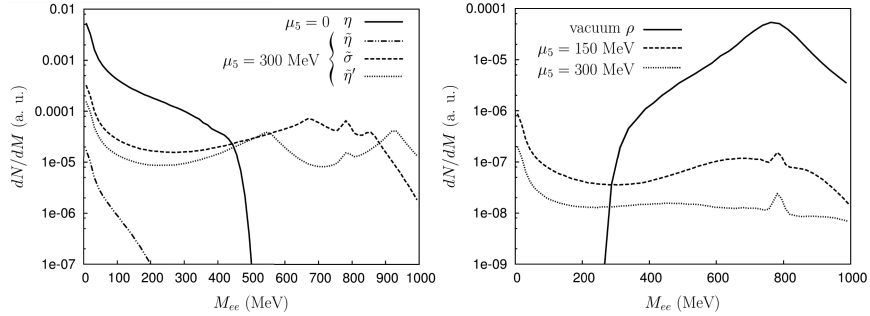


FIGURE 4. Left: the production from the three new eigenstates is compared to the one from η decays (that is the only relevant one if undistorted vacuum properties are assumed). The latter is depleted but there is a net enhancement at larger invariant masses. Right: the total dilepton production from the discussed mechanism is shown for two values of μ_5 and compared to the vacuum ρ peak as a reference. However be warned that beyond $\mu_5 \sim 100$ MeV the effective lagrangian technique shows clear signs of failure.

ρ meson broadening

P -odd effects introduce broadening in a totally natural manner. Because masses depend on the polarization, the original Breit-Wigner actually splits in three different peaks. This is clearly shown in Fig. 5 There is a similar effect for the ω meson.

It is of course very tempting to compare this ‘automatic’ broadening with the very precise experimental results obtained by the NA60 collaboration from dimuon production. And indeed the peculiar shape of the ρ spectrum measured by NA60 is grossly reproduced with amazingly little effort. We used our best guess from the available published data by the NA60 collaboration itself to implement the experimental cuts. As the reader can see, we have not attempted to superimpose the two plots because the NA60 data is not properly acceptance corrected. We have tried hard to obtain more information to make a meaningful comparison but found the collaboration to be unresponsive.

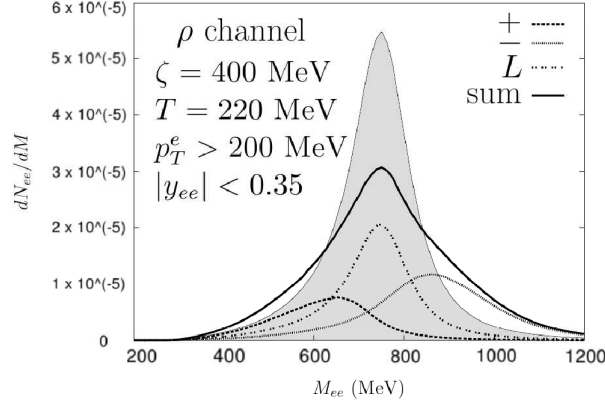


FIGURE 5. Polarization splitting in the ρ spectral function with local parity breaking $\zeta = 400$ MeV ($\mu_5 = 290$ MeV) compared with $\zeta = 0$ (shaded region). Note that the temperature is an effective or ‘boosted’ one so it is not surprising that it is actually larger than the deconfining temperature.

In any case the comparison is tantalizing. Within the assumption of local parity breaking only one parameter is fitted—the value of μ_5 ; the best fit is obtained for values of μ_5 close to the constituent mass. Incidentally these values are preferred in NJL type analysis for a stable parity breaking phase to exist[20]

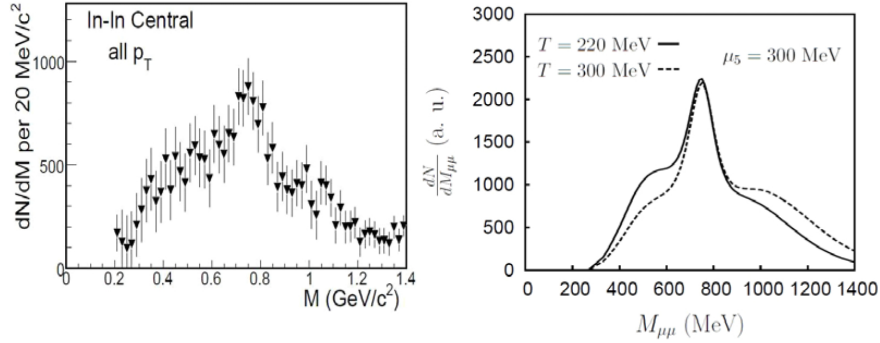


FIGURE 6. Left: NA60 results for the ρ spectral line-shape as a function of the dimuon invariant mass. Right: the analogous quantity obtained from assuming local parity breaking with a value of $\mu_5 = 300$ and two values for the effective temperature

In-medium vector meson decays $V \rightarrow \ell^+ \ell^-$

We shall be even briefer here, as this particular point has already been reported and discussed in several conferences. We will now include both the ρ and ω mesons and consider the corresponding distortions in their respective spectra for dilepton production. The results are shown in Fig. 7.

The associated enhancement of the dilepton yield could (at least partly) explain the anomalous enhancement seen by PHENIX and STAR.

OBSERVABLES SENSITIVE TO P-ODD EFFECTS

One of the unambiguous signals of P -odd effects is the separation between polarizations. Is there any way to study these decays in order to separate the different polarizations and thus confirm or rule out the presence of local parity breaking in HIC?

It is well known that the angular distribution of leptons carries information on the polarization of the decaying meson. However, current angular distribution studies are not thought to detect possible P -odd effects.

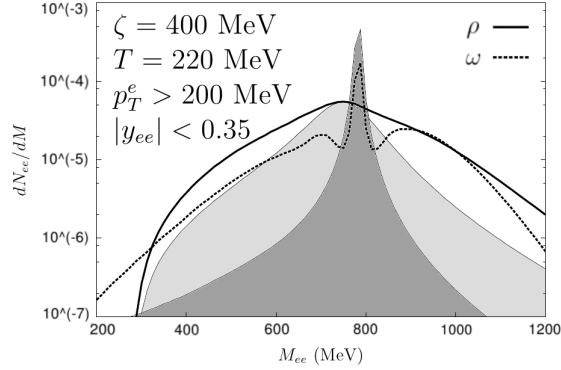


FIGURE 7. In-medium ρ and ω channels (solid and dashed line) and their vacuum contributions (light and dark shaded regions) for $\mu_5 = 290$ MeV.

We will instead define two angles as described in Fig. 8. In order to isolate the transverse polarizations, we will perform different cuts for each angle and study the variations of the ρ (and ω) spectral function. The results are given in the next figures.

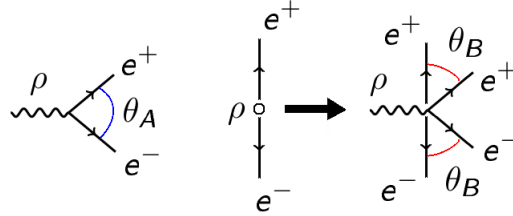


FIGURE 8. Case A: θ_A is the angle between the two outgoing leptons in the laboratory frame. Case B: θ_B is the angle between one of the two outgoing leptons in the laboratory frame and the same lepton in the dilepton rest frame.

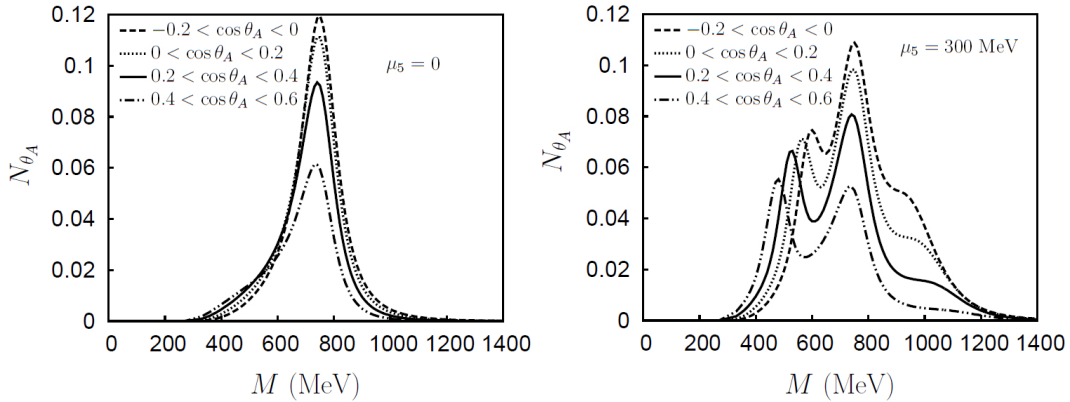


FIGURE 9. Angle θ_A between the two outgoing leptons in the laboratory frame. ρ spectral function depending on the dielectron invariant mass M in vacuum ($\mu_5 = 0$) and in a parity-breaking medium with $\mu_5 = 300$ MeV for different ranges of θ_A .

A quite visible secondary peak appears in a P -odd medium! Note however that due to the cuts (needed to make the secondary peak visible) there is an important reduction of the number of events: the vacuum peak shows at most about 10% of the events one would expect without any cut in θ_A .

If the secondary peak is found for a particular angular coverage, its position would be an unambiguous measurement of the effective or mean value for μ_5 . This value of course need not be the same for each HIC. It need not even be uniform. One only needs that domains are sufficiently large for the ρ meson to decay in the medium. The vacuum ρ peak hides the secondary one for $\mu_5 \simeq 100$ MeV or below due to its large width. For ω , all the peaks are visible.

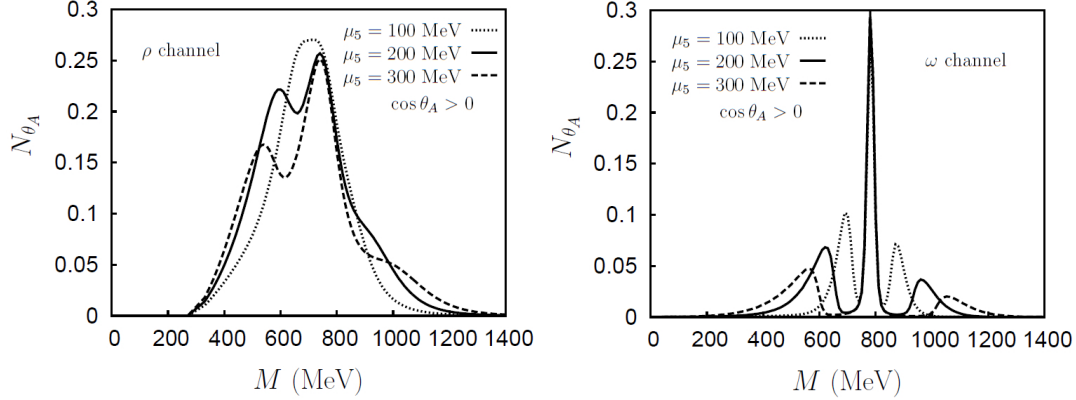


FIGURE 10. ρ and ω spectral functions depending on the invariant mass M and integrating $\cos \theta_A \geq 0$ for $\mu_5 = 100, 200$ and 300 MeV.

We also present the combination of the ρ and ω channels. In this case, the total production is normalized to PHENIX data. It may seem questionable to assume that the ω decay inside the firewall (and this is one reason to carefully separate both contributions). However, local parity breaking should increase substantially the number of omega mesons decaying. For one thing there are the by now familiar parity considerations; parity is no longer a conserved number in such a medium. On the other hand, due to the different dispersion relations a number of ω mesons cannot leave the medium[22] and this obviously increases the probability of a decay inside the hadronic phase of the fireball.

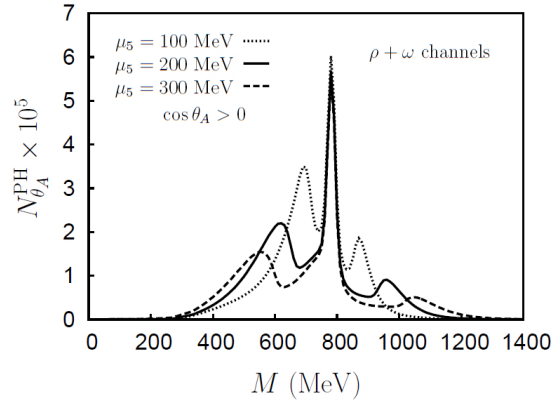


FIGURE 11. Combination of ρ and ω spectral functions depending on the invariant mass M and integrating $\cos \theta_A \geq 0$ for $\mu_5 = 100, 200$ and 300 MeV.

Now we turn to angle B. The discussion is rather similar. See [21] for more details. The main difference with θ_A is a slightly smaller number of events. The rest of the analysis is completely equivalent.

CONCLUSIONS AND OUTLOOK

Let us first list the main results of our work:

- P - and CP -odd effects not forbidden by any physical principle in QCD at finite density or high temperature, particularly out of equilibrium.
- Topological fluctuations transmit their influence to hadronic physics via an axial chemical potential for light quarks only.
- P - breaking leads to unexpected modifications of the in-medium properties of scalar and vector mesons.

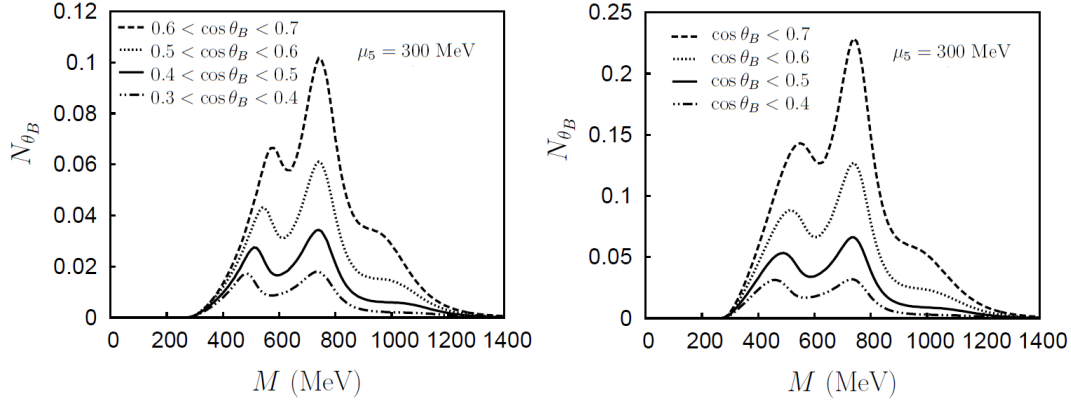


FIGURE 12. Angle θ_B between one of the two outgoing leptons in the laboratory frame and the same lepton in the dilepton rest frame. ρ spectral function depending on the invariant mass M for different ranges of θ_B for fixed $\mu_5 = 300$ MeV.

- Local parity breaking has an influence on the observed dilepton spectrum in the low-mass region of PHENIX and STAR.
- Some observables may allow us to establish P - and CP -breaking unambiguously.

Perhaps it is fair to end with some criticisms. To begin with, the possibility that domains with a non-vanishing chiral charge form and that they grow to a sufficiently large size is of course unproven. Without this assumption there could be no local parity breaking nor any of the effects discussed in this presentation. However, we think that it may be possible to understand this problem (and give an answer) within the glasma picture in the framework of relativistic hydrodynamics. On the other hand, the situation regarding Dalitz decays is clearly unsatisfactory. We have reasons to think that they may be substantially enhanced (and help explain e.g. the STAR data) but it seems very difficult to study this issue as the effective lagrangian approach as discussed here is not good at all.

All in all, the possibility that signals of parity breaking can be detected in a strong interaction context in HIC is truly fascinating. But detecting it is very challenging. We have proposed here a couple of observables that could possibly yield a signal if local parity breaking is present. Experimental collaborations should definitely check this possibility.

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