EFFECTS OF DIFFERENTIAL TAXATION ON FACTOR ACCUMULATION AND GROWTH¹

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Abstract: In this paper we try to analyze the role of fiscal policy in fostering a higher participation of the different production factors in the human capital production sector in the long-run. Introducing a tax on physical capital and differentiating both a tax on raw labor wage and a tax on skills or human capital we also attempt to present a way to influence inequality as measured by the skill premium, thus trying to relate the increase in human capital with the decrease in income inequality. We will do that in the context of a non-scale growth model.

The model here is capable to alter the shares of private factors devoted to each of the two production sectors, final output and human capital, and affect inequality in a different way according to the different tax changes. The simulation results derived in the paper show how a human capital (skills) tax cut, which could be interpreted as a reduction in progressivity, ends up increasing both the shares of labor and physical capital devoted to the production of knowledge and decreasing inequality. Moreover, a raw labor wage tax decrease, which could also be interpreted as an increase in the progressivity of the system, increases the share of labor devoted to the production of final output and increases inequality. Finally, a physical capital tax decrease reduces the share of physical capital devoted to the production of knowledge and allows for a lower inequality value. Nevertheless, none of the various types of taxes ends up changing the share of human capital in the knowledge production, which will deserve our future attention.

Key words: human capital, inequality, tax reform.

JEL Classification: J31, O40
**Resum:** En aquest article hem tractat d’analitzar el paper que la política fiscal pot tenir a l’hora d’estimular la participació dels diversos factors de producció en l’elaboració de capital humà a llarg termini. Amb la introducció d’un impost sobre el capital físic i la diferenciació de dos impostos, un sobre el sou procedent del treball o esforç físic, i un altre procedent del capital humà o de les diverses habilitats adquirides, també s’ha tractat de presentar una possible via per tal d’influir en la desigualtat, que hem mesurat mitjançant el *skill premium*. D’aquesta manera es vol relacionar un increment en el capital humà amb una davallada en la desigualtat a nivell de renda. Aquesta anàlisi s’ha dut a terme en el context dels models del tipus no escalar.

El model és capaç de canviar la participació que els factors privats de producció tenen en cadascun dels dos sectors productius de què es compon l’economia, producció de béns finals i de capital humà, així com d’influïr en la desigualtat de manera diferent segons es tracti d’un canvi impositiu o d’un altre. Els resultats fruit dels diferents exercicis de simulació que s’han realitzat ens mostren com una reducció de l’impost sobre el capital humà, que podria ésser interpretada com una davallada de la progressivitat del sistema, acaba incrementant tant la participació de l’esforç físic com del capital físic en la producció de coneixement o capital humà, així com també exercint una reducció de la desigualtat. Pel que fa a l’impost sobre el treball físic, val a dir que una reducció del mateix, que pot interpretar-se com un increment de la progressivitat, comporta un increment de la participació del treball físic en la producció de béns finals, així com un augment de la desigualtat. Finalment, una reducció de la imposició sobre el capital físic es tradueix en un decrement de la participació del capital físic en la producció de capital humà alhora que permet una davallada final de la desigualtat. Tanmateix, cap dels canvis impositius és capaç de canviar la participació del capital humà en la producció del mateix, fet que és mereixedor d’un estudi posterior.
1. Introduction

Some economists see education and human capital acquisition as a good investment since it increases wages. Others, however, confer it a consumption value. Actually, most countries’ personal tax codes treat household spending on human capital as consumption. In any case, studies on the effects of government tax policies on human capital accumulation are still quite limited. Conventional wisdom in the human capital literature (Ben-Porah, 1967; Boskin, 1974) tends to suggest that income taxes do not encourage human capital accumulation. Some authors find that labor income taxation may either have no effect or a positive effect on human capital accumulation. Boskin (1975) showed that since labor income taxation reduces both the return and cost of human capital investment by the same proportion, it has no effect on human capital accumulation. Heckman (1976) challenged this view arguing that an income tax depresses the interest rates and thus the costs of borrowing, hence encouraging human capital investment. He also showed that (physical) capital income taxation and labor income taxation might have completely different effects on human capital accumulation. The first one could have a positive influence on human capital based on the fact that taxation on physical capital encourages a substitution to human capital.

One interesting aspect to take into account is the presence of uncertainty in the return to human capital investment. Eaton and Rosen (1980) did so and found that a labor tax may improve welfare by decreasing risk. They showed that without uncertainty labor income taxation has no effect on human capital, but in
the presence of uncertainty labor income taxation may increase human capital. Later on, Lucas (1990) showed that income taxation lowers the return to human capital and reduces the incentive to accumulate human capital, reducing the hours worked. This reduction decreases the return to human capital investment. Stokey and Rebelo (1995) tend to confirm Lucas (1990) results in the sense that if human capital’s share is large in all sectors and if the production of human capital is lightly taxed, then taxing returns in the physical capital sector will have modest growth effects.

Considering time as the principal input producing human capital, the primary cost of investing in human capital is forgone wages. Then, income taxation would reduce the net wage, which are both the returns on human capital investment as well as its primary cost. Taking this into account, most initial studies did not find a significant effect of taxation on human capital since they considered that taxation of wage income reduced the return and cost of human capital by more or less the same proportion. However, Trostel (1993) considers that time is not the only input in the production of human capital, and includes some other inputs whose cost is not reduced by taxation, such as tuition. He finds that if taxation reduces only part of the cost of investing in human capital, then the cost reduction is lower than the return reduction, a fact that translates a tax increase into a lower accumulation of human capital. His results can be summarized saying that most of the long-run impact of taxation on effective labor supply occurs through human capital. Also, that capital income taxation has a small negative effect on human capital because of the fall in the gross wage rate. Thus, human capital accumulation is reduced mainly because of the wage tax component. More specifically, he found that a one-percent increase in the
income tax rate would cause the human capital stock to decline by 0.39% in the long-run.

Some authors have also analyzed the effects caused by replacing one kind of tax by a different one on the long-run value of human capital. Davies and Whalley (1991) found small effects on human capital coming from replacing an income tax with a consumption tax because capital adjusts such that the after-tax interest rate returns to approximately its original value. Dupor et al. (1996) also found that switching from an income tax to a consumption tax largely increases physical capital accumulation, but has only slightly positive effects on human capital. On the other hand, Ortigueira (1998), studied the implications of taxation policies in an endogenous growth model, concluding that the transitional period turns out to be highly relevant in evaluating the welfare cost of a tax reform. He also concludes that once the initial ratio of physical-human capital is fixed, the investment rate in human capital depends on the magnitude of the tax rate, which has a permanent effect on the level of the economy.

Also, the way government sets aside tax revenues may influence the long-run effects of taxation on capital accumulation. In this sense, Lin (1998) shows that with the tax revenues being used to compensate the individuals who pay taxes, an increase in the tax rate has no effect on human capital. However, if the tax revenues are consumed by the government, an increase in the labor income tax rate raises the real interest rate, lowers the present discounted value of the future income, reduces time allocated toward education, and decreases human capital.
In this paper we will try to analyze the way to foster a higher participation of the different production factors in the human capital production sector in the long-run by making fiscal policy quite specific, that is, adding various types of taxes. It seems that changing from an income tax to a wage tax is welfare improving, although a wage tax seems to increase wealth inequality, compared to the distributional neutrality of a consumption tax, as some authors have pointed out (see for instance Pecorino, 1994; Perroni, 1995, 1997; Felder, 1997). Taking this into account, we will examine the different influence that various types of taxes may exert on human capital accumulation as well as on inequality and economic growth in a non-scale growth model. In any case, the results derived in the paper attract attention to the fact that decreasing taxes on human and physical capital may end up positively influencing the accumulation of human capital and increasing the percentage of raw labor and physical capital devoted to the production of knowledge. But they attract attention especially to the fact that none of the various types of taxes ends up changing the share of human capital in the knowledge production.

The structure of the paper is as follows. Section 2 briefly reviews the literature on taxation and human capital, with special attention to progressivity and its likely negative influence on human capital accumulation. Section 3 presents the non-scale model with specific taxation. Section 4 describes the general equilibrium and stationary states, and section 5 undertakes an analysis of the influence of different tax changes on inequality and some other economic variables. Finally, section 6 concludes.
2. Progressivity and human capital

Some of the main concerns about fiscal policy that exist nowadays are how economics should be used to evaluate and design tax changes; which tax changes are needed to restore the generational imbalance between young and elderly; specially in the US, some authors, like Karoly (1994) or Zee (1999) claim for a need of larger progressivity in order to reduce inequality. Karoly relies on a more progressive taxation system whenever some conditions are met, such as a certain degree of egalitarism on the welfare function, or on the distribution of endowments, or the level of responsiveness of labor supply to changes in the after tax wage rate that would be determining the value of efficiency costs. In this line, Zee looks at progressive taxation as a means of increasing the revenue to finance targeted transfers to the poor and lessening inequality programs. He mainly addresses the government problem of maximizing a social welfare function, which displays some degree of income inequality aversion. According to him richer people in poorer countries should be taxed heavier than richer people in richer countries.

In any case, a vast part of the literature on taxes and human capital considers that progressive tax schedules are a tax on human capital accumulation, (Trostel, 1993) since they reduce the after tax value of the future returns to education by more than it reduces the after tax value of the foregone labor market earnings. Hence, they may dissuade agents from investing in human capital.

However, most of the analyses of capital income taxation have focused on the taxation of physical and financial capital, with lower attention to human capital
taxation. Conventional income tax is said to ignore changes in the value of human capital over time and to tax only realizations in the form of wages (Kaplow, 1996). The inclusion of human capital has arisen several questions, mainly trying to answer which is the impact on skill formation of proposals such as to switch from progressive taxes to flat income and consumption taxes. According to Boskin (1975), any human capital investment that increases future earnings enough to drive the taxpayer into a higher tax bracket may decrease the present value of the depreciation allowance to the present value of the incremental tax liability. So, investments that are profitable at the current tax rate may not be so when taking account of the increased future tax rate. He considers that the progressive tax rate structure of the personal income tax probably creates a disincentive to accumulate human capital. This disincentive might be more severe for secondary workers in two-earner families whose incremental incomes from human capital investment may generate a large increase in marginal tax rates.

Moreover, a tax which lowers the after-tax rental rate on human capital, given the rate of interest and the price of any purchased inputs in human capital production, will decrease human capital investment. Boskin also considers important to note that the long-run supply of labor and the sensitivity of human capital to its return do affect the incidence of the tax. Boskin and Shoven (1980) challenge Schultz view, which mainly argued that the U.S. tax system discriminated against human capital investment, by underlining that the main point in Shultz view based on the lack of deductibility of expenditures on human capital was not correct, since the most important costs of human capital were foregone earnings, rather than tuition payments. However, if education caused a
significant movement across tax brackets, then progressivity could be an important aspect to take into account in order for the agents to make their education decisions. According to Heckman et al. (1998) that is what happens, a progressive wage tax reduces the incentive to accumulate skills since human capital promotes earnings growth and moves persons to higher tax brackets. As a result, marginal returns on future earnings are reduced more than marginal costs of schooling. They analyze the effects of moving to a flat tax, eliminating progressivity in wages and stimulating skill formation. Their results show how the aggregate stock of high-school human capital declines, while the amount of college human capital increases resulting from a rise in college enrollment.

On the other hand, Bovenberg and van Ewijk (1997) show that the introduction of overlapping generations induce non-neutrality of progressive taxation with respect to the decision to invest in human capital. They consider that with overlapping generations, progressive taxes decrease the expected growth of after-tax wages by transferring resources to future generations, which reduces the marginal value of accumulating additional human capital, and hence the incentives to invest. In fact, a progressive tax system transfers resources from the richer, older generations to the poorer, new ones, thereby reducing the growth of after-tax wages and thus harming the incentives to learn. According to Bovenberg and van Ewijk, without government intervention, intergenerational spillovers of human capital imply that households do not invest enough in human capital. Hence, by reducing the incentive to further investment, a progressive tax may exacerbate the distortions associated with those spillovers. Gordon and Tchilinguirian (1998) argue that the tax environment for the average production worker who invests tends to be slightly more advantageous than the tax
environment for the top-bracket investor, but that progressive tax codes do not translate neatly into progressive investment incentives. They refer to Sweden as one major example whose tax system creates a kind of education trap at income levels close to or below those of the average production worker, creating strong disincentives to invest in education, but it tends to be neutral for higher incomes. Thus, they argue that lowering progressivity could be a way to enhance incentives to invest in education, at least for medium income earners.

3. Non-scale model. Individual optimization

Consider an economy that comprises \( N \) individuals. The exogenous rate of population growth is constant at \( n \). Each individual \( i \) produces output \( Y_i \) using capital stock, \( K_i \), skills, \( H_i \) and public services provided by the government, \( G \). This output production sector is subject to positive externalities arising from the aggregate stocks of physical capital, \( K \), human capital, \( H \), as well as government spending, \( G \), according to the Cobb-Douglas production function:

\[
Y_i = \alpha_i \Theta^{b_X} [\psi H_i]^{b_H} \left[ \phi K_i \right]^{b_K} K^{c_K} H^{c_H} G^{c_G} \tag{1a}
\]

The individuals also produce new human capital, \( J_i \), in another sector, also subject to positive externalities arising from the aggregate stock of physical
capital, $K$, and human capital, $H$, as shown by the following Cobb-Douglas production function:

$$J_i = a_j(1-\theta)^{\alpha}[(1-\psi)H_i]^{\gamma}[(1-\phi)K_i]^{\kappa}K^{f_k}H^{f_H} \quad (1b)$$

Both production functions exhibit increasing returns in the private production factors and the externalities. All factors have positive marginal values, thus the only restrictions on productive elasticities are the following ones:

$$1 > \sigma_K > 0; \ 1 > \sigma_H > 0; \ c_G > 0;$$

$$1 > \eta_K > 0; \ 1 > \eta_H > 0;$$

The public good is available equally to each individual, independently of the usage of others. The constants $\alpha_j$, $a_j$ represent exogenous technological shift factors to the production functions, while $b_j$, $c_j$, $e_j$, $f_j$ are the respective productive elasticities. Besides, each individual is endowed with a unit of labor, $\theta$ of which is allocated to the production of new output and $(1-\theta)$ to the production of new human capital. In addition, he allocates a fraction $\psi$ of his current human capital, $H_i$, to the production of final output, and the balance $(1-\psi)$ to the accumulation of further human capital. Likewise, he allocates a fraction $\phi$ of his physical
capital, $\kappa$, to the production of final output and the rest $(1-\phi)$ to the human capital sector.

All agents in the economy are assumed to be identical so that aggregate and individual quantities are related by:

$$Y \equiv NY_i, \quad K \equiv NK_i, \quad H \equiv NH_i$$

We also assume that the government sets its aggregate expenditure level, $G$, as a constant fraction, $g$, of aggregate output, $Y$, while government services derived by the individual are proportional to individual output, in accordance with:

$$G = gY = gNY_i; \quad G_S = gY$$

Hence, any further expansion in government expenditure will be modeled by an increase in the output share, $g$.

Substituting (3) into (1a) we can rewrite the production function as:

$$Y_i = a_F \theta^{b_F/(1-c_0)} [\mu H_i]^{p_{H_i}/(1-c_0)} [\phi K_i]^{p_{K_i}/(1-c_0)} K^{c_{K_i}/(1-c_0)} H^{c_{H_i}/(1-c_0)} N^{c_{N_i}/(1-c_0)}$$
where \( a_r \equiv \left( \alpha, g^{(r)} \right)^{1/c_r} \).

The rate at which the individual accumulates the two types of capital is described by:

\[
\dot{K}_i = \left[ (1 - \tau_k) r_k \phi - n - \delta_k \right] K_i + (1 - \tau_w) w_n \theta + (1 - \tau_h) r_h H_i - C_i - T_i
\]

\[(4a)\]

\[
\dot{H}_i = a_j (1 - \theta)^\gamma \left[ (1 - \phi) K_i \right] f_\theta \left[ (1 - \psi) H_i \right] f_\psi K_i H_i - (\delta_H + n) H_i
\]

\[(4b)\]

where:

\[
r_k = \frac{\partial Y_i}{\partial (\phi K_i)} = b_k \frac{Y_i}{\phi K_i} = b_k \frac{Y}{\phi K}
\]

\[(5a)\]

\[
w_n = \frac{\partial Y_i}{\partial \theta} = b_n \frac{Y_i}{\theta} = b_n \frac{Y}{\theta N}
\]

\[(5b)\]

\[
r_H = \frac{\partial Y_i}{\partial (\psi H_i)} = b_H \frac{Y_i}{\psi H_i} = b_H \frac{Y}{\psi H}
\]

\[(5c)\]

According to (4a), raw labor wage is taxed at the rate \( \tau_w \), capital is taxed at \( \tau_k \), and skills are taxed at \( \tau_h \). In addition, we allow for lump-sum taxation, \( T_i \).
The representative agent in the economy chooses individual consumption, $C_i$, the sectoral allocation of labor, physical and human capital, and the rates of physical and human capital accumulation to maximize his intertemporal utility function:

$$\frac{1}{1-\gamma} \int_0^\infty (C_i)^{1-\gamma} e^{-\rho t} dt \quad \rho > 0; \quad \gamma > 0$$

where $\rho$ denotes the constant rate of time preference. The constant elasticity utility function implies a constant elasticity of substitution equal to $\frac{1}{\gamma}$. The optimization will be subject to the production functions (1a)-(1b) and the accumulation constraints, (4a) - (4b). Note that in making its decisions, the household takes $r_K, w_N, r_H$ as given, though these are determined in equilibrium as in (5a)-(5c). Also, each agent takes $G$ and the aggregate physical and human capital as given. The optimization is to maximize:

$$\frac{C_i^{1-\gamma}}{1-\gamma} e^{-\rho t_i} + \mu e^{-\rho t_i} \left[ (1-\tau_k) r_K (\phi K_i) + (1-\tau_w) w_N \theta + (1-\tau_h) r_H (\psi H_i) - C_i - T_i - (n + \delta_K) K_i - \dot{K}_i \right]$$

$$+ \mu e^{-\rho t_i} \left[ a_j (1-\theta) \nu \left[ (1-\phi) K_j \right] \kappa \left[ (1-\psi) H_j \right] F_{H} K^f H^{l_H} - (\delta_H + n) H_i - \dot{H}_i \right]$$
The following first order and transversality conditions are obtained:

\[ C_i = v_i \]  
\[ v_i (1 - \tau_w) w_N = \mu_i e_N \frac{J_i}{1 - \theta} \]  
\[ v_i (1 - \tau_h) r_H H_i = \mu_i e_H \frac{J_i}{1 - \psi} \]  
\[ v_i (1 - \tau_k) r_K K_i = \mu_i e_K \frac{J_i}{1 - \phi} \]  
\[ r_K (1 - \tau_k) \phi - (\delta_K + n) + \frac{\mu_i}{v_i} e_K \frac{J_i}{K_i} = \rho - \frac{v_i}{v_i} \]  
\[ r_H (1 - \tau_h) \psi \frac{v_i}{\mu_i} - (\delta_H + n) + e_H \frac{J_i}{H_i} = \rho - \frac{\mu_i}{\mu_i} \]  
\[ \lim_{t \to \infty} v_i K_i e^{-\rho t} = \lim_{t \to \infty} \mu_i H_i e^{-\rho t} = 0 \]

where \( v_i, \mu_i \) are the respective shadow values of physical capital and human capital.
4. The aggregate economy

To derive the behavior of the aggregate economy we first sum (1a’) and (1b) over the $N$ individuals in the economy. We may express the resulting quantities in terms of the aggregates:

$$Y = a_k \theta^{s_N \phi^{s_K} \psi^{s_H}} K^{\sigma_K} H^{\sigma_H} N^{\sigma_N}$$  \hspace{1cm} (8a)$$

$$J = a_j (1 - \theta)^{e_N} (1 - \phi)^{e_K} (1 - \psi)^{e_H} K^{\eta_K} H^{\eta_H} N^{\eta_N}$$  \hspace{1cm} (8b)$$

where:

$$s_N = \frac{b_N}{1 - c_G}; \quad s_K = \frac{b_K}{1 - c_G}; \quad s_H = \frac{b_H}{1 - c_G}$$

$$\sigma_K = \frac{b_K + c_K}{1 - c_G}; \quad \sigma_H = \frac{b_H + c_H}{1 - c_G}; \quad \sigma_N = \frac{1}{1 - c_G} - \frac{b_H}{1 - c_G} - \frac{b_K}{1 - c_G}$$

$$\eta_K \equiv e_K + f_K; \quad \eta_H \equiv e_H + f_H; \quad \eta_N \equiv 1 - e_H - e_K;$$

We will assume that the government finances its expenditure in accordance with a balanced budget, which aggregated over $N$ individuals, can be expressed as:

$$\tau_w w_N \theta N + \tau_k r_K N K_i + \tau_h r_H \psi N H_i + NT_i = g N Y_i$$  \hspace{1cm} (9)$$
or, in terms of the aggregate quantities, \( Y = NY_i \), \( H = NH_i \), \( K = NK_i \), \( T = NT_i \):

\[
\tau_w w_N j_k N + \tau_k r_N K + \tau_h r_H H + T = gY \tag{9'}
\]

To complete the macroeconomic equilibrium, we must consider the aggregate accumulation of physical and human capital. To do this, note that:

\[
\dot{K} = N \dot{K}_i + nK \tag{10a}
\]

\[
\dot{H} = N \dot{H}_i + nH \tag{10b}
\]

Multiplying the individual accumulation equations (4a) and (4b) by \( N \) and combining with the government budget constraint (9'), aggregate physical and human capital in the economy are accumulated according to the product market equilibrium conditions:

\[
\dot{K} = (1 - g)Y - C - \delta_k K \tag{10a}
\]

\[
\dot{H} = J - \delta_h H \tag{10b}
\]

where \( Y, J \) are defined in (8a), (8b), above.
4.1 Balanced equilibrium behavior

According to the stylized empirical facts (Romer 1986), we assume that the output/capital ratio, $Y/K$, is constant. Thus, taking the differentials of the production functions (8a) and (8b), and solving, we obtain:

$$
\hat{H} = \left[ \frac{\eta_N (1 - \sigma_K) + \sigma_N \eta_K}{(1 - \eta_H)(1 - \sigma_K) - \sigma_H \eta_K} \right] \equiv \beta_h n \quad (11a)
$$

$$
\hat{K} = \hat{Y} = \hat{C} = \left[ \frac{\sigma_N (1 - \eta_H) + \sigma_H \eta_N}{(1 - \eta_H)(1 - \sigma_K) - \sigma_H \eta_K} \right] \equiv \beta_k n \quad (11b)
$$

and thus per capita growth rate of output (capital) is:

$$
\hat{Y} - n = \left[ (1 - \eta_H) (\sigma_N + \sigma_K - 1) + \sigma_H (\eta_H + \eta_N + \eta_K - 1) \right] \frac{n}{(1 - \eta_H)(1 - \sigma_K) - \sigma_H \eta_K} \quad (11c)
$$

4.2 Dynamics of a two-sector model

To derive the equilibrium dynamics around the balanced growth path we define the following stationary variables:
\( y \equiv Y/N^{\beta_k} \); \( k \equiv K/N^{\beta_k} \); \( c \equiv C/N^{\beta_k} \); \( h \equiv H/N^{\beta_H} \); \( j \equiv J/N^{\beta_H} \); \( q \equiv \nu/\mu N^{(\beta_H - \beta_k)} \).

For convenience, we shall refer to \( y, k, c, \) and \( h \) as *scale-adjusted* quantities. This allows us to rewrite scale-adjusted output and human capital as:

\[
y = a_F \theta^{s_N} \phi^{s_K} \psi^{s_H} h^{\sigma_H} k^{\sigma_K} \tag{12a}
\]

\[
j = a_J (1-\theta)^{\epsilon_N} (1-\psi)^{\epsilon_H} (1-\phi)^{\epsilon_K} h^{\eta_H} k^{\eta_K} \tag{12b}
\]

The optimality conditions then enable the dynamics to be expressed in terms of these scale-adjusted variables, as follows. First, substituting (12a) and (12b) into the labor allocation condition, (7b), the human capital allocation condition, (7c), and the physical capital allocation condition, (7d), yields the three relationships:

\[
(1-\tau_w) a_F q b_N \theta^{s_N} \psi^{s_H} h^{\sigma_H} k^{\sigma_K} = a_J e_N (1-\theta)^{\epsilon_N} (1-\psi)^{\epsilon_H} (1-\phi)^{\epsilon_K} h^{\eta_H} k^{\eta_K} \tag{13a}
\]

\[
(1-\tau_h) a_F q b_H \theta^{s_N} \psi^{s_H} h^{\sigma_H} k^{\sigma_K} = a_J e_H (1-\theta)^{\epsilon_N} (1-\psi)^{\epsilon_H} (1-\phi)^{\epsilon_K} h^{\eta_H} k^{\eta_K} \tag{13b}
\]

\[
(1-\tau_k) a_F q b_K \theta^{s_N} \psi^{s_H} h^{\sigma_H} k^{\sigma_K} = a_J e_K (1-\theta)^{\epsilon_N} (1-\psi)^{\epsilon_H} (1-\phi)^{\epsilon_K} h^{\eta_H} k^{\eta_K} \tag{13c}
\]

In principle, we can solve these three relationships for the allocation of labor, human capital, and physical capital across sectors:
\[
\theta = \theta((1-\tau_w), (1-\tau_h), (1-\tau_k), q, h, k) \quad (14a)
\]
\[
\psi = \psi((1-\tau_w), (1-\tau_h), (1-\tau_k), q, h, k) \quad (14b)
\]
\[
\phi = \phi((1-\tau_w), (1-\tau_h), (1-\tau_k), q, h, k) \quad (14c)
\]

Using the optimality conditions, the dynamics of the system can be expressed in terms of the redefined stationary variables by:

\[
\dot{k} = k \left[ (1-g)a_k \theta \phi \psi h^\alpha k^{\sigma_k - 1} \frac{c}{k}, -\delta_k - \beta_k n \right] \quad (15a)
\]
\[
\dot{h} = h \left[ a_j (1-\theta)^e \phi (1-\psi)^d \psi h^{\eta_h - 1} k^{\eta_k} - \delta_h - \beta_h n \right] \quad (15b)
\]
\[
\dot{q} = q \left[ \left( \frac{e \phi}{1-\psi} \right) a_j \theta \phi \psi h^{\eta_h - 1} k^{\eta_k} \right] - \left[ (1-\tau_k) \left( a_k \theta \phi \psi h^{\eta_h - 1} k^{\eta_k} \right) - n(\beta_h - \beta_k) - (\delta_h - \delta_k) \right] \quad (15c)
\]
\[
\dot{c} = \frac{c}{\gamma} \left[ \left( \frac{b_K}{\phi} \right) (1 - \tau_k) \left( a_{k} \theta \phi \psi \phi \psi h \phi h^{-1} k \right) - (\rho + \delta_k) + \left[ \gamma (1 - \beta_K) - 1 \right] n \right]
\]  

(15d)

The steady state to this system, denoted by "\~" superscripts, can be summarized by:

\[
\frac{(1 - g) \tilde{y}}{\tilde{k}} - \frac{\tilde{c}}{\tilde{k}} = \beta_k n + \delta_k
\]  

(16a)

\[
\frac{\tilde{j}}{\tilde{h}} = \beta_H n + \delta_H
\]  

(16b)

\[
\frac{\tilde{j}}{\tilde{h}} \left( \frac{e_H}{1 - \psi} \right) - \delta_H - \beta_H n = (1 - \tau_k) \frac{\tilde{y}}{\tilde{k}} \frac{b_K}{\phi} - \delta_k - \beta_K n
\]  

(16c)

\[
(1 - \tau_k) \frac{\tilde{y}}{\tilde{k}} \frac{b_K}{\phi} - \delta_k - \beta_K n = \rho + (1 - \gamma)(1 - \beta_K) n
\]  

(16d)

plus allocation conditions:

\[
\tilde{q} (1 - \tau_w) \frac{b_N}{\theta} \tilde{y} = \frac{e_N}{1 - \theta} \tilde{j}
\]  

(17a)
These seven equations determine the equilibrium as follows:

1. $\frac{\bar{y}}{h}$ is determined by (16b)

2. Given $\frac{\bar{y}}{h}$, (16d) jointly with (16c) determine $\psi$, which is independent of taxes.

3. Given $\psi$, equation (17c) determines $\bar{\theta}$, and given $\bar{\theta}$, equation (17b) determines $\bar{\phi}$.

4. Given $\frac{\bar{y}}{h}$, $\psi$ and $\bar{\phi}$, (16c) determines $\frac{\bar{v}}{k}$.

5. Given $\frac{\bar{y}}{k}$, (16a) determines $\frac{c}{k}$.

6. Given $\frac{\bar{y}}{k}$, $\frac{\bar{j}}{h}$ and $\bar{\theta}$,$\bar{\phi}$,$\psi$, the two scale-adjusted production functions determine the stocks of human and physical capital, $\bar{h}, \bar{k}$ and therefore $y, \bar{j}, \bar{c}$.

7. Finally, (17a) determines $q$.
The key point to observe is that the steady-state equilibrium growth rates as well as the long-run allocation of human capital are independent of any of the tax rates. However, the equilibrium sectoral asset allocations, $\theta$ and $\phi$, are both influenced by $\tau_h$. Besides, $\theta$ is influenced by $\tau_w$ and $\phi$ is influenced by $\tau_k$. Moreover, it is interesting to know that the long-run allocation of human capital to the manufacturing sector, $\psi$, is not influenced by any tax change.

5. Simulations and wage premium responses to tax reforms

In this section we will use the different simulations in order to analyze a way to foster a higher participation of the different productive factors in the human capital production sector in the long-run. Besides, we are also interested in seeking for a possible way to reduce inequality.

We will measure income inequality in terms of the wage premium of skilled to unskilled workers, $w_R$, defined by:

$$w_R \equiv \frac{w_H}{w_N} = \frac{w_N + r_H}{w_N} = 1 + \frac{r_H}{w_N} = 1 + \frac{\partial Y}{\partial (\psi H)} = 1 + \frac{b_H}{b_N} \frac{\theta}{\psi} \frac{N}{H} > 1$$

(18)

where $w_N, r_H$ are the marginal products of unskilled workers and skills, respectively, representing the returns to raw labor and the returns to skills.
It is interesting to note that the presence of both a tax on raw labor wage, $w_N$, and a tax on skills return, $r_H$, could be seen as an easy way to introduce progressivity in the model, since unskilled workers will be taxed exclusively on their base salary, $(\tau_w)$, whereas skilled workers will be taxed both on their base salary, $(\tau_w)$, as well as on the returns of their skills, $(\tau_h)$. Hence, we can interpret $\tau_w$ and $\tau_h$ as the two tax brackets in a progressive tax system.

*Table 1* reports the values we employ for our fundamental parameters. These values are generally consistent with those suggested by previous calibration exercises (Lucas, 1988; Jones, 1995; Ortigueira and Santos, 1997). In these first simulations, externalities are set to zero in both sectors, except for the ones coming from government spending $(g)$. Thus, in this paper we will start from a situation where both production functions exhibit constant returns to scale in the private factors of production.

<table>
<thead>
<tr>
<th>Table 1. Benchmark parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production parameters</td>
</tr>
<tr>
<td>Preference parameters</td>
</tr>
<tr>
<td>Depreciation and population</td>
</tr>
<tr>
<td>Fiscal policy parameters</td>
</tr>
</tbody>
</table>

The model adopts the following key benchmark equilibrium values, as reported in *table 2*. The share of labor allocated to the production of final output is about
86%, the share of physical capital allocated to production is 94% and about 15% of the skills are used in the education sector. The implied equilibrium output-capital ratio is 0.35, and the consumption-output ratio is 0.66, both of which are highly plausible.

<table>
<thead>
<tr>
<th>$\tau_w$</th>
<th>$\tau_k$</th>
<th>$\tau_h$</th>
<th>$\theta$</th>
<th>$\phi$</th>
<th>$\psi$</th>
<th>$Y \tilde{K}$</th>
<th>$C \tilde{Y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30%</td>
<td>20%</td>
<td>35%</td>
<td>0.8594</td>
<td>0.9372</td>
<td>0.8473</td>
<td>0.3461</td>
<td>0.6591</td>
</tr>
</tbody>
</table>

Given that the equilibrium dynamics are generated by a fourth-order system, we know from Eicher and Turnovsky (1999) that the transition may be characterized by significant non-monotonicites in the state variables. This implies that both the starting point of the economy and the size of the tax cuts may have a certain influence on the actual transition to the new steady state. Hence, from now on, we will explore how successful this model is in representing a more specific fiscal policy thanks to the inclusion of various types of taxes, and also in explaining the evolution of inequality and the allocation of different productive factors after each fiscal shock. In order to do that, we will undertake three different tax cuts, a human capital tax cut, a raw labor wage one, and a physical capital one, and we will analyze the economic implications of each of them. We will start analyzing the dynamics predicted by the model after a human capital tax cut from 35% to 30%. The model predicts a new steady state as reported in table 3.
Table 3. Tax reform (human capital) equilibrium values

<table>
<thead>
<tr>
<th>$\tau_w$</th>
<th>$\tau_k$</th>
<th>$\tau_h$</th>
<th>$\tilde{\theta}$</th>
<th>$\tilde{\phi}$</th>
<th>$\tilde{\psi}$</th>
<th>$Y \tilde{\ell} K$</th>
<th>$C \tilde{\ell} Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30%</td>
<td>20%</td>
<td>30%</td>
<td>0.8502</td>
<td>0.9327</td>
<td>0.8473</td>
<td>0.3444</td>
<td>0.6581</td>
</tr>
</tbody>
</table>

Note that this experiment can be seen as a reduction in the progressivity of the system. More specifically, we have ended up equalizing both the raw labor wage and the skills wage, hence eliminating progressivity somehow.

This steady state differs from the benchmark both in the share of labor and physical capital allocation as well as in the capital output and the consumption output ratios. Although the growth rates of the endogenous factors and the share of human capital allocated across sectors change only during the transition, the share of labor allocated to the final output (manufacturing) sector ends up decreasing by 1% and the share of physical capital by 0.5%. The output to capital ratio declines by about 0.5% and the consumption to output ratio by less, 0.15%. The phase diagram in figure 1A shows that the human capital tax cut initially enhances the accumulation of human capital at the expense of physical capital. Figure 1C shows that scale adjusted per capita human capital growth rises to above 2% during the early stages after which it falls rapidly down towards its long-run steady state. Scale adjusted per physical capital capita growth, however, has a completely different pattern. After its initial decline it slightly overshoots its long-run growth rate and eventually rises the return of investing in physical capital sufficiently relative to the return on human capital. The initial accumulation of human capital attracts more resources devoted to the
knowledge production sector and away from the output production sector. We can see that from the initial decrease in the shares of labor, physical and human capital in the manufacturing sector, as shown in the appendix A.1. In addition, under the assumption \( \sigma_h < \eta_h \), the initial increase in knowledge further serves to drive the allocation of factors to the human capital sector. Over time, as the growth rate of human capital decreases and the growth rate of physical capital increases, the allocation of factors to the human capital sector declines (the allocation to the manufacturing sector increases). It does so in such a way that we end up with the same human capital share in manufacturing as the one we had previous to the tax cut.

If we now analyze the evolution of the income inequality, regarding (18), it is quite intuitive to think that the initial large decrease in the shares of raw labor and human capital devoted to the output production sector quite offset each other, and it is the initial big increase in human capital the one that initially drives the skill premium down. Recalling (13) and (14), we see that on impact a decrease in \( \tau_h \) has two effects on employment and skills in the final output sector, \( \theta, \psi \). First, given \( q \), it increases the after-tax relative price of final output, \( q(1-\tau_h) \), thereby increasing \( \theta \) and \( \psi \). But at the same time, it reduces the before-tax relative price \( q \) causing an offsetting reduction in \( \theta \) and \( \psi \). Over time, labor and skills will continue to move in response to the changing shadow value, as well as to the changing relative stocks of physical and human capital. Moreover, during the early phases of the adjustment \( q \) may continue to decline, thereby offsetting the effect of the accumulating human capital stock.
Figure 1. Evolution of the economy after a human capital tax cut
Figure 1.C. Adjustment dynamics

Figure 1.D. Skill premium

Figure 1(ctd.) Evolution of the economy after a human capital tax cut
Furthermore, as the growth rate of human capital slows down, the skill premium starts increasing. Then, it remains for several periods below the path followed in a situation with no tax decrease, thus inducing a lower value for inequality. In sum, devoting more labor and physical capital resources to the production of skills relative to the production of final output seems to increase the base wage received by an unskilled worker relative to the skills return.

The second experiment consists in a raw labor wage tax cut from 30% to 25%. The model predicts a new steady state as shown in table 4. We could see this case as opposed to the previous change in the sense that by reducing the raw labor wage tax we end up increasing progressivity.

<table>
<thead>
<tr>
<th>Table 4. Tax reform (raw labor wage) equilibrium values.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_w$</td>
</tr>
<tr>
<td>25%</td>
</tr>
</tbody>
</table>

This steady state differs from the benchmark only in the share of labor allocation but neither in the capital output nor in the consumption output ratios. Whereas the growth rates of the endogenous factors and the shares of capital and human capital allocated across sectors change only during the transition, the share of raw labor allocated to the final output sector increases by 1%.
Figure 2.A. Growth rates of output, capital, and human capital

Figure 2.B. Growth rate of physical capital

Figure 2. Evolution of the economy after a raw labor wage tax cut
Figure 2.C. Adjustment dynamics

Figure 2.D. Skill premium

Figure 2(ctd.) Evolution of the economy after a raw labor wage tax cut
The phase diagram in figure 2A shows how the raw labor wage tax cut initially fosters the accumulation of physical capital as can also be seen from the evolution of adjusted per capita physical and human capital rates of growth in figure 2C. Adjusted per capita physical capital growth rate rises above 2% immediately after the tax decrease and it goes down rapidly. However, the evolution of human capital follows a decreasing path until reaching its new long-run value.

When looking at the evolution of the skill premium, as before, the initial large decrease in $\theta$ and $\psi$ tend to offset each other and hence it is the initial large decline in human capital the main factor driving the increase in the skill premium immediately after the wage tax decrease. After some periods of adjustment the skill premium starts decreasing for a long time. Yet, it remains above the path followed by the skill premium simulated without any tax change for several periods. Thus, devoting more labor resources to the production of output we may end up decreasing the raw labor wage relative to the skills return, and hence increasing income inequality.

The third experiment consists in a physical capital tax decrease from 20% to 15%. The model predicts a new steady state as shown in table 5.

This steady state differs from the benchmark one in the share of physical capital allocation as well as in the capital output and the consumption output ratios. In this case, the share of physical capital allocated to the final output (manufacturing) sector increases by 0.4%, the output to capital ratio declines by
about 5.5% and the consumption to output ratio by 1.7%. Hence, as the tax on physical capital falls, the accumulation of physical capital increases, since saving has become more attractive.

<table>
<thead>
<tr>
<th>$\tau_w$</th>
<th>$\tau_k$</th>
<th>$\tau_h$</th>
<th>$\bar{\theta}$</th>
<th>$\bar{\phi}$</th>
<th>$\bar{\psi}$</th>
<th>$Y/K$</th>
<th>$C/Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30%</td>
<td>15%</td>
<td>35%</td>
<td>0.8594</td>
<td>0.9407</td>
<td>0.8473</td>
<td>0.3270</td>
<td>0.6480</td>
</tr>
</tbody>
</table>

According to the phase diagram in figure 3A, a physical capital tax decrease starts stimulating the accumulation of physical capital at the expense of human capital accumulation. Figure 3C shows how during the immediate periods following the physical capital tax decrease, scale adjusted per capita capital growth rises to around 2%, then, it starts going down towards its long-run value. With respect to scale adjusted per capita human capital growth rate, it starts being negative at the very beginning followed by a slight but continuous increase. Given this, we can say that the initial accumulation of physical capital attracts more resources devoted to the output production sector, and the shares of labor, physical and human capital in the final good production sector face a large initial increase. Over time, as the growth rate of physical capital decreases and the growth rate of human capital increases, the allocation of factors to the output production sector declines. Eventually, the shares of labor and human capital undershoot their long-run value, but they finally end up with the same value they had previous to the shock, being the share of physical capital the only one that finishes with a higher value in the manufacturing sector. In sum, we can see from
*figure 3A* that physical capital taxation has a small and negative effect on human capital, in line with Trostel (1993).

*Figure 3.A. Growth rates of output, capital, and human capital*

*Figure 3.B. Growth rate of physical capital*

*Figure 3. Evolution of the economy after a physical capital tax cut*
Figure 3.C. Adjustment dynamics

Figure 3.D. Skill premium

Figure 3 (ctd.) Evolution of the economy after a physical capital tax cut
Under this situation, the skill premium shows a short and fast initial increase, coinciding with a quite large and also fast initial decrease in human capital. Next, it starts decreasing during some periods following a subsequent increase in human capital. Even though during the first periods following the physical capital tax cut the skill premium remains above the skill premium associated to a situation with no physical capital tax change, it very soon decreases below the no-tax change skill premium value, remaining like that for several periods. This allows us to say that a physical capital tax cut leads to a long-run decrease in inequality. In sum, devoting more physical capital resources to the production of output relative to the production of human capital allows us to come up with an income inequality that ends up following a lower trajectory than the one followed without the tax cut.

6. Conclusion

In this paper we attempt to present a way to influence inequality as measured by the skill premium. In the context of non-scale models, we have introduced various types of taxes in order to make fiscal policy more specific. To be more precise, we have introduced a tax on physical capital, a tax on skills and a tax on raw labor wage. The model here is capable to alter the shares of private factors devoted to each of the two production sectors, final output and human capital, and affect inequality in a different way according to the different tax changes. We have also tried to detail the impact of fiscal policy on the transitional dynamics. In fact, the presence of capital and raw labor in the human capital
technology opens a route for the influence of different types of taxes on the long-run allocation of labor, physical and human capital, among the knowledge and output production sectors.

A human capital (skills) tax cut, which could be interpreted as a reduction in progressivity, ends up increasing both the shares of labor and physical capital devoted to the production of knowledge and decreasing inequality by lowering the path followed by the skill premium. Given that the net returns on skills are larger after the tax decrease, we can think that the human capital tax cut may foster the relative production of human capital and it does so by devoting a higher percentage of the productive factors, \( N \) and \( K \) to it. Moreover, a raw labor wage tax decrease, which could also be interpreted as an increase in the progressivity of the system, increases the share of labor devoted to the production of final output and increases the path followed by the skill premium. Since the net return on raw labor wage is larger than before the tax decrease, this reduction in the wage tax may disincentive the relative production of human capital. Finally, a physical capital tax decrease reduces the share of physical capital devoted to the production of knowledge and allows for a lower inequality value than the one achieved without changing the fiscal policy. However, it is interesting to note that none of the taxes we have introduced influences the long-run allocation of human capital to the two production sectors, quite a provocative result. We might have to introduce a tax or a subsidy directly on the production of human capital to have such an effect here. Nevertheless, even if a subsidy may be more efficient than direct government expenditures, it may undesirably alter the distribution of tax burdens. Besides, the choice of the optimal rate of subsidy requires balancing efficiency and distributional equity as said by Aaron and
Boskin (1980). These authors go on saying that in an economy in which individuals have different wage rates and different tastes, a subsidy may be optimal in some income classes and demographic groups but not in others. In the presence of externalities, for instance, it may be best to concentrate subsidies on groups with high price elasticities. On the other hand, educational investment subsidies, as said by Steuerle (1996), may operate like investment tax credits for physical capital investment, that is, favoring short-lived over long-lived capital. Lin and Russo (1999) also suggest that in many countries that have attempted to encourage R&D with subsidies and have paid for them by raising taxes, those subsidies may have discouraged rather than encouraged growth. Extrapolating this result to human capital, we should be aware of the pernicious consequences of subsidization when implementing any kind of subsidy.

One final remark would be that once a progressivity reduction has taken place with the subsequent increase in human capital accumulation and thus some efficiency gains, government could use some complementary instruments aimed at redistributing those efficiency gains across generations, increasing the number of households who benefit from it. But then it comes the problem of subsidization again.

Finally, a broad discussion on how changes in externalities, government expenditures, etc. will affect the transition, growth, inequality and so on, using the formulation in this paper as benchmark is in our immediate future agenda. In any case, the framework presented in this paper could be a likely avenue for government policy to influence long-run inequality in a non-scale economy. The
present model and its simulative results provide an operational framework within which future policy inequality issues can be addressed.
References


Appendix. Graphical evolution of the shares of labor, physical capital and human capital in the manufacturing sector after a tax change.

A.1. Human capital tax decrease from 35% to 30%.
A.2. Raw labor wage tax decrease from 30% to 25%.
A.3. Physical capital tax decrease from 20% to 15%.