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# Convergence and Inter-Distributional Dynamics among the Spanish Provinces. A Non-parametric Density Estimation Approach

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# CONVERGENCE AND INTER-DISTRIBUTIONAL DYNAMICS AMONG THE SPANISH PROVINCES. A NON-PARAMETRIC DENSITY ESTIMATION APPROACH [a b]

Leone Leonida[c], Daniel Montolio[d]

ABSTRACT: The purpose of this paper is to investigate the dynamics of the per capita GDP over the period 1961-1997 for the Spanish provinces. To avoid issues linked to the cross-sectional regression and the time series approaches, a non-parametric density estimation approach is instead used. The main goal is to show that Spanish provinces had a convergence dynamics during years, but a divergent one in the last decades. After a period of convergence during the sixties and part of the seventies, evidence in favour of intra-groups convergence (or polarization of income) is found during the eighties, and a starting process of divergence and increasing polarization among groups (clustering) is found for the last period analysed (1991-1997). Moreover, the inter-distributional dynamics are analysed for the Spanish provinces, presenting how the provinces have evolved during this period.

*JEL Classification*: C14, O40, O41
*Key words*: Convergence, Polarization, Inter-Distributional Dynamics, Non-parametric Density Estimation.

RESUM: Aquest treball investiga la dinàmica del PIB per càpita per les províncies espanyoles pel període 1961-1997. Per evitar problemes relacionats amb les regressions de caire transversal i temporal, s'utilitza una aproximació no paramètrica per l'estimació de la distribució de la riquesa. L'objectiu principal es mostrar que les províncies espanyoles han tingut una dinàmica convergent durant els primers anys de la sèrie, però una dinàmica divergent en els darrers anys analitzats. Després d'un període de convergència durant els seixanta i part del setanta, es troba evidencia empírica a favor de convergència entre diferents grups de renda (*polarització de la renda*) durant els vuitanta, i una accentuació del procés de polarització de la riquesa entre províncies (*clustering*) i una incipient dinàmica divergència per l'últim període analitzat (1991-1997). Finalment, s'estudia la dinàmica inter-distribucional entre les províncies espanyoles, mostrant com aquestes han evolucionat dins la distribució de la renda en el període analitzat.

*Paraules Clau*: Convergència, Polarització, Dinàmica Inter-Distribucional, Estimació No Paramètric.

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## 1 Introduction

Even if economic growth has been, and it is, object of a huge quantity of studies, economists are far from understanding which are its main sources.

During the middle 80's, the endogenous growth theories (see, i.e., Romer (1986), Lucas (1988), Aghion and Howitt (1998)) and the ensuing conflict with "classical" models *a là* Solow (1956) have given new impulse to empirical analysis of cross-country growth determinants (i.e. Solow (1994), Barro and Sala-i-Martín (1996)).

The key question in this debate is whether a set of economies (countries, regions, provinces) starting from different income (or product) levels will tend to converge, provided the possibility of controlling for structural differences, to the same per-capita income level.

Among others, Quah (1993b, 1996c), Canova and Marcet (1995) and Jones (1997a,b) have raised some critics to the parametric or classical approach to the convergence issue. Since only the first two moments of the distribution are involved in this kind of analysis, the approach itself is uninformative about the dynamics of the entire income distribution.

By estimating non-parametrically the world income distribution, Quah (1993b, 1996c) found a "twin peaks" shape of the per-capita income densities for the world countries. Estimation of the steady state distribution show that this "twin peak" feature is a not a transitory issue, unless very unrealistic assumption are made (Jones (1997a,b)).

We can find various studies that try to assess, using the so-called "convergence equation", whether there has been convergence for the Spanish case. Much of this work, however, has been focused at the regional level; the provincial case has not been deeply studied. Therefore, our contribution is twofold.

Firstly, we want to fill this gap. Pioneering works were Dolado et. al. (1994) where the convergence issue was mainly analysed for the Spanish provinces, and García-Greciano and Raymond (1994) who studied regional convergence in Spain[1]. However, in the writers' opinion, the issue has still to be defined and studied.

Secondly, our contribution is also methodological, the analysis for the Spanish case did not take advantage of the tools recently developed in the convergence literature. This paper goes back to the original issue on convergence, trying to assess the original one: are the Spanish provinces converging or not?

In the following essay, some of the tools developed in the convergence debate are

[1] These works were followed by other regional studies such as De la Fuente (1994, 1996), García-Greciano et. al. (1995), Mas et. al. (1994, 1995, 1998), Cuadrado et. al. (1999), Gorostiaga (1999), Salas (1999), García-Greciano and Raymond (1999), among others. Using different specifications and econometric tools, convergence among Spanish regions has been a common result in these works.

used in order to investigate the dynamics of the per-capita Gross Domestic Product (GDP) over the period 1961-1997 for the Spanish provinces.

To avoid issues linked to the cross-sectional regression and the time series approaches, a non-parametric density estimation approach is used. This kind of analysis overcomes at least two important shortcomings of standard tests usually used in the convergence debate: they make use only of the first few moments of the distributions and, in the case of rejections of the convergence hypothesis no information is usually given.

The main goal is to show that Spain had a convergence dynamics during years, but a divergent one in the last decades. After a period of convergence during the sixties and part of the seventies, evidence in favour of intra-groups convergence (or polarization of income) is found during the eighties, and a starting process of divergence and increasing polarization among groups (clustering) is found for the last period analysed (1991-1997).

The rest of the paper is organised as follows. Section 2 revises the theoretical and empirical debate on convergence. Section 3 is an overview about non-parametric density estimation and issues arising when this technique is used. Section 4 describes the data set used for the empirical analysis. Section 5 deals with the empirical results on polarisation and convergence. Finally, section 6 sketches some conclusions.

## 2 Convergence: Theoretical and Empirical Debate

The neoclassical growth models that were initially developed by authors such as Ramsey (1928), Solow (1956), Cass (1965) or Koopmans (1965) have a very well known result: "The per-capita rate tends to be inversely related to the starting level of output or income per person" Barro and Sala-i-Martín (1992). This classical approach to convergence comes from solving the maximization problem of the representative, infinite-horizon household in the optimal growth model[2].

The convergence idea underlying the neoclassical growth models is that "if two economies have the same parameters of preferences and technology, then the key result is that the initially poorer economy tends to grow faster in per-capita terms" Barro and Sala-i-Martín (1992). This idea has been called *The Convergence Hypothesis* in the neoclassical growth models.

Sala-i-Martín introduced a wide-used terminology to describe the convergence hypothesis. The author defined "β-convergence in a cross-section of economies if we find a negative relation between the growth rate of income per capita and the initial level of income. In other words, we say that there is β-convergence if poor economies tend to grow faster than wealthy ones" Sala-i-Martín (1996).

[2] For a detailed explanation of the model and its analytical solution see, for instance, Blanchard and Fischer (1989) or Barro and Sala-i-Martín (1991).

This β-convergence concept was extended in two different ways: absolute β-convergence and conditional β-convergence, that together with σ-convergence, are the main concepts used to analyse this important issue.

*a) The Absolute β-Convergence*

The empirical implications of the neoclassical model are clear. The classical approach to this issue, in large part due to Baumol (1986) and Barro and Sala-i-Martín (1991, 1992), is to estimate cross-sectionally the measured growth rates on initial level of income.

During the eighties, the first estimations of the absolute β-convergence were not successful, denying one of the more important results of the neoclassical models (for instance, see DeLong (1988)). This empirical result is not surprising, because one of the main points when estimating the absolute β-convergence is that the estimate does not take into account structural differences among economies.

The main assumption is, hence, that all the economies we are analysing have the same preferences, technology, and tastes, but this hypothesis is not tested.

*b) The Conditional β-Convergence*.

However, when researchers try to analyse convergence among different countries it is very difficult to have the same level of preferences and technology.

This drawback in the estimation of the classical growth theory was overcome by the idea developed by authors such as Barro and Sala-i-Martín (1992), Mankiw, Romer and Weil (1992), Galor (1996) or Durlauf (1996) among others, called *conditional β-convergence*.

The main conclusion is that one needs to take into account not only differences in preferences and technologies for different countries, but also differences in human capital, government policy, natural resources, etc. Therefore, it is important to realise that the catching up process of poor economies will happen among countries that are similar in preferences, technologies, rates of population growth, government policies, etc. Therefore, similar economies *except* for the initial level of capital per worker are expected to converge to the same steady state (and hence, to one another): and this is the definition of conditional β-convergence[3].

This hypothesis of convergence was extensively tested[4], reaching the conclusion

[3] It is important to clarify that "the neoclassical growth model leads to the conditional convergence hypothesis rather than to the absolute one" Galor (1996). However, the first estimations of the convergence hypothesis that rejected the validity of the absolute β-convergence were used to reject the neoclassical growth models as the framework for the study of economic growth, giving the chance for the endogenous growth theory to be extensively developed.

[4] See for instance, Barro and Sala-i-Martín (1991, 1992), Mankiw, Romer and Weil (1992) for a cross section approach, or Knight et. al (1993)and Islam (1995) for a panel data approach.

that the empirical evidence supports the neoclassical model once one controls for differences in preferences, technology and institutions. One of the most striking results was that “countries appear to approach their own steady state at a fairly uniform rate of, say, 2 percent per year” Jones (1997)[5].

The conditional β-convergence issue has been also analysed for the Spanish case. For instance, Dolado et. al. (1994) studied the β-convergence in the Spanish provinces and regions for the 1955-1989 period, using the same data source as we use in this work[6], but with a shorter data span. The main results found are that for the overall period there is evidence in favour of conditional convergence (they introduce regional and sectorial variables (“proxies”), to have into account the possibility of different steady states across provinces or regions) at a rate of 4.4%, higher than the 2% rate usually found in the empirical literature. Moreover, introducing more variables to control for exogenous characteristics of each province/region they found that the introduction of the investment rate in physical capital, and a proxy variable for migrations increases the convergence rate up to 6%.

This increase in the convergence rate when introducing more exogenous and relevant characteristics in the “convergence equation” is also found in Gorostiaga (1999) for the Spanish case. In Gorostiaga’s work the β-convergence rate increases up to 18%, much higher than the rate found by other authors but consistent with studies, which realised that the “slow” convergence rate is maybe due an incorrect specification of the model, and therefore, they use different econometric tools to tackle the convergence issue[7]. The main conclusion of her article is that “the poorest regions are growing faster but not necessarily to catch-up the richest regions, but to reach its own steady state” Gorostiaga (1999).

Both articles highlight the importance of the existence of significant regional fixed effects that must be taken into account for a good estimation of the convergence rate in the Spanish case[8].

The specific characteristics of the Spanish case are pointed out in various works. For instance, the results obtained in the estimation of the β-convergence by Dolado et. al. (1994) for all the period analysed (6% rate of convergence) do not hold when they consider different sub-periods in the sample. They found that during the period 1964-1977, there is no empirical evidence in favour of conditional β-convergence, showing that in that period the Spanish regions and provinces were not approaching its own steady state.

In this work, the analysis of different sub-samples will be easily done because of the

[5] This “mythical” 2 percent speed of convergence was initially found by Barro and Sala-i-Martín (1991).
[6] As we will see in Section 4 of this work, the observations in this data base are biannual. Hence, it makes the estimation and the interpretation of the results different than in the case of having annual data.
[7] See Islam (1995) and Evans and Karras (1996) for the USA case, Canova and Marcet (1995) for the European regions, and Raymond and García-Greciano (1994) and De la Fuente (1996) for the Spanish case.
[8] Therefore, the OLS approach is clearly misleading, at least, for the Spanish case.

advantages of the non-parametric method of estimation. Basically, we will analyse not only how the shape of the distribution changed between two different points in time, but also we can study which provinces are moving and where they are going to set in the distribution (inter-distribution dynamics). Therefore, it will be interesting to compare our results (globally and for different sub-periods) with the results found in the convergence literature for the Spanish case.

*c) The σ-Convergence.*

With the conditional convergence idea, it is impossible to test for overall convergence among economies. It is only possible to see if the observations are converging to their own steady state or not. However, other measures to study converge across economies have been used. For instance, to investigate if two economies are converging or not, it can be tested if the dispersion of the real income per capita across groups of economies tends to fall over time. This is what is called *σ-convergence*[9].

The σ-convergence issue has been also broadly studied for the Spanish case. For instance, Dolado et. al. (1994) found that for the 1955-1964 period there is a clear path of σ-convergence among the Spanish provinces. For the period 1964-1977, the authors found that there is neither σ-convergence nor conditional β-convergence. Finally, and more interestingly, during the period 1977-1989 there is an increase in the variance of the logarithm of the GVA per capita, indicating that we could speak about divergence in the σ-convergence sense.

This result is also confirmed by García-Greciano and Raymond (1999). Their study focuses in the GVA per capita in the Spanish regions for the period 1955-1997. The main results are that during the period 1955-1979 there is what can be called σ-convergence (i.e. a decrease in the regional disparities among the Spanish regions). However, for the period 1979-1997 they found a sharp slowdown of the convergence process.

### 2.1 Critics to the "Classical Approach"

Recently, some researchers have underlined that the approach explained so far is misleading in some ways. For instance, Quah (1993b, 1996c) criticizes one of the main results of the convergence theory: poor and rich economies all appear to be converging toward each other at a stable, uniform rate of 2% per year. "The idea here is that such consistency might only reflect something mechanical and independent of the economic structure of growth" Quah (1996b).

When testing for conditional convergence, researchers remove part of the

[9] For more details see Sala-i-Martín (1996)

heterogeneity across countries, however, it is not likely that after conditioning, all significant differences in economic growth across economies are removed, especially when we analyse diverse geographical and time samples. Moreover, Quah (1996a,b) shows that even if the underlying structures of economic growth across economies have enough uniformity to produce this stable result, this can be merely a statistical invariance.

Moreover, Quah (1993b) not only examines the 2% speed of convergence, repeatedly found in the literature, but also revises the β- and σ-convergence approach itself. Using the idea of the Galton's Fallacy[10] the author shows that calculating a cross-section regression to explain time-averaged growth rates is inadequate to determine how the distribution of the income per-capita across economies evolves through time. The author shows, moreover, that a negative cross-section regression coefficient on initial levels is consistent with absence of convergence in the sense of each country eventually becoming as rich as all the others (β-convergence).

Quah (1996a) also pointed out some drawbacks of the σ-convergence approach. The main point is that even $\sigma_t^2$ can be unchanged through time, showing no converge or no divergence in this sense, "the economies underlying the cross-section could still be moving about within the invariant distribution". Therefore, the σ-convergence approach fails to explain inter and intra-distributional dynamics. A very important concept if we want to analyse persistence of income disparities over time (see Quah (1996a,b)). "The σ-convergence analysis alone is not sufficient to study convergence unless more information is gained on how units move within the distribution" Bianchi (1997).

We can find more problems linked to the regression approach in the analysis of convergence[11]: aggregation problems, economic interpretation of the coefficients (Durlauf (1996)), measurement problems in the poor part of the world (Daveri (1996)), and robustness of results (Durlauf (1996), Solow (1994)). This last issue is particulate important: a slight change in the period of analysis (or in the sample) often gives completely different results in the size and the sign of the estimated parameters (Levine and Renelt (1992), Solow (1994), Ben-David (1994)).

The main drawback of the cross-section estimation approach to the convergence issue is that we cannot figure out the dynamics of the underlying distribution of per-capita income. "Cross-section regressions can represent only average behaviour, not the behaviour of an entire distribution" Quah (1996a).

Therefore, it seems more adequate to estimate the distribution of the per-capita income for different years, to analyse the shape and to see if there is any tendency to collapse toward a unimodal distribution, provided that the initial one is bimodal, or the

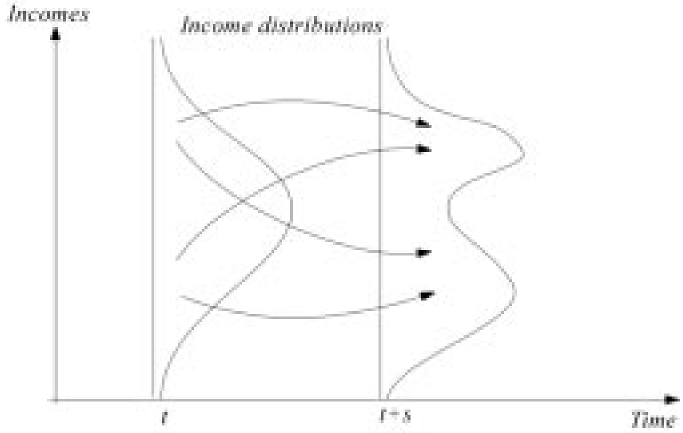
[10] For more detail see Quah (1993b).
[11] For a good interpretation and problems in interpreting the cross-section and time series tests on convergence see Bernard and Durlauf (1996).

other way round.

Figure 2.1 represents an income distribution at time t and another (possible) at time t+s. If the distribution collapses from a unimodal to a bimodal one (emerging twin peaks), we find intra-convergence in groups of per-capita GDP but divergence with other group(s). On the other hand, if the distribution collapses from a bimodal to a unimodal, economies are said to converge over time.

This approach allows us to look for inter and intra-distribution dynamics in which we can combine macroeconomic theories of growth and microeconomic models of cross-sectional interaction. Therefore, analysing the distribution itself of the income per-capita across countries, regions, or provinces. We will study at the same time both shape and mobility dynamics[12].

Figure 2.1. Emerging Twin-Peaks.




Using this non-parametric approach, and the same data set investigated by other researchers (Summer and Heston (1991)), Quah (1996a, 1997) showed that there is no evidence in favour of convergence among countries in the world. The author estimated the densities for some years and he found a fluctuating bimodal distribution.

Durlauf and Johnson (1994) found evidence for a multi-modal behaviour of the world income distribution. Bianchi (1997) builds up a test for multimodality and rejects unimodality in favour of bimodality in the world distribution of income as well.

Moreover, in Quah (1997) and Durlauf and Johnson (1994) we find empirical evidence in favour of the club convergence hypothesis[13], that is when "per-capita incomes of countries that are identical in their structural characteristics converge to one another in

[12] In one hand, studying the shape of the distribution we will analyse if there is *convergence* or *divergence*, if more than two peaks emerge in the distribution (*stratification*) or if countries are catching up with one another but only within particular subgroups (*convergence clubs*). In the other hand, studying the mobility dynamics of a the distribution will allow us to examine if rich countries at *t* are still rich at *t+s* (*persistence*), if some poor countries at *t+s* had begun rich (*churning* or *mobility*) or/and if some groups of these economies originally close together in the middle class have separated because a process of divergence (*separability*).
[13] Or evidence in favour of polarisation, persistent poverty and clustering.

the long-run provided that their initial conditions are similar as well" Galor (1996).

This is the approach used in this paper to analyse the convergence issue among the Spanish provinces, and it is deeply explained in the following section.

## 3 Methodology

In this section, we will describe a straightforward way to analyse the cross-section distribution of the provincial per-capita gross domestic product (GDP) over time. The main tool we are going to use is a non-parametric density estimation approach. This framework will allow us not only to test for convergence on the Spanish provinces, but also to analyse the dynamics of the entire distribution. Therefore, we will explain if Spanish provinces are converging or not, and how they have evolved through time; this issue cannot be studied in the conventional analysis.

The main reason to use this approach is the advantages that the study of the shape of the entire distribution gives us. Moreover, this approach overcomes the problems of the parametric regressions mentioned above. The main gain is that we will not impose any particular form on the underlying density as in the parametric approach. Since we want to investigate the shape of the entire distribution, it is not useful to impose any particular form on the underlying density. Instead, the methodology used in this article only requires that the underlying densities are sufficiently smoothed.

### 3.1 A Direct Analysis for Convergence: Issues in Non-parametric Density Estimation

There are many kinds of methods that can be used to estimate the density of a given data[14]. However, we will use the Kernel estimator. A Kernel function is defined as:

$$\int_{-\infty}^{\infty} K(x)dx = 1 \tag{3.1}$$

We can define a broad class of density estimators (the Ronsenblatt-Parzen Kernel density estimator) as:

$$\hat{f}(x) = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{h}K\left(\frac{x-x_i}{h}\right) \tag{3.2}$$

Where $N$ is the number of observations in the sample, $h$ is the bandwidth of the interval chosen (also called the smoothing parameter), $K$ refers to a Kernel weighting function, and $x_i$ is the $i$th observation of the sample.

The density function is estimated using $N$ observations $(x_1,...,x_N)$, the density for any

[14] For a good survey of different estimation methods, see Silverman (1986).

$x$ (our variable of interest, the relative per-capita income) is estimated at a point $x=x_0$ as a weighted sum of all the observations of $1/N$, where the weight for the $i$th observation $x_i$ is given by the weighting function (the kernel in this case) evaluated at ($x_i$-$x_0$).

### 3.1.1 Choosing the Kernel Function

As we have pointed out, the Kernel function is used to weight each point of the distribution. However, the weight for the $i$th observation is given by the height of the standard normal density function if a "normal" (or Gaussian) density is used.

However, many Kernel functions can be used, each of them with different advantages (efficiency, smoothing power, etc.). All positive functions of this distance that integrate to unity (because is a way to assign weights to observations) could play this role. Therefore, a theoretical issue arises: which Kernel is the optimal one?

Normally Kernel density estimators are locally smooth, so "the choice of Kernel turns out to be relatively unimportant" Johnston and Dinardo (1997). In this work, the different estimations are realized with the Gaussian (or "normal") Kernel weighting function[15]. The main reason to use this Kernel is that is the easiest to calculate, and permits the automatic calculation of an optimal bandwidth, as we will see in the next section.

### 3.1.2 Choosing the Optimal Bandwidth

More crucial than the election of the Kernel function, is to choose the bandwidth ($h$). The magnitude of $h$ is indeed going to determine "which observations we are looking at" Kennedy (1996). If $h$ is chosen "too small", the kernel is going to assign non-negligible weight only to the observations very close to $x_0$, with the result that the estimated density function is going to be under-smoothed and uninformative. On the other hand, if $h$ is chosen "too large" the kernel will assign a non-negligible weight even to observations very far from $x_0$, over-smoothing the estimated density function and therefore losing crucial information about the "true" shape of the distribution[16].

It turns out that the bandwidth choice will be the crucial issue in the effective estimation of the density function of the Spanish provincial per-capita GDP.

A first way of deciding the bandwidth magnitude is to "judge the right bandwidth by the "eyeball" method, that is, whatever looks appropriate to the eye" Johnston and Dinardo (1997). Silverman (1986) also proposes to choose the bandwidth that is most in accordance with "one's prior ideas about the density". Although this is not a formal approach, it can be

[15] The Gaussian (or "normal") Kernel is defined as: $K(x) = \frac{1}{\sqrt{2\pi}} e^{-0.5x^2}$ (3.3)

[16] There is a trade-off between bias and variance. For more details, see Silverman (1986).

useful to produce a reasonable range of possible bandwidths.

A more formal way of deciding the relevant bandwidth is to use “automatic bandwidth selectors”. Although this approach has some drawbacks, is the method we are going to use in this paper. Other authors concerned about testing on convergence, like Quah (1997), have used this method with successful results.

There are many automatic bandwidth selectors. In this paper, density estimations will be run using the *Jones’ optimal bandwidth* and the *Least-Squares Cross-Validation* methods[17].

Obviously, these procedures will be repeated for the estimation of the density for each year. Therefore, each year will have a different optimal bandwidth. Because we are interested in comparing densities for different years and see the evolution of the overall distribution through time, we cannot compare two estimated distributions with different smoothing parameters. “When it is desired to compare several density estimates, meaningful comparison (of densities with comparable “features”) can only be done when de same amount of smoothing is done for each curve” Marron and Schmitz (1992).

Given that our data has the same scale for all the relevant years analysed, and the same size (52 provinces), we can use a quite simple way that will allow us to compare the estimated densities for different years. This simple method takes the average of the Jones’ bandwidths or the average of the Cross-Validated Bandwidths for the individual years. Applying either one or the other average, we can compare across different years.

The main results presented in Section 5 are obtained using the average of the Jones’ bandwidth in the kernel estimation. However, in the appendix (figure A.1) the same estimates are shown with the average of the Least-Squares Cross-Validation bandwidths.

### 3.2 Bivariate Kernel Density Estimation and Inter-Distributional Dynamics

It is possible to study the inter-distributional dynamics in the Spanish case using a multivariate Kernel (Quah (1997)). This analysis will enrich the results obtained using the non-parametric approach explained so far.

The minimum number of cross-sectional observations required for the bivariate estimation is 20 (see Silverman (1986)). Given that 52 are the Spanish provinces, we can perfectly use this approach for the provincial case. However, given that regions are the sum of some provinces[18], analysing the dynamics of the provinces through time, we will implicitly study the evolution of the regions. Moreover, the use of provinces is also more

[17] For more details, see Silverman (1986).

[18] There are 7 regions that have only one province (uniprovincial): Asturias, Baleares, Cantabria, Madrid, Múrcia, Navarra and La Rioja. Therefore, for these units the provincial analysis is at the same time the regional analysis of the evolution through time.

informative given that there are inequalities *inside* regions, with this level of desaggregation we will study how the inequalities of income have evolved inside each region, and of course, for the overall country.

Therefore, this approach will complete the study of the shape of the relative per-capita GDP distribution, focusing in which provinces have increased or decreased their position in the income distribution through time.

A multivariate kernel studies the join distribution for *d* years. The definition of the univariate kernel estimate can be easily adapted for the multivariate case. The non-parametric density estimate, with kernel *K* and bandwidth *h*, is defined as:

$$\hat{f}(\mathbf{x}) = \frac{1}{Nh^d} \sum_{i=1}^{N} K\left\{ \frac{1}{h} \left( \mathbf{x} - \mathbf{X}_i \right) \right\} \tag{3.4}$$

In which the kernel function *K(x)* is now a function defined for a vector of variables x of dimension d. The weights of this function are given by:

$$\int_{R^d} K(\mathbf{x}) d\mathbf{x} = 1 \tag{3.5}$$

Conveniently, *K* is chosen from a family of densities that are radially symmetric. As an example, we will use the normal multivariate density function:

$$K(\mathbf{x}) = (2\pi)^{-d/2} e^{-0.5\mathbf{x}^{\mathrm{T}}\mathbf{x}} \tag{3.6}$$

Of course, all the previous consideration about the election of the bandwidth and the kernel function are still valid. The interesting point for our analysis is that using this method for two years (*d*=2), we will have an estimation of the joint distribution that will give us not only the dynamics of the modes in the distribution of income, but also the dynamics of the provinces during this period.

## 4 Data Description: Variables and Data Sources

The variable we are going to use to investigate the existence of convergence among the Spanish provinces, and its dynamics properties, is the per-capita Gross Domestic Product (GDP) for the period 1961 - 1997 for the 52 provinces (including the Spanish cities in the north of Africa: Ceuta and Melilla).

This variable is one of the more comparable indexes across different economies (regions or provinces) and time. Moreover, per-capita GDP is one of the most common measures of wealth of an area, and hence, a good instrument to investigate the convergence process across different economies.

The main data source is the recently published data base elaborated by the

Fundación Banco Bilbao Vizcaya (Fundación BBV (1999)), which allow us to have homogeneous series from 1955 to 1997[19].

We have to point out that the possibility of using this new data base permit us to extent the period of investigation to very recent years (1997), while others works on convergence for the Spanish case (mainly using a parametric approach) used data until the beginning of the nineties. This point, as we will see later, will be crucial when interpreting our results and comparing with previous studies.

In order to avoid the bias that the inflation could cause in our analysis, we use the series at 1990 constant prices.

Following Quah (1997), we have calculated the ratio between per-capita GDP in each year and for each province and the average per-capita GDP for Spain[20]. This normalization is very useful in two aspects. Firstly, it is an easy way to abstract from Spain's total growth and fluctuations. Secondly, since the average for Spain is equal to one, the data assumes the form of dispersion around the mean and it is possible to make direct comparisons across years.

## 4.1 A First Look to the Data

The elevated number of provinces 52 makes impossible to plot in an informative graph the relevant series we will use in this work. However, we can analyse a related variable that has been used as a measure of converge among economies: the variance of the per-capita GDP.

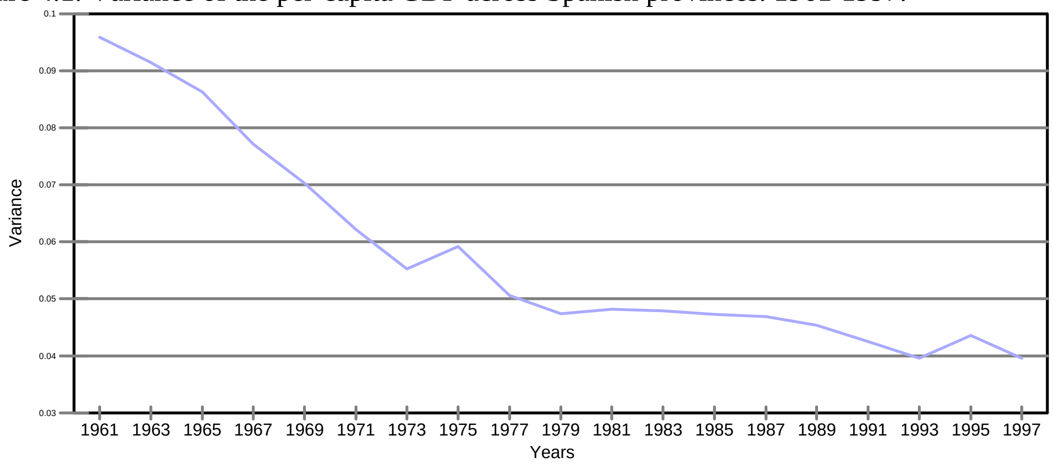
The evolution of the variance is shown in figure 4.1

[19] This data base calculates the GDP every two years (1955, 1957, 1959, etc.), but for our purposes this characteristic does not affect at all the results obtained with the non-parametric approach, mainly because we are analysing the density estimate at each point of time. Therefore, we can choose which individual years we want to study. Our election has been the first year of each decade and the last available year in our data base (1961, 1971, 1981, 1991 and 1997). In this way we can analyse how the overall distribution evolves during each decade, and more important, in recent years. Moreover, we think that 10 years is enough time to see how the distribution evolves through time, and to highlight its important features. Nevertheless, we conducted the analysis for all the years in our data base, and we have finally chosen the most significant years: the ones that can shed light into the convergence issue in Spain.

[20] More precisely the normalization of the GDP per capita gives us the *relative GDP per capita* (starred in the formula) in the following way:

$$\left(\frac{GDP}{Pop}\right)^*_{it} = \frac{\dfrac{GDP_{it}}{Pop_{it}}}{\dfrac{GDP_{Spa,t}}{Pop_{Spa,t}}} \quad \{i = 1,...,52(\textit{provinces}) \text{ and } t = 1961,...,1997$$

Figure 4.1. Variance of the per-capita GDP across Spanish provinces. 1961-1997.




From 1961 to 1973 the variance for provinces decreased, this means that the dispersion of the relative per-capita GDP is decreasing, showing a period of decrease in the inequalities in Spain (σ-convergence), or a process of clustering among provinces.

However, for the 1973-1981period the evolution is quite bizarre, increasing first and decreasing afterwards. After 1981, the value of the variance stabilizes around a certain value, and seems to increase a bit for the beginning of the eighties and for the beginning of the nineties. This could indicate that the during the eighties and nineties the variance stopped its convergence pattern, and instead, it started to have a more random behaviour, stabilizing around one value, but also increasing in some periods, indicating the possibility of the beginning of a process of divergence.

Although this is a first and rough approximation to the convergence issue, these results will be analysed in depth and more detail in the next section.

## 5 Empirical Results

In this section, we will study the density estimates of the relative per-capita GDP distribution for the Spanish provinces.

What we want to analyse is the shape of the entire distribution of the per-capita GDP for each year we are interested. Kernel smoothed estimates of the income distribution for 5 years (1961, 1971, 1981, 1991 and 1997) are shown in figure 5.1. These density estimations are obtained using a Gaussian kernel, and the Jones' automatic bandwidth selector.

As we have said in the methodology, in order to make comparisons between years we have calculated the mean of the Jones' bandwidth for each year and re-estimated the density again taking this average as a common window width. If we would not have done this, the comparison of the density distributions for each year, with its own bandwidth, had not been possible.

### 5.1 The Shapes of the Distributions

Before starting to analyse our results, and since the bandwidth magnitude could determine how many modes the estimated distribution display, in table 5.1 we report the results regarding the Cross-Validation (or optimal), the Jones and the critical bandwidths for unimodality, bimodality and trimodality (critical bandwidths for each switching regime).

The shapes of the different distributions estimated for the Spanish provinces have changed a lot for the various years used in the non-parametric estimation. In 1961, we can see that there are at least three significant modes; this is confirmed with the comparison of the Jones' and the critical bandwidth (third and fourth column in table 5.1). The Jones' bandwidth is smaller than the critical one for bimodality, but bigger than the critical bandwidth for three modes.

Table 5.1. Bandwidths for the density estimates of the Spanish provinces.

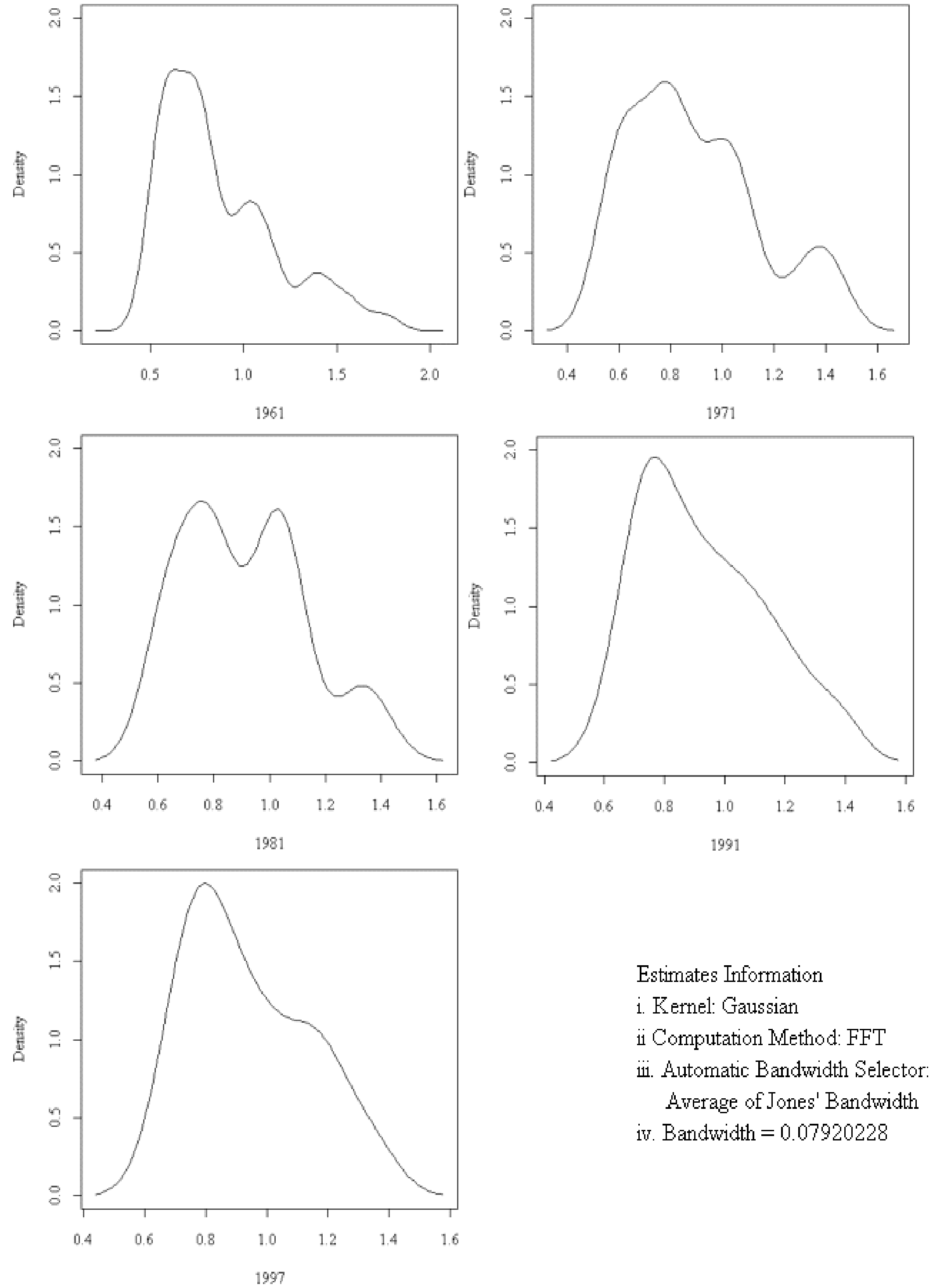
| **Year** | | **1961** | **1971** | **1981** | **1991** | **1997** |
|---|---|---|---|---|---|---|
| ***h*-Jones** | | 0.086 | 0.096 | 0.052 | 0.079 | 0.080 |
| ***h*-Cross Validation** | | 0.041 | 0.055 | 0.046 | 0.081 | 0.090 |
| ***h*-Critical** | *Unimodality* | 0.110 | 0.115 | 0.115 | 0.057 | 0.072 |
| | *Bimodality* | 0.097 | 0.083 | 0.093 | 0.040 | 0.038 |
| | *Trimodality* | 0.076 | 0.070 | 0.047 | 0.038 | 0.032 |

In the density estimate of the 1971, the Jones' bandwidth is greater than the critical one for bimodality, so the distribution is bimodal. This is confirmed if we look at the shape of this distribution: we can clearly see two modes at points 1.0 and 1.4 of the income distribution. However, a third mode (not significant) seems to appear close to the first one.

In 1981, the comparison of bandwidths tells us that the distribution in that year seems to be trimodal again, but the modes now are different than in 1971, and also different to the 1961 estimated distribution. Instead, in 1991 the comparisons of the window widths tell us that the distribution is unimodal because the Jones' is greater than the critical bandwidth for unimodality. Finally, in 1997 we have that the Jones' bandwidth is again greater than the critical one for unimodality, concluding that in 1997 the distribution is unimodal.

At this point, we can highlight that the difference between the critical bandwidth that implies a change between one and two modes in the estimated distribution and the Jones' bandwidth is less in 1997 than in 1991.Therefore, the distribution seems to evolve to a bimodal again.

Figure 5.1. Kernel Density Estimates (Gaussian) of the Relative per-capita GDP with the average of the Jones Bandwidth for the Spanish Provinces (*h*=0.07920228). 1961–1997.



The critical bandwidth for unimodality has increased in 1997 with respect to 1991; this implies that in 1997 the Jones' bandwidth is close to the critical bandwidth. Hence, we could conclude that in this year the unimodality is not very strong (or the bimodality is not rejected as clearly as before), this is confirmed if we look at the density estimate of the income relative GDP distribution in 1997. In this estimated density a second mode is coming, but is not still very significant.

Having decided the shape of the distribution, one can focus on the convergence issue again. This is going to be studied in the following subsection.

### 5.2 Convergence among the Spanish Provinces

As we have pointed out before, not a lot of research focused in the converge issue using Spanish provinces as the aggregation level, instead we can find many references dealing with the regional level. Therefore, we have to be careful when comparing our results with previous studies, because our methodology is different from the usual parametric approach. Moreover, although regions are aggregations of provinces (except for regions that are uniprovincial), we can find inequalities *inside* regions; this fact can make differ the conclusions drawn from these two different approaches.

In 1961, the distribution shows clearly three modes. Provinces were grouped around 0.7, 1 and 1.5 of the income distribution (Spanish average=1). Therefore, in the initial period of our analysis there were three groups of provinces: one group that can be considered as "poor provinces" (below the average), another grouped around the Spanish average, that we can call "middle class provinces", and finally a third group of "rich provinces" (above the average)[21]

From 1961 to 1971, we can see a period of convergence across provinces. First of all, the poor mode is "moving to the right", it means that the average value of the per-capita GDP in that mode is increasing (from 0.7 to 0.8) and approaching the middle class mode (in 1.0), being both modes very close, but still the poor mode is having enough observations to be significant. Secondly, the rich provinces are loosing, and now they are grouped around 1.4 of the income distribution. Therefore, for this initial period we can speak about overall convergence (the modes are less distant), result also found by Dolado et al. (1994) in their study of the convergence issue for the Spanish provinces. Our analysis, however, shows that a process of clustering of poor and middle class regions characterizes this convergence pattern.

Between 1971 and 1981, there is a process of polarization of income among the Spanish provinces. During this period, the convergence process observed during 1961-1971

[21] In the next subsection, we will analyse, using the bivariate kernel density estimation, which provinces are in each mode and how they have evolved during the period analysed.

stopped, and instead in 1981 we find two clear and distant modes, one below the average (0.8) and the other on the average (1.09), and the rich still losing and approaching the middle class provinces (*stratification*). This result was also pointed out by Dolado et. al. (1994).

For the 1961-1981 period we can highlight that rich provinces have lost positions in the distribution of income, but they still create a separate mode (showing *persistence*), implying that in that period in Spain we could find few provinces very rich. The poor provinces increase from 0.7 to 0.8 in the income distribution, but they do not catch up the middle class provinces. The new and important mode in 1981 around 1 is created by rich provinces losing a lot, and some of the poor provinces catching up, forming all together a prominent middle class mode (*mobility* of some provinces). We will analyse these inter-distributional dynamics in the next subsection.

From table 5.1 we know that the estimated 1991 distribution is unimodal, and it is mainly concentrated around 0.8. However, we can see that there is a little bump around 1.1 (of course, not enough important to be considered a significant mode). The 1981-1991 evolution of the income distribution can be understood as a process of some middle class provinces losing and being part of the poor mode, and some growing and approaching the rich mode: the middle class mode vanishes (this process is shown in figure 5.2 with the comparison of 1981 and 1997 density estimates). The provinces remaining at 1.1, that start to catch up rich provinces (that at the same time were losing), are not enough to create a mode in the estimated distribution.

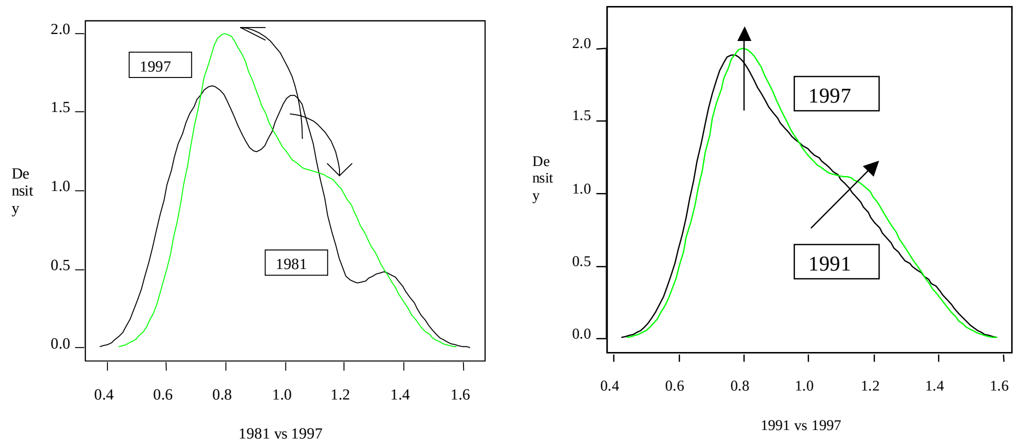
Finally, the 1991-1997 period shows a starting process of polarization of income and divergence. The 1997 distribution is also unimodal (as explained in section 5.1 bimodality is less strongly rejected); in this case, clearly a second mode is coming around 1.2. The provinces are grouped in two levels of income: 0.8 (below average) and 1.2 (above average) implying *polarization* of income (showed in figure 5.2a).

The *divergence* process can be seen in figure 5.2b. Provinces with income above the average are not only creating a significant mode but also separating from the mode of the provinces with an income below the average. Therefore, starting to diverge (the distance between modes is greater in 1997 than in 1991).

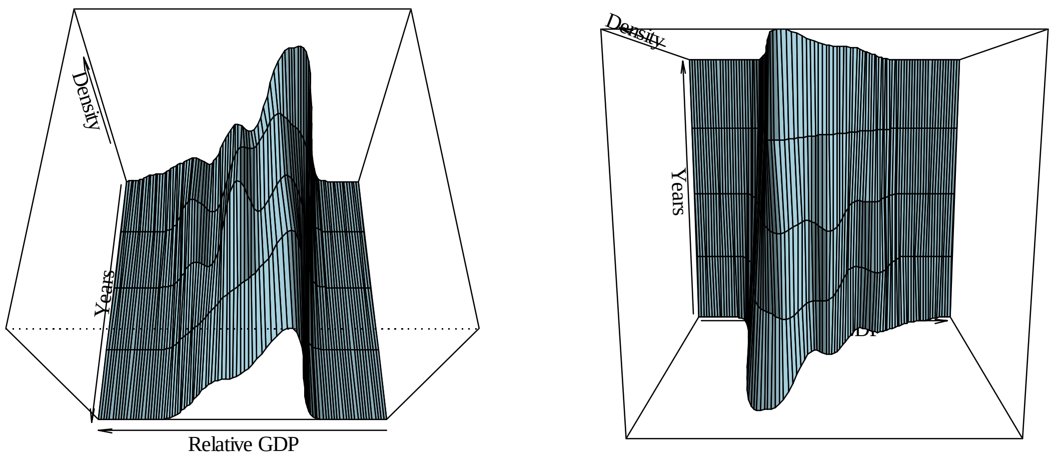
Looking to the 3D plot reported in figures 5.3a and 5.3b, we have in one picture what has happened in the last 36 years to the income distribution of income among provinces.

We can conclude that provinces from 1961 to 1981 have evolved from a bizarre distribution to a polarized distribution with two modes, mainly because rich provinces were losing more than poor or middle class provinces catching up.

Figure 5.2. a) Comparison between 1981 and 1997 Density Estimates for the Spanish Provinces. Polarization of Income. b) Comparison between 1991 and 1997. Divergence.

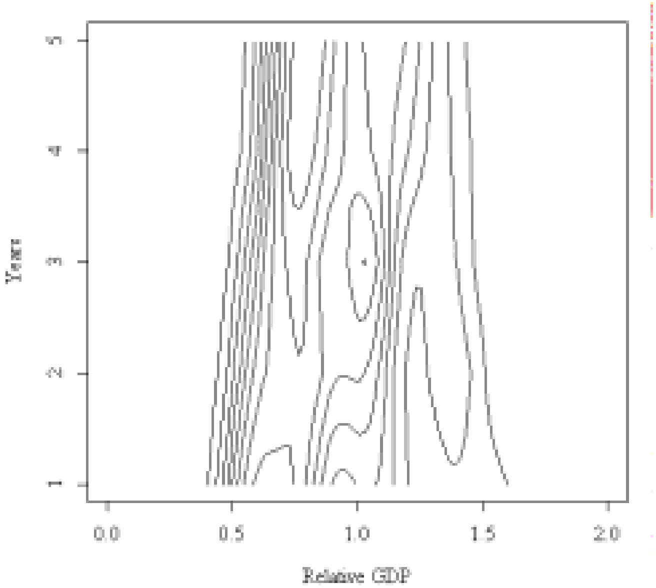



Figures 5.3a and 5.3b. Density Estimations of the Relative per-capita GDP across Spanish Provinces (all 5 years). Two different perspectives.




However from 1981 to 1991 starts a process of polarization of income, the middle class mode start to vanish: some of the provinces forming this mode start to be rich and some move to the poor mode. This process of polarization is maybe clearer in figure 5.4; in this contour plot after 1981 (called 3 in the vertical axis) the distribution seems be characterized by this movement of the middle class provinces. In 1997, the process of polarization is evident, and another mode is clearly coming and separating from the other mode.

Figure 5.4. Contour plot of the density estimates for the Spanish provinces. (All 5 years).




Note: in the vertical axis 1, 2, 3, 4, and 5 correspond to 1961, 1971, 1981, 1991 and 1997 respectively.

## 5.3 Inter-distributional Dynamics

This section will be devoted to the study of the bivariate kernel estimates. Figure 5.5 reports the bivariate kernel estimates for each decade (1961-1971, 1971-1981, 1981-1991 and 1991-1997) and for the overall period (1961-1997). For clarity reasons we have not introduced the names of the provinces (each point in the figure). However, we will explain the dynamics of the provinces through time. For a better understanding of which regions we are referring, in the appendix (figures A.2, A.3, A.4 and A.5) we present clearer graphs for the different decades, in those figures we change the range to introduce the names of the provinces and to show how poor, middle class and rich provinces have evolved through time.

### 5.3.1 Bivariate Kernel Density Estimation

Figures such as 5.5 (or figures A.2 to A.5 in the appendix) show the contour plot of the bivariate density estimation of the join distribution for two different years, and give us the dynamics between these two particular years. To interpret the figures we can recall Quah (1997): “if most of the graph were concentrated along the 45-degree diagonal, the elements in the distributions remain where they began. If, in contrast, most of the mass in the graph were rotated 90 degrees counter-clockwise from that 45-degree diagonal, the substantial overtaking occurs".

In others words, all the points below the 45-degree line correspond to provinces that have lost from the year represented in the x-axis with respect the year in the y-axis, the argument is the opposite if we look at points above the 45-degree line.

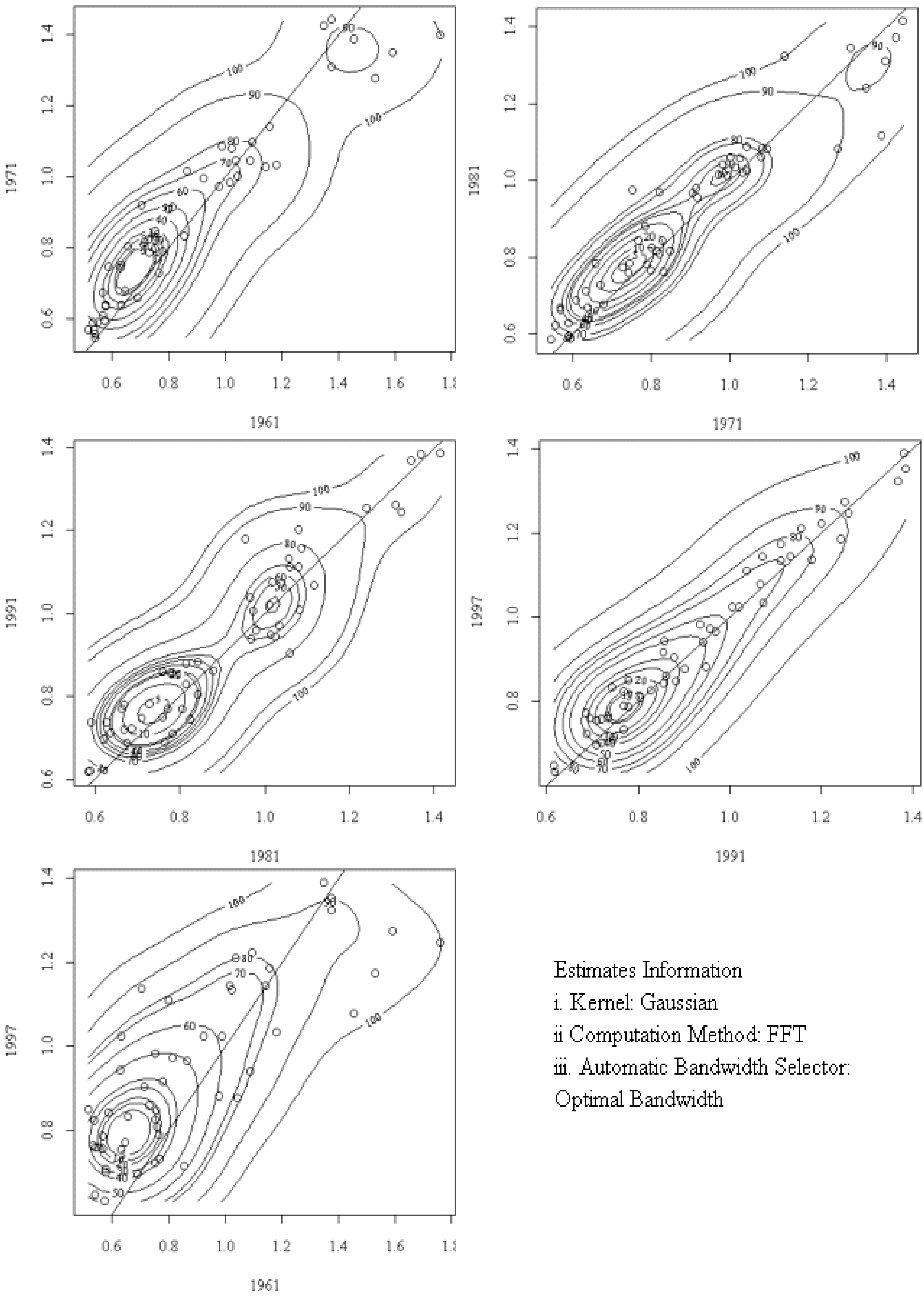
The main results can be summarized as follows. In the initial year of the period 1961-1971 (see figure A.2), some provinces (Almería, Ávila, Badajoz, Cáceres, Cuenca, Ciudad Real, Granada, Jaén, Lugo and Orense) were poor, with an income around half of the Spanish average. However, during this period the main pattern observed among below average provinces is growth, except for Ceuta, Melilla and Sevilla that lost positions in the income distribution, and for Badajoz, Granada, León and Salamanca that at the end of the period remained in the same position as in 1961. Therefore, if in 1961 poor provinces were grouped around a mode of 0.6, in 1971 they are grouped around 0.7 of the income distribution.

During this period, as we have analysed before, there is convergence between poor and middle class provinces. If poor provinces are growing, middle class (with a mode round 1 of the income distribution) are losing positions as well. Alicante, Asturias, Cantabria, Castellón, Valencia and Zaragoza lost positions, La Rioja, Navarra and Tarragona remained unchanged and only Huesca, Lleida, Las Palmas de Gran Canaria and Valladolid grew. Therefore, after 10 years (looking from the y-axis) we find middle class and poor provinces more concentrated and nearly creating a unique mode.

If in 1961 there were provinces with an income half of the Spanish average, we can also found Madrid with an income 1.76 the Spanish average, Barcelona with 1.6, and Vizcaya with 1.53 of relative income. This indicated the huge inequalities existing among Spanish provinces during the sixties. However, these three provinces lost a lot during the decade, and they end up in 1971 with a relative income around 1.3. Girona and Guipúzcoa also lost positions and only Álava and Baleares grew slightly. Therefore, in 1971 rich provinces are grouped in a mode around 1.4.

However, this process of convergence changed in the following decade (1971-1981). Rich provinces such as Baleares, Barcelona and Madrid lost positions in the income distribution. It is interesting to see how the three Basque Provinces lost a lot during this period, especially Vizcaya and Guipúzcoa. This means that these provinces are approaching the middle class provinces in fact the estimated "rich mode" in 1981 is less prominent than in 1971 (less observations).

Figure 5.5. Contour plot of the Bivariate Kernel Density Estimates. Spanish Provinces. 1961-1997.




Estimates Information

i. Kernel: Gaussian

ii Computation Method: FFT

iii. Automatic Bandwidth Selector: Optimal Bandwidth

More interesting is to analyse the clear process of divergence found between 1971 and 1981. The density estimation shows that in 1981 there are two clear modes: one for the poor and another for the regions with income around the Spanish average. Now, with the bivariate kernel analysis we can see how in the second panel of figure A.3 provinces with relative per-capita GDP are above the 45-degree line, this means that this provinces grew from 1971 to 1981 and for this reason we cannot find the same pattern of convergence found in the previous period. Figure A.3 shows us that provinces such as Alicante, Asturias, Burgos, Castellón, Guadalajara, La Rioja, Las Palmas de Gran Canaria, Santa Cruz de Tenerife, Valladolid and Zaragoza, grew and separated from the poor mode.

Therefore, the 1971-1981 was not only a period of growth for middle class provinces, but also a period of polarization of income, because poor provinces, although growing, they are not catching up middle class provinces, they just group in a slightly higher level of income. This result was also found by Dolado et al. (1994).

The second panel of figure A.4 shows that during the 1981-1991 decade some middle class provinces are growing a lot (Burgos, Castellón, Guadalajara, La Rioja, Lleida, Navarra, Teruel, Valencia, Vizcaya and Zaragoza), but others are losing at the same time (Alicante, Asturias, Cantabria, Guipúzcoa, Huesca, Las Palmas de Gran Canaria, Palencia and Santa Cruz de Tenerife). This is what we have called before polarization of income or how the middle class vanishes. The middle class provinces, instead of remaining close to their value of income[22] they split and move in two opposite directions (see figure 5.2). At the same time, rich provinces are less numerous than before (1981): Álava, Baleares, Barcelona, Girona, Madrid and Tarragona (province that caught up rich provinces), therefore they are not enough to create one mode as happened before.

The prominent mode around 0.8 of the distribution estimated for 1991 is also formed for provinces below the average that grew. Over 27 regions below the average in 1981 (around 0.7) 20 grew and in 1991 reach an income of 0.8 of the Spanish average. Provinces such as Cáceres, Ciudad Real, Orense, Segovia, among others, grew and catch up the middle class provinces that were losing; creating the prominent mode in 1991. Hence, the overall picture for this period is a unimodal density estimate in 1991: the middle class mode in 1981 vanishes, making the mode below the average higher (meaning more observations concentrated around that point of the distribution) and forming a not clear mode above the average of income per capita (there is only a bump).

The final, and very interesting, period studied (1991-1997) shows that another mode is clearly coming (around 1.2), proving polarization and divergence of income. The bivariate analysis sheds more light into this issue.

If in 1991 we could observe six very rich provinces, in 1997 we find only three:

[22] That could be their steady state value.

Álava, Baleares and Girona (around 1.35), and four more group around 1.23: Barcelona, Madrid, Navarra (that before was a middle class province but during this period has catch up rich provinces) and Tarragona. These second group, together with provinces that grew a lot from the vanished middle class (Burgos, Castellón, Lleida, Guipúzcoa, Huesca, La Rioja, Teruel, Vizcaya, Valladolid and Zaragoza), are creating a new mode[23]. Hence, these provinces have grown enough to join the rich provinces (that at the same time have lost positions) and to start to create a new mode around 1.2. This clearly implies divergence of some provinces from the rest[24]. The divergence and polarization patterns are also obvious if we look at the bivariate kernel density for province below the average. They are, all of them, very close to the 45-degree line. It means that a high percentage of the Spanish provinces (64%) have remained in the same position for the last period analysed. In other words, they have found their *basin of attraction* (Quah (1997)) at a low-income level (0.8 of the Spanish average). Therefore, many provinces are grouped around a low level of relative income, while others provinces are growing and separating in a higher mode.

## 6 Conclusions

Some important conclusions, for the policy maker, can be drawn from the above analysis although the complexity of the convergence issue among the Spanish provinces. However, the non-parametric approach used in this paper has shed some light into this issue.

First, during the sixties there was a period of convergence between provinces below the Spanish average and middle class provinces (around the average). At the same time, rich provinces were loosing positions but creating a distant and very significant mode. This result completes previous studies that found convergence during this period; we can conclude that the convergence process observed during the sixties was mainly caused by middle class and poor provinces converging. In others words, we can speak about both convergence and clustering dynamics during this period.

The seventies was a period where the convergence process stopped. First, some middle class provinces grew and separate from poor provinces that did not grow enough to joint them. Second, rich regions keep loosing position in the income distribution, and some of them approach the middle class mode.

During the eighties, the middle class mode estimated at the beginning of the period (1981) vanished. In one hand, some middle class provinces lost positions and joint the poor provinces that grew and grouped in a higher mode. In the other hand, some middle class

[23] It is worth mentioning that all the provinces that are creating this new mode are in the north of Spain.

[24] We can also speak about divergence because the tendency of this new mode seems to separate more from the mode below the average, especially if we compare 1991 and 1997 density estimates. “The bump” in 1991 was around 1.1 and the coming mode in 1997 is around 1.2.

provinces grew enough to approach the rich provinces (grouped in a much lower mode than in 1961). These results have also found in others works dealing with the convergence issue in the Spanish case.

The final and interesting period, the nineties, shows us how the main mode, below the average, remained unchanged, and how provinces from the vanished middle class have finally catch up rich regions, creating a new mode around 1.2 of the income distribution. Therefore, this implies not only a process of polarization of income: we can find to separated groups of provinces, one below the average (around 0.8) and another above the Spanish average (around 1.2), but also a process of starting divergence: the two modes seem to start to separate. In this new mode we can find provinces located, basically, in the north of Spain, implying that maybe the pattern of polarization of income observed during the nineties is separating north from south.

The specific characteristics of the evolution through time of the Spanish provinces makes the topic quite interesting for further research to determine the main causes of the evolution pattern observed among Spanish provinces.

# 7 Appendix

Table A.1. Spanish Provinces

| | | | |
|---|---|---|---|
| 1. Álava | 14. Castellón | 27. Lleida | 40. Segovia |
| 2. Albacete | 15. Ciudad Real | 28. Lugo | 41. Sevilla |
| 3. Alicante | 16. Córdoba | 29. Madrid | 42. Soria |
| 4. Almería | 17. Coruña (A) | 30. Málaga | 43. Tarragona |
| 5. Asturias | 18. Cuenca | 31. Murcia | 44 Teruel |
| 6. Ávila | 19. Girona | 32. Navarra | 45. Toledo |
| 7. Badajoz | 20. Granada | 33. Orense | 46. Valencia |
| 8. Baleares | 21. Guadalajara | 34. Palencia | 47. Valladolid |
| 9. Barcelona | 22. Guipúzcoa | 35. Palmas (Las) | 48. Vizcaya |
| 10. Burgos | 23. Huelva | 36. Pontevedra | 49. Zamora |
| 11. Cáceres | 24. Huesca | 37. Rioja (La) | 50. Zaragoza |
| 12. Cádiz | 25. Jaén | 38. Salamanca | 51. Ceuta |
| 13. Cantabria | 26. León | 39. Sta. C. Tenerife | 52. Melilla |

Figure A.1. Kernel Density Estimates (Gaussian) of the Relative per-capita GDP with the Least-Squares Cross-Validation Bandwidth for the Spanish Provinces ($h$ = 0.0627815). 1961 – 1997.

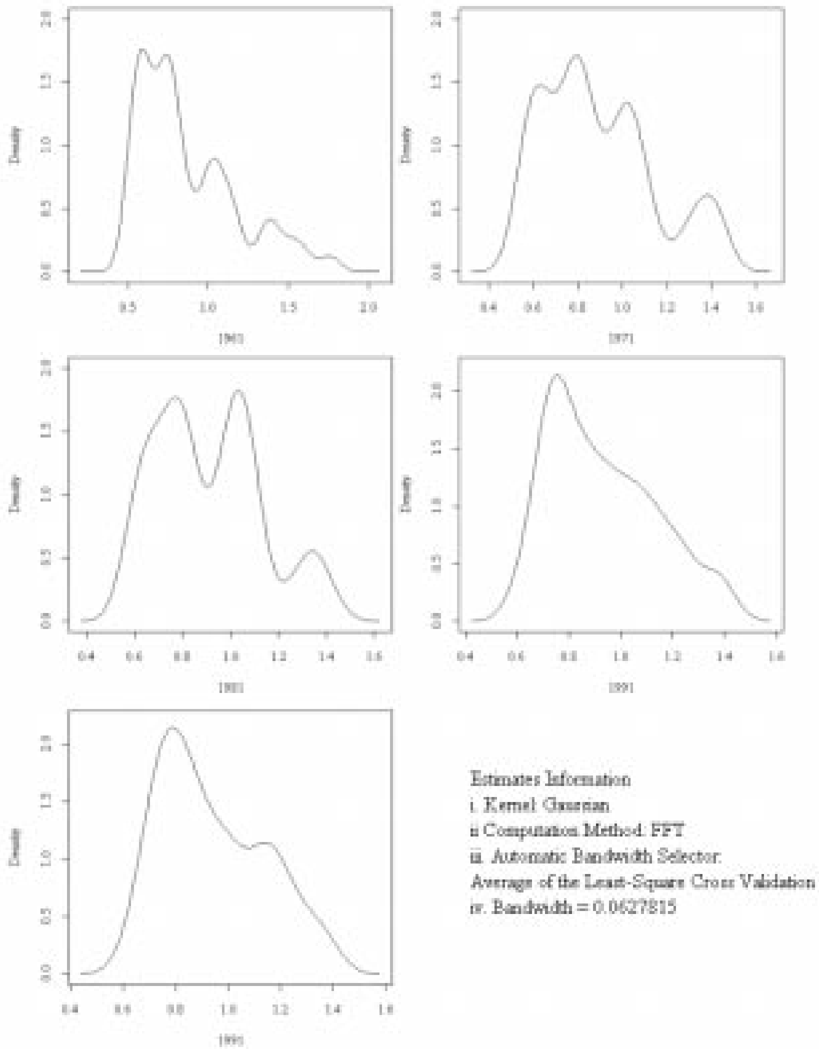


Figure A.2 Bivariate Kernel Density Estimates with provinces' names (1961-1971).

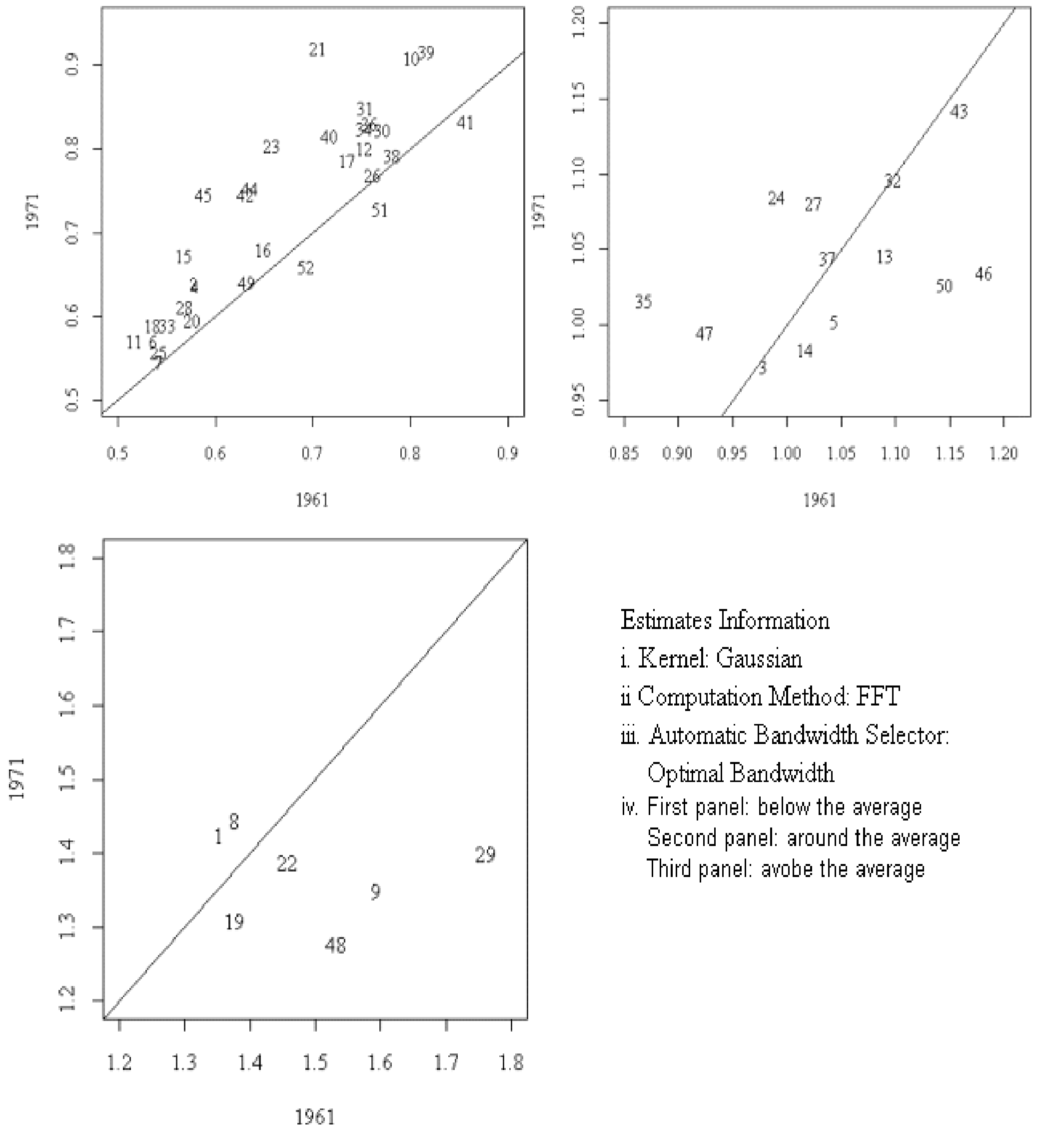


Figure A.3. Bivariate Kernel Density Estimates with provinces' names (1971-1981).

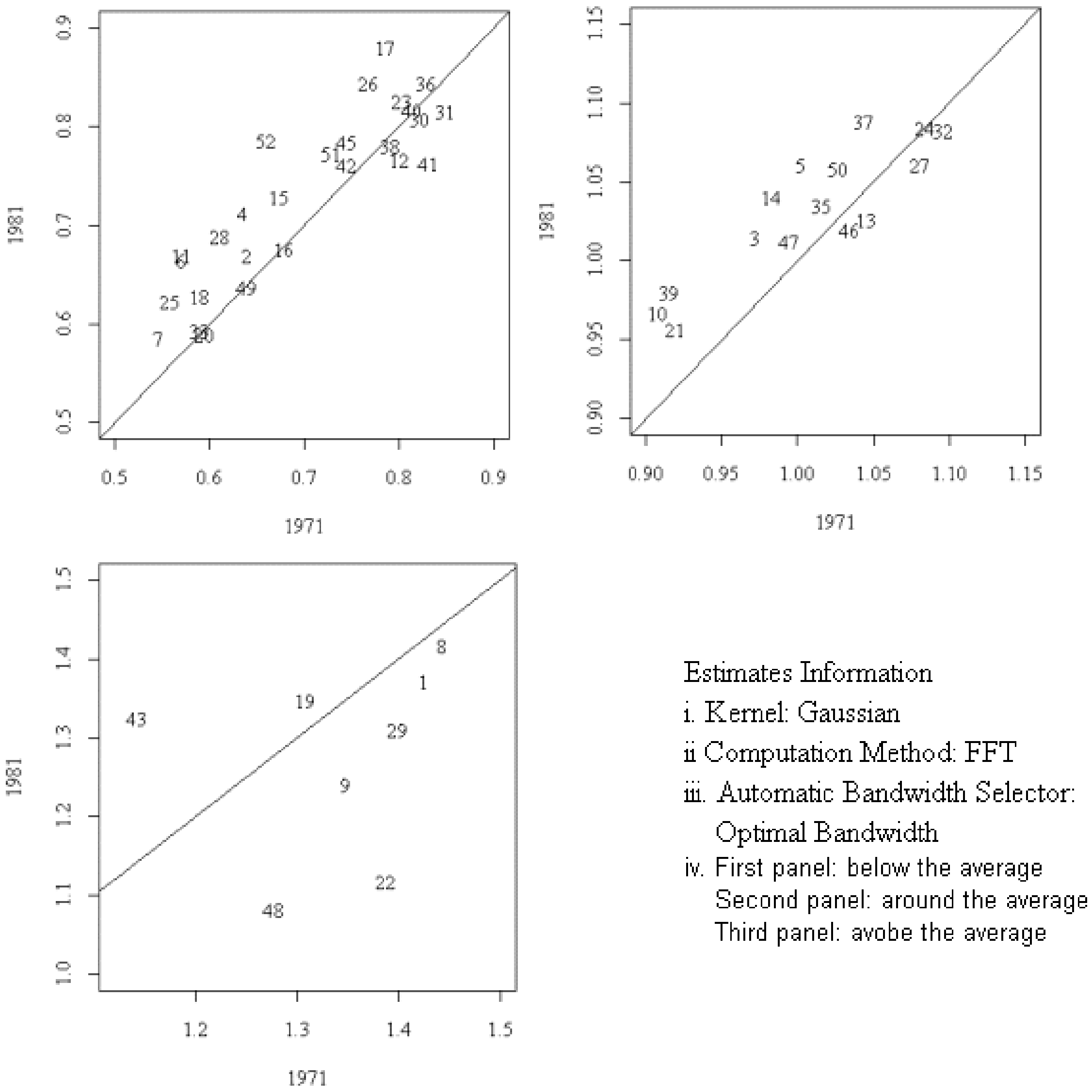


Figure A.4. Bivariate Kernel Density Estimates with provinces' names (1981-1991).

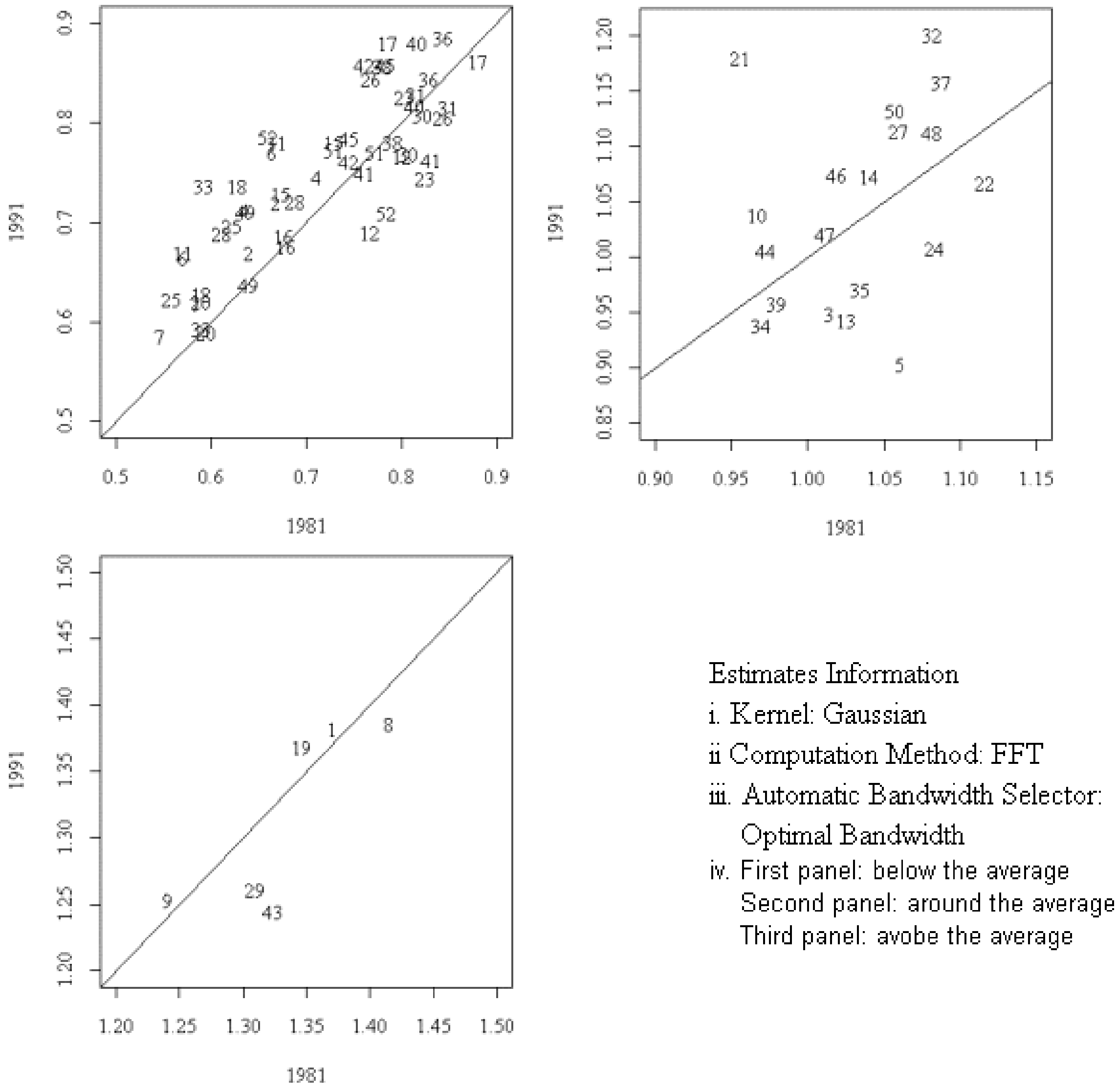
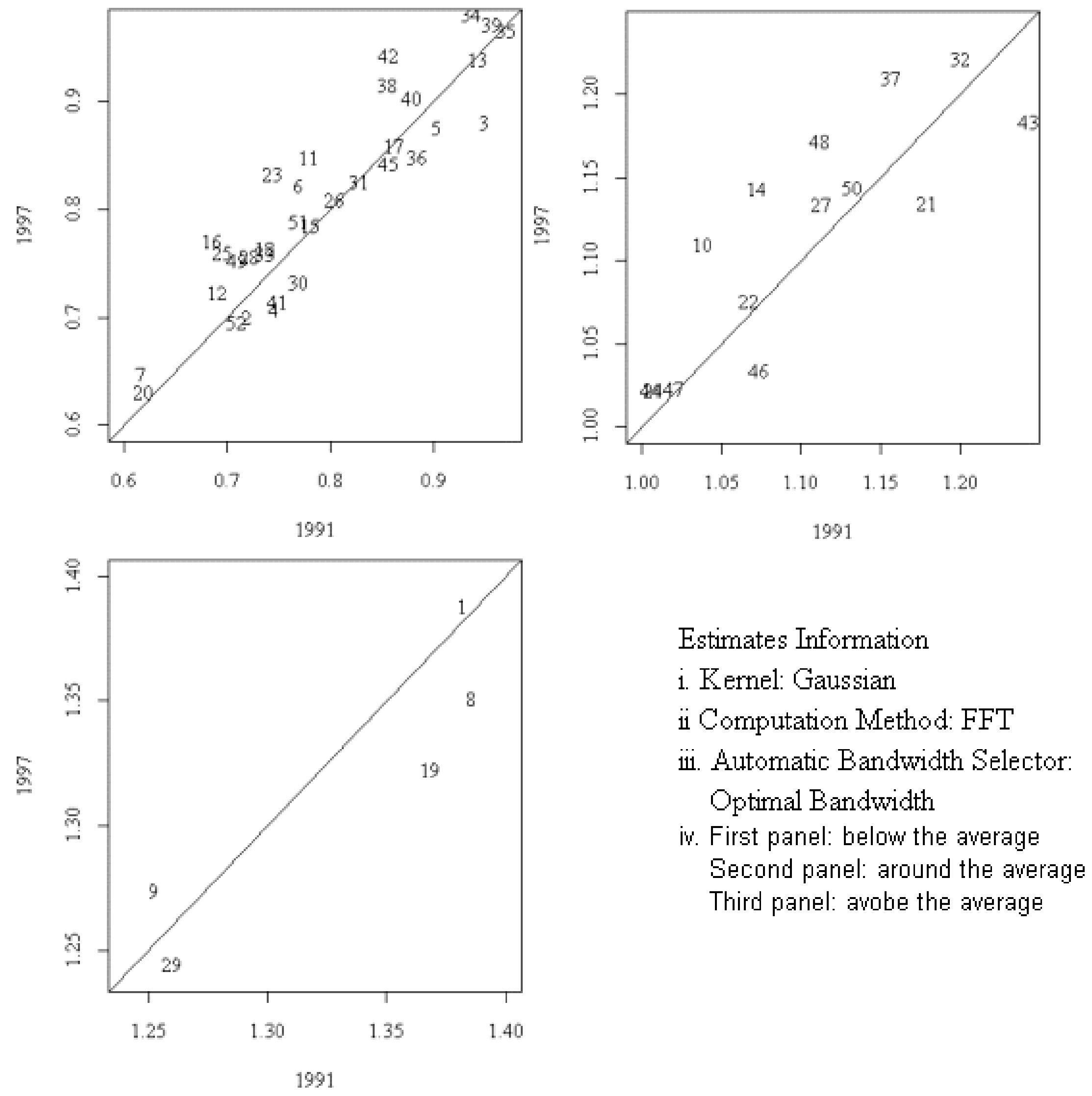


Figure A.5. Bivariate Kernel Density Estimates with provinces' names (1991-1997).



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