TAX REFORMS AND INEQUALITY:
THEORETICAL AND EMPIRICAL IMPLICATIONS

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Abstract: In this paper we examine the effect of tax policy on the relationship between inequality and growth in a two-sector non-scale model. With non-scale models, the long-run equilibrium growth rate is determined by technological parameters and it is independent of macroeconomic policy instruments. However, this fact does not imply that fiscal policy is unimportant for long-run economic performance. It indeed has important effects on the different levels of key economic variables such as per capita stock of capital and output. Hence, although the economy grows at the same rate across steady states, the bases for economic growth may be different.

The model has three essential features. First, we explicitly model skill accumulation, second, we introduce government finance into the production function, and we introduce an income tax to mirror the fiscal events of the 1980’s and 1990’s in the US. The fact that the non-scale model is associated with higher order dynamics enables it to replicate the distinctly non-linear nature of inequality in the US with relative ease. The results derived in this paper attract attention to the fact that the non-scale growth model does not only fit the US data well for the long-run (Jones, 1995b) but also that it possesses unique abilities in explaining short term fluctuations of the economy. It is shown that during transition the response of the relative simulated wage to changes in the tax code is rather non-monotonic, quite in accordance to the US inequality pattern in the 1980’s and early 1990’s.

More specifically, we have analyzed in detail the dynamics following the simulation of an isolated tax decrease and an isolated tax increase. So, after a tax decrease the skill premium follows a lower trajectory than the one it would follow without a tax decrease. Hence we are able to reduce inequality for several periods after the fiscal shock. On the contrary, following a tax increase, the evolution of the skill premium remains above the trajectory carried on by the skill premium under a situation with no tax increase. Consequently, a tax increase would imply a higher level of inequality in the economy.

Key words: human capital, inequality, tax reform.
JEL Classification: J31, O40
**Resum:** En aquest article s’examina l’efecte de la política impositiva en la relació entre desigualtat i creixement en el marc d’un model non-scale amb dos sectors. Amb models del tipus non-scale, l’equilibri de llarg termini ve determinat pels paràmetres tecnològics i és independent dels instruments de política macroeconòmica. Ara bé, aquesta situació no implica que la política fiscal no intervingui en l’activitat econòmica en el llarg termini. De fet, té importants efectes en els nivells de variables econòmiques tals com l’estoc de capital i la producció. Així doncs, tot i que l’economia creix a la mateixa taxa en els diferents estats estacionaris, les bases pel creixement econòmic poden diferir.

El model té tres aspectes essencials que el caracteritzen. En primer lloc, hem modelat l’acumulació d’habilitats; en segon, hem introduït el govern a la funció de producció; finalment, hem incorporat un impost sobre la renda per tal de reproduir els canvis fiscals que es produiren als Estats Units durant els anys vuitanta i noranta. El fet que els models non-scale es trobin associats amb dinàmiques d’ordre elevat ens permet replicar la naturalesa de la desigualtat americana de manera força acurada. Així, els resultats obtinguts en aquest article ens mostren com el nostre model no només capta l’evolució en el llarg termini (Jones, 1995b), sinó que també és capaç d’explicar les fluctuacions econòmiques en el curt termini. De fet, es mostra com durant la transició, la resposta del *skill premium* simulat envers els canvis impositius és no-monotònica, seguint el patró de la desigualtat americana dels anys vuitanta i noranta.

Més concretament, hem analitzat amb un cert detall la dinàmica fruit de la simulació d’un increment i d’una reducció impositives aïllades, tot i comprovant que després d’una davallada impositiva, el *skill premium* segueix una trajectòria inferior a la seguida sense el decrement impositiu. Vist així, es pot dir que la desigualtat minva durant els períodes que segueixen el canvi fiscal. Per altra banda, després d’un increment impositiu, el *skill premium* roman per sobre de la trajectòria prèvia a l’augment. Per tant, un increment impositiu implicaria un nivell de desigualtat superior a l’existent fins aleshores a l’economia.
1. Introduction

During the last decade, literature on tax reforms has centered on their macroeconomic growth effects. Even though empirical studies come up with low values for the influence of taxes on growth, theory has shown that small tax changes can have a large cumulative effect on long-run growth. Studies on taxation show mainly how higher taxes tend to discourage investment rates (Auerbach and Hassett, 1991; Cummins et al., 1994, 1996) as well as labor supply of individuals (Hausman, 1985; Triest, 1990; Mariger, 1995; Eissa, 1996) and productivity growth, the latter one by attenuating R&D. Some findings show that working hours and labor force participation are only mildly responsive to tax changes. On the other hand, in order to have lower taxes, some authors advise of the need to keep the economy as close as possible to its full employment potential. Lately, some authors have used endogenous growth models to simulate the effects of tax reforms on economic growth (Auerbach, 1996; Gale, 1996), finding that a decrease in the distorting effects of the current tax structure may lead to a permanent increase in economic growth mainly by enhancing national saving rates, thereby increasing the investment rate. In any case, when changing taxes, literature puts some emphasis on analyzing the transitional cost distribution, which could be crucial for policymakers.

Moreover, one of the interesting points to analyze is the welfare impact of taxes. Actually, the welfare impact of unanticipated tax changes is shown to depend on the mechanism underlying the production of knowledge, as well as the complementarity between knowledge and physical capital. In general, the growth effects stemming from policy actions do not impinge on all sectors of the economy equally. Some groups benefit more than others do, causing an inevitable change in the income distribution and income inequality. Studies on how changes in taxes and transfers tend to reinforce market trends in inequality and poverty have been undertaken throughout the literature. Taxes may increase growth if they finance public services, buy may decrease growth when used to redistribute income between classes and try to decrease inequality, as said by Chang (1998). Since inequality induces more redistribution, Perotti (1993), Persson and Tabellini (1994) or Benabou (1996) say that it would lead to lower economic growth through higher distortion and lower investment.
Several papers on political economy of growth have tried to reconcile this idea by introducing the decision on taxes depending upon the choice of the median voter.

In this paper we examine the effect of tax policy on the relationship between inequality and growth in a two-sector non-scale model. The model has three essential features. First, we explicitly model skill accumulation, second, we introduce government finance into the production function, and we introduce an income tax to mirror the fiscal events of the 1980’s mainly. The fact that the non-scale model is associated with higher order dynamics enables it to replicate the distinctly non-linear nature of inequality in the US with relative ease. ¹

The results derived in the paper draw attention to the fact that the non-scale growth model not only fits the US data well for the long run (as first shown by Jones, 1995), but also that our parameterization of the model is able to track important aspects of the short-run evolution of the economy. Specifically, we show that the initial phase of the transitional adjustment of the skill premium in response to changes in the tax code is inherently nonlinear, a feature that closely follows the US skill premium experienced in the 1980s and early 1990’s.

Our simulations also relate well to the literature on the macroeconomic effects of tax reforms. Studies examining tax reforms show that higher taxes tend to discourage investment rates (Auerbach and Hassett, 1991; Cummins et al., 1994, 1996) and that income taxes seems to increase wealth inequality, compared to the distributional neutrality of a consumption tax, as some authors have pointed out (Perroni, 1995; Felder, 1997). The empirical studies show that the impact of tax changes depends on the mechanism underlying the production of knowledge, as well as the complementarity between knowledge and physical capital, a feature also shared by our model.

We choose as our benchmark the dramatic changes in the US tax code in the early 1980s, and the ensuing, significant changes in the skill premium. Any growth model that seeks to

¹ Cross-country evidence on the effect of growth on inequality and on that of inequality on growth is inconclusive (see Anand and Kanbur, 1993, Deininger and Squire, 1998, and Forbes, 2000), while historical
speak to the discussion on policy, taxation, and inequality must be able to replicate key events in the data to render its implications relevant. A key feature of the data in the 1980s was the non-monotonic nature of the response of the skill premium to changes in the tax code. This is a transition that is difficult for a conventional one-sector growth model or a two-sector endogenous growth model to explain. This is because in either case the transitional adjustment path is a one-dimensional locus and therefore can generate only monotonic adjustments. In our simulations, using actual tax rates for the 1980s, we find that the model replicates both key steady-state variables of the economy as well as tracking the short-run non-linear transition path of the skill premium fairly closely.

The previous literature on the topic is separated into two strands: the growth and inequality literature and the growth and taxation models. Existing explanations for the observed patterns of inequality have been focused on sectoral mobility2 (from agriculture to industry), capital market imperfections3, socio political (in)stability and redistribution4, or skill-biased technical change5. In contrast, the tax reform literature centered on macroeconomic variables, such as employment and accumulation. These studies show how higher taxes discourage investment rates (e.g., Auerbach and Hassett, 1991) or labor supply (e.g.; Eissa, 1996). Models that actually correlate changes in the tax structure with changes in growth (e.g., Auerbach, 1996), focus on the distorting effects of current tax structure and not on the productive services that the government might finance – to the benefit of economic growth.

The structure of the paper is as follows. Section 2 presents the non-scale model with endogenous skill formation. Section 3 describes the general equilibrium and stationary states,

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2 See Kuznets (1953).
3 See Galor and Zeira (1993).
and section 4 outlines how Tax Reforms in the 1980’s correlate both with the predictions of
the model and the evolution of real inequality. Section 5 concludes.

2. A Two-sector non-scale economy

We begin by outlining the structure of a two-sector non-scale model that features exogenous
population growth and endogenously growing physical and human capital. We focus on a
decentralized economy in which population, \( N \), is assumed to grow at the steady rate
\( \frac{\dot{N}}{N} = n \). The government plays a simple role. It taxes final income and uses the proceeds
to finance the purchases of a productive input that increases productivity in the final output
sector.

2.1. Individual agents

Individual \( i \) produces final output, \( Y_i \), and gross additions to human capital, \( J_i \), in separate
sectors, each subject to externalities according to the Cobb-Douglas production functions:

\[
Y_i = \alpha_i \theta H_i^{\psi} [\phi K_i]^{\rho} K^{\omega} H^{\sigma} G^{\tau} \\
J_i = \alpha_i (1 - \theta) H_i^{\psi} [1 - \psi] K_i^{\rho} [(1 - \phi) K_i]^{\rho} K^{\omega} H^{\sigma}
\]

(1a) \hspace{1cm} (1b)

Each individual is endowed with a unit of labor, \( \theta \) of which is allocated to the production of
new output and \( (1-\theta) \) to the production of new human capital. In addition, he allocates a
fraction \( \psi \) of his current human capital, \( H_i \), to the production of final output, and the
balance \( (1-\psi) \) to the accumulation of further human capital. Likewise, he allocates a
fraction \( \phi \) of his physical capital, \( K_i \), to the production of final output and the rest \( (1-\phi) \) to
the human capital sector. The production of new output is subject to positive externalities
arising from the aggregate stock of physical capital, \( K \), human capital, \( H \), as well as
government spending, \( G \), on a productive input, which is in the form of a public good.
Government services are modeled as being non-rival and non-excludable, constituting a pure
public good in the sense of Samuelson (1954). The production of new human capital is subject to similar externalities from the aggregate stocks of physical and human capital. The constants $\alpha$, $a_j$ represent exogenous technological shift factors to the production functions, while $b_i$, $c_i$, $e_i$, $f_i$ are the respective productive elasticities.

We assume that all agents in the economy are identical so that aggregate and individual quantities are related by

$$Y = NY_i, \quad K = NK_i, \quad H = NH_i$$

(2)

Government expenditure is proportional to the aggregate output, in accordance with:

$$G = gY = gNY_i$$

(3)

and substituting (3) into (1a) we can rewrite the production function as:

$$Y_i = a_F \theta^{b_Y (1-c_y)} H_i^{b_H (1-c_H)} K_i^{b_K (1-c_K)} H_i^{c_K, (1-c_K)} K_i^{c_K, (1-c_K)} N_i^{c_H, (1-c_H)}$$

(1a’)

where $a_F \equiv (\alpha_F g^{c_F})^{1/(1-c_F)}$.

The rate at which the individual accumulates the two types of capital is described by:

$$\dot{K}_i = (1 - \tau_y)Y_i - C_i - T_i - (\delta_K + n)K_i$$

(4a)

$$\dot{H}_i = J_i - (\delta_H + n)H_i$$

(4b)

According to (4a), income is taxed at the rate $\tau_y$, and in addition we allow for lump-sum taxation, $T_i$. The individual’s net rate of accumulation of both types of capital must allow for the rate of the respective rate of depreciation $\delta_K$, $\delta_H$, and the need to equip the growing population with both types of capital.
The individual agent is assumed to maximize the intertemporal utility function:

$$\frac{1}{1-\gamma} \int_0^\infty (C_t)^{-\gamma} e^{-\rho t} dt \quad \rho > 0; \quad \gamma > 0$$

subject to the production functions (1a)-(1b), the accumulation constraints, (4a)-(4b), and the usual initial conditions. His decision variables are: (i) the rate of consumption, $C_i$; (ii) the sectoral allocation of labor, $\theta$, human capital, $\psi$, and physical capital, $\phi$; and (iii) the rates of accumulation of physical and human capital. The optimality conditions to this problem can be summarized as follows:

$$C_t^{-\gamma} = v_i$$

Equation (6a) equates the marginal utility of consumption to the shadow value of capital. Equations (6b), (6c) and (6d) determine the sectoral allocations of labor, human capital, and physical capital such that their respective after-tax marginal products are equated across sectors:

$$v_i b_n (1-\tau_n) \frac{Y_n}{\theta} = \mu_n e_n \frac{J_n}{1-\theta}$$

$$v_i b_h (1-\tau_h) \frac{Y_h}{\psi} = \mu_h e_h \frac{J_h}{1-\psi}$$

$$v_i b_k (1-\tau_k) \frac{Y_k}{\phi} = \mu_k e_k \frac{J_k}{1-\phi}$$

Together with the transversality conditions:

$$\lim_{t \to \infty} v_i K_i e^{-\rho t} = \lim_{t \to \infty} \mu_i H_i e^{-\rho t} = 0$$

where $v_i$, $\mu_i$ are the respective shadow values of physical capital and human capital.

Equation (6a) equates the marginal utility of consumption to the shadow value of capital. Equations (6b), (6c) and (6d) determine the sectoral allocations of labor, human capital, and physical capital such that their respective after-tax marginal products are equated across sectors.
sectors. Equation (6e) equates the marginal return to investing in an additional unit of physical capital to the return on consumption, both measured in terms of units of final output. Analogously, (6f) equates the marginal return to human capital to the return on consumption, both expressed in units of human capital. In both cases the return to the asset reflects the fact that the additional unit will be allocated.

2.2. The aggregate economy

To derive the behavior of the aggregate economy we first sum (1a’) and (1b) over the N individuals in the economy. We may express the resulting quantities in terms of the aggregates:

\[ Y = a_F \phi^x \psi^k K^{\sigma_K} H^{\sigma_H} N^{\sigma_N} \] (7a)

\[ J = a_J (1 - \theta)^{\psi} (1 - \phi)^{\psi} \left(1 - \psi\right)^{\gamma} K^{\eta_K} H^{\eta_H} N^{\eta_N} \] (7b)

where:

\[ s_N = \frac{b_N}{1 - c_G}; \quad s_K = \frac{b_K}{1 - c_G}; \quad s_H = \frac{b_H}{1 - c_G} \]

\[ \sigma_K = \frac{b_K + c_K}{1 - c_G}; \quad \sigma_H = \frac{b_H + c_H}{1 - c_G}; \quad \sigma_N = \frac{1 - b_K}{1 - c_G} - \frac{b_H}{1 - c_G} - \frac{b_K}{1 - c_G} \]

\[ \eta_K = e_K + f_K; \quad \eta_H = e_H + f_H; \quad \eta_N = 1 - e_H - e_K; \]

Next, we introduce the government and consider the aggregate accumulation equations. We will assume that the government finances its expenditure in accordance with a balanced budget, which aggregated over N individuals, can be expressed as:

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6This technology is somewhat more general than Jones (1995a), who specifies \( \sigma_K = \sigma_N = 1 - \sigma_K = \sigma \). Eicher and Turnovsky (1999) discuss the balanced-growth characteristics of a general two-sector technology, in which physical capital enters the production function of new knowledge. The Romer model corresponds to the case \( \sigma_K = \sigma_N = 1 - \sigma_K, \eta_K = \eta_N = 1 \).
\[ \tau_i N Y_i + NT_i = gNY_i \]  

(8)

or, in terms of the aggregate quantities:

\[ \tau Y + T = gY \]  

(8’)

Summing (4a) and (4b) over the individuals in the economy, and applying the government budget constraint (8’), the aggregate rates of capital accumulation can be expressed as:

\[ \dot{K} = (1-g)Y - C - \delta_k K \]  

(9a)

\[ \dot{H} = J - \delta_H H \]  

(9b)

where \( Y, J \) are defined in (7a), (7b), above.

2.3. Balanced growth equilibrium

Before describing the dynamics, we characterize the balanced growth equilibrium. This is defined to be a growth path along which all variables grow at constant, but possibly different, rates. In accordance with the stylized empirical facts (Romer 1986), we assume that the output/capital ratio, \( Y/K \), is constant. A key feature of the non-scale model is that the equilibrium percentage growth rates of human and physical capital, \( \hat{H} \) and \( \hat{K} \), respectively are determined entirely by production conditions. Taking the differentials of the production functions (9a) and (9b), and solving, we obtain:

\[ \dot{H} = \left[ \frac{\eta \nu (1 - \sigma_k) + \sigma_H \eta_H}{(1 - \eta_H)(1 - \sigma_k) - \sigma_H \eta_H} \right] n = \beta_H n \]  

(10a)

\[ \dot{K} = \dot{Y} = C = \left[ \frac{\sigma_h (1 - \eta_H) + \sigma_H \eta_H}{(1 - \eta_H)(1 - \sigma_k) - \sigma_H \eta_H} \right] n = \beta_K n \]  

(10b)
and thus the per capita growth rate of output (capital) is:

\[ \hat{y} - n = \frac{\left[ (1 - \eta_H) \left( \sigma_H + \sigma_K - 1 \right) + \sigma_H \left( \eta_H + \eta_K - 1 \right) \right] n}{(1 - \eta_H)(1 - \sigma_K) - \sigma_H \eta_K} \]  

(10c)

It is evident from (10a) and (10b) that the magnitudes of the relative sectoral growth rates depend upon the assumed production elasticities in conjunction with the population growth rate. Equation (10c) implies that countries converge to identical output per capita growth rates if either their production technologies are identical, or their production functions exhibit constant returns to scale. If production technologies differ across countries, growth rates exhibit conditional convergence. Jones (1995a) specifies output to be constant returns to scale in physical capital and knowledge-adjusted labor, \( AN \). The striking feature of the equilibrium growth rate in the Jones model is that per capita consumption, per capita output and capital, and technology all grow at a common rate determined by: (i) the growth rate of labor, and (ii) the elasticities of labor and knowledge in the knowledge-producing sector alone. Any characteristic of the final output sector is irrelevant. This contrasts with the determination of per capita output (capital) growth rates in (10c), which in general depend upon the technological parameters in both sectors. It contrasts even more sharply with the general production function considered by Eicher and Turnovsky (1999), in which case the growth rate of technology also depends upon output elasticities.

The fact that (10c) implies that the per capita growth rate of output in general depends upon the population growth rate is somewhat controversial. Jones (1995a) provides an extensive discussion of the relevance of this class of growth model, in light of the mixed evidence on the correlation between population growth and output growth (see e.g. Barro and Sala-i-Martín 1995). He emphasizes that this aspect of the model, which is a direct consequence of non-constant returns to scale, relies foremost on the creation of new technology, and on the growth of effective researchers. A zero effect of population on output growth is consistent with either (i) increasing returns to scale in one sector, offset by appropriate decreasing returns to scale in the other; or (ii) constant returns to scale in both sectors. In particular, scale models of growth attain non-zero growth in the absence of population growth by imposing constant returns to scale in the accumulated factors, \( K \) and \( H \). Thus AK or Lucas-
Romer type growth models require the knife-edge assumption that
\[ \eta_H + \eta_K = 1, \text{ and } \sigma_H + \sigma_K = 1. \]

Finally, the possibly differential equilibrium growth rates of physical capital and knowledge are reflected in the growth rates of their respective shadow values, \( \nu, \mu \). Using the optimality conditions (6), they can be shown to grow in accordance with (where since agents are identical we drop the subscript \( i \)):

\[ \dot{\nu} - \dot{\mu} = (\beta_H - \beta_K)n \quad (10d) \]

3. Dynamics of a two-sector model

To derive the equilibrium dynamics about the balanced growth path we define the following stationary variables:

\[ y \equiv Y/N^{\beta_H}; \quad k \equiv K/N^{\beta_K}; \quad c \equiv C/N^{\beta_K}; \quad h \equiv H/N^{\beta_H}; \quad j \equiv J/N^{\beta_H}; \quad q \equiv \nu N^{\beta_H - \beta_K}. \]

For convenience, we shall refer to \( y, k, c, \) and \( h \) as scale-adjusted quantities.\(^7\) This allows us to rewrite scale-adjusted output and human capital as:

\[ y = a y'^{\phi_H} \psi_H^{\eta_H} h^{\sigma_H} k^{\eta_K} \quad (11a) \]
\[ j = a j(1 - \theta)^{\psi_H} (1 - \psi)^{\eta_H} (1 - \phi)^{\eta_K} h^{\eta_H} k^{\eta_K} \quad (11b) \]

It is important to specify the dynamics in terms of scale-adjusted variables, since being stationary, long-run changes reflect the accumulated effects of policy changes.

The optimality conditions then enable the dynamics to be expressed in terms of these scale-adjusted variables, as follows. First, substituting (11a) and (11b) into the labor allocation

\(^7\)Under constant returns to scale these scale-adjusted quantities are just regular per capita quantities.
condition, (6b), the human capital allocation condition, (6c), and the physical capital allocation condition, (6d), yields the three relationships:

\[
(1 - \tau_y) a_j q b_n \theta^u \psi^a \phi^s h^u k^e = a_j e_n (1 - \theta)^u (1 - \psi)^u (1 - \phi)^u h^u k^e \tag{12a}
\]

\[
(1 - \tau_y) a_j q b_n \theta^u \psi^a \phi^s h^u k^{e-1} = a_j e_{n-1} (1 - \theta)^u (1 - \psi)^u (1 - \phi)^u h^{e-1} k^e \tag{12b}
\]

\[
(1 - \tau_y) a_j q b_k \theta^u \psi^a \phi^s h^u k^{e-1} = a_j e_{k-1} (1 - \theta)^u (1 - \psi)^u (1 - \phi)^u h^{e-1} k^{e-1} \tag{12c}
\]

In principle, we can solve these three relationships for the allocation of labor, human capital, and physical capital across sectors:

\[
\theta = \theta(q(1 - \tau_y), h, k); \quad \partial \theta / \partial q > 0, \quad \text{sgn}(\partial \theta / \partial h) = \text{sgn}(\sigma_h - \eta_h), \quad \text{sgn}(\partial \theta / \partial k) = \text{sgn}(\sigma_k - \eta_k) \tag{13a}
\]

\[
\psi = \psi(q(1 - \tau_y), h, k); \quad \partial \psi / \partial q > 0, \quad \text{sgn}(\partial \psi / \partial h) = \text{sgn}(\sigma_h - \eta_h), \quad \text{sgn}(\partial \psi / \partial k) = \text{sgn}(\sigma_k - \eta_k) \tag{13b}
\]

\[
\phi = \phi(q(1 - \tau_y), h, k); \quad \partial \phi / \partial q > 0, \quad \text{sgn}(\partial \phi / \partial h) = \text{sgn}(\sigma_h - \eta_h), \quad \text{sgn}(\partial \phi / \partial k) = \text{sgn}(\sigma_k - \eta_k) \tag{13c}
\]

Intuitively, an increase in the relative value of physical capital, \(q\), attracts resources to the output (capital-producing) sector; labor, human capital, and physical capital therefore move from human capital production to final output production. An increase in the stock of either form of capital raises the productivity of both sectors in proportion to an amount that depends upon the respective productive elasticity. Resources will therefore move toward the sector in which that form of capital has the greater production elasticity (is more productive).

Using the optimality conditions, the dynamics of the system can be expressed in terms of the redefined stationary variables by:

\[
\dot{k} = k \left[ (1 - g) a_j \theta^u \phi^s \psi^a h^u k^{e-1} - \frac{c}{k} - \delta - \beta n \right] \tag{14a}
\]

\[
\dot{h} = h \left[ a_j (1 - \theta)^u (1 - \phi)^u h^{u-1} k^{e-1} - \delta - \beta n \right] \tag{14b}
\]
where \( \theta(\cdot), \psi(\cdot), \phi(\cdot) \) are determined by (13). To the extent that we are interested in the per capita growth rates of physical and human capital, they are given by

\[
\dot{K}/K = \dot{k}/k + (\beta_K - 1)n; \quad \dot{H}/H = \dot{h}/h + (\beta_H - 1)n.
\]

The steady state to this system, denoted by "\( \sim \)" superscripts, can be summarized by:

\[
\frac{(1-g)\bar{y}}{\bar{k}} - \frac{\bar{c}}{\bar{k}} = \beta_K n + \delta_K \quad \text{(15a)}
\]

\[
\frac{\bar{J}}{\bar{h}} = \beta_H n + \delta_H \quad \text{(15b)}
\]

\[
\frac{\bar{J}}{\bar{h}} \left( \frac{e_H}{1-\psi} \right) - \delta_H - \beta_H n = (1-\tau_y) \frac{\bar{y}}{\bar{k}} \frac{b_K}{\phi} - \delta_K - \beta_K n \quad \text{(15c)}
\]

\[
(1-\tau_y) \frac{\bar{y}}{\bar{k}} \frac{b_K}{\phi} - \delta_K - \beta_K n = \rho + (1-\gamma)(1-\beta_K)n \quad \text{(15d)}
\]

together with the two production functions, (11a)-(11b), and the sectoral allocation conditions (12).

These nine equations determine the steady-state equilibrium in the following sequential manner. First, equation (15b) yields the gross equilibrium growth rate of human capital (knowledge), \( \frac{\bar{J}}{\bar{h}} = \frac{\dot{J}}{\dot{h}} + \dot{\bar{J}}/\dot{\bar{h}}, \) in terms of the returns to scale, \( \beta_H, \) and the rates of population growth and depreciation. Next, given \( \frac{\bar{J}}{\bar{h}}, \) equations (15c) and (15d) jointly determine the sectoral allocation of human capital, \( \bar{\psi}, \) such that the net rates of return to investing in physical capital and human capital are equalized and equal to the rate of return on
consumption. Having determined the human capital allocation, \( \tilde{\psi} \), dividing (12a) and (12c), respectively, by (12b) yields the corresponding sectoral allocation of labor, \( \tilde{\theta} \), and physical capital, \( \tilde{\phi} \). With the value for physical capital, \( \tilde{\phi} \), (15d) determines the output-capital ratio. Having obtained the output-capital ratio, (15a) determines the consumption-capital ratio consistent with the growth rate of capital necessary to equip the growing labor force and replace depreciation. Also, given \( \tilde{\theta}, \tilde{\psi}, \tilde{\phi}, \hat{y}/\tilde{k} \) and \( \hat{j}/\tilde{h} \), the scale-adjusted production functions together determine the stocks of physical capital, \( \tilde{k} \), and human capital, \( \tilde{h} \). Finally, having derived \( \tilde{\theta}, \tilde{\psi}, \tilde{\phi}, \tilde{h}, \tilde{k} \), any of the three sectoral allocation conditions determine the long-run relative shadow value equilibrium of the two assets, \( \tilde{q} \), as well as the final production value, \( \tilde{y} \).

A detailed characterization of the transitional dynamics of our model is provided in the Appendix. The dynamics characterizing our analysis are based on the linearization of the fourth order system (14). In order for the dynamics to describe a unique stable adjustment path, we require that the number of unstable roots equal the number of jump variables (2). Unfortunately, the system is too complex to yield intuitive formal stability conditions that ensure well-behaved saddle-point behavior. Hence, we shall assume that this condition is met, as indeed it is in all of our numerical simulations. For the purposes of this paper, the analysis of changes in tax regimes on inequality, using simulation analysis, the formal stability conditions are of only secondary importance.

3.1. The impact of public policy on inequality

Our concern is to analyze the dynamic response of the economy to changes in income tax rates. We seek to go beyond qualitative results and simulate variations in tax rates that reflect the significant policy events that occurred in the United States during the 1980s. To understand and interpret the adjustments in the economy in response to tax changes, it is useful to first derive the qualitative nature of the long-run equilibrium.
An immediate manifestation of the non-scale characteristic of the model as seen from equations (10a, 10b) is that the steady-state equilibrium growth rates, of both capital goods are independent of the tax rate $\tau_y$ [see Jones 1995, and Eicher and Turnovsky (1999)]. This is a major advantage of the model, since the evidence shows no lasting impact of tax policy on long run growth (Stokey and Rebelo 1991, Easterly and Rebele, 1993, Jones, 1995). The fact that long run growth is not affected by public policy does not rule out substantial effects of tax policy on economic variables in the short and intermediate term. The magnitude of such effects depends on the speed of adjustment in the economy. We will address this point further in our simulations.

Since taxes do not influence the long run growth the equilibrium sectoral asset allocations, $\bar{\varphi}$, $\psi_0$, and $\phi_0$, are also independent of the tax rate. From (15d) we know that a lower tax rate reduces only the output-capital ratio in the long run, which in turn leads to a lower consumption-capital ratio, in order for the equilibrium growth rate of final output to remain unchanged. The sectoral allocation may vary significantly during the transition, to impact the skill premium, as shown in our simulations.

The impact on the steady-state scale adjusted stocks of physical and human capital are, respectively:

$$\frac{\partial k}{\partial \tau_y} = \left(\frac{\eta_H - 1}{1 - \tau_y \left[ 1 - \sigma_K \right] \left[ 1 - \eta_H \right] - \sigma_H \eta_K} \right) < 0 \quad (16a)$$

$$\frac{\partial h}{\partial \tau_y} = -\left(\frac{\eta_K}{1 - \tau_y \left[ 1 - \sigma_K \right] \left[ 1 - \eta_H \right] - \sigma_H \eta_K} \right) < 0 \quad (16b)$$

Since one of our goals is to reproduce the real evolution of income inequality as measured by the skill premium, $w_R$, in order to get a manageable formulation coming out from our model, we have differentiated between the base wage as a reward to raw labor and the reward to skills. Hence,
\[ w_r = \frac{w_H}{w_N} = \frac{w_N + r_H}{w_N} = 1 + \frac{r_H}{w_N} = 1 + \frac{\partial Y}{\partial \psi H} \left( \frac{H}{\delta(\theta N)} \right) = 1 + \frac{b_H}{b_N} \theta N H > 1 \] (4.17)

where \( w_N \) is the marginal product of unskilled workers (raw labor) and \( r_H \), the marginal product of human capital, represents the returns to skills. Essentially, the skill premium is the base wage received for raw labor plus the marginal product derived from skills in the final goods sector, scaled by the return to raw labor.

The short-run adjustments in the skill premium operate through the short-run allocations of raw labor \( \theta \) and human capital \( \psi \), as well as the skill adjustment, \( h \). Indeed, in the short-run, a tax decrease will initially attract resources to the output producing sector. If raw labor is more sectorally mobile than human capital (i.e. \( \theta \) responds more intensely than \( \psi \)), then the tax cut will be associated with a sharp initial short-run decline in the skill premium, which becomes milder as time goes by before ultimately declining over time.

The differential adjustment in the long and short-run of the skill premium is novel to the literature. It requires a non-linear transition path that is particular to this strand of growth models. Previously the transition speed was constant throughout the entire adjustment, now not only the speed of adjustment of different variables may vary at different rates over time, but also the variable does not need to transition in a monotonic fashion throughout.

4. Response of Skill Premia to Tax Reforms

In this section we report the results of simulating the model. One key question of the growth literature is if the theoretical policy guidance is derived from models that are empirically relevant. Hence the purpose of the simulations in this section is twofold. First we would like to confirm that reasonable parameter values generate plausible steady-state values to key economic variables. This would confirm whether this class of models is capable of
generating growth rates of output and skills, as well as allocations of resources to the two sectors that are reasonable, given observed real world quantities.

Our second purpose is to gain insights into to the qualitative nature of the transition path followed by the economy in response to a tax shock. If our benchmark economy replicates transition paths that are similar to those observed in the data, our simulations can be taken as policy guidance. We will ask how well these transition paths implied by the model correlate with to actual observed changes in inequality following changes in the US tax code. As mentioned above, our benchmark is the dramatic change in the tax code in the US in the 1980’s.

It is important to note that the changes over time of the skill premium operate through the allocation of labor and skills, \( \theta \), \( \psi \), and specially through the evolution of human capital, \( h \). An increase in \( \theta \) (decrease in the number of raw labor employed in the production of knowledge) raises the skill premium, whereas an increase in \( \psi \) (decrease in the number of skills employed in the production of human capital) decreases it.

\textit{Table 1} reports the values we employ for our fundamental parameters. These values are generally consistent with those suggested by previous calibration exercises (Lucas, 1988; Jones, 1995a; Ortigueira and Santos, 1997), (to the extent that they appear in these models). In these first simulations, externalities are set to zero in both sectors, except for the ones coming from government spending, \( G \).

<table>
<thead>
<tr>
<th>Table 1. Benchmark parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production parameters</td>
</tr>
<tr>
<td>( \alpha_r = 2.1; \alpha_k = 2; \sigma_k = s_k = 0.4687; \sigma_k = s_k = 0.3723; \sigma_h = s_h = 0.2127; )</td>
</tr>
<tr>
<td>( \alpha_r = 1; \eta_k = e_s = 0.50; \eta_k = e_s = 0.20; \eta_k = e_s = 0.3; )</td>
</tr>
<tr>
<td>( \epsilon_w = 0.04; b_w = 0.45; b_k = 0.35; b_h = 0.2 )</td>
</tr>
<tr>
<td>Preference parameters</td>
</tr>
<tr>
<td>( \rho = 0.04, \gamma = 2.5 )</td>
</tr>
<tr>
<td>Depreciation and population parameters</td>
</tr>
<tr>
<td>( \delta_k = 0.05, \delta_h = 0.05, n = 0.015 )</td>
</tr>
<tr>
<td>Fiscal policy parameters</td>
</tr>
<tr>
<td>( g = 0.15, \tau_{r(1994)} = 11.654% )</td>
</tr>
</tbody>
</table>
Both production functions exhibit constant returns to scale in the private factors of production, although the output production function exhibits increasing returns to scale when we introduce the government externality. The rate of time preference is 0.04 and the intertemporal elasticity of substitution is 0.4. Both capital goods depreciate at 5% and the population growth rate is 1.5%. We assume that the government expenditure devoted to production is 15%.

The key policy variable we consider is the income tax rate and the experiment we analyze and try to replicate are the short-run effects of the accumulated tax cut that occurred in the United States between 1980-1995. One of the main drawbacks we face when analyzing taxation effects is the one that comes out from the absence of suitable measures of aggregate taxation. Following Mendoza et al. (1994) as an alternative suggested by Lucas (1990b) and Razin and Sadka (1993), we have used effective tax rates, which relate the effective overall tax burden to the main income sources. More specifically, the effective tax ratio on total household income, $\tau_y$, is calculated as in Mendoza et al. (1994) and Carey and Tchilinguirian (2000) as follows:

$$\tau_y = \frac{1100}{OSPUE + PEI + W}$$

where:

- **1100**: taxes on income, profit and capital gains of individuals or households;
- **OSPUE**: unincorporated surplus of private unincorporated enterprises;
- **PEI**: households’ property and entrepreneurial income;
- **W**: wages and salaries of dependent employment.

Our benchmark income tax rate is 11.654 percent, corresponding to the effective US tax rate in 1984. The model replicates key benchmark equilibrium values of the US economy, as reported in **table 2**:

---

8 Tax revenue data come from OECD Revenue Statistics; income sources are represented by the variables from OECD National Accounts.
Table 2. Benchmark equilibrium values

<table>
<thead>
<tr>
<th>τ_γ</th>
<th>̂θ</th>
<th>̂ϕ</th>
<th>̂ψ</th>
<th>Y̅ / K</th>
<th>C̅ / Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.654</td>
<td>0.8575</td>
<td>0.9213</td>
<td>0.8168</td>
<td>0.3081</td>
<td>0.6355</td>
</tr>
</tbody>
</table>

The share of labor allocated to the production of final output is about 86%, the share of capital allocated to production is 92 % and about 18% of the skills are used in the education sector. The implied equilibrium output-capital ratio is 0.31, and the consumption-output ratio is 0.63, both of which are highly plausible. Although less information on the sectoral allocation of assets exists, we feel that these fractions are plausible as well, so that in all, the model manages to replicate all key variables with relative ease given the standard set of assumptions on the fundamental parameters.

Given that the equilibrium dynamics are generated by a fourth-order system, we know from Eicher and Turnovsky (1999) that the transition may be characterized by significant non-monotonicites in the state variables. This implies that both the origin of the economy and the size of the tax cut may have an influence on the actual transition to the new steady state. This result, only established in simulations in Eicher and Turnovsky (1999), can be explored here using data from US tax cuts and skill premium movements.

The key issue here is the response of the economy to changes in the underlying policy parameters. We chose to focus on the US economy, where inequality data is available in the most complete format, and because the US underwent significant changes in its tax code, especially during the Reagan Years 1980-88. We will undertake our analysis for this period and up to mid nineties. *Figure 1* clearly shows the legacy of the Reagan administration, specially concentrated in his first mandate, with substantial tax cuts in 1980-81 from 15.152% to 14.054%, then to 12.872% in 1982, 12.354% in 1983, 11.654% in 1984 and finally 11.031% in 1985.
The time period of the Reagan tax cuts is especially opportune, since skill premium did not move much in the mid to last 1970’s, until the Reagan tax reforms were instituted. Then a clear long-run upward trend set in, with specific decreases in the years 1981, 1984, 1987 and 1991; see figure 2. Clearly the effect of taxes on skill premia is a function of time lags and an array of interrelated economic factors. However, the model developed above incorporates key economic variables, such as investment in education and physical capital. Hence we would like to explore how successful the model would be in explaining the marked trend in skill premia, which coincides with US tax changes during the same period of time.
We will start analyzing the dynamics predicted by the model after both a tax decrease and a tax increase. Decreasing from 11.654% in 1984 to 11.031% in 1985, the model predicts a new steady state as reported in table 3.

Table 3. Tax reform (decrease) equilibrium values

<table>
<thead>
<tr>
<th>τy</th>
<th>0.8575</th>
<th>0.9213</th>
<th>0.8168</th>
<th>0.3060</th>
<th>0.6340</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.031</td>
<td>0.8575</td>
<td>0.9213</td>
<td>0.8168</td>
<td>0.3060</td>
<td>0.6340</td>
</tr>
</tbody>
</table>

This steady state differs from the previous one (for 1984 taxes of 11.654 percent) only in the capital output and the consumption output ratios. The growth rates of the endogenous factors and of the shares of the factors allocated across sectors change only during the transition. The output to capital ratio declines by about 0.7% and the consumption to output ratio by less, 0.24%. Hence, as the tax on income falls, capital accumulation increases, as saving has become more attractive. As capital accumulation increases, so does human capital accumulation to maintain the parity in the shadow values of the endogenous variables, since the marginal product of skills increases with the accumulation of capital.

The adjustment of the skill premium shows that it falls down to its long-run value faster than without the tax decrease. We can see that coming from the rapid initial decrease, which slows down as time goes by. In any case, the interesting part is how a tax decrease is able to change the trajectory of the skill premium accelerating its initial decline, thus inequality is further reduced than without the tax decrease for several periods. After the policy change, the skill premium evolves according to:

---

9 The simulations are based on the Mathematica algorithm developed by Eicher and Turnovsky (2001).
The key feature of the model driving the impact of tax policy on our measure of income inequality, $w_h$, as pointed out before, is the adjustment of human capital, jointly with the response of labor allocation, $\theta$, and skills allocation, $\psi$, across the sectors; see (17). We can think of the long-run values of the share of labor and skills in manufacturing as the ones corresponding to full employment potential. Recalling (13), we see that on impact a decrease in $\tau_y$ has two effects on employment and skills in the final output sector; $\theta$, $\psi$. First, given $q$, it increases the after-tax relative price of final output, $q(1 - \tau_y)$, thereby increasing $\theta$ and $\psi$. But at the same time, it reduces the before-tax relative price $q$ causing an offsetting reduction in $\theta$ and $\psi$. Over time, labor and skills will continue to move in response to the changing shadow value, and relative stocks of physical and human capital. During the early phases of the adjustment $q$ may continue to decline, thereby offsetting the effect of the accumulating capital stock. Yet, the impacts on $\theta$ and $\psi$ rather offset each other, hence most of the adjustment comes from the skill adjustment, $h$.

The second experiment is similar in nature but has opposite impacts on the US economy. Increasing from 11.581% in 1993 to 11.962% in 1994, the model predicts a new steady state as shown in table 4.
Table 4. Tax reform (increase) equilibrium values.

<table>
<thead>
<tr>
<th>$\tau_y$</th>
<th>$\tilde{\theta}$</th>
<th>$\tilde{\phi}$</th>
<th>$\tilde{\psi}$</th>
<th>$Y \tilde{K}$</th>
<th>$C \tilde{Y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.962</td>
<td>0.8575</td>
<td>0.9213</td>
<td>0.8168</td>
<td>0.3091</td>
<td>0.6362</td>
</tr>
</tbody>
</table>

The changes to the steady state are the opposite to the ones above. Again, more interesting than the mere comparison of stationary states is the analysis of the transition. The graph of the skill premium evolution clearly indicates an adjustment where skill premium initially goes up for one period and then begins to go down. Note that for several periods after the tax increase, the evolution of the new skill premium remains above the evolution of the skill premium prevailing before the tax increase, thus a tax increase brings an increase in inequality into the economy.

Such an adjustment depicted here is impossible to replicate with most growth models even in transition, because the dimensionality of the dynamics does not allow for such non-linearity (see Eicher and Turnovsky, 1999). Given the non-scale structure of the model and the fourth order system, transitions are also characterized by a two-dimensional manifold, and hence reversals in the trend of the skill premium can possibly be replicated by the model.

Let’s now have a look at the temporary evolution of the skill premia. Figure 4.5 shows both real and simulated skill premia together. In order to come out with the data for the skill...
premia using our model, we have proceeded undertaking continuous simulations, one per year corresponding to the different annual tax values. We have plugged the yearly effective income taxes corresponding to the period 1980-1995 in our model taking as initial values for capital and skills for each new tax change the ones after one year of validity of the previous tax. This has allowed us to get the corresponding yearly value for skill premia using the Mathematica algorithm previously mentioned.

Comparing both skill premia, the main feature is that simulated skill premia replicate the increasing long-run trend of inequality. Going into more detail, simulated skill premia seem to follow the non-monotonic movements of the real skill premia with some exceptions, which deserve a comment. Real inequality decreased in 1987, when taxes scarcely moved. Besides, the evolution of inequality during the period 1989-91 followed an opposite pattern to the movement in taxes. These two exceptions are likely to show the fact that during those years, aspects other than taxes, such as economical or institutional, not captured in this model simply using tax changes, influenced inequality more than taxes did. Moreover, after the tax decrease in 1982, we see how both real and simulated inequality increased. This could be showing the lack of effectiveness or the anticipation of the agents after some consecutive tax decreases (1980, 1981).
Figure 6 compares the evolution of simulated skill premia with real skill premia two periods ahead, that is, with taxes having a two-period lagged effect on (simulated) skill premia.

Now, we can see how our simulated series predicts the 1982-1994 evolution of the real skill premia better. The exception is the period 1986-87, where the model seems not to predict correctly all the non-monotonicities.

In sum, despite the simplicity of the model, we can get some good insights on the influence of taxes on inequality. Generally speaking, it seems that various consecutive tax changes of the same sign, specially decreases, may not have the desired initial negative effect on inequality, either because agents anticipated the changes and thus these changes have already been incorporated in their behavior (changes lose effectiveness), or because there are some lagged effects. Besides, it could happen that several tax decreases were not quite credible. In any case, there seems to be a mild response to large tax changes, specially to tax decreases, confirming some of the results in the literature.

As we have previously said, it is also worth noting that, according to the data, inequality, as measured by skill premia, shows a growing long-run trend, which, according to our skill premium modelization, can be due to different factors: changes in raw labor and skills
elasticities in the output production sector, long-run changes in the share of labor and skills in the manufacturing sector, as well as to changes in the ratio \( N/H \) (see (17)). Besides, a better performance of the economy due to, for instance, higher productivity, could be associated to what is called skilled-biased technical change, leading to increasing long-run inequality (Krugman, 1994; Wood, 1994; Manacorda and Petrongolo, 1996, 1999; Mortensen and Pissarides, 1999). Using a non-scale model leads us to the abandonment of the first explanation, though.

One additional aspect subject to analysis would be the response of output to tax changes. Intuitively it seems that tax decreases should translate into larger output values through higher physical and human capital accumulation, and thus tax increases into lower output increases. A large part of the literature tends to argue that taxes, if any, exert a negative influence on growth. The positive effect of a tax cut on growth is likely to be higher than the negative effect caused by a reduction in the shares of capital and labor also induced immediately after the tax cut. As time goes by, the positive effect is reinforced with the reverse trend of these shares, which start increasing, thus impinging a higher growth rate on the economy. This is what seems to be confirmed here both using real data as well as our simulated data. Simulated output has been obtained in a similar way as we obtained the skill premium values. Results, as shown in figure 7, using both real and simulated output data tend to confirm the likely presence of a negative relation between (one-period lagged) taxes and growth, which would show, on the one side, that fiscal policy may be rather effective, and, on the other side, the burden that taxes may impinge on the economic performance of an economy when not appropriately designed.
When comparing the evolution of both simulated and real growth, as shown in *figure 4.8*, we can appreciate how the model is capable to capture most of the movements in the growth rate with the main exception of the years 1983-85. We have to note, though, that there seems to be a one-period lag in the period 1982-1990. In any case, the simulated growth mimics the movement of real growth quite closely.
5. Conclusion

This paper presents an alternative explanation for skill premium inequality based on the rich dynamics of a non-scale growth model with endogenous skill formation. In this sense, the model is capable to replicate the unusual skill premium dynamics of the United States economy in the 1980s and half 1990’s. The paper provides thus additional evidence of the relevance of non-scale models, indicating that they not only predict the long-run dynamics of industrialized countries with remarkable ease, but also indicating that they allow for the non-linear rich dynamics that are necessary to explain the skill premium movements that took place within more than one decade in the US economy.

More specifically, we have analyzed in detail the dynamics following the simulation of an isolated tax decrease and an isolated tax increase. The results we got are in line with the expected ones. After a tax decrease the skill premium follows a lower trajectory than the one it would follow without a tax decrease. Hence we are able to reduce inequality. On the contrary, following a tax increase, the evolution of the skill premium remains above the trajectory carried on by the skill premium under a situation with no tax increase. Consequently, a tax increase would imply a higher level of inequality in the economy.

The dynamics of the simulated skill premium mimicked the real US skill premium dynamics quite nicely. However, one should bear in mind that it might take some time for an economy to wholly absorb policy changes. In this sense, the need for some periods of time to recover would probably have offset the different tax cuts that took place consecutively throughout the period 1981-85. This fact could give us some insights about the lagged effects of fiscal policy, and therefore about the likely need to separate two consecutive tax changes in order to capture, at least partially, the effects of both of them. This seems to be confirmed when using a two-period lagged influence of taxes on simulated skill premia, since these last ones show a closer adjustment to the real values, especially for the period comprising 1982-1994. Problems seem to show up again in the first half of the eighties, and thus coming after the various large consecutive tax cuts, which reinforces our previous conclusions.
On the other hand, the evolution of inequality shows a long-run growing trend, which could be due to the joint effect of tax increases and the mild response of the economy to tax decreases. These last ones, despite being numerous, could have lost some effectiveness either as a result of agents anticipation or given a lower credibility of the agents on tax decreases.

With non-scale models, the long-run equilibrium growth rate is determined by technological parameters and it is independent of macroeconomic policy instruments. However, this fact does not imply that fiscal policy is unimportant for long-run economic performance. It indeed has important effects on the different levels of key economic variables such as per capita stock of capital and output. Hence, although the economy grows at the same rate across steady states, the bases for economic growth may be different.

Finally, while the model predicts rather correctly the skill premium dynamics in the period under analysis, as well as the strong increase in investment during that period, it does not capture the strong increase in consumption observed over that period, especially relative to the level of output. The 1980’s showed a marked decline in the savings rate, which runs counter to most economic theories, including Ricardian equivalence. Consumers in our model do not have the option to accumulate debt, which became a crucial issue in the 1980’s, as credit cards and consumer loans became more prevalent via banking regulations. The inclusion of fiscal and personal debt is left for future research. Besides, it would be interesting to seek for a further reduction in inequality (measured by skill premia), which, in the context of non-scale models with constant elasticity parameters would mainly lead us to look for a way to increase the stock of human capital in the long-run relative to the population size, maybe allowing for the presence of some externalities in the production of skills. Linked to this one, another interesting aspect to analyze would be how to foster a higher participation of the different production factors in the human capital production sector in the long-run. The introduction of various types of taxes in order to make fiscal policy more specific could be a way to pursue this further goal.
Appendix 1. Characterization of transitional dynamics

Henceforth we assume that the stability properties are ensured so that we can denote the two stable roots by $\mu_1$ and $\mu_2$. The key variables of interest are physical capital, and technology. The generic form of the stable solution for these variables is given by:

$$k(t) - \tilde{k} = B_1 e^{\nu_1 t} B_2 e^{\nu_2 t}$$  \hspace{1cm} (18a)

$$h(t) - \tilde{h} = B_1 v_{21} e^{\nu_1 t} B_2 v_{22} e^{\nu_2 t}$$  \hspace{1cm} (18b)

where $B_1, B_2$ are constants and the vector $\left(1 \ v_{2i} \ v_{3i} \ v_{4i}\right)$, $i = 1, 2$ (where the prime denotes vector transpose) is the normalized eigenvector associated with the stable eigenvalue, $\mu_i$. The constants, $B_1, B_2$, appearing in the solution (18) are obtained from initial conditions, and depend upon the specific shocks. Thus suppose that the economy starts out with given initial stocks of capital and knowledge, $k_0, h_0$ and through some policy shock converges to $\tilde{k}, \tilde{h}$. Setting $t=0$ in (18a), (18b) and letting $d\tilde{k} \equiv \tilde{k} - k_0$, $d\tilde{h} \equiv \tilde{h} - h_0$, $B_1, B_2$ are given by:

$$B_1 = \frac{\nu_{22} d\tilde{k}}{v_{22} - v_{21}}; \quad B_2 = \frac{v_{21} d\tilde{k} - d\tilde{h}}{v_{22} - v_{21}}$$  \hspace{1cm} (4.19)

When studying the dynamics, we are interested in characterizing the slope along the transitional path in $h$-$k$ space. In general, this is given by:

$$\frac{dh}{dk} \to_{t \to \infty} \nu_{21}$$

and is time varying. Note that since, $0 > \mu_1 > \mu_2$ as $t \to \infty$ this converges to the new steady state along the direction $(dh/dk)_{t \to \infty} = \nu_{21}$, for all shocks. The initial direction of motion is obtained by setting $t = 0$ in (18) and depends upon the source of the shock. It is convenient to express the dynamics of the state variables in phase-space form:
\[
\begin{pmatrix}
  k \\
  h
\end{pmatrix} =
\begin{pmatrix}
  \frac{\mu_1 v_{22} - \mu_2 v_{21}}{v_{21} - v_{22}} & \frac{\mu_2 - \mu_1}{v_{21} - v_{22}} \\
  \frac{v_{22} - v_{21}}{v_{21} - v_{22}} & \frac{v_{22} - v_{21}}{\mu_2 v_{22} - \mu_1 v_{21}}
\end{pmatrix}
\begin{pmatrix}
  k - \bar{k} \\
  h - \bar{h}
\end{pmatrix}
\tag{4.21}
\]

By construction, the trace of the matrix in (21) equals \( \mu_1 + \mu_2 < 0 \) and the determinant equals \( \mu_1 \mu_2 > 0 \), so that (21) describes a stable node. The dynamics expressed in (18) and (21) are in terms of the scale-adjusted quantities, from which the growth rates of per capita capital and knowledge can be derived.\(^{10}\)

Equations (18a) and (18b) highlight the fact that with the transition path in \( k \) and \( h \) being governed by two stable eigenvalues, the speeds of adjustment for capital and skills are neither constant nor equal over time. In addition, with output being determined by capital and knowledge, the transition of output is also not constant over time, but a simple composite of the transition characteristics of \( h \) and \( k \) as determined in (21).

\(^{10}\)Note that the representation of the transitional dynamics in \( h-k \) space takes full account of the feedbacks arising from the jump variables, \( q \) and \( c \); these are incorporated in the two eigenvalues.
Appendix 2. Graphical evolution of the shares of labor, physical capital and human capital in the manufacturing sector after a tax change.

A.2.2. Tax increase from 11.581% to 11.962% (1993-1994)