The Role of Unions in an Endogenous Growth Model with Human Capital[*]

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Abstract

In this paper we study the relationship between unions and growth in a two-sector overlapping generations model with altruism and human capital. This relationship depends on the interaction between the technology in the sector that produces human capital, the degree of unionization of the economy and the operativeness of the bequest motive.

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1 Introduction

The purpose of this paper is to investigate the relationship between growth and the role of unions in a two-sector overlapping generations (OLG hereinafter) model with altruistic agents where endogenous growth is driven by the accumulation of human capital.

There are two branches of the literature to which our paper relates. The first one combines altruism and endogenous growth based on human capital accumulation as, for example, Glomm and Ravikumar (1992). These authors present an OLG model where each parent has a bequest motive and values the quality of education passed on to the offspring. Eckstein and Zilcha (1994) develop a model where the human capital of the children is determined by the percentage of the leisure time that parents devote to their offspring. In a recent contribution, Zhang (1996) emphasizes the importance of the units of goods invested by the parent, the units of the parent's time spent, and the human capital of the parent on the human capital of the child.

The second branch of the literature addresses specifically the problem of unemployment in the context of growth models in unionized economies, but without taking altruism into account.[1,2] Bean and Pissarides (1993) develop a search model where matching frictions create unemployment. They find out that the relationship between growth and the relative bargaining power of workers' is ambiguous, being the reason that the shift in income from entrepreneurs to workers could be compensated by an increase in savings, and therefore in the growth rate. The same result is achieved by de la Croix and Licandro (1995) in a model with irreversible decisions; they conclude that a raise in union power produces crowding-out of physical capital, but at the same time it raises the firm's value, and the physical capital as well.

This paper departs from these two approaches in the following sense. We develope a two-sector OLG model with intergenerational altruism and unions, where endogeneous growth is generated by human capital accumulation in the educational sector. The presence of altruism, in the sense that parents of a given generation will get utility from the utility of their offspring as the dynastic utility in Barro (1974), allows us to generate different results depending on the operativeness of the bequest motive[3].

This framework allows two possible situations: in the first one, the production of human capital is given by a linear technology on human capital

[1] Aghion and Howitt (1994) study the interaction between unemployment and growth in a one sector model, whereas van Schaik and de Groot (1995) analyse the same question in a two sector model. None of them includes unions.

[2] Other models, that emphasize the effect of unionisation on innovation and growth, are Ulph and Ulph (1994) and Acemoglu (1997).

[3] As an example, Caballé (1998) shows how economic growth rates change depending on the operativeness of the bequest motive, which in turn depends on the tax structure.

as in Uzawa (1965) or Lucas (1988) and thus, there is no physical capital in the educational sector. For this reason, we can analyse two economies: in the first one, the walrasian case, we face a competitive economy without unions, whereas in the second one there are unions in the sector that produces consumption goods. We find that the rate of growth is higher in the unionized economy than in the walrasian one, and this holds no matter the operativeness of the bequest motive. Higher wages in the unionized sector have two effects: to increase the demand for physical capital and to decrease the quantity of labour hired in this sector. Unemployed labour in the consumption goods sector causes wages to decrease in the educational sector and thus higher returns from human capital investment. The amount of labour hired increases implying a higher production of human capital and, hence, a higher rate of growth. It is also worth to emphasize that the level of altruism that makes the physical bequest motive to be or not operative is the same in both the walrasian and the unionized economies. This means that the degree of unionization of the economy does not affect the individual's decision about leaving physical bequests.

In contrast with the first situation, in the second one physical capital enters the production function for human capital accumulation as an input. This allows four possible cases: in the first one, the walrasian case, we face a competitive economy without unions, very much in contrast with the last one, where we find unions in each sector; there are two other similar, intermediate cases, with unions in one sector and none in the other (partially unionized economies). We show that, with strictly positive bequests, the rate of growth is higher in the walrasian case than in the completely unionized economy. The reason is that higher wages in both sectors in the fully unionized economy cause the total demand for labour to decrease, creating unemployment. The resulting fall in production more than offsets the increase in wages. However, the same conclusion might be reversed comparing the walrasian case to the partially unionized economies, that is, a partially unionized economy may grow faster than a walrasian economy. Higher wages in the unionized sector cause the demand for labour to decrease, but there is no unemployment because lower wages in the non-unionized sector increase the demand for labour. The overall effect on the rate of growth is then ambiguous. In turn, when bequests are inoperative, we get a different result, in the sense that even fully unionized economies can grow at higher rates than competitive ones. When bequests are inoperative, parents devote more resources to their childrens' education, and since there are higher wages in both sectors in the fully unionized economy, these two effects might offset the fall in production due to the unemployment caused by the existence of unions, compared to the walrasian economy with full employment but lower wages. In this context, the level of altruism that makes the physical bequest motive to be or not operative is different for the walrasian and the fully unionized economies. Thus, given a certain level of altruism, the unionization degree affects the rate of growth of the economy via two channels: the direct effect on human capital investment and the indirect effect on the operativeness of the bequest motive.

In the next section we present the basic model. Linear technology in the educational sector is discussed in section 3, whereas section 4 studies the introduction of physical capital in this sector. Section 5 concludes. All proofs and computations can be found in the Appendix.

2 The basic model

We construct a two-sector OLG model with constant population, whose mass is normalized to one, and where agents live for three periods. In the first period individuals obtain education bequest from their parents, which endows them with a human capital level h_t. They are endowed with one unit of labour time that will be supplied inelastically in the second period, where they also receive a physical bequest b_t from their parents and have a child. Then they decide about consumption C_t in that period, the provision of education or human capital of their offspring h_{t+1}, and savings s_t. In the third period they distribute the return from savings between consumption C_{t+1} and bequests to their offspring b_{t+1}. The utility of an individual born at $t-1$ is

$$V_{t-1} = \ln C_t^{t-1} + \beta \ln C_{t+1}^{t-1} + \rho V_t; \tag{1}$$

where the superscript denotes the generation to which individuals belong, V_t is the utility of their offspring, and $\beta > 0$. The parameter $\rho > 0$ is the altruism factor.

The unit of labor time inelastically supplied by an individual at time t becomes h_t efficiency units of labor. Thus, the agent's labour income in the second period is $w_t^a h_t$, where w_t^a is the average wage. Since there are two sectors in the economy and the wage paid in each sector may be different, individuals take the wage as the sum of the time they work in each sector times the wage earned in that sector. Note that w_t^a is the average wage across firms, but each and every worker receives precisely w_t^a. Education is provided through schools or specific educational price-taker firms. We normalize the price of consumption goods to one. Thus, consumer's budget constraints are:

$$C_t + p_t h_{t+1} + s_t = w_t^a h_t + b_t; \tag{2}$$

$$s_t (1 + r_{t+1}) = C_{t+1} + b_{t+1}; \tag{3}$$

where $p_t h_{t+1}$ is the total expenditure on the education of the offspring, p_t is the price in goods of each unit of human capital acquired by the offspring in the school at period t, and r_{t+1} is the interest rate at $t+1$. Maximizing

(1) subject to (2), (3) and $b_{t+1} \geq 0$ is equivalent to solving the following problem:

$$V_{t-1}^{*}(h_t; b_t) = \max_{\{s_t; h_{t+1}; b_{t+1}\}} \{ \ln (w_t^a h_t + b_t - s_t - p_t h_{t+1})$$

$$+ \beta \ln [s_t (1 + r_{t+1}) - b_{t+1}] + \rho V_t^{*} (h_{t+1}; b_{t+1}) \} ; \quad b_{t+1} \geq 0: \tag{4}$$

In order to ensure that V_{t-1}^{*} is bounded above and the solution we will find is the optimal one, we asssume $\rho < 1$.

There is a consumption goods sector in which each firm produces according to the production function $Y_t = F(K_t; L_t) = K_t^{\gamma_1} L_t^{1-\gamma_1}$ where $\gamma_1 \in (0; 1)$, K_t is the capital employed by the firm and $L_t = h_t N_t$ is the efficient labor which in turn equals the human capital of workers' h_t times the time they are hired by the firm N_t. Since firms are able to observe the level of skill of each worker and are price-takers, from the maximization problem of the firm we have that factors are paid their marginal products:

$$1 + r_t = \gamma_1 K_t^{\gamma_1 - 1} (h_t N_t)^{1-\gamma_1} ; \tag{5}$$

$$w_t = (1 - \gamma_1) K_t^{\gamma_1} (h_t N_t)^{-\gamma_1} ; \tag{6}$$

where w_t is the wage paid for one efficiency unit of labor by the firm. From the above equalities we get

$$\frac{w_t}{1 + r_t} = \frac{1 - \gamma_1}{\gamma_1} \left(\frac{K_t}{h_t N_t} \right) ; \tag{7}$$

and the optimal capital-labour ratio

$$k_t^d = \left(\frac{w_t}{1 - \gamma_1} \right)^{\frac{1}{\gamma_1}} ; \tag{8}$$

whereas the firm's optimal labour demand is, as follows from (8):

$$h_t N_t^d = K_t^d \left(\frac{1 - \gamma_1}{w_t} \right)^{\frac{1}{\gamma_1}} : \tag{9}$$

This basic framework is shared by the two situations we discuss below. The first one is analysed in section 3 and presents a model where there is a linear technology in the human capital accumulation function. The second situation is presented in section 4, where physical capital enters the production function for human capital accumulation as an input.

3 Linear technology in the educational sector

We assume the existence of an educational sector, composed by price-taking firms. We can interpret it as if individuals contracted the service of one educational firm or school in order that their offspring receives a certain human capital level or education. Each educational firm maximizes profits $p_t h_{t+1} - \hat{w}_t h_t \overline{N}_t$ taking into account the way human capital accumulates (or its "production" function) $h_{t+1} = \theta \; h_t \overline{N}_t$, where $\overline{N}_t$ is the time workers are hired by the firm, $\hat{w}_t$ is the wage paid for one efficiency unit of labor by the firm, and θ is a positive parameter. Since the technology in this sector is linear, each firm hires a positive and finite quantity of labour if the following condition holds:

$$\hat{w}_t = \theta p_t; \tag{10}$$

We consider two cases: in the first one, without unions, the wage per efficient unit of labour must be equal in both sectors for all firms. Besides, all labor supplied is employed and unemployment does not exist. Thus $N_t + \overline{N}_t = 1$[4] and $w_t = \hat{w}_t$, which implies:

$$(1 - \gamma_1) K_t^{\gamma_1} (h_t N_t)^{-\gamma_1} = \theta p_t: \tag{11}$$

In the second case, where there are unions in the consumption goods sector, we assume one union per firm which invests in physical capital before the wage bargaining takes place. Thus, the capital stock will be considered as a constant by unions and firms when negotiating. In each period the union sets the wage taking into account that it will affect the quantity of labor employed only in its firm. Thus, the union cares about employment and compares the wage with the income a worker can get if negotiation fails (v_t). The union maximizes $(w_t - v_t) h_t N_t$ subject to the labor demand given by (9). The optimal condition is given by

$$w_t = \frac{v_t}{1 - \gamma_1}: \tag{12}$$

Analogous to Barth and Zweimüller (1995), we assume that in equilibrium v_t is the expected income that a worker may get over all the economy, i.e. the average income $v_t = w_t^a = w_t N_t + \hat{w}_t \overline{N}_t$. Substituting into (12) for v_t yields

$$\hat{w}_t \overline{N}_t = w_t (1 - \gamma_1 - N_t): \tag{13}$$

Since the educational sector is not unionized, the wage in this sector is that inducing full employment, i.e. $N_t + \overline{N}_t = 1$.

[4] Note that both the population mass and the labour time endowment of the individual are normalized to one.

The following relationships are valid no matter whether we study the walrasian or the unionized economy. First, the amount saved by generation t equals the stock of physical capital at t + 1:

$$s_t = K_{t+1}. \tag{14}$$

Second,

$$w_t^a = w_t N_t + \overline{w}_t \overline{N}_t; \tag{15}$$

that is, expected wage is equal to the sum of the time they work in each sector times the wage earned in that sector.[5]

The problem for consumers is to solve (4). The first order conditions for s_t and b_{t+1} are given by equations (16) and (17) respectively, and for h_{t+1} are (18W) and (18U) in the walrasian and the unionized economies respectively (see the Appendix):

$$\frac{1}{C_t^{t-1}} = \frac{\beta(1 + r_{t+1})}{C_{t+1}^{t-1}}; \tag{16}$$

$$\frac{\beta}{C_{t+1}^{t-1}} \geq \frac{\rho}{C_{t+1}^{t}} \qquad \text{with equality if } b_{t+1} > 0; \tag{17}$$

$$\frac{p_t h_{t+1}}{\rho C_t^{t-1}} = \frac{(1-\gamma_1) w_{t+1} h_{t+1}}{C_{t+1}^t} + \frac{\beta s_{t+1}(1-\gamma_1)(1+r_{t+2})}{C_{t+2}^t}; \tag{18W}$$

$$\frac{p_t h_{t+1}}{\rho C_t^{t-1}} = \frac{(1-\gamma_1)^2 w_{t+1} h_{t+1}}{C_{t+1}^t} + \frac{\beta s_{t+1}(1-\gamma_1)(1+r_{t+2})}{C_{t+2}^t}. \tag{18U}$$

When bequests are operative, the derivation of the steady-state rates of growth are as follows. Using (18W) and (18U), (16) and the fact that in steady-state $\frac{C_{t+1}^t}{C_t^{t-1}} = g$, we get

$$\frac{g p_t h_{t+1}}{\rho} = (1 - \gamma_1) w_{t+1} h_{t+1} + (1 - \gamma_1) s_{t+1}; \tag{19W}$$

$$\frac{g p_t h_{t+1}}{\rho} = (1 - \gamma_1)^2 w_{t+1} h_{t+1} + (1 - \gamma_1) s_{t+1}. \tag{19U}$$

Introducing (7) and (14) into (19W) and (19U), and using (10):

$$\frac{g \overline{w}_t h_{t+1}}{\theta\rho} - (1 - \gamma_1) w_{t+1} h_{t+1} = \gamma_1 \frac{w_{t+2} N_{t+2} h_{t+2}}{1 + r_{t+2}}; \tag{20W}$$

$$\frac{g \overline{w}_t h_{t+1}}{\theta\rho} - (1 - \gamma_1)^2 w_{t+1} h_{t+1} = \gamma_1 \frac{w_{t+2} N_{t+2} h_{t+2}}{1 + r_{t+2}}. \tag{20U}$$

[5] Since there are constant returns to scale, we identify the labor demanded by one firm as the total amount of labor demanded by all firms in the sector.

In the walrasian equilibrium we have $w_t = \overline{w}_t$ and $N_t = 1 - \overline{N}_t$. Besides, if we evaluate in steady state, use from the human capital technology $\frac{g^W}{\theta} = \overline{N}^W$ and divide by h_{t+1}, we can rearrange (20W) as

$$\frac{g^W}{1+r} = \frac{\frac{\overline{N}^W}{\rho} - (1-\gamma_1)}{\gamma_1 \left(1 - \overline{N}^W\right)}. \tag{21}$$

From (16) and (17) we can obtain

$$\frac{g^W}{1+r} = \rho. \tag{22}$$

Equations (21), (22) and the production function for human capital accumulation allow us to get

$$g^W = \theta \overline{N}^W = \frac{\theta\rho\left[(1-\gamma_1) + \rho\gamma_1\right]}{1+\rho^2\gamma_1}. \tag{23}$$

The process of derivation of the steady-state rate of growth in the unionized economy (g^U) is similar to the one described above. The only difference is that $w_t \left(\overline{N}_t - \gamma_1\right) = \overline{w}_t \overline{N}_t$ so that (23) becomes:

$$g^U = \theta \overline{N}^U = \frac{\theta\left[\rho(1-\gamma_1)^2 + \gamma_1(1+\rho)^2\right]}{1+\rho^2\gamma_1}. \tag{24}$$

In the case of inoperative bequests, $b_{t+1} = 0$ and (17) holds with strict inequality. Combining (2), (3) and (16), we get

$$C_t = \frac{w_t^{a} h_t - p_t h_{t+1}}{1+\beta}. \tag{25}$$

Substituting (25) into (2) for C_t gives

$$s_t = \frac{(w_t^{a} h_t - p_t h_{t+1})\beta}{1+\beta}. \tag{26}$$

Substituting (7), (14) and (15) into (26), and using (10) and the rule of human capital accumulation yields

$$\frac{\gamma_1}{1-\gamma_1}\frac{g_C^W}{1+r_{t+1}} = \frac{\beta}{1+\beta}. \tag{27}$$

Combining (27), (21) and the production function for human capital accumulation, we obtain

$$g_C^W = \theta \overline{N}_C^W = \frac{\theta\rho(1-\gamma_1)(1+2\beta)}{(1+\beta)+\beta\rho(1-\gamma_1)}. \tag{28}$$

The process of derivation for the case of the unionized economy is similar to the one described above. The only difference is that $w_t\left(\bar{N}_t - \gamma_1\right) = \bar{w}_t \bar{N}_t$ so that (28) becomes

$$g_C^U = \theta \bar{N}_C^U = \frac{\theta\left[\rho(1-\gamma_1)(\beta + (1-\gamma_1)(1+\beta)) + (1+\beta)\gamma_1\right]}{(1+\beta) + \beta\rho(1-\gamma_1)}. \tag{29}$$

We aim now at finding the conditions under which parents leave positive physical bequests to their offsprings in both the walrasian and the unionized cases.

Define $\bar{\rho}_W$ and $\bar{\rho}_U$ as the levels of altruism such that the non-negativity constraint is just binding in equilibrium, i.e. $b = 0$ and (17) holds with equality in the walrasian and the unionized economies respectively. Then from equations (22) and (27), and noting that both of them are valid in the unionized and the walrasian cases, we have

$$\bar{\rho}_W = \bar{\rho}_U = \left(\frac{1-\gamma_1}{\gamma_1}\right)\left(\frac{\beta}{1+\beta}\right) = \bar{\rho}. \tag{30}$$

The fact that the critical level of altruism is the same in both the walrasian and the unionized economies means that the degree of unionization of the economy does not affect the individual's decision about leaving physical bequests.

In principle, we should expect that agents are more likely to leave bequests when ρ is high, and viceversa, and this is in fact stated in the next propositions.

Proposition 1 *If $\rho \leq \bar{\rho}$ then $b = 0$ in the balanced growth path of both the walrasian and the unionized economies.*

Proposition 2 *If $\rho > \bar{\rho}$ then $b > 0$ in the balanced growth path of both the walrasian and the unionized economies.*

What the above propositions tell us is that when individuals care little about their children, they leave them zero physical bequests. In turn, when individuals' concern about their children is high, bequests will be positive.

The comparison between the rates of growth in both cases is summarized in the following proposition.

Proposition 3 *(a) When bequests are operative (i.e. $\rho > \bar{\rho}$), then $g^U > g^W$.*
(b) When bequests are inoperative (i.e. $\rho \leq \bar{\rho}$), then $g_C^U > g_C^W$.

Proof. Direct comparison between (23) and (24) and between (28) and (29). ■

Higher wages in the unionized sector has two effects: to increase the demand for physical capital and to decrease the quantity of labour hired in this sector. Unemployed labour in the consumption goods sector causes wages to decrease in the educational sector and thus higher returns from human capital investment. The amount of labour hired increases, implying a higher production of human capital and, hence, a higher rate of growth.

4 Physical capital in the educational sector

In this section we introduce a different rule of human capital accumulation in the educational sector. Each educational firm maximizes profits $p_t h_{t+1} - \hat{w}_t h_t \overline{N}_t - (1 + r_t) \hat{K}_t$, taking into account the way human capital accumulates $h_{t+1} = \theta \hat{K}_t^{\gamma_2} \left(h_t \overline{N}_t \right)^{1-\gamma_2}$, where $\hat{K}_t$ is the capital employed by the firm, $\overline{N}_t$ is the time workers are hired by the firm, $\gamma_2 \in (0; 1)$, $\hat{w}_t$ is the wage paid for one efficiency unit of labor by the firm, and θ is a positive parameter. The optimality conditions are:

$$1 + r_t = \gamma_2 \theta p_t \hat{K}_t^{\gamma_2 - 1} \left(h_t \overline{N}_t \right)^{1-\gamma_2}; \tag{31}$$

$$\hat{w}_t = (1 - \gamma_2) \theta p_t \hat{K}_t^{\gamma_2} \left(h_t \overline{N}_t \right)^{-\gamma_2}; \tag{32}$$

from where

$$\frac{\overline{w}_t}{1 + r_t} = \frac{1 - \gamma_2}{\gamma_2} \left(\frac{\overline{K}_t}{h_t \overline{N}_t} \right): \tag{33}$$

The optimal capital-labour ratio is

$$\overline{k}_t^d = \left[\frac{\overline{w}_t}{\theta p_t (1 - \gamma_2)} \right]^{\frac{1}{\gamma_2}}; \tag{34}$$

whereas the firm's optimal labour demand is, as follows from (34):

$$h_t \overline{N}_t^d = \overline{K}_t^d \left[\frac{\theta p_t (1 - \gamma_2)}{\overline{w}_t} \right]^{\frac{1}{\gamma_2}}: \tag{35}$$

The following relationships are valid no matter which of the four types of equilibria we study. First, we have expected wage as defined in (15). Second, the amount saved by generation t equals the stock of physical capital at $t+1$:

$$s_t = K_{t+1} + \overline{K}_{t+1}: \tag{36}$$

Finally, by arbitrage the interest rate must be equal in both sectors. Therefore, from (5) and (31),

$$\gamma_1 K_t^{\gamma_1 - 1} (h_t N_t)^{1-\gamma_1} = \gamma_2 \theta p_t \hat{K}_t^{\gamma_2 - 1} \left(h_t \overline{N}_t \right)^{1-\gamma_2}: \tag{37}$$

From the consumer's maximization problem (4), we obtain the first order conditions (16) and (17) for s_t and b_{t+1} respectively, and the following equation for h_{t+1} (see the Appendix).

$$\frac{p_t h_{t+1}}{\rho C_t^{t-1}} = \frac{(1-\gamma_1) w_{t+1} N_{t+1} h_{t+1} + (1-\gamma_2)\overline{w}_{t+1}\overline{N}_{t+1} h_{t+1}}{C_{t+1}^{t}} + \frac{\beta s_{t+1}(1-\gamma_2)^2(1+r_{t+2})}{C_{t+2}^{t}}. \quad (38)$$

We analyse four possible situations: walrasian equilibrium (no unions), one union in one of the sectors and none in the other, and finally, a fully unionized economy (unions in both sectors). We assume that firms invest in capital before the wage bargaining takes place. Thus, the capital stock will be considered as a constant by unions and firms when negotiating.

Let us analyse now the derivation of the steady-state rates of growth when bequests are operative in each of the four cases. In the walrasian case, without unions, the wage per efficient unit of labour must be equal in both sectors for all firms. Besides, all labor supplied is employed and unemployment does not exist. Thus $N_t + \overline{N}_t = 1$ and $w_t = \overline{w}_t$:

$$(1-\gamma_1) K_t^{\gamma_1} (h_t N_t)^{-\gamma_1} = (1-\gamma_2)\theta p_t \overline{K}_t^{\gamma_2}\left(h_t \overline{N}_t\right)^{-\gamma_2}. \quad (39)$$

Combining this equality and (37) we obtain that

$$K_t = \frac{1-\gamma_2}{\gamma_2}\frac{\gamma_1}{1-\gamma_1}\frac{N_t}{\overline{N}_t}\overline{K}_t, \quad (40)$$

which combined with (39) gives

$$\left(\frac{\overline{K}_t}{h_t \overline{N}_t}\right)^{\gamma_1-\gamma_2} = \left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_1}\left(\frac{1-\gamma_2}{1-\gamma_1}\right)^{1-\gamma_1}\theta p_t. \quad (41)$$

From (41) and after some manipulations (see the Appendix) we get the expression for the steady-state rate of growth of this economy $G^W = \frac{h_{t+1}}{h_t} = G^W(\theta, \rho, \gamma_1, \gamma_2)$.

In the second case, with unions in the consumption goods sector, the bargaining process is the same than in the previous section. Thus, (12) and (13) are still valid. Besides, since the educational sector is not unionized, the wage in this sector is that inducing full employment, i.e. $N_t + \overline{N}_t = 1$. Plugging (6) and (32) into (13) for w_t and $\overline{w}_t$ respectively:

$$(1-\gamma_1) K_t^{\gamma_1} (h_t N_t)^{-\gamma_1} = (1-\gamma_2)\theta p_t \overline{K}_t^{\gamma_2}\left(h_t \overline{N}_t\right)^{-\gamma_2}\frac{\overline{N}_t}{\overline{N}_t-\gamma_1}, \quad (42)$$

which combined with (37) gives

$$K_t = \frac{\gamma_1}{\gamma_2}\frac{(1-\gamma_2)}{(1-\gamma_1)}\frac{\overline{N}_t}{\overline{N}_t-\gamma_1}\overline{K}_t. \quad (43)$$

Combining the last equality and (42) results in

$$\left(\frac{\tilde{K}_t}{h_t\overline{N}_t}\right)^{\gamma_1-\gamma_2}=\left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_1}\left(\frac{1-\gamma_2}{1-\gamma_1}\right)^{(1-\gamma_1)}\theta p_t\left(\frac{\overline{N}_t}{\overline{N}_t-\gamma_1}\right)^{1-\gamma_1}. \tag{44}$$

The derivation of the steady-state rate of growth $G^G=G^G(\theta;\rho;\gamma_1;\gamma_2)$ from the last equation is straightforward (see the Appendix).

Proposition 4 Given $(\rho;\gamma_1;\gamma_2)\in\Omega=(0;1)\times(0;1)\times(0;1)$, there exist two disjoint open subsets with positive measure, Ω_1 and Ω_2, such that $\Omega_1\cup\Omega_2=\Omega$. If $(\rho;\gamma_1;\gamma_2)\in\Omega_1$ then $G^W>G^G$ and if $(\rho;\gamma_1;\gamma_2)\in\Omega_2$ then $G^W\leq G^G$.

Higher wages in the consumption goods sector have two effects: to increase the demand for physical capital and to decrease the quantity of labour hired in this sector. Unemployed labour in the consumption goods sector causes wages to decrease in the educational sector, and thus the amount of labour hired increases and the demand for physical capital decreases. The overall effect on the returns from human capital investment depends on the elasticities of productive inputs.

In the third case, with unions in the educational sector, the union maximizes $(\tilde{w}_t-v_t)\,h_t\overline{N}_t$ subject to the labor demand of the firm, given by (35), and the outcome is

$$\tilde{w}_t=\frac{v_t}{1-\gamma_2}. \tag{45}$$

Using $v_t=w_t^a$, (45) becomes

$$w_tN_t=\tilde{w}_t\left(1-\gamma_2-\overline{N}_t\right). \tag{46}$$

Since the consumption goods sector is not unionized, there is full employment, i.e. $N_t+\overline{N}_t=1$. From this last equality and substituting (6) and (32) into (46) for w_t and $\overline{w}_t$ respectively:

$$(1-\gamma_1)\,K_t^{\gamma_1}(h_tN_t)^{-\gamma_1}=(1-\gamma_2)\,\theta p_t\tilde{K}_t^{\gamma_2}\left(h_t\overline{N}_t\right)^{-\gamma_2}\frac{1-\gamma_2-\overline{N}_t}{1-\overline{N}_t}; \tag{47}$$

which combined with (37) gives

$$K_t=\frac{\gamma_1(1-\gamma_2)}{\gamma_2(1-\gamma_1)}\frac{1-\gamma_2-\overline{N}_t}{\overline{N}_t}\tilde{K}_t. \tag{48}$$

Combining(47) and (48), we obtain

$$\left(\frac{\tilde{K}_t}{h_t\overline{N}_t}\right)^{\gamma_1-\gamma_2}=\left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_1}\left(\frac{1-\gamma_2}{1-\gamma_1}\right)^{(1-\gamma_1)}\theta p_t\left(\frac{1-\gamma_2-\overline{N}_t}{1-\overline{N}_t}\right)^{1-\gamma_1}; \tag{49}$$

from where we get the steady-state rate of growth $G^E = G^E(\theta; \rho; \gamma_1; \gamma_2)$ (see the Appendix).

Proposition 5 Given $(\rho; \gamma_1; \gamma_2) \in \Omega = (0;1) \times (0;1) \times (0;1)$, there exist two disjoint open subsets with positive measure, Ω_1 and Ω_2, such that $\Omega_1 \cup \Omega_2 = \Omega$. If $(\rho; \gamma_1; \gamma_2) \in \Omega_1$ then $G^W > G^E$ and if $(\rho; \gamma_1; \gamma_2) \in \Omega_2$ then $G^W \le G^E$.

Higher wages in the educational sector have two effects: to increase the demand for physical capital and to decrease the quantity of labour hired in this sector. Unemployed labour in the educational sector causes wages to decrease in the consumption goods sector, and thus the amount of labour hired increases and the demand for physical capital decreases. The overall effect on the returns from human capital investment depends on the elasticities of productive inputs.

Finally, the fourth case presents a situation where all firms in both sectors are unionized. The bargaining process is the same than the one described above for the partially unionized economies. Thus, we can combine equations (12) and (45) to get

$$w_t(1-\gamma_1) = \tilde{w}_t(1-\gamma_2); \tag{50}$$

and substituting it into (46) we obtain

$$N_t = \frac{1-\gamma_1}{1-\gamma_2}\left(1-\gamma_2-\overline{N}_t\right): \tag{51}$$

From (50), (6) and (32) we have

$$(1-\gamma_1)^2 K_t^{\gamma_1}(h_t N_t)^{-\gamma_1} = (1-\gamma_2)^2 \theta p_t \tilde{K}_t^{\gamma_2}\left(h_t \overline{N}_t\right)^{-\gamma_2}: \tag{52}$$

Combining this equality and (37) we obtain

$$K_t = \frac{\gamma_1 (1-\gamma_2)^2}{\gamma_2 (1-\gamma_1)^2} \frac{N_t}{\overline{N}_t} \tilde{K}_t: \tag{53}$$

From the last equality, substituting in (52) for $\frac{K_t}{N_t}$ gives

$$\left(\frac{\tilde{K}_t}{h_t \overline{N}_t}\right)^{\gamma_1-\gamma_2} = \left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_1}\left(\frac{1-\gamma_2}{1-\gamma_1}\right)^{2(1-\gamma_1)} \theta p_t; \tag{54}$$

from where we get the steady-state rate of growth $G^U = G^U(\theta; \rho; \gamma_1; \gamma_2)$ (see the Appendix).

Proposition 6 $G^W > G^U$.

The existence of unions in both sectors increases wages, and thus the demand for labour in both sectors decreases and the demand for capital increases. However, the fall in production due to the lower amount of labour employed offsets the former effect, which in turn implies that the growth rate of the fully unionized economy is unambiguously lower.

Let us consider now the situation in which bequests are inoperative, i.e. $b_{t+1} = 0$. Unfortunately, the problem is not easily tractable, and we focus on the particular parameter configuration $\gamma_1 = \gamma_2 = \gamma$, and the walrasian and the fully unionized economies[6]. Though we lose information about steady-state growth rates, this simplifying assumption still will allow us to see how the results differ. The most important one indicates that the steady-state rate of growth of the fully unionized economy can be higher than the walrasian steady-state growth rate. This result is in contrast with the findings in proposition 6 because we obtain that even fully unionized economies can grow faster than competitive economies.

It is interesting to characterize the case $b_{t+1} = 0$ in terms of the altruism factor ρ given the intuition that relates this parameter and the "willingness" to leave bequests. Solving for the critical levels of altruism (see the Appendix), we obtain $\rho^*_W = \rho^*_W(\gamma, \beta)$ and $\rho^*_U = \rho^*_U(\gamma, \beta)$ for the walrasian and the fully unionized economies, respectively.

Lemma 7 Assume $\gamma_1 = \gamma_2 = \gamma$. Then $\rho^*_U > \rho^*_W$.

In principle, we should expect that agents are more likely to leave bequests when ρ is high, and viceversa.

Proposition 8 Assume $\gamma_1 = \gamma_2 = \gamma$. If $\rho \leq \rho^*_W$ then $b = 0$ in the balanced growth path of the walrasian economy.

Proposition 9 Assume $\gamma_1 = \gamma_2 = \gamma$. If $\rho > \rho^*_W$ then $b > 0$ in the balanced growth path of the walrasian economy.

What proposition 8 tells us is that when individuals care little about the utility of their children, they leave zero physical bequests. Whereas, when individuals' concern about their children's utility is high, bequests will be positive. The same results apply for the fully unionized economy. When bequests are inoperative, the solution to the consumer's maximization problem yields (see the Appendix) $G^W_C = G^W_C(\theta, \gamma, \rho, \beta)$ and $G^U_C = G^U_C(\theta, \gamma, \rho, \beta)$, where the subindex C denotes that is evaluated when bequests are zero.

Proposition 10 When bequests are inoperative (i.e. $\rho < \rho^*_W$) and $\gamma_1 = \gamma_2 = \gamma$, then, given $(\gamma, \rho, \beta) \in \Theta = (0,1) \times (0,1) \times (0,1)$, there exist two

[6] Even the simplifying assumption $\gamma_1 = \gamma_2 = \gamma$, we skip the analysis in the cases in which there is a union in one of the sectors and none in the other given the untractability of the algebra.

disjoint open subsets with positive measure, Ω_1 and Ω_2, such that $\Omega_1 \cup \Omega_2 = \Omega$. If $(\gamma;\rho;\beta) \in \Omega_1$ then $G_C^W > G_C^U$ and if $(\gamma;\rho;\beta) \in \Omega_2$ then $G_C^W \le G_C^U$.

The intuition behind this result is that when the economy is fully unionized, wages in both sectors increase, which in turn implies lower returns from human capital investment. At the same time, we have zero bequests, implying that agents increase the proportion of their income they devote to their childrens' education. Hence, the overall effect on the rate of growth is ambiguous.

5 Concluding Remarks

In this paper we study the relationship between growth and the role of unions in a two-sector OLG model with altruistic agents where endogeneous growth is generated by human capital accumulation in the educational sector.

This framework allows two possible situations: in the first one, the production of human capital is given by a linear technology on human capital and thus, there is no physical capital in the educational sector. For this reason, we can analyse two economies: in the first one, the walrasian case, we face a competitive economy without unions, whereas in the second one there are unions in the sector that produces consumption goods. We find that the rate of growth is higher in the unionized economy than in the walrasian one, and this holds no matter the operativeness of the bequest motive. It is also worth to emphasize that the level of altruism that makes the physical bequest motive to be or not operative is the same in both the walrasian and the unionized economies. This means that the degree of unionization of the economy does not affect the individual's decision about leaving physical bequests.

In contrast with the first situation, in the second one physical capital enters the production function for human capital accumulation as an input. This allows four possible cases: in the first one, the walrasian case, we face a competitive economy without unions, very much in contrast with the last one, where we find unions in each sector; there are two other similar, intermediate cases, with unions in one sector and none in the other (partially unionized economies). We show that, with strictly positive bequests, the rate of growth is higher in the walrasian case than in the completely unionized economy. However, the same conclusion might be reversed comparing the walrasian case to the partially unionized economies, that is, a partially unionized economy may grow faster than a walrasian economy. In turn, when bequests are inoperative, we get a different result, in the sense that even fully unionized economies can grow at higher rates than competitive ones. In this context, the level of altruism that makes the physical bequest motive to be operative or not is different for the walrasian and the fully unionized economies. Thus, given a certain level of altruism, the unionization degree affects the rate of growth of the economy via two channels: the direct effect on human capital investment and the indirect effect on the operativeness of the bequest motive.

Appendix

- Consumer's problem in the case of linear technology in the educational sector

We explain how to get equations (16), (17), (18W) and (18U). The FOC for s_t (16) is straightforward from (4). To obtain the FOC for b_{t+1}, (17), we differentiate (4) to get

$$\frac{\beta}{C_{t+1}^{t-1}} = \rho \left(\frac{\partial V_t^*(h_{t+1}; b_{t+1})}{\partial b_{t+1}} \right) : \tag{A.1}$$

Using the envelope theorem in (4) shifted one period ahead, we have

$$\frac{\partial V_t^*(h_{t+1}; b_{t+1})}{\partial b_{t+1}} = \frac{1}{C_{t+1}^{t}} : \tag{A.2}$$

Equation(A.2) into (A.1) for $\frac{\partial V_t^*(h_{t+1}; b_{t+1})}{\partial b_{t+1}}$ yields the FOC (17). To obtain (18W) and (18U), the FOC for h_{t+1}, we differentiate (4) to obtain

$$\frac{p_t}{C_t^{t-1}} = \rho \left(\frac{\partial V_t^*(h_{t+1}; b_{t+1})}{\partial h_{t+1}} \right) : \tag{A.3}$$

Using the envelope theorem in (4) shifted one period ahead, we get

$$\frac{\partial V_t^*(h_{t+1}; b_{t+1})}{\partial h_{t+1}} = \frac{w_{t+1}^a + h_{t+1}\frac{\partial w_{t+1}^a}{\partial h_{t+1}} - p_{t+1}\frac{\partial h_{t+2}}{\partial h_{t+1}}}{C_{t+1}^t} + \frac{\beta s_{t+1}\frac{\partial(1+r_{t+2})}{\partial h_{t+2}}\frac{\partial h_{t+2}}{\partial h_{t+1}}}{C_{t+2}^t}$$

$$+\rho \frac{\partial V_{t+1}^*(h_{t+2}; b_{t+2})}{\partial h_{t+2}} \frac{\partial h_{t+2}}{\partial h_{t+1}} : \tag{A.4}$$

From the human capital accumulation function,

$$\frac{\partial h_{t+2}}{\partial h_{t+1}} = \theta \overline{N}_{t+1} = \frac{h_{t+2}}{h_{t+1}} : \tag{A.5}$$

In the walrasian case, $w_t = \overline{w}_t$ and $N_t = 1 - \overline{N}_t$. From firms' profit maximizing conditions, plugging (6) into (15) and differentiating with respect to h_{t+1} yields

$$\frac{\partial w_{t+1}^a}{\partial h_{t+1}} = -\gamma_1 (1 - \gamma_1) K_{t+1}^{\gamma_1} N_{t+1}^{-\gamma_1} h_{t+1}^{-(1+\gamma_1)} = \frac{-\gamma_1 w_{t+1}}{h_{t+1}} : \tag{A.6W}$$

In the unionized economy, $w_t \left(\overline{N}_t - \gamma_1 \right) = \overline{w}_t \overline{N}_t$ and $N_t = 1 - \overline{N}_t$. This, together with (6) into (15) and differentiating with respect to h_{t+1} yields

$$\frac{\partial w_{t+1}^a}{\partial h_{t+1}} = -\gamma_1 (1 - \gamma_1)^2 K_{t+1}^{\gamma_1} N_{t+1}^{-\gamma_1} h_{t+1}^{-(1+\gamma_1)} = \frac{-\gamma_1 (1 - \gamma_1) w_{t+1}}{h_{t+1}} : \tag{A.6U}$$

From (5) we can obtain

$$\frac{\partial(1+r_{t+2})}{\partial h_{t+2}} = \gamma_1(1-\gamma_1)K_{t+2}^{\gamma_1-1}(N_{t+2}h_{t+2})^{1-\gamma_1}\frac{1}{h_{t+2}} = (1-\gamma_1)\frac{1+r_{t+2}}{h_{t+2}}. \tag{A.7}$$

Plugging equations (15), (A.5), (A.6W), (A.7) and (A.3) shifted one period ahead into (A.4) yields

$$\frac{\partial V_t^*(h_{t+1}; b_{t+1})}{\partial h_{t+1}} = \frac{(1-\gamma_1)w_{t+1}}{C_{t+1}^t} + \frac{\beta(1-\gamma_1)s_{t+1}\frac{1+r_{t+2}}{h_{t+1}}}{C_{t+2}^t}; \tag{A.8W}$$

and (A.8W) into (A.3) for $\frac{\partial V_t^*(h_{t+1}; b_{t+1})}{\partial h_{t+1}}$ gives the FOC (18W). Plugging equations (15), (A.5), (A.6U), (A.7) and (A.3) shifted one period ahead into (A.4) yields

$$\frac{\partial V_t^*(h_{t+1}; b_{t+1})}{\partial h_{t+1}} = \frac{(1-\gamma_1)^2 w_{t+1}}{C_{t+1}^t} + \frac{\beta(1-\gamma_1)s_{t+1}\frac{1+r_{t+2}}{h_{t+1}}}{C_{t+2}^t}; \tag{A.8U}$$

and (A.8U) into (A.3) for $\frac{\partial V_t^*(h_{t+1}; b_{t+1})}{\partial h_{t+1}}$ gives the FOC (18U).

• Proof of Proposition 1

The proof of the next two propositions is done below for the walrasian case. We skip the proof in the unionized case because of its similarity. Before proceeding with the proof, we will rewrite (16) and (17) in a more convenient way. From (16), (2), (3) and (14) we get

$$K_{t+1} = s_t = \frac{\beta}{1+\beta}(w_t^a h_t + b_t - p_t h_{t+1}) + \frac{b_{t+1}}{\beta(1+r_{t+1})}. \tag{A.9}$$

From (A.9), use $w_t^a = w_t = \overline{w}_t$, equations (5), (6), (10) and the rule of human capital accumulation, and divide by $h_{t+1}N_{t+1}$ to obtain in steady state

$$k = \frac{\beta(1-\gamma_1)}{1+\beta}\frac{k^{\gamma_1}}{g} + \frac{\beta}{1+\beta}\frac{b_t}{h_{t+1}N} + \frac{b_{t+1}}{\beta\gamma_1 h_{t+1}N}k^{1-\gamma_1}. \tag{A.10}$$

where k is the capital-efficient labour ratio, $g = g^W$ and $N = N^W$ but we supress the superindex for an easier treatment. From (16), (17) and (5), we get in steady state

$$g \geq \rho\gamma_1 k^{\gamma_1 - 1}. \tag{A.11}$$

To prove the proposition we proceed by contradiction (see Caballé 1995). Suppose the proposition is false, that is, $\rho \leq \bar{\rho}$ and $b^c > 0$. Then the equilibrium triplet $(g_c; k_c; b^c)$ must satisfy

$$k_c = \frac{\beta(1-\gamma_1)}{1+\beta}\frac{k_c^{\gamma_1}}{g_c} + \frac{\beta}{1+\beta}\frac{b_t^c}{h_{t+1}N_c} + \frac{b_{t+1}^c}{\beta\gamma_1 h_{t+1}N_c}k_c^{1-\gamma_1}. \tag{A.12}$$

and (A.11) holds with equality.

Define $\bar{k}$ as the unique positive solution when the non-negativity constraint is just binding in equilibrium, i.e., $b_i = 0$ for all i and (A.11) holds with equality. Hence, the associated savings function (A.10) becomes

$$\bar{k} = \frac{\beta(1-\gamma_1)}{1+\beta}\frac{\bar{k}^{\gamma_1}}{\bar{g}}. \tag{A.13}$$

Given that $b_i^c > 0$ for all i, from (A.12) and (A.13) we have

$$\frac{k_c^{\gamma_1-1}}{g_c} < \frac{\bar{k}^{\gamma_1-1}}{\bar{g}}.$$

Since $\rho \leq \bar{\rho}$, from (A.11) we get

$$\frac{k_c^{\gamma_1-1}}{g_c} \geq \frac{\bar{k}^{\gamma_1-1}}{\bar{g}},$$

which yields a contradiction. Therefore, (g_c, k_c) cannot characterize a balanced equilibrium path with positive physical bequests.

• Proof of Proposition 2

Suppose the proposition is false, i.e., $\rho > \bar{\rho}$ and $b_i^c = 0$ for all i. Then (A.11) holds with inequality:

$$\frac{1}{\gamma_1\rho} \geq \frac{k_c^{\gamma_1-1}}{g_c}. \tag{A.14}$$

By the definition of $\bar{\rho}$ we get

$$\frac{1}{\gamma_1\bar{\rho}} = \frac{\bar{k}^{\gamma_1-1}}{\bar{g}}. \tag{A.15}$$

Clearly, since $\rho > \bar{\rho}$, from (A.14) and (A.15) it follows that

$$g_c k_c^{1-\gamma_1} > \bar{g}\bar{k}^{1-\gamma_1}. \tag{A.16}$$

Since $b_i^c = 0$ for all i, from (A.10), we have

$$k_c = \frac{\beta(1-\gamma_1)}{1+\beta}\frac{k_c^{\gamma_1}}{g_c}. \tag{A.17}$$

From (A.13) and (A.17) we obtain

$$g_c k_c^{1-\gamma_1} = \bar{g}\bar{k}^{1-\gamma_1},$$

which contradicts (A.16).

- Consumer's problem in the case of physical capital in the educational sector

From problem (4), the first order conditions (FOC's) for s_t, b_{t+1} and h_{t+1} are given by equations (16), (17) and (38) respectively. The derivation of (38), the FOC for h_{t+1}, is as follows. From the human capital accumulation function,

$$\frac{\partial h_{t+2}}{\partial h_{t+1}} = (1-\gamma_2)\frac{h_{t+2}}{h_{t+1}}. \tag{A.18}$$

From firms' profit maximizing conditions, plugging (6) and (32) into (15) and differentiating with respect to h_{t+1} yields

$$\begin{aligned}\frac{\partial w^a_{t+1}}{\partial h_{t+1}} = & -\gamma_1(1-\gamma_1)K_{t+1}^{\gamma_1}N_{t+1}^{-\gamma_1}h_{t+1}^{-(1+\gamma_1)}N_{t+1} \\ & -\gamma_2(1-\gamma_2)\theta p_{t+1}\overline{K}_{t+1}^{\gamma_2}\overline{N}_{t+1}^{-\gamma_2}h_{t+2}^{-(1+\gamma_2)}\overline{N}_{t+1} \\ = & -\gamma_1 w_{t+1}\frac{N_{t+1}}{h_{t+1}}-\gamma_2\overline{w}_{t+1}\frac{\overline{N}_{t+1}}{h_{t+1}} = \frac{1}{h_{t+1}}\left(-\gamma_1 w_{t+1}N_{t+1}-\gamma_2\overline{w}_{t+1}\overline{N}_{t+1}\right);\end{aligned} \tag{A.19}$$

and from (31),

$$\frac{\partial(1+r_{t+2})}{\partial h_{t+2}} = (1-\gamma_2)\frac{1+r_{t+2}}{h_{t+2}}. \tag{A.20}$$

Equations (A.18),(A.19),(A.20) and (A.3) shifted one period ahead into (A.4) yields

$$\begin{aligned}\frac{\partial V_t^*(h_{t+1}, b_{t+1})}{\partial h_{t+1}} = & \frac{(1-\gamma_1)w_{t+1}N_{t+1}+(1-\gamma_2)\overline{w}_{t+1}\overline{N}_{t+1}-(1-\gamma_2)p_{t+1}\frac{h_{t+2}}{h_{t+1}}}{C^t_{t+1}} \\ & +\beta\frac{s_{t+1}(1-\gamma_2)^2\frac{1+r_{t+2}}{h_{t+1}}}{C^t_{t+2}}+\frac{p_{t+1}}{C^t_{t+1}}(1-\gamma_2)\frac{h_{t+2}}{h_{t+1}},\end{aligned} \tag{A.21}$$

and (A.21) into (A.3) for $\frac{\partial V_t^*(h_{t+1}, b_{t+1})}{\partial h_{t+1}}$ gives the FOC (38). Now we rearrange (38) to get a tractable equation; from (16) and (17) we get

$$\frac{1}{C_t^{t-1}} = \frac{\beta(1+r_{t+1})}{C_{t+1}^{t-1}} = \frac{\rho(1+r_{t+1})}{C^t_{t+1}}. \tag{A.22}$$

Plugging (16) shifted one period ahead and (A.22) into (38) and after rearranging

$$p_t = \frac{1}{1+r_{t+1}}\left\{\left[(1-\gamma_1)w_{t+1}N_{t+1}+(1-\gamma_2)\overline{w}_{t+1}\overline{N}_{t+1}\right]+(1-\gamma_2)^2\frac{s_{t+1}}{h_{t+1}}\right\}. \tag{A.23}$$

Now we try to get rid of s_{t+1} in (A.23). Combining equations (7), (33) and (36)

$$\frac{w_{t+1}}{1+r_{t+1}} = \frac{1-\gamma_1}{\gamma_1}K_{t+1}(h_{t+1}N_{t+1})^{-1} = \frac{1-\gamma_1}{\gamma_1}\left(s_t - \overline{K}_{t+1}\right)(h_{t+1}N_{t+1})^{-1}$$

$$= \frac{1-\gamma_1}{\gamma_1}\left(s_t - \frac{\gamma_2}{1-\gamma_2}\frac{\overline{w}_{t+1}}{1+r_{t+1}}h_{t+1}\overline{N}_{t+1}\right)(h_{t+1}N_{t+1})^{-1};$$

implying

$$\frac{\gamma_1}{1-\gamma_1}\frac{w_{t+1}}{1+r_{t+1}}h_{t+1}N_{t+1} + \frac{\gamma_2}{1-\gamma_2}\frac{\overline{w}_{t+1}}{1+r_{t+1}}h_{t+1}\overline{N}_{t+1} = s_t. \tag{A.24}$$

Substituting (A.24) into (A.23) for s_{t+1} yields

$$p_t = \frac{1}{1+r_{t+1}}\Bigg\{\Big[(1-\gamma_1)w_{t+1}N_{t+1} + (1-\gamma_2)\overline{w}_{t+1}\overline{N}_{t+1}\Big]$$

$$+ (1-\gamma_2)^2\frac{h_{t+2}}{h_{t+1}}\frac{1}{1+r_{t+2}}\left[\frac{\gamma_1}{1-\gamma_1}w_{t+2}N_{t+2} + \frac{\gamma_2}{1-\gamma_2}\overline{w}_{t+2}\overline{N}_{t+2}\right]\Bigg\} \tag{A.25}$$

- Derivation of the steady-state rate of growth G^W

In the competitive equilibrium we have $w_t = \overline{w}_t$ and $N_t = 1 - \overline{N}_t$. Besides, $1 + r_{t+1} = \frac{G^W}{\rho}$ from (A.22), and we can rearrange (A.25) as

$$p_t = \frac{1-\gamma_2}{\gamma_2}\frac{\overline{K}_{t+1}}{h_{t+1}\overline{N}_{t+1}}\Big[\overline{N}_{t+1}(\gamma_1-\gamma_2) + (1-\gamma_1)\Big]$$

$$+\frac{\rho(1-\gamma_2)^2}{\gamma_2(1-\gamma_1)}\frac{\overline{K}_{t+2}}{h_{t+2}\overline{N}_{t+2}}\Big[\overline{N}_{t+2}(\gamma_2-\gamma_1) + \gamma_1(1-\gamma_2)\Big]. \tag{A.26}$$

Substituting (A.26) into (41) for p_t and using $\frac{\overline{K}_t}{h_t\overline{N}_t} = \left(\frac{h_{t+1}}{h_t}\frac{1}{\mu\overline{N}_t}\right)^{\frac{1}{\gamma_2}}$ from the human capital accumulation function, we obtain

$$\left(\frac{h_{t+1}}{h_t\mu\overline{N}_t}\right)^{\frac{\gamma_1-\gamma_2}{\gamma_2}} = \left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_1}\left(\frac{1-\gamma_2}{1-\gamma_1}\right)^{1-\gamma_1}\frac{1-\gamma_2}{\gamma_2}\mu\Bigg\{\left(\frac{h_{t+2}}{h_{t+1}\mu\overline{N}_{t+1}}\right)^{\frac{1}{\gamma_2}}[\overline{N}_{t+1}(\gamma_1-\gamma_2)$$

$$+(1-\gamma_1)] + \frac{\rho(1-\gamma_2)}{(1-\gamma_1)}\left(\frac{h_{t+3}}{h_{t+2}\mu\overline{N}_{t+2}}\right)^{\frac{1}{\gamma_2}}\Big[\overline{N}_{t+2}(\gamma_2-\gamma_1) + \gamma_1(1-\gamma_2)\Big]\Bigg\}. \tag{A.27}$$

In the balanced growth path, the steady-state growth rate is given by $\frac{h_{t+1}}{h_t} = G^W$. Thus, (A.27) becomes

$$\left(\frac{G^W}{\mu\overline{N}}\right)^{\frac{\gamma_1-\gamma_2-1}{\gamma_2}} = \left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_1}\left(\frac{1-\gamma_2}{1-\gamma_1}\right)^{1-\gamma_1}\frac{1-\gamma_2}{\gamma_2}\mu\Big\{\Big[\overline{N}(\gamma_1-\gamma_2) + (1-\gamma_1)\Big]$$

$$+\frac{\rho(1-\gamma_2)}{(1-\gamma_1)}\left[\overline{N}(\gamma_2-\gamma_1)+\gamma_1(1-\gamma_2)\right]\Big\}. \tag{A.28}$$

Now, in order to find $\overline{N}^W$ in steady-state, from (A.23) and (A.24) , we get

$$\frac{1}{(1-\gamma_2)^2}\left\{p_{t-1}h_t(1+r_t)-h_t\left[(1-\gamma_1)w_tN_t+(1-\gamma_2)\overline{w}_t\overline{N}_t\right]\right\}$$
$$=\frac{h_{t+1}}{1+r_{t+1}}\left[\frac{\gamma_1}{1-\gamma_1}w_{t+1}N_{t+1}+\frac{\gamma_2}{1-\gamma_2}\overline{w}_{t+1}\overline{N}_{t+1}\right]. \tag{A.29}$$

From the human capital accumulation function and (32), we have that $p_{t-1}h_t=\frac{\overline{w}_{t-1}h_{t-1}\overline{N}_{t-1}}{1-\gamma_2}$; hence, (A.29) becomes

$$\frac{1}{(1-\gamma_2)^2}\left\{\frac{\overline{w}_{t-1}h_{t-1}\overline{N}_{t-1}}{1-\gamma_2}(1+r_t)-h_t\left[(1-\gamma_1)w_tN_t+(1-\gamma_2)\overline{w}_t\overline{N}_t\right]\right\}$$
$$=\frac{h_{t+1}}{1+r_{t+1}}\left[\frac{\gamma_1}{1-\gamma_1}w_{t+1}N_{t+1}+\frac{\gamma_2}{1-\gamma_2}\overline{w}_{t+1}\overline{N}_{t+1}\right]. \tag{A.30}$$

Introducing $w_t=\overline{w}_t=w$, $N_t=1-\overline{N}_t=1-\overline{N}^W$, and $1+r_t=\frac{G^W}{\rho}$ for all t, and dividing by h_{t-1}

$$\overline{N}^W=\frac{\rho(1-\gamma_2)\left[(1-\gamma_1)^2+\rho\gamma_1(1-\gamma_2)^2\right]}{(1-\gamma_1)+\rho(\gamma_1-\gamma_2)^2(1-\gamma_2)}. \tag{A.31}$$

Equations (A.28) and (A.31) define the rate of growth in this economy $G^W=G^W(\theta,\rho,\gamma_1,\gamma_2)$.

- Derivation of the steady-state rate of growth G^G

From equation (13) we have $w_tN_t=\frac{\overline{w}_t\overline{N}_t}{\overline{N}_t-\gamma_1}\left(1-\overline{N}_t\right)$, so that (A.26) becomes:

$$p_t=\frac{1-\gamma_2}{\gamma_2}\frac{\overline{K}_{t+1}}{h_{t+1}\overline{N}_{t+1}}\frac{\overline{N}_{t+1}}{\overline{N}_{t+1}-\gamma_1}\left[\overline{N}_{t+1}(\gamma_1-\gamma_2)+1-2\gamma_1+\gamma_1\gamma_2\right]$$
$$+\frac{\rho(1-\gamma_2)^2}{\gamma_2(1-\gamma_1)}\frac{\overline{K}_{t+2}}{h_{t+2}\overline{N}_{t+2}}\frac{\overline{N}_{t+2}}{\overline{N}_{t+2}-\gamma_1}\left[\overline{N}_{t+2}(\gamma_2-\gamma_1)+\gamma_1(1-2\gamma_2+\gamma_1\gamma_2)\right] \tag{A.32}$$

Substituting (A.32) into (44) for p_t and after some manipulations yields, in steady-state:

$$\left(\frac{G^G}{\theta\overline{N}^G}\right)^{\frac{\gamma_1-\gamma_2-1}{\gamma_2}}=\left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_1}\left(\frac{1-\gamma_2}{1-\gamma_1}\right)^{1-\gamma_1}\frac{1-\gamma_2}{\gamma_2}\theta\left\{\left[\overline{N}^G(\gamma_1-\gamma_2)+1-2\gamma_1+\gamma_1\gamma_2\right]\right.$$
$$\left.+\frac{\rho(1-\gamma_2)}{(1-\gamma_1)}\left[\overline{N}^G(\gamma_2-\gamma_1)+\gamma_1(1-2\gamma_2+\gamma_1\gamma_2)\right]\right\}\left(\frac{\overline{N}^G}{\overline{N}^G-\gamma_1}\right)^{2-\gamma_1} \tag{A.33}$$

Reasoning as in the previous case to find out $\overline{N}^G$ in steady-state:

$$\overline{N}^G = \frac{\rho(1-\gamma_2)[(1-\gamma_1)(1-2\gamma_1+\gamma_1\gamma_2)+\rho\gamma_1(1-\gamma_2)(1-2\gamma_2+\gamma_1\gamma_2)]}{(1-\gamma_1)+\rho(\gamma_1-\gamma_2)(1-\gamma_2)[\rho(1-\gamma_2)-(1-\gamma_1)]} \qquad \text{(A.34)}$$

Equations (A.33) and (A.34) define the rate of growth in this economy $G^G = G^G(\theta;\rho;\gamma_1;\gamma_2)$.

- Proof of Proposition 4

Substituting (A.31) into (A.28) and (A.34) into (A.33) for $\overline{N}^W$ and $\overline{N}^G$ respectively would give us G^W and G^G as a function of the parameters of the model, and direct comparison between these two rates of growth yields that $(\rho;\gamma_1;\gamma_2) \in \Omega_1$ if

$$\left(\frac{\rho(1-\gamma_2)\left[(1-\gamma_1)^2+\rho\gamma_1(1-\gamma_2)^2\right]}{\gamma_1(1-\gamma_1)+\rho(1-\gamma_2)[(1-\gamma_1)(1-2\gamma_1+\gamma_1\gamma_2)+\rho\gamma_1(1-\gamma_2)(1-2\gamma_2+\gamma_1\gamma_2)]}\right)^{\frac{1-\gamma_2}{\gamma_2}} > (1-\gamma_1).$$

Equivalently, $(\rho;\gamma_1;\gamma_2) \in \Omega_2$ if the above inequality does not hold.

- Derivation of the steady-state rate of growth G^E

The derivation is similar to that of G^W, getting:

$$\left(\frac{G^E}{\theta\overline{N}^E}\right)^{\frac{\gamma_1-\gamma_2-1}{\gamma_2}} = \left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_1}\left(\frac{1-\gamma_2}{1-\gamma_1}\right)^{1-\gamma_1}\left(\frac{1-\gamma_2}{\gamma_2}\right)\left(\frac{1-\gamma_2-\overline{N}^E}{1-\overline{N}^E}\right)^{1-\gamma_1}\theta(1-\gamma_2)$$
$$\left\{\left[\overline{N}^E(\gamma_1-\gamma_2)+(1-\gamma_1)\right]+\frac{\rho(1-\gamma_2)}{(1-\gamma_1)}\left[\overline{N}^E(\gamma_2-\gamma_1)+\gamma_1(1-\gamma_2)\right]\right\}; \qquad \text{(A.35)}$$

where

$$\overline{N}^E = (1-\gamma_2)\,\overline{N}^W: \qquad \text{(A.36)}$$

Equations (A.35), (A.36) and (A.31) define the rate of growth of this economy $G^E = G^E(\theta;\rho;\gamma_1;\gamma_2)$.

- Proof of Proposition 5

Comparing (A.35) to (A.28) using (A.36) and (A.31) to get rid of $\overline{N}^E$ and $\overline{N}^W$ would yield that $(\rho;\gamma_1;\gamma_2) \in \Lambda_1$ if

$$\left(\frac{\rho(1-\gamma_2)\{(\gamma_1-\gamma_2)[\rho(1-\gamma_2)-(1-\gamma_1)]-[(1-\gamma_1)^2+\rho\gamma_1(1-\gamma_2)^2]\}}{(1-\gamma_1)+\rho(1-\gamma_2)\{(\gamma_1-\gamma_2)[\rho(1-\gamma_2)-(1-\gamma_1)]-(1-\gamma_2)\left[(1-\gamma_1)^2+\rho\gamma_1(1-\gamma_2)^2\right]\}}\right)^{\frac{\gamma_2}{1-\gamma_2}} > (1-\gamma_2).$$

Equivalently, $(\rho;\gamma_1;\gamma_2) \in \Lambda_2$ if the above inequality does not hold.

- Derivation of the steady-state rate of growth G^U

The derivation follows the reasoning for that of G^W, getting:

$$\left(\frac{G^U}{\theta\overline{N}^U}\right)^{\frac{\gamma_1-\gamma_2-1}{\gamma_2}} = \left(\frac{\gamma_2}{\gamma_1}\right)^{\gamma_1}\left(\frac{1-\gamma_2}{1-\gamma_1}\right)^{2(1-\gamma_1)}\frac{1-\gamma_2}{\gamma_2}\theta\left\{\left[\overline{N}^U(\gamma_1-\gamma_2)\right.\right.$$

$$+ (1-\gamma_1)(1-\gamma_2)] + \frac{\rho(1-\gamma_2)}{(1-\gamma_1)}\Big[\overline{N}^U(\gamma_2-\gamma_1)+\gamma_1(1-\gamma_2)\Big]\Big\}, \quad (A.37)$$

where

$$\overline{N}^U = (1-\gamma_2)\overline{N}^W. \quad (A.38)$$

Equations (A.37), (A.38) and (A.31) define the rate of growth of this economy $G^U = G^U(\theta,\rho,\gamma_1,\gamma_2)$.

Note that there exists unemployment in this case, given by

$$U = 1-\overline{N}^U - N^U = \gamma_1 + \frac{\rho(\gamma_1-\gamma_2)(1-\gamma_2)^2\left[(1-\gamma_1)^2+\rho\gamma_1(1-\gamma_2)^2\right]}{(1-\gamma_1)+\rho(\gamma_1-\gamma_2)^2(1-\gamma_2)}. \quad (A.39)$$

- Proof of Proposition 6

Directly comparing (A.37) and (A.28) using (A.38) and (A.31).

- Derivation of the critical levels of altruism ρ^*_W and ρ^*_U and proof of Lemma 7

Consider the limiting case in which $b_{t+1} = 0$ but the first order condition (17) is just binding. Let us restrict our attention to the steady-state of the walrasian case. Equation (A.22) implies

$$1+r^* = \frac{G^*}{\rho}, \quad (A.40)$$

where the star denotes that is evaluated when bequests are zero. From (5), (40) and the human capital accumulation function, we obtain

$$1+r^* = \gamma\left(\frac{K}{h\overline{N}}\right)^{\gamma-1} = \gamma\left(\frac{G^*}{\theta\overline{N}^W}\right)^{\frac{\gamma-1}{\gamma}}. \quad (A.41)$$

Combining (A.40) and (A.41) we get

$$\rho\gamma = \left(\frac{G^*}{\left[\theta\overline{N}^W\right]^{1-\gamma}}\right)^{\frac{1}{\gamma}}. \quad (A.42)$$

Combining equations (A.41) and, from the derivation of the steady-state growth rates (A.58), we obtain

$$\left(\frac{G^*}{\left[\theta\overline{N}^W\right]^{1-\gamma}}\right)^{\frac{1}{\gamma}} = \frac{1-\gamma-\overline{N}^W}{1+\frac{1}{\beta}}. \quad (A.43)$$

From equations (A.42), (A.43) and, from the derivation of the steady-state growth rates (A.61), we get, solving for the critical level of altruism[7]

$$\rho^*_W = \frac{-\left[\gamma(1+\beta)+\beta(1-\gamma)^2\right]+\sqrt{\left[\gamma(1+\beta)+\beta(1-\gamma)^2\right]^2+4\beta^2\gamma(1-\gamma)^3}}{2\beta\gamma(1-\gamma)^2}.$$

A similar reasoning in the case of the fully unionized economy would give

$$\rho^*_U = \frac{-\left[\gamma(1+\beta)+\beta(1-\gamma)^2\right]+\sqrt{\left[\gamma(1+\beta)+\beta(1-\gamma)^2\right]^2+4\beta^2\gamma(1-\gamma)^2}}{2\beta\gamma(1-\gamma)^2}$$

• Proof of Proposition 8

Noting that $k = \bar{k}$, from equations (2), (3), (16), (32), (36), (40) shifted one period ahead, the rule for human capital accumulation and dividing by $h_{t+1}N_{t+1}$ we can obtain:

$$\frac{Gk^{-\gamma}}{\theta} + \frac{1+\beta}{\beta}Gk^{1-\gamma} = 1-\gamma + \frac{b_t}{h_{t+1}}Gk^{-\gamma} + \frac{1+\beta}{\beta^2}\frac{b_{t+1}}{(1+r_{t+1})\,h_{t+1}}Gk^{-\gamma}. \tag{A.44}$$

From (15), (16) and (17) we get:

$$G \geq \rho\gamma k^{\gamma-1}; \tag{A.45}$$

From (5) and (A.59), we can derive:

$$\frac{1}{(1-\gamma)^2\rho} = \frac{1+Gk^{1-\gamma}}{\bar{N}}. \tag{A.46}$$

To prove the proposition, we proceed by contradiction. Suppose not, that is, $\rho_s \leq \rho^*$ and $b^s > 0$. From (A.45) we have that

$$G^s k_s^{1-\gamma} \leq G^* k_*^{1-\gamma}; \tag{A.47}$$

where k_* is the unique positive solution when the non-negativity constraint is just binding in equilibrium, i.e., $b_i = 0$ for all i and (A.45) holds with equality. The savings function associated to k_* (A.44) becomes

$$\frac{G^* k_*^{-\gamma}}{\theta} + \frac{1+\beta}{\beta}G^* k_*^{1-\gamma} = 1-\gamma. \tag{A.48}$$

Since $\rho_s \leq \rho^*$; from (A.44) and (A.48) we get

$$\frac{G^s k_s^{-\gamma}}{\theta} + \frac{1+\beta}{\beta}G^s k_s^{1-\gamma} > \frac{G^* k_*^{-\gamma}}{\theta} + \frac{1+\beta}{\beta}G^* k_*^{1-\gamma}. \tag{A.49}$$

[7] We are left with a second degree equation in ρ, from where we pick the positive solution, given the assumption that $\rho > 0$.

From (A.47) and (A.49), and since from the human capital accumulation function $\frac{Gk^{-\gamma}}{\theta} = \bar{N}$, it follows

$$\bar{N}_s > \bar{N}_*. \tag{A.50}$$

Now, given that $\rho_s \leq \rho^*$, from (A.46) and (A.47) we have

$$\bar{N}_s \leq \bar{N}_*, \tag{A.51}$$

which contradicts (A.50).

• Proof of Proposition 9

Suppose the proposition is false, that is, $\rho_s > \rho^*$ and $b_i = 0$ for all i. Then (A.44) becomes

$$\frac{G^s k_s^{-\gamma}}{\theta} + \frac{1+\beta}{\beta} G^s k_s^{1-\gamma} = 1 - \gamma. \tag{A.52}$$

From (A.45) and the definition of k_*, we have that

$$G^s k_s^{1-\gamma} > G^* k_*^{1-\gamma}. \tag{A.53}$$

From (A.48), (A.52) and (A.53) we obtain, since $\frac{Gk^{-\gamma}}{\theta} = \bar{N}$,

$$\bar{N}_s < \bar{N}_*. \tag{A.54}$$

Now, from (A.46) and given that $\rho_s > \rho^*$,

$$\frac{1 + G^s k_s^{1-\gamma}}{\bar{N}_s} < \frac{1 + G^* k_*^{1-\gamma}}{\bar{N}_*}. \tag{A.55}$$

From (A.53) and (A.55) we have

$$\bar{N}_s > \bar{N}_*,$$

which contradicts (A.54).

• Derivation of the steady-state growth rates when $b_{t+1} = 0$

In the case of inoperative bequests, $b_{t+1} = 0$ and (18) holds with strict inequality. Combining (2), (3) and (16), we get

$$C_t = \frac{w_t^a h_t - p_t h_{t+1}}{1+\beta}. \tag{A.56}$$

Substituting (A.56) into (2) for C_t gives

$$s_t = \frac{(w_t^a h_t - p_t h_{t+1})\beta}{1+\beta}. \tag{A.57}$$

Substituting (15) and (A.24) into (A.57) for w_t^a and s_t respectively, and since $\gamma_1 = \gamma_2 = \gamma$ yields, after some manipulations,

$$\frac{\gamma}{1-\gamma}\frac{G_C^W}{1+r_{t+1}} = \frac{1-\frac{\overline{N}^W}{1-\gamma}}{1+\frac{1}{\beta}}. \tag{A.58}$$

From (38) and (16), noting that $\frac{C^t_{t+1}}{C^{t-1}_t} = G_C^W$, substituting for s_t from (A.24), and using $p_t h_{t+1} = \frac{\overline{w}_t h_t \overline{N}_t}{1-\gamma_2}$, $\gamma_1 = \gamma_2 = \gamma$, $w_t = \overline{w}_t = w$ for all t, $N_t = 1 - \overline{N}_t$ and dividing by h_t,

$$\frac{\gamma}{1-\gamma}\frac{G_C^W}{1+r_{t+1}} = \frac{1}{(1-\gamma)^2}\left(\frac{\overline{N}^W}{\rho(1-\gamma)} - (1-\gamma)\right). \tag{A.59}$$

Solving (A.58) and (A.59) for G_C^W and $\overline{N}^W$, using (5) and noting that $k = \overline{k}$, yields

$$G_C^W = (\rho\theta)^{1-\gamma}(1-\gamma)^{2-\gamma}\frac{\{\beta[1-\rho(1-\gamma)]\}^{\gamma}\{1+\beta+\beta(1-\gamma)\}^{1-\gamma}}{1+\beta+\rho\beta(1-\gamma)^2}; \tag{A.60}$$

$$\overline{N}^W = \frac{\rho(1-\gamma)^2[1+\beta+\beta(1-\gamma)]}{1+\beta+\rho\beta(1-\gamma)^2}. \tag{A.61}$$

The same reasoning for the case of the fully unionized economy would give us

$$G_C^U = (\rho\theta)^{1-\gamma}(1-\gamma)^{3(1-\gamma)}\frac{\left\{\beta\left[1-\rho(1-\gamma)^2\right]\right\}^{\gamma}\{1+2\beta\}^{1-\gamma}}{1+\beta+\rho\beta(1-\gamma)^2}; \tag{A.62}$$

$$\overline{N}^U = \frac{\rho(1-\gamma)^3(1+2\beta)}{1+\beta+\rho\beta(1-\gamma)^2}. \tag{A.63}$$

- Proof of Proposition 10

  - Comparing (A.60) and (A.62) we obtain that $(\rho;\gamma_1;\gamma_2) \in \Phi_1$ if $\left(\frac{1+2\beta-\gamma\beta}{1+2\beta}\right)^{1-\gamma}\left[\frac{1-\rho(1-\gamma)}{1-\rho(1-\gamma)^2}\right]^{\gamma} - (1-\gamma)^{1-2\gamma} > 0$. Equivalently, $(\rho;\gamma_1;\gamma_2) \in \Phi_2$ if the above inequality does not hold.

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