Firms’ operational costs, market entry and growth*

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Abstract

The industrial organizational literature identiﬁes operational costs as being an important determinant of industry evolution over time; however, it also shows that they can be endogenous and time-dependent. In this paper, we analyze the effects of endogenous and time-dependent operational costs on economic activity and, hence, on economic growth. We show that the particular nature of these costs determines the way in which the overall number of ﬁrms grows, which ultimately determines the pattern of economic growth. Our analysis differs from other approaches in that (i) a new ﬁrm is associated with the creation of a new product in such a way that a planned expenditure of resources is required (e.g., R&D), and (ii) an accumulation law for the growth of the number of ﬁrms is assumed. Hence, we show that growth can occur endogenously in an economy without any speciﬁc growth generating sector.

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1 Introduction

Dynamic models of industry evolution suggest that operational costs (also known as fixed or sunk costs) are an important determinant of firm behavior and, hence, of industry evolution over time. Since operational costs play a central role in determining the equilibrium structure of any industry, it follows that they are also an important determinant of economic activity and, hence, of economic growth. This relationship has been widely studied in the macroeconomic literature for the case of a constant operational cost, measured in terms of final goods and independent of the volume of output (see Matsuyama, 1995, among many others). However, since the publication of Sutton’s (1991) influential work, the industrial organization literature has shown that operational costs can be endogenous and time-dependent; see Shaked and Sutton (1987), Sutton (1989), Cabral (1995), Davies and Lyons (1996), Lyons, Matraves and Moffatt (2001), Amir and Lambson (2003), Vasconcelos (2006) and Ellickson (2007). In particular, Sutton (1991) focuses on advertising outlays as the premier type of endogenous operational costs, showing that there is a positive correlation between market concentration and operational costs. Not only are advertising and marketing costs associated with the generation of many products (see Spence, 1976), but they represent more than $280 billion (2006) in the United States alone, well above 2% of its gross domestic product (see Doraszelski and Markovich, 2007). In this paper, we analyze the effects of these endogenous and time-dependent operational costs on economic activity and, hence, on economic growth.

We analyze a monopolistically competitive economy characterized by the presence of both agglomeration economies and free market entry. In this economy, operational costs determine the number of firms, which in turn determines the level of aggregate production. Some operational costs depend exclusively on the technological or institutional characteristics of the economy, such as capital requirements or government regulations. Others, however, are affected by external variables, such as the particular market structure of the economy, including advertising or brand costs. Moreover, the endogenous evolution of the market might affect these costs via learning or imitation processes. As Kim (1997, 2004) argues, it is reasonable to think that operational costs and firm size are positively related, and that operational costs are therefore affected by production. Firm size is captured by Carlson, Fisher and Giammarino (2004) by making operational costs dependent on the firm’s level of capital. Thus, the exact form of the operational costs depends on their specific nature or origin, but at the same time it also seems to be related to the market conditions. This line of thinking is in accordance with Rivera-Batiz and Romer (1991), who point out that “Theoretical arguments dating from Adam Smith’s analysis of the pin factory have emphasized the potential importance of fixed costs and the extent of the market (p. 531)”.

The importance of agglomeration economies for growth has been broadly documented. For example, Nickell (1996) shows that an increase in the number of firms is associated with a higher rate of total factor productivity growth. Álvarez-Peláez and Groth (2005),
likewise, highlight the importance of the returns to specialization on growth. Ciccone and Hall (1996) find that doubling employment density increases average labor productivity by 6 percent, whereas Davis, Fisher and Whited (2014) estimate that the impact of local agglomeration on the growth rate is about 10 percent.

We show that, in the presence of agglomeration economies, the economy experiences growth when it becomes more capital intensive. Moreover, this outcome occurs when the number of firms grows. That is, competition drives growth, as Nickell (1996) has shown empirically. Thus, any mechanism whose effects result in an increase in the number of firms causes agglomeration economies to induce growth. In particular, two variables can affect entry in our economy. On the one hand, (exogenous) population growth increases the productivity of capital and, hence, profits. This promotes new entries. On the other hand, when operational costs are dependent on market conditions, both the number and the size of the firms may impact them. This would affect profits and so potentially sustain a continuous flow of new entries. Whereas population growth may originate semi endogenous growth, endogenous operational costs can originate endogenous growth. Specifically, we show that when operational costs are positively related to the industry’s firm size, represented by the capital level of the industry’s firms, endogenous growth occurs. Moreover, since there is a positive correlation between market concentration and operational costs (Sutton, 1991) and an increased number of competitors in the same industry is positively correlated with productivity growth (Nickell, 1996), the only functional form in our model that can simultaneously explain both types of evidence is the operational cost dependent on capital level.

The literature has pointed out the importance of market entry for production and trade; see Romer (1990), Jones (1995), Bilbiie, Ghironi and Melitz (2012), and Lewis and Poilly (2012) among many others. In these papers, a planned expenditure of resources is required in order to create a new firm (either R&D expenditure or specific labor dedicated to create new firms), which determines the operational costs and, in turn, aggregate production. In this paper, we analyze the other possible causality: that the evolution of the operational costs determines the number of firms, which in turn determines aggregate production. Therefore, we show that growth can occur endogenously in an economy without any specific sector devoted to generating growth.

Schumpeterian models of growth (see Aghion and Howitt, 1992) show a negative relation between competition and growth, since an increase in competition decreases monopoly rents, which in turn decreases innovators and, hence, growth. Subsequent variants of models of this type, such as Aghion, Dewatripont and Rey’s (1997), give ambiguous predictions on the relation between competition and growth. However, a positive correlation between competition and productivity growth has been shown empirically. While Geroski (1995) and Blundell, Griffith and Van Reenen (1999) observe a positive relation between competition and innovative activity, Nickell (1996) presents evidence that an increased number of competitors in the same industry is positively correlated with productivity growth. In our paper, an increase in the number of firms affects operational costs, which in turn
raises productivity. Since in our economy a new firm means a new product, and given that Bernard, Redding and Schott (2010) show that product creation (in new and existing firms) accounts for almost 50 percent of output in a five-year interval, competition could explain an important part of income growth.

In the next section, we present a reduced-form model that allows us to identify clearly the causes of growth in an economy with free entry. In section 3, we study an economy with a competitive final sector and a monopolistically competitive intermediate sector with free entry. Section 4 analyzes the relationships among the variables that affect growth and illustrates the importance of operational costs for determining different patterns of growth through different examples. In section 5 we discuss the results, and section 6 concludes.

2 A reduced-form model of growth

The relationship between market structure and operational costs is complex, since the former affects the latter, which in turn impacts on operational costs. In order to present our results, we describe below a reduced-form model to show how this interaction might induce growth in an economy where a monopolistically competitive sector produces intermediate goods that are used in a competitive sector.

Consider an economy with a unique final good $Y_t$ that is produced by competitive firms using a continuum of intermediate goods (that are not perfect substitutes). In each period $t$, a final goods firm maximizes

$$Y_t - \int_0^{z_t} p_i x_i dt,$$

where $x_{it}$ and $p_{it}$ are the quantity and price of the intermediate good $i$ in period $t$, respectively, and $z_t$ is the number of intermediate goods in period $t$, which is taken as given by the final goods firms. We have normalized the final goods price to one.

Each intermediate firm produces at most one good. In order to operate, these firms have to pay an operational cost $\psi_t$. Since intermediate goods are not perfect substitutes, these firms face a downward sloping demand curve which grants them some degree of market power. This implies that, after maximizing profits and paying for inputs, the profits function $\pi_{it}$ can be written as

$$\pi_{it} = \eta p_{it} x_{it} - \psi_t,$$

where $\eta$ measures the degree of market power or mark-up, and $\eta p_{it} x_{it}$ are monopoly rents. We have assumed that the operational cost is measured in terms of the final good.

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1 We assume that the operational cost lies completely outside the scope of the firm’s decision taking. In the next section, we study the case where the operational cost is partially dependent on the firm’s decision taking.
Since there is perfect competition in the final goods sector, in a symmetric equilibrium where all firms produce the same quantity \( x_{it} = x_t \), set the same price \( p_{it} = p_t \), and have the same profits \( \pi_{it} = \pi_t \), we have

\[
Y_t = z_t p_t x_t.
\]

(3)

Suppose there is free entry in the intermediate goods sector. Thus, in each period new intermediate goods producers may enter and produce a new variety. The total number of firms \( z_t \) is determined by the zero-profit condition. Applying this condition to Eq.(3), the gross domestic product \( GDP_t \) can be written as

\[
GDP_t = Y_t - z_t \psi_t = \left( \frac{1 - \eta}{\eta} \right) z_t \psi_t.
\]

(4)

Total population \( N_t \) grows at a constant rate, so that \( N_{t+1}/N_t = n \). Hence, assuming a constant mark-up \( \eta \), per capita GDP growth is

\[
\gamma_{gdp_{t+1}} = \gamma_{z_{t+1}} \gamma_{\psi_{t+1}} \frac{n}{\gamma_{ht_{t+1}}},
\]

where \( gdp_{t+1} = GDP_{t+1}/N_{t+1} \), and \( \gamma_{ht_{t+1}} = h_{t+1}/h_t \) denotes the growth in variable \( h \) between \( t + 1 \) and \( t \).

Eq.(5) relates growth to the evolution of population, operational costs and number of firms. This relationship can be interpreted in different ways. A first approach, taken by Romer (1990), Jones (1995), Bilbiie, Ghironi and Melitz (2012), and Lewis and Poilly (2012) among others, considers that a planned expenditure of resources is required (e.g., R&D) in order to create a firm. Then, it is the evolution of the number of firms \( \gamma_{z_{t+1}} \) that determines the evolution of the operational costs \( \gamma_{\psi_{t+1}} \) and, hence, growth \( \gamma_{gdp_{t+1}} \).

A second approach, developed in this paper, analyzes the opposite direction. That is, it is the evolution of the operational costs \( \gamma_{\psi_{t+1}} \) that determines the growth in the number of firms \( \gamma_{z_{t+1}} \) and, hence, growth \( \gamma_{gdp_{t+1}} \). Thus, free entry in the intermediate goods sector makes \( \gamma_{z_t} \) dependent on the particular specification and evolution of \( \psi_t \).

3 Market economy

We construct an economy with returns to specialization or agglomeration economies to analyze how new firm entries determine the economy’s growth rate. We build on the model proposed by Coto-Martínez, Garriga and Sánchez-Losada (2007), where the number of firms is endogenous. We introduce in that model the possibility of both population growth and operational costs being dependent on market conditions. The economy has two sectors: an intermediate goods sector with monopolistically competitive firms, and a competitive final goods sector where firms combine the intermediate goods à la Dixit-Stiglitz, but we separate the agglomeration economies from the monopolistic mark-up, as proposed by Dixit and Stiglitz (1975), Ethier (1982) and Benassy (1996).

\footnote{We discuss this claim in section 5.}
**Final goods production:** There is a unique final good which is produced by competitive firms through a continuum of intermediate goods, with the following technology (as in Benassy, 1996):

\[
Y_t = \left( z_t^{v(1-\eta)-\eta} \int_0^{z_t} x_{it}^{1-\eta} di \right)^{\frac{1}{1-\eta}}, \quad \eta \in (0, 1), \ v \geq 0, \quad (6)
\]

where \(\eta\) is the inverse of the elasticity of the demand for each intermediate good. In a symmetric equilibrium, all firms in the intermediate goods sector will produce the same output level \(x_t\) and, thus, aggregate output will be \(Y_t = z_t^{v+1} x_t\). Then, the elasticity of output with respect to the number of firms \(z_t\) is given by the degree of *agglomeration economies*, \(v\). This parameter measures the intensity with which society benefits from increasing economic density. As a result, an increase in the number of intermediate goods improves the total factor productivity of the final goods technology. This formulation allows us to separate the effect of the mark-up from that of the agglomeration economies.

The representative firm in the final goods sector takes the number of intermediate goods as given and solves

\[
\max_{\{x_{it}\}} z_t^{(1-\eta)-\eta} \left( \int_0^{z_t} x_{it}^{1-\eta} di \right)^{\frac{1}{1-\eta}} - \int_0^{z_t} p_{it} x_{it} di, \quad (7)
\]

from which the demand function for each intermediate input is

\[
x_{it} = (p_{it})^{-\frac{1}{\eta}} z_t^{\frac{1}{\eta}(1-\eta)} Y_t, \quad (8)
\]

**Intermediate goods production:** Each intermediate goods firm solves

\[
\max_{\{p_{it}, K_{it}, L_{it}\}} \pi_{it} = p_{it} x_{it} - (1 + r_t) K_{it} - w_t L_{it} - \psi_{it}, \quad (9)
\]

subject to the final goods sector demand, Eq.(8); where \(x_{it} = K_{it}^{1-\alpha} L_{it}^\alpha\), \(K_{it}\) and \(L_{it}\) are the capital and labor used by firm \(i\), respectively, \(\alpha \in (0, 1)\), \(w_t\) is the wage, and \(r_t\) is the interest rate. We have assumed that there is complete capital depreciation. By allowing the operational cost to depend (partially) on the firm’s decisions, the associated first-order conditions of the firm’s problem yield

\[
1 + r_t = p_{it} (1 - \eta) (1 - \alpha) K_{it}^{-\alpha} L_{it}^\alpha - \psi_{K_{it}}, \quad (10)
\]

and

\[
w_t = p_{it} (1 - \eta) \alpha K_{it}^{1-\alpha} L_{it}^{-1} - \psi_{L_{it}}, \quad (11)
\]

where \(\psi_h\) denotes the partial derivative of \(\psi_i\) with respect to \(h\).

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3We maintain the notation from the previous section.
4The conventional formulation established by Dixit and Stiglitz (1977) corresponds to the case \(v = \eta / (1 - \eta)\), where there is a one-to-one relationship between the market power and the degree of agglomeration economies.
In a symmetric equilibrium, where \( K_{it} = K_t \) and \( L_{it} = L_t \), final output is equal to

\[
Y_t = z_t^{v+1} K_t^{1-\alpha} L_t^\alpha = z_t^{v+1} x_t,
\]

and the price, by substituting Eq. (12) into Eq. (8), is

\[
p_t = z_t^\nu.
\]

Since there is free entry in the intermediate goods sector, each firm has zero profits. Thus, using Eq. (13), we obtain

\[
\eta z_t^\nu K_t^{1-\alpha} L_t^\alpha = \psi_t - \psi_{K_t} K_t - \psi_{L_t} L_t.
\]

In economics of this type, the entry of a new firm into the market has two direct and opposite effects on the incumbent firms: a complementary effect and a business-stealing effect. The entry of a new firm increases the aggregate productivity, which in turn increases the incumbent firms’ demand. This is the complementary effect. At the same time, the presence of a new intermediate firm increases competition, which decreases the incumbent firms’ demand. This is the business-stealing effect. Although neither of these effects are considered by entrants, they ultimately determine both the number and the size of the firms. Moreover, two variables may affect entry. On the one hand, (exogenous) population growth increases the productivity of capital and, hence, profits. This promotes new entries. On the other hand, when operational costs are dependent on market conditions, both the number and the size of firms may impact them. This would affect profits and so potentially sustain a continuous flow of new entries. We refer to this effect as the general equilibrium effect.

In our economy, the presence of agglomeration economies \( \nu > 0 \) means that the entry of a new firm reduces the relative price between final output and intermediate goods \( 1/p_t = z_t^{-\nu} \), thus, making entry more profitable. However, new entries allow the final goods producers to choose from among a greater variety of inputs, which decreases the demand of the incumbent firms. Thus, the interaction between the complementary effect and the business-stealing effect would determine both the number and the size of the firms. But this interaction can originate a general equilibrium effect, as it may impact the operational cost \( \psi_t \).

**Consumers:** We assume Solow individuals: at any period \( t \), each individual \( j \) saves a constant fraction of her income \( R_i^j \) and is endowed with one unit of labor that she supplies inelastically. Therefore, savings for individual \( j \) are

\[
S_i^j = sR_i^j,
\]

where \( s \in (0, 1) \) is the constant propensity to save.\(^5\)

\(^5\)Litina and Palivos (2010) characterize the class of production and utility functions that makes the infinite horizon model isomorphic to that of Solow; that is, where a constant propensity to save arises. In particular, the Cobb-Douglas production function used in this paper belongs to this set.
Labor market clearing condition: In equilibrium, labor demand and supply coincide; that is,

\[ z_t L_t = N_t. \] (16)

As population grows at a constant rate, we have

\[ \gamma_{z_{t+1}} \gamma_{L_{t+1}} = \gamma_{N_{t+1}} = n. \] (17)

Capital market clearing condition: The amount saved by individuals at \( t \) equals the stock of physical capital at \( t + 1 \); that is,

\[ \int_0^{N_t} S_t^t dj = \int_0^{N_t} R_t^t dj = z_{t+1} K_{t+1}. \] (18)

Noting that \( \int_0^{N_t} R_t^t dj = w_t N_t + (1 + r_t) z_t K_t = z_t [w_t L_t + (1 + r_t) K_t] = (1 - \eta) z_t^{v+1} K_t^{1-\alpha} L_t^\alpha - z_t (\psi_K K_t + \psi_L L_t) \), where we have used the definition of the individuals income and Eqs (10), (11), (13) and (16), the previous equation becomes

\[ s \left[ (1 - \eta) z_t^{v+1} K_t^{1-\alpha} L_t^\alpha - z_t (\psi_K K_t + \psi_L L_t) \right] = z_{t+1} K_{t+1}, \] (19)

or, using Eq.(14),

\[ s GDP_t = s (Y_t - z_t \psi_t) = z_{t+1} K_{t+1}. \] (20)

Balanced Growth Path (BGP): The dynamics of the model can be reduced to the capital accumulation Eq.(19) and the free entry condition Eq.(14), which using Eqs (16) and (17) can be written as

\[ \eta n \frac{\gamma_{K_{t+1}}}{\gamma_{L_{t+1}}} = s \left[ (1 - \eta) \frac{\psi_t}{K_t} - \psi_K - \psi_L \frac{L_t}{K_t} \right]. \] (21)

and

\[ \eta N_t^{\psi} K_t^{1-\alpha} L_t^\alpha = \psi_t \frac{L_t}{K_t} - \psi_K - \psi_L \frac{L_t}{K_t}. \] (22)

Thus, for a positive BGP to exist, the right-hand side of Eq.(21) must be asymptotically a positive constant.

4 Operational costs and growth

We next analyze how both population growth and endogenous operational costs can explain the existence of the general equilibrium effect (i.e., whether the number and the size of the firms affect the operational costs) and, therefore, different patterns of growth. In particular, we show that growth is explained by increasing competition among intermediate firms, measured in terms of market share \( 1/z \). This result is consistent with Nickell (1996), where competition is positively correlated to total factor productivity growth.
In order to study the impact of operational costs on growth patterns, henceforth we consider the operational cost function as belonging to the family of Cobb-Douglas functions; that is, \( \psi_t = \psi (z_t, k_t, l_t, z_{t-1}, k_{t-1}, l_{t-1}, \ldots) \cdot K_t^a L_t^b \), where \( k_t \) and \( l_t \) denote the average level of capital and labor per firm, respectively. Then, using Eqs (16) and (17), Eqs (21) and (22) can be written as

\[
\eta \gamma_{K_{t+1}} \gamma_{z_{t+1}} = s \left[ 1 - a - b \right] \frac{\psi_t}{K_t} \tag{23}
\]

and

\[
\eta z_t^a K_t^{-\alpha} L_t^\alpha = [1 - a - b] \frac{\psi_t}{K_t}, \tag{24}
\]

from where we have to assume that \( a + b < 1 - \eta \) for a BGP to exist. When the right-hand side of Eq.(23) is constant, the right-hand side of Eq.(24) is also constant. This implies that in any BGP the ratio \( \psi_t/K_t \) must be constant; that is,

\[
\gamma_{\psi} = \gamma_K. \tag{25}
\]

Moreover, from Eqs (24) and (25) we obtain

\[
\gamma_{z}^v = \gamma_{\psi} \gamma_K^{-1} \gamma_{z}^\alpha = \gamma_{\psi} \gamma_{L}^{-\alpha} = \gamma_{K/L}^\alpha. \tag{26}
\]

From Eqs (16) and (20) we also have \( gdp_t = z_{t+1} K_{t+1}/sz_t L_t \), which implies\(^6\)

\[
\gamma_{gdp} = \gamma_K \gamma_{L}^{-1} = \gamma_{K/L}. \tag{27}
\]

Hence, from the previous equations we have

\[
\gamma_{gdp} = \gamma_z^s. \tag{28}
\]

The economy experiences positive growth when it becomes more capital intensive; i.e., \( \gamma_{K/L} > 1 \). Moreover, \( \gamma_{K/L} > 1 \) turns out to be the case when \( \gamma_{z} > 1 \). That is, competition drives growth. Note that the previous equation clearly informs us about the positive effect of agglomeration economies on GDP per capita growth; that is, a greater number of firms is the source of growth. Thus, any mechanism whose effects are translated into \( \gamma_{z} > 1 \) will induce positive growth.

Depending on the exact form of \( \psi_t \), the economy may experience different patterns of growth. A useful point of departure is given by considering the familiar setting in which the level of operational costs is treated as a constant; that is, \( \gamma_{\psi_t} = 1 \) for all \( t \). Then, from Eq.(25) we have \( \gamma_K = 1 \) and, therefore, using Eqs (17) and (26) we obtain \( \gamma_z = n^{\alpha/(\alpha - v)} \). Hence, Eq.(28) gives

\[
\gamma_{gdp} = n^{v/(\alpha - v)}. \tag{29}
\]

Thus, it is clear that when \( v < \alpha \) the mechanism that activates the agglomeration economies is population growth. That is, semi endogenous growth occurs, which is a

\(^6\)Note that Eqs (17), (25) and (27) give Eq.(5).
standard result in variety-based growth models.\textsuperscript{7} When population does not grow, \( n = 1 \), there is no mechanism to activate the agglomeration economies. Hence, the number of firms remains unchanged and, consequently, the economy converges to a steady state, as in Coto-Martínez et al. (2007). Therefore, when operational costs are constant, population growth is required to activate the agglomeration economies and promote growth. In the presence of agglomeration economies \( v > 0 \), the complementary effect reinforces entry. As a consequence, since the operational cost is constant, a new entry implies that each firm will produce less quantity but at a higher price. As a result, firms hire less labor\textsuperscript{8} and become more capital intensive, implying a higher labor productivity, which in turn makes output per capita grow. In contrast, in the absence of agglomeration economies \( v = 0 \), the economy converges to a steady state where population growth only varies the absolute size of the economy. The complementary effect is exactly compensated by the business-stealing effect. Hence, population growth acts as the mechanism that continuously increases the number of firms in the economy and, through the agglomeration economies, induces growth. Note that, since the operational cost is constant, there is no general equilibrium effect.

It should be noticed that a constant operational cost is sufficient to obtain semi endogenous growth, but this condition is far from necessary. In fact, many specifications of the operational cost function can be obtained where population growth is also required to obtain growth. By way of example, consider a situation where operational costs are positively related to the capital intensiveness of the industry’s firms, such that \( \psi_t = \psi \left( \frac{K_t}{L_t} \right)^\mu \left( \frac{k_{t-1}}{l_{t-1}} \right)^{1-\mu} \), where \( \psi \) is a positive constant. In this case, since in equilibrium \( k_{t-1} = K_{t-1} \) and \( l_{t-1} = L_{t-1} \), we have \( \gamma_{\psi} = \gamma_K \gamma_{L}^{-1} \), implying, from Eq.(25), that \( \gamma_L = 1 \).\textsuperscript{9} Then, Eqs (17), (26) and (28) yield \( \gamma_z = n, \gamma_K = n^v/\alpha \) and

\[
\gamma_{gdp} = n^\frac{v}{\alpha}.
\]

In this case, each intermediate firm produces the same quantity but at a higher price and, at the same time, they become more capital intensive. When population does not grow, there is no mechanism to activate the agglomeration economies and, then, the economy converges to a steady state as in the case of constant operational costs.

Although \( n > 1 \) is required in the previous examples to obtain growth, it may also be the case that it does not suffice. If operational costs are related to aggregate capital, such that \( \psi_t = \psi \left( z_t k_t \right)^\mu \left( z_{t-1} k_{t-1} \right)^{1-\mu} \), where \( \mu \in [0,1] \), we have \( \gamma_{\psi} = \gamma_K \gamma_{z} \), implying, from Eqs (25) and (28), that \( \gamma_z = 1 \) and

\[
\gamma_{gdp} = 1.
\]

Although population grows, agglomeration economies cannot be activated when the number of firms remains constant. As a consequence, growth cannot occur.

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\textsuperscript{7}This same result would also be obtained by Coto-Martínez et al. (2007) if allowing for population growth.

\textsuperscript{8}From Eq.(17) it is direct to check that \( \gamma_L = n^{-v/(\alpha-v)} \).

\textsuperscript{9}Note that the case \( \psi_t = \psi \left( K_t L_t \right)^\mu \left( k_{t-1} l_{t-1} \right)^{1-\mu} \) would yield the same result.
Our point is that endogenous growth can also be obtained from the competition promoted by the existence of endogenous operational costs. Noting that, from Eq. (25), $\gamma_\psi = \gamma_K$ is required for a BGP to exist, an immediate specification of an operational cost function satisfying this condition would be $\psi_t = \psi K^\mu k^{-1}_{t-1}$. This specification resembles the quality cost function proposed by Peretto (2007) in a Schumpeterian growth model, $\psi_t = \psi Z^\mu Z^{1-\mu}$, where firms decide on the amount of R&D that affects the quality of their product $Z_t$, and $Z$ is the average quality of the products in the industry.$^{11}$ Also, Romer (1990) assumes that total factor productivity is increasing in the capital stock of the economy (justified in terms of a learning-by-doing process). The crucial element in the specification of $\psi_t = \psi K^\mu k^{-1}_{t-1}$ is that, when translating into growth rates, it does not add any restriction on $\gamma_z$ and $\gamma_L$, since it satisfies the condition for a BGP to exist, $\gamma_\psi = \gamma_K$ (properties of this type are usually called knife-edge conditions). In this case, in the BGP where $k_t = K_t$, and since the left-hand side of Eq. (24) must be constant in a BGP, using Eq. (17) we can write Eqs (23) and (24) as $\gamma_K = 2^{-\mu} / [s (1 - \eta - \mu) \psi] / \eta \gamma_z$ and $\gamma_K = \gamma_z (v - \alpha) / \alpha n$. Thus, solving these two equations and using Eq. (28) we obtain

$$\gamma_{gdp} = \left( \frac{\eta}{s \psi (1 - \eta - \mu)} \right)^{n(2-\mu)} n^{(1-\mu)/(\nu(2-\mu))}$$

implying that, when $v < \alpha (1 - \mu) / (2 - \mu)$ and $\psi < \eta / s (1 - \mu - \eta)$, there is endogenous growth, which is enhanced as population grows. A sufficiently low unit operational cost is needed for new firms entering the market. Otherwise, operational costs cannot be compensated for by monopoly rents. Now, the mechanism that induces growth through new entries is the endogenous evolution of the operational costs. The rationale is that, in a situation where each firm produces less quantity but at a higher price, the general equilibrium effect causes operational costs to fall. As a consequence, the number of firms grows. Note that the existence of a complementary effect that reinforces entry is still necessary to obtain endogenous growth. Otherwise, the general equilibrium effect cannot be spread into a higher capital intensity.

5 Discussion

In order to shed light on the precise mechanisms that induce growth in our economy, we next discuss some aspects that are present in the models, ours included, in which economic growth is dependent on growth in the number of firms.

$^{10}$ Different operational cost functions, such as $\psi_t = \psi K^\mu k^{-1}_{t-1}$, $\psi_t = \psi K^\mu k^{-1}_{t-1} (L_t/L_{t-1})^\nu$, $\psi_t = \psi K^\mu k^{-1}_{t-1} (L_t/L_{t-1})^\nu$, and $\psi_t = \psi K^\mu k^{-1}_{t-1} (z_t/z_{t-1})^\nu$, give the same qualitative conclusions.

$^{11}$ In this case, however, $Z_t$ affects directly and positively the demand of the firm, which contrasts with our variety-based growth model where operational costs affect the demand of firms indirectly through the complementary effect.
5.1 The nature of the operational cost

In all the examples discussed (and in most of the papers cited), operational costs are measured in terms of the final good. Alternatively, they might be measured in terms of the intermediate good, as in Kim (1997, 2004). This enables us to identify a different mechanism that induces growth and so clarify the causes of growth.

The basic difference with the situation considered in the previous section is that now the operational cost is linearly affected by the intermediate good price. In particular, the (net) intermediate good production of a firm $i$ is now given by $K_{it}^{1-\alpha}L_{it}^\alpha - \psi_t$. Replicating the calculations presented in the previous section, the equations characterizing the BGP reduce to

$$\eta \gamma_{K_{t+1}} = s (1-\eta) z_t^v \left( \frac{\psi_t}{K_t} - \psi_{K_t} - \psi_{L_t} \frac{L_t}{K_t} \right)$$

(33)

and

$$\eta K_t^{-\alpha} L_t^\alpha = \frac{\psi_t}{K_t} - (1-\eta) \left( \psi_{K_t} + \psi_{L_t} \frac{L_t}{K_t} \right).$$

(34)

Using Eq.(17) and the Cobb-Douglas operational cost function, the previous equations can be written as

$$\eta \gamma_{K_{t+1}} \gamma_{z_{t+1}} = s (1-\eta) (1-a-b) z_t^v \frac{\psi_t}{K_t}$$

(35)

and

$$\eta K_t^{-\alpha} L_t^\alpha = [1 - (1-\eta) (a+b)] \frac{\psi_t}{K_t}.$$  

(36)

Since the right-hand side of Eq.(35) has to be constant for a BGP to exist, we have

$$\gamma_K = \gamma_z^v \gamma_\psi.$$  

(37)

From Eqs (17) and (36), we obtain

$$\gamma_K^{1-\alpha} n^\alpha \gamma_z^{-\alpha} = \gamma_\psi,$$

(38)

and combining the two previous equations yields

$$\gamma_K^\alpha = n^\alpha \gamma_z^{v-\alpha}.$$  

(39)

Noting that now $GDP_t = z_t^{v+1} \left( K_t^{1-\alpha} L_t^\alpha - \psi_t \right)$, using Eq.(36) we have $\eta GDP_t = (1-\eta) (1-a-b) z_t^{v+1} \psi_t$, from where

$$\gamma_{gdp} = n^{-1} \gamma_z^{v+1} \gamma_\psi.$$  

(40)

Then, combining Eqs (17), (37), (39) and (40), we obtain

$$\gamma_{gdp} = \gamma_z^v = \gamma_K/L.$$  

(41)

Note that, since the nature of the operational cost has changed, Eq.(5) does not hold.
Although the relationship between GDP per capita and the number of firms remains the same as in the previous section, the capital accumulation condition Eq.(37) now implies that $\gamma_z = 1$ when $\gamma_K = \gamma_\psi$. Thus, an operating cost function of the form $\psi_t = \psi K_t^\mu k_{t-1}^{1-\mu}$ induces no growth even if population growth is positive. That is,

$$\gamma_{gdp} = 1,$$

implying $\gamma_K = n = \gamma_L$. Output per capita does not grow. This happens because now the complementary effect does not reinforce entry, since the relative price of the operational costs is independent of $z_t$. Consequently, the general equilibrium effect cannot be extended into higher capital intensity and, therefore, the number of firms remains unchanged even if there are agglomeration economies. This shows that when the general equilibrium effect fails to induce a complementary effect that reinforces entry, growth is not possible. Moreover, as the general equilibrium effect does not affect the price of intermediate goods since $\gamma_z = 1$, then the capital-labor ratio and, therefore, the productivity of capital remain unchanged. Hence, population growth does not necessarily promote entry, meaning that it may be absorbed by the existing firms.

Although the general equilibrium effect becomes inoperative when operational costs are measured in terms of intermediate goods, population growth can activate the agglomeration economies. However, it may also be the case that it does not suffice. We illustrate this claim by considering the same examples as in the previous section. Under constant operational costs, however, from the capital accumulation condition Eq.(37) and the free entry condition Eq.(38) we obtain $\gamma_K = n^{v\alpha/(\alpha-v(1-\alpha))}$ and $\gamma_z = n^{\alpha/(\alpha-v(1-\alpha))}$, which combined with Eq.(41) gives

$$\gamma_{gdp} = n^{\alpha-v(1-\alpha)}.$$  

We have the same qualitative growth rates as those in the previous section, with the exception that now capital per firm increases. Population growth induces a continuous growth of firms so that the agglomeration economies are activated.

For the other operational costs specifications analyzed in the previous section, we find similar results. In particular, if operational costs are $\psi_t = \psi (z_t k_t)^\mu (z_{t-1} k_{t-1})^{1-\mu}$, we have $\gamma_\psi = \gamma_z \gamma_K$. Hence, using Eqs (17), (37) and (38), we obtain $\gamma_z = 1$, $\gamma_L = n$ and $\gamma_K = n$, yielding $\gamma_{gdp} = 1$. Thus, we obtain the same qualitative growth rates as those in the previous section. In the case where $\psi_t = \psi (K_t/L_t)^\mu (k_{t-1}/l_{t-1})^{1-\mu}$, we have $\gamma_\psi = \gamma_K \gamma_L^{-1}$ and, using Eqs (17), (37) and (38), we obtain $\gamma_z = n^{1/(1+v)}$, $\gamma_K = n^{(1+\alpha)v/\alpha(1+v)}$ and $\gamma_L = n^{v/(1+v)}$, yielding $\gamma_{gdp} = n^{v/\alpha(1+v)}$. Again, we have the same qualitative growth rates as those in the previous section, with the exception that now labor per firm increases.

### 5.2 Growth of the number of firms as a determinant of the operational costs

We turn now to the models where a planned expenditure of resources (say R&D) is required in order to create a firm; that is, when the growth of the number of firms implicitly
determines the evolution of the operational costs and, hence, growth. There are two main differences between these models and ours. The first is that, by construction, in these models we have $\gamma_\varepsilon > 1$, so that growth is imposed in the BGP. The second is that, since the expenditure is intentional, the firm becomes an asset and the operational cost forms part of the GDP in payment of this asset. We now show how the two best known papers in this literature, Romer (1990) and Jones (1995), can be adapted to our Eq.(5). In particular, we adapt both their R&D production functions and their specific implicit operational costs (price of patents).

First, consider a situation where the number of firms evolves as

$$z_{t+1} - z_t = \delta L_{z,t} z_t,$$  \hspace{1cm} (44)

where $L_{z,t}$ is a given fraction of the total population dedicated to developing new firms, and $\delta$ is a positive constant. The evolution of the number of firms causes operational costs (price of a patent) to be constant in the BGP (Romer, 1990, p. S90); that is, $\psi_t = \psi$. Then, rewriting Eq.(5) yields

$$\gamma_{gdp_{t+1}} = \frac{1 + \delta L_{z,t}}{n}.$$  \hspace{1cm} (45)

Constant population is required for a BGP to exist; that is, $L_{z,t}$ must be fixed. If there is population growth, then growth is not balanced, since $L_{z,t}$ in Eq.(45) is permanently growing whereas $n$ is a constant. Moreover, without population growth, the country with the biggest $L_{z,t}$ would have the highest per capita GDP growth. This is the reason why this type of model is said to have scale effects. Note that the growth rate coincides with that in Romer (1990).

Second, consider a situation where the number of firms evolves as

$$z_{t+1} - z_t = \delta L_{z,t} z_t^{1+\lambda},$$  \hspace{1cm} (46)

where $l_{z,t}$ is an externality, and $\phi$ and $\lambda$ are constants. The parameter $\phi$ captures the fact that the number of current firms can affect either positively or negatively the emergence of new firms. Assuming that in aggregate $l_{z,t} = L_{z,t}$, the growth of the number of firms is

$$\gamma_{z_{t+1}} = \left(1 + \delta L_{z,t}^{\lambda} z_t^{\phi - 1}\right).$$  \hspace{1cm} (47)

Population cannot be constant in a BGP, since the growth of the number of firms $z_t$ does not allow the factor $1 + \delta L_{z,t}^{\lambda} z_t^{\phi - 1}$ to be constant. This factor remains constant if $\gamma_\varepsilon = n^{\lambda/(1-\phi)}$. Moreover, in the BGP, the evolution of the number of firms causes the operational cost to take the form $\psi_t = \psi N_t$, so that rewriting Eq.(5) yields

$$\gamma_{gdp} = n^{\lambda/(1-\phi)}.$$  \hspace{1cm} (48)

13The price of a patent in the BGP can be easily derived from Jones (1995). From (A15), $\gamma_w = \gamma_{PA} + \gamma_A - \gamma_{LA}$. Similarly, from the BGP we obtain $\gamma_{\varepsilon} - \gamma_{LA} = \gamma_A - \gamma_{LA} = \lambda n/(1 - \phi) - n$. Dividing (A2) by $L$ we obtain $w/L = \alpha y/L \gamma_A$ and, therefore, $\gamma_w = n + \gamma_y - \gamma_{LY} = n + \lambda n/(1 - \phi) - n$, since in any BGP $\gamma_{LY} = \lambda n/(1 - \phi) - n$. Then, we have $n = \gamma_{PA}$, that is, the price of a patent increases at the same rate as population in the BGP.
Hence, we have semi endogenous growth. When population does not grow, the number of firms collapses into a constant and, then, growth is not possible. Note that this growth rate coincides with that in Jones (1995).

6 Concluding remarks

The main contribution of this paper has been to stress the importance of endogenous and time-dependent operational costs for growth. In an economy with both agglomeration economies and free market entry, we have analyzed the relevance of the relationship between operational costs and market structure in determining growth. In particular, we find that economic growth is positively correlated with the number of firms. We show that the economy experiences positive growth as it becomes more capital intensive. Moreover, this outcome occurs when the number of firms grows. That is, competition drives growth, as Nickell (1996) has shown empirically. Thus, any mechanism whose effects result in an increase in the number of firms causes agglomeration economies to induce growth. In particular, two variables can affect entry in our economy. On the one hand, (exogenous) population growth increases the productivity of capital and, hence, profits. This promotes new entries. On the other hand, when operational costs are dependent on market conditions, both the number and the size of firms may impact them. This would affect profits and so potentially sustain a continuous flow of new entries. Whereas population growth may originate semi endogenous growth, endogenous operational costs may originate endogenous growth. Specifically, we show that when operational costs are positively related to the industry’s firm size, represented by the capital level of the industry’s firms, endogenous growth occurs.

This said, it is our contention that the relationship between technology and market structure needs to be more closely addressed to fully understand some of the growth mechanisms. Likewise, factors such as human capital or public infrastructure could alter operational costs. A study of the endogeneity of the mark-up, different mark-ups for different sectors, or different market structures, would also shed more light on this relationship.
References


