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User cost of capital with delayed investment grants

Jorge Navas* and Jesús Marín-Solano

Adreça correspondència:

Dept. Matemàtica econòmica, financera i actuarial
Facultat de Ciències Econòmiques i Empresarials
Universitat de Barcelona
Avda. Diagonal 690
08034 Barcelona
* e-mails: jnavas@ub.edu

Abstract: The usual assumption when considering investment grants is that grant payments are automatic when investments are undertaken. However, evidence from case studies shows that there can exist some time lag until funds are received by granted firms. In this paper the effects of delays in grant payments on the optimal investment policy of the firm are analyzed. It is shown how these delays lead not only to a higher financing cost but to an effective reduction in the investment grant rate, and in some cases, how benefits from investment grants could be canceled due to interactions with tax effects.

Key words: User cost of capital, Delayed grant payments
JEL classification: D92, H32, C61

Resumen: El supuesto habitual cuando se consideran subvenciones a la inversión es que los pagos asociados se reciben automática e instantáneamente cuando se emprenden las inversiones. Sin embargo la evidencia empírica muestra que frecuentemente existen retrasos en el cobro de las subvenciones. En este trabajo se analizan los efectos de estos retrasos en la política óptima de la empresa, y se demuestra que éstos conllevan no únicamente un mayor coste financiero sino una reducción efectiva de la subvención, y que en algunos casos ésta puede ser totalmente absorbida a consecuencia de los efectos fiscales.

Palabras claves: Coste de capital, Retrasos en el pago de subvenciones
Clasificación JEL: D92, H32, C61

1 Introduction

Among the set of government policies to stimulate capital accumulation, fiscal policies are being used with the aim of reducing the user cost of capital (Jorgenson, 1963 and Hall and Jorgenson, 1967). Several works in the tradition of the neoclassical theory of investment (e.g., Ruane, 1982, and Van Loon, 1983) have studied the effects of financial incentives on the investment policy of firms in the form of grants related to assets. These incentives are usually paid out on a project-by-project basis (a firm qualifying for them should purchase, construct or otherwise acquire long-term assets) and are typically available only for specific regions or industries.

The usual assumption made in the literature when considering investment grants is that, when an investment is undertaken, the firm automatically receives the associated funds (or they are paid directly to the equipment supplier by the granting institution) and therefore only a fraction of the investment expenditure is needed. However, it is not unusual for there to be some time lag between investment and the reception of funds, leading the firm to face the whole investment expenditure and, some time later, receive the grant payment. In the European Commission's Synthesis Report (1999) case studies for several European regions are analyzed. For instance, in the Spanish region of Castilla-La Mancha, delays of up to 6 months are reported (and after a substantial reduction from recent past) between completion of the investment and grant payment, and actions such as up-front payments to help cash flow are quoted as innovative examples of good practice.

The aim of this paper is to analyze the effects of delayed grant payments on the optimal investment policy of firms through the effects on the user cost of capital. We discuss and solve a simple model that includes a fixed time delay for grants associated to current investment. It is shown that not only a financing cost due to advancing the amount of the grant payment arises, but a reduction in the effective grant proportional to the time lag. Moreover, interactions with corporate income tax could cancel all the benefits from the grant. In addition, a detailed analysis of the dynamics illustrates the effects of the existence of delays on the speed of convergence of the firm to the optimal size.

The mathematical techniques involved in the resolution of the delayed model are those of optimal control methods of hereditary systems (or systems with aftereffect). In general, these problems are much more complicated than when the assumption of the immediate response is made. In fact, although this topic has been the object of much interest in the

engineering field, delayed response problems in economics are often modeled as immediate response problems. As we will show, our model falls within the kind of delayed problems which can be handled in a nonstandard but easy way.

The paper is organized as follows: in Section 2 the model describing the lag is introduced as a delayed optimal control problem. In Section 3 we solve the delayed optimal control problem, deriving an expression for the user cost of capital. Section 4 analyzes and compares the cost of capital obtained for the delayed case, with both the non-delayed case and the non-granted case. In Section 5 we extend the model in order to break the linearity by introducing investment adjustment costs, and the effects of delays on the optimal investment policy are analyzed. Finally, in Section 6, a discussion of the results obtained is given, and generalizations of the model are suggested.

2 The model

In most of the models in the literature, automatic eligibility (approval of the grant and consequent right acquisition and grant recognition) and instantaneous grant payment are assumed. We differentiate these two instants by introducing a time lag of τ time units from the eligibility moment until the grant is received by the firm. Note that the distinction between eligibility and grant payment only matters when taxation is considered. Without corporate taxation, the relevant moment is that of the payment of the grant.

Consider a firm that maximizes its value to the shareholders,

$$J = \int_0^\infty D(t)e^{-it}dt \ , \tag{1}$$

where $D(t)$ is the variable representing the dividend payout, and i is the time preference rate of the shareholders. The state of the firm is described by the capital goods stock $K(t)$, and the firm can decide on the dividend payout, $D(t)$, and investment, $I(t)$.

Let $S(K)$ be the earnings function with the usual concavity assumptions ($S(K) > 0$, $dS/dK > 0$, $d^2S/dK^2 < 0$), a the true economic and tax deductible depreciation rate, f the profit tax rate and g the investment grant rate. With respect to the rules governing the tax base, there exists a great variation among countries, so that a general rule for taxation of grants cannot be drawn. In order to determine the corporate income tax, two methods of presentation in the balance sheet of grants related to assets are commonly used: the gross method and the net method. Under the gross method, the grant is accounted for as deferred income, whereas under the net method the grant is deducted in arriving at the

carrying amount of the asset. In any case, both methods lead to a depreciation amount related to the non-granted portion of the capital goods, that is, $(1-g)\,aK\,(t)$, where price of capital assets is taken to be unity and income recognition (gross method) follows the economic depreciation rate. According to the accrual basis of accounting, we assume that the gross method is applied, income is recognized at the economic depreciation rate, and a proportion γ of the grant is considered taxable income, so that for $\gamma = 1$ the tax rule is similar to the accounting rule, and, for $\gamma = 0$, investment grants are not subject to taxation.[1] Then, profit taxes, $T(K(t))$, are[2]

$$T(K(t)) = f \cdot [S\,(K\,(t)) - (1-\gamma g)\,aK\,(t)]\,. \tag{2}$$

Assuming that there is no debt, and no cash account is considered in the model, operating cash flow is allocated between dividends and investment:

$$S\,(K\,(t)) - T(K(t)) + gI\,(t-\tau) = D(t) + I(t), \tag{3}$$

where current cash from grants comes from investments undertaken τ periods before, due to the delay in grant payments.

The dynamics of the state variable is described by the following first order differential equation:

$$\dot{K}\,(t) = I\,(t) - aK\,(t)\,, \qquad K(0) = K_0\,. \tag{4}$$

We will also make the usual assumption that, under stationary market conditions, there exists a desired firm size that maximizes (1), which is represented by the optimal capital stock K_s, i.e.,

$$\lim_{t\to\infty} K(t) = K_s\,. \tag{5}$$

Finally, as every action (investment) performed at a moment t will affect the system at t and $t+\tau$, we will assume the (otherwise reasonable) condition that $I(t)$ is a given function for $t \in [-\tau, 0)$; that is, before the beginning of the planning horizon the value of $I(t)$ is a data of the problem. For instance, if we think of the firm as a new project, then

[1] This framework is not as restrictive as it might seem. In most EU countries, tax regulations consider it the rule, or it is included as an option. See, for instance, the Paper of the Accounting Advisory Forum on Government Grants XV/312/91 rev.3 (1995): 'http://ec.europa.eu/internal_market/accounting/otherdocs_en.htm'

[2] Note that, under the above assumptions, corporate taxes do not depend on the time delay, since the legal financial structure of the assets at a given time is not affected by the delayed grant payments due to the automatic recognition of the grant.

$I(t) = 0$ for $t \in [-\tau, 0)$, since before the beginning of the planning horizon no investment activity is carried out.

3 User cost of capital

The problem stated above is a nonstandard linear optimal control problem, in the sense that investment has a twofold effect: it appears as a current and as a delayed variable in the model. Note that the delayed variable is $I(t-\tau)$, where τ is finite and discrete,[3] which simplifies greatly the problem. In order to derive the optimality conditions, we follow an approach similar to the one described in Kamien and Schwartz (1991), ch. 19 (see also Budelis and Bryson, 1970).

If we write (1) as

$$J = \int_0^\infty L dt \ , \tag{6}$$

where $L = D(t)e^{-it}$, using (3) and (4), and denoting the delayed variables by $K_\tau(t) = K(t-\tau)$ (and $\dot{K}_\tau(t) = \dot{K}(t-\tau)$), we can express L as

$$L(K, \dot{K}, K_\tau, \dot{K}_\tau, t) = \left[S(K) - T(K) + g\left(\dot{K}_\tau + aK_\tau\right) - \left(\dot{K} + aK\right)\right] e^{-it} \ . \tag{7}$$

A necessary condition for the maximization of (6) is that the first variation of J vanishes, i.e. $dJ = 0$.[4] We get

$$\begin{aligned} dJ &= \int_0^\infty \left(\frac{\partial L}{\partial K}\delta K + \frac{\partial L}{\partial \dot{K}}\delta \dot{K} + \frac{\partial L}{\partial K_\tau}\delta K_\tau + \frac{\partial L}{\partial \dot{K}_\tau}\delta \dot{K}_\tau\right) dt = \\ &= \int_0^\infty \left(\frac{\partial L}{\partial K}\delta K + \frac{\partial L}{\partial \dot{K}}\delta \dot{K}\right) dt + \int_0^\infty \left(\frac{\partial L}{\partial K_\tau}\delta K_\tau + \frac{\partial L}{\partial \dot{K}_\tau}\delta \dot{K}_\tau\right) dt. \end{aligned}$$

Since we are assuming that the activity prior to the beginning of the planning horizon is known at $t = 0$ (i.e., $I(t)$ and $K(t)$ are given for $t \in [-\tau, 0)$), then δK_τ and $\delta \dot{K}_\tau$ vanish for $t < \tau$. Therefore, the second integral in the relation above can be rewritten as

$$\begin{aligned} \int_0^\infty \left(\frac{\partial L}{\partial K_\tau}\delta K_\tau + \frac{\partial L}{\partial \dot{K}_\tau}\delta \dot{K}_\tau\right) dt &= \int_\tau^\infty \left(\frac{\partial L}{\partial K_\tau}\delta K_\tau + \frac{\partial L}{\partial \dot{K}_\tau}\delta \dot{K}_\tau\right) dt = \\ &= \int_0^\infty \left(\left.\frac{\partial L}{\partial K_\tau}\right|_{t+\tau} \delta K + \left.\frac{\partial L}{\partial \dot{K}_\tau}\right|_{t+\tau} \delta \dot{K}\right) dt \end{aligned}$$

[3] General hereditary systems consider the case in which the delay is continuous and it is distributed along a time interval, which can be finite or infinite.

[4] Since L is concave in K ($S(K)$ is concave) and linear in $\dot{K}$, this condition is also sufficient for the existence of a maximum of (6).

(after doing the change of variables $t \to t - \tau$). Hence,

$$dJ = \int_0^\infty \left(\frac{\partial L}{\partial K} + \left. \frac{\partial L}{\partial K_\tau} \right|_{t+\tau} \right) \delta K dt + \int_0^\infty \left(\frac{\partial L}{\partial \dot{K}} + \left. \frac{\partial L}{\partial \dot{K}_\tau} \right|_{t+\tau} \right) \delta \dot{K} dt. \tag{8}$$

Integrating by parts the second integral in (8), we obtain

$$\int_0^\infty \left(\frac{\partial L}{\partial \dot{K}} + \left. \frac{\partial L}{\partial \dot{K}_\tau} \right|_{t+\tau} \right) \delta \dot{K} dt = \lim_{b \to \infty} \left(\frac{\partial L}{\partial \dot{K}} + \left. \frac{\partial L}{\partial \dot{K}_\tau} \right|_{t+\tau} \right) \delta K(t) \Big|_0^b -$$

$$- \int_0^\infty \delta K \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{K}} + \left. \frac{\partial L}{\partial \dot{K}_\tau} \right|_{t+\tau} \right) dt. \tag{9}$$

Since $K(0) = K_0$, then $\delta K(0) = 0$. In a similar way, $\lim_{t\to\infty} K(t) = K_s$ (the steady state capital goods stock is also fixed), so $\lim_{t\to\infty} \delta K(t) = 0$, and the term $\lim_{b\to\infty} |\cdot|$ inside (9) vanishes. Therefore, if we substitute (9) in (8), we obtain

$$dJ = \int_0^\infty \left[\left(\frac{\partial L}{\partial K} + \left. \frac{\partial L}{\partial K_\tau} \right|_{t+\tau} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{K}} + \left. \frac{\partial L}{\partial \dot{K}_\tau} \right|_{t+\tau} \right) \right) \delta K \right] dt.$$

Then, the (necessary) criticality condition $dJ = 0$ is satisfied if, and only if, the following Euler-Lagrange equation is fulfilled,

$$\frac{\partial L}{\partial K} + \left. \frac{\partial L}{\partial K_\tau} \right|_{t+\tau} = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{K}} + \left. \frac{\partial L}{\partial \dot{K}_\tau} \right|_{t+\tau} \right) . \tag{10}$$

Using (7) and (2) we get

$$\frac{\partial L}{\partial K} = \left[\frac{dS}{dK} - f \left(\frac{dS}{dK} - a(1 - \gamma g) \right) - a \right] e^{-it} ,$$

$$\left. \frac{\partial L}{\partial K_\tau} \right|_{t+\tau} = age^{-i(t+\tau)} ,$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{K}} \right) = ie^{-it} ,$$

$$\frac{d}{dt} \left(\left. \frac{\partial L}{\partial \dot{K}_\tau} \right|_{t+\tau} \right) = -ige^{-i(t+\tau)} .$$

Therefore, from the Euler-Lagrange equation above, and rearranging terms, the optimal capital stock K_s must satisfy the following equation,

$$\left. \frac{dS}{dK} \right|_{K=K_s} = c_\tau = \frac{i}{1-f} \left(1 - ge^{-i\tau} \right) + a \left(1 - \frac{e^{-i\tau} - f\gamma}{1-f} g \right) . \tag{11}$$

The right-hand side of equation (11) represents the marginal cost of one unit of capital goods, c_τ, which characterizes the optimal capital goods stock in the steady state.

4 Effects of delayed grant payments

In this Section we analyze the expression for the user cost of capital derived above. In order to make comparisons, let us first analyze, taking into account the assumptions made here, the well-known expression for the cost of capital in the non-delayed case,[5] c_0, which is obtained from equation (11) by taking ($\tau = 0$):

$$c_0 = \frac{i}{1-f}(1-g) + a\left(1 - \frac{g}{1-f} + \frac{f\gamma}{1-f}g\right). \tag{12}$$

Equation (12) can be divided into two terms. The first one expresses the cost of equity per unit of capital goods. Note that the time preference rate of the shareholders has been transformed into a marginal rate of return to equity before tax payments of $i/(1-f)$, taking into account that dividend payout cannot be offset against corporate income tax. The second term expresses the cost due to economic depreciation. Here we must differentiate two cases: whether investment grants are considered taxable income ($\gamma = 1$) or not ($\gamma = 0$). For $\gamma = 1$ the cost due to economic depreciation is $a(1-g)$, that is, the true economic depreciation rate of the non-granted investment. For $\gamma = 0$ this cost reduces to $a\left[1 - g/\left(1-f\right)\right]$, since the tax liability due to the recognition of the investment grant as current income, $(ag\left[(f\gamma)/\left(1-f\right)\right])$, vanishes.

Let $c = i/\left(1-f\right) + a$ be the user cost of capital when there are no investment grants ($g = 0$). Rearranging equation (12), we have

$$c_0 = c - g\left(\frac{i + \left(1 - f\gamma\right)a}{1-f}\right).$$

Since $f\gamma < 1$, it holds (according to intuition) that $c_0 < c$. Note that the previous inequality is always satisfied independently of the value for γ, i.e., the investment grant reduces the cost of capital whether it is considered taxable income or not.

Returning to the general expression for the user cost of capital, c_τ, in (11) it can be seen that the higher the time delay until the grant payment the higher the corresponding user cost of capital ($\partial c_\tau / \partial \tau > 0$). Moreover, the user cost of capital depends positively on the discount rate of the shareholders i. Therefore, firms facing high interest rates from their shareholders are more sensitive to delays.

Comparing equations (11) and (12), we can observe that whereas the tax liability due to recognition of the grant as current income remains unaffected (since it is independent of

[5] See for instance, Hilten et al. (1993), ch. 6, for a model with investment grants.

the payment moment), the financing and economic components of the user cost increase because of a reduction in the effective investment grant rate. This reduction equals its discounted value at the time preference rate of the shareholders, from g to $ge^{-i\tau}$. This can be explained by the fact that a loss arises due to the need of advancing the grant component of the investment expenditure for an amount of $g(1-e^{-i\tau})$ valued at the moment when the investment is undertaken. Because of this loss the firm faces two new costs: the financing cost, $ig(1-e^{-i\tau})/(1-f)$, and the corresponding cost of economic depreciation $ag\,(1-e^{-i\tau})\,/\,(1-f)$.

Finally we compare the user cost of capital for the delayed case with the non-granted case. Proceeding as before, we obtain

$$c_\tau = c - \left(\frac{ige^{-i\tau} + (e^{-i\tau} - f\gamma)\, ga}{1-f} \right). \tag{13}$$

Note that whereas for non-taxable grants ($\gamma = 0$) the cost of capital is always lower than the non-granted case, i.e., ($c > c_\tau$), in case that investments grants are subject to taxation ($\gamma = 1$), from (13) we have that $c < c_\tau$ when $(i+a) < fae^{i\tau}$. Hence, in this case the investment grant becomes a disincentive for the firm since the tax liability due to the grant offsets its benefits. Although this negative effect is not likely to occur for usual values of the parameters involved, it could arise in situations of high profit tax rates and/or large delays in the grant payments.

5 Investment adjustment costs

The simple setting analyzed so far, despite allowing us to study the effects of delayed grant payments, presents some well-known drawbacks: it leads to an instantaneous adjustment to the desired stock of capital K_s, and there is no investment function as such.[6] In order to avoid this unrealistic immediate adjustment process, two extensions have been made in the literature. The first one is to consider the existence of financing limits (e.g., Leland, 1972, or Van Loon, 1983). By also adding a lower bound for the control variable, for instance, assuming irreversible investments, Arrow (1968), the control variable becomes bounded below and above. Therefore, although the structure of the model remains linear, the instantaneous adjustment to K_s is avoided, and the problem becomes one of the bang-bang type. The second approach incorporates investment adjustment costs (e.g.,

[6] A detailed analysis can be found in Takayama (1985).

Eisner and Strotz, 1963, Gould, 1968, Treadway, 1969 or Nickell, 1978), in order to break the linearity of the model. The term 'adjustment costs' refers[7] to costs due to the investment expenditures of the firm, for instance those due to temporary decrease of productivity as a consequence of reorganization of the production line or retraining of workers. An usual specification for the adjustment cost function is to consider that it is a convex and increasing function of investments (net or gross), reflecting the fact that, on average, adjustment costs are higher, the greater the investment rate. Later, Kort (1988) studies a model which incorporates both financial constraints and adjustment costs, and characterizes the optimal investment policy of the firm by means of the net present value of marginal investments.

In this section, we extend the model introduced in Section 2 by assuming that there exist convex adjustment costs associated to the investment process, and analyze changes in the optimal investment policy due to delayed grant payments.

Consider that the adjustment cost function, $U(I)$, is a strictly convex function of gross investments, i.e., $U(I) > 0, U'(I) > 0$ and $U''(I) > 0$ for all $I(t) > 0$, $U'(0) \geq 0$, and $U(0) = 0$. Assume that a fraction β of the adjustment costs are assumed to be tax deductible, so that, profit taxes are

$$T(K(t)) = f \cdot \left[S\left(K(t)\right) - (1 - \gamma g)\, aK(t) - \beta U(I(t))\right].$$

Now, equation (3) transforms into

$$D(t) + I(t) + U(I(t)) = S\left(K\left(t\right)\right) - T(K(t)) + gI(t - \tau).$$

In order to solve the problem, define the functional (6) for the extended model with

$$L(K, \dot{K}, K_\tau, \dot{K}_\tau, t) = \Big[S(K) - T(K) + g\left(\dot{K}_\tau + aK_\tau\right) - \left(\dot{K} + aK\right) - U(K, \dot{K})\Big]\, e^{-it}. \quad (14)$$

Following the same steps as before, the criticality condition $dJ = 0$ is satisfied if, and only if, expression (10) holds with $L(\cdot)$ defined as in (14). For more specific results, and following Eisner and Strotz (1963), let us assume that both $S(K)$ and $U(I)$ functions are quadratic. Then, consider as the earnings function

$$S(K) = AK - \frac{B}{2}K^2, \quad (15)$$

[7]For a survey see Hilten et al. (1993), ch. 5.

where A and B are two positive constants. $S(K)$ is an increasing and strictly concave function on the relevant interval for K. With respect to the adjustment cost function, we take

$$U(I) = \frac{\omega}{2} I^2 \quad \text{i.e.} \quad U(\dot{K} + aK) = \frac{\omega}{2}(\dot{K} + aK)^2, \text{ with } \omega > 0. \tag{16}$$

Now, after substituting expressions (15) and (16) into (10), we obtain the following second-order linear non-homogeneous differential equation to be satisfied by the optimal capital goods stock,

$$(1-\beta f)\omega\ddot{K} - (a+i)(1-\beta f)\omega\dot{K} - [(1-f)B + (a+i)a(1-\beta f)\omega]K = \\ (a+i)(1-ge^{-i\tau}) - f(1-\gamma g)a - (1-f)A. \tag{17}$$

Solving (17), applying initial condition (4), and assuming that there exists a desired firm size K_s that maximizes (6), the optimal path for the capital goods stock is[8]

$$K^*(t) = (K_0 - K_s)e^{\alpha t} + K_s, \tag{18}$$

where α is the negative root of the characteristic equation of the homogeneous part of (17), i.e.,

$$\alpha = \frac{(a+i)}{2} - \frac{1}{2}\sqrt{(a+i)^2 + 4\left((a+i)a + \frac{(1-f)B}{(1-\beta f)\omega}\right)},$$

and K_s is the desired firm size

$$K_s = -\frac{(a+i)(1-ge^{-i\tau}) - f(1-\gamma g)a - (1-f)A}{(1-\beta f)\omega(a+i)a + (1-f)B}.$$

For the problem to be sensible from an economic point of view, it is assumed that $K_s > 0$. Note that the firm size never reaches the value K_s, except for the unlikely case in which $K_0 = K_s$. In any other case, the optimal path for the capital goods stock is to approach asymptotically K_s.

Next, the optimal investment policy during this path can easily be derived. From (4) and the derivative of (18), we have that

$$I^*(t) = (a+\alpha)(K_0 - K_s)e^{\alpha t} + aK_s. \tag{19}$$

When the initial size of the firm stands below K_s, the optimal investment policy is to invest proportionally to the existing gap between K_0 and K_s, with a decreasing factor

[8] For a detailed derivation of the solution see, for instance, Chiang (1992).

with respect to time, plus the economic depreciation that would correspond to the optimal firm size, so that the firm grows monotonically toward K_s. On other hand, if the initial firm size is higher than K_s, the optimal investment policy is to reduce the current firm size approaching the desired one. In this case, $K^*(t)$ is a decreasing function on time, that is, net investment is always negative.

Finally, in order to see the effects of delayed grant payments, we differentiate expressions (18) and (19) with respect to τ to obtain

$$\frac{\partial K^*(t)}{\partial \tau} = -(1 - e^{\alpha t})\frac{i(a+i)ge^{-i\tau}}{(1-\beta f)\omega(a+i)a + (1-f)B} < 0$$

and

$$\frac{\partial I^*(t)}{\partial \tau} = -(a(1 - e^{\alpha t}) - \alpha e^{\alpha t})\frac{i(a+i)ge^{-i\tau}}{(1-\beta f)\omega(a+i)a + (1-f)B} < 0.$$

Therefore, for a similar initial firm size, not only the desired capital goods stock, K_s, is negatively affected by delayed grant payments ($\partial K_s/\partial \tau < 0$), but also the optimal capital goods stock as well as the optimal investment rate, at every instant of the planning horizon, are lower the larger the delay. Investment grants reduce the marginal cost of acquiring a new unit of capital goods through reducing the price of capital goods. As shown in Kort (1988), the optimal investment policy during the planning horizon is characterized by the equality between marginal earnings and marginal costs of investment. As delays in grant payments reduce the effective grant rate, they increase marginal costs of capital goods. Hence, since marginal earnings are a decreasing function of $K(t)$, the optimal firm size is achieved at lower levels of $K(t)$. Note that this fact also leads to a permanent lower operating cash flow, so that in case where any liquidity constraint were to be incorporated in the model, negative effects of delayed grant payments would be intensified.

6 Concluding remarks

In this paper the expression for the user cost of capital is derived when grant payments suffer some delay from the moment of investment. It is shown that this time lag leads to a reduction in the effective investment grant rate, which equals its discounted value at the time preference rate of the shareholders, while the tax liability remains unchanged. Finally, it is pointed out that, under certain values of the relevant parameters, benefits from investment grants could reverse and result in a higher cost of capital.

When the model is extended introducing investment adjustment costs, and optimal capital goods stock and optimal investment rate paths are derived for the adjustment process to the desired firm size K_s, it is further shown that delays in grant payments also have a negative effect on the dynamics of the firm.

There are interesting extensions for this study. For instance, due to the fact that the time delay τ is unknown in many real cases, it may be more realistic to consider the time delay τ not as fixed, but taking different values (maybe a continuum of them) with certain probabilities. Nevertheless, the simple analysis carried out here draws attention to the negative effects of delayed grant payments, and stress the need for government and granting institutions to take measures to reduce time limits for granted firms in order to obtain final payments, or to consider counteracting actions such as up-front payments.

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