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Leisure Time and the Sectoral Composition of Employment

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Abstract: We observe the following patterns in the US economy during the period 1965-2015: (i) the rise of the service sector, (ii) the increase in leisure time, and (iii) the increase in recreational services. To display the last pattern, we measure the fraction of the value added of the service sector explained by the consumption of recreational services, and we show that it increases during this period. We explain these three patterns of structural change in a multisector growth model, where leisure time and the consumption of recreational services are complements. We show that this complementarity introduces a mechanism of structural change that contributes to explain the rise of the service sector and that also affects the labor supply. We measure the reduction in employment due to a tax increase to illustrate the effect on the labor supply of this mechanism.

JEL Codes: O41, O47.

Keywords: Sectoral composition, Leisure, Non-homothetic preferences, Elasticity of substitution, Biased technological progress.

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1. Introduction

We observe two important patterns of structural change during the last fifty years. The first one is the large shift in employment and production from the goods to the service sector. Figure 1 illustrates this pattern for the US economy, during the period 1965-2015. In 1965, only 55% of workers were employed in the service sector, whereas 77% were employed in this sector in 2015. Figure 1 also shows a similar pattern for the shares of value added. The recent multisector growth literature has explained these patterns of structural change as the result of income effects (Kongsamut, Rebelo and Xie, 2001) or price effects (Acemoglu and Guerrieri, 2008; and Ngai and Pissarides, 2007). More recently, this literature has argued that the significant increase of the service sector can only be explained by combining both types of effects (Boppart, 2014; Dennis and Iscan, 2009; Foellmi and Zweimuller, 2008; and Herrendorf, Rogerson, and Valentinyi, 2013). Herrendorf, Rogerson, and Valentinyi (2014) offers an exhaustive review of this literature and shows that this process of structural change is not specific to the US, but it is quite a general feature.

The second pattern is the change in the uses of time. Using survey data, Aguiar and Hurst (2007) and Ramey and Francis (2009) obtain the evolution of the uses of time in the US economy during the second half of the last century, and they outline the increase in leisure time. Duernecker and Herrendorf (2015) show similar patterns for France, Germany and United Kingdom, among other countries. This pattern for the US economy is also illustrated in Figure 1, where it is shown that leisure increases from 46% of total time in 1965 to 54% in 2015.¹

The increase in leisure time is mainly explained by an income effect due to non-homothetic preferences (Duernecker and Herrendorf, 2015; and Restuccia and Vandenbroucke, 2013 and 2014). Note that this explanation is entirely independent of the multisectoral structure of the economy. In fact, there are few papers relating the rise of the service sector with changes in the uses of time. Examples are the papers by Buera and Kaboski (2012), Gollin, Parente, and Rogerson (2004), Moro, Moslehi and Tanaka (2015), Ngai and Pissarides (2008), and Rogerson (2008). In these papers, the relationship between the service sector and the uses of time is based on home production and its different substitutability with the market production of the different sectors. More precisely, the reduction in home production causes the increase in the employment share of the service sector because home production is a better substitute for services than for the goods produced in the other sectors. The relationship between uses of time and the service sector is also obtained by Greenwood and Vandenbroucke (2005) and Ngai and Pissarides (2008), who introduce leisure activities that combine leisure time with durable goods produced in the manufacturing sector.² Again, the different substitutability of these activities with the market production of the different sectors contributes to explain the increase in the employment share of the service sector.

¹Appendix B explains in detail the construction of the time series displayed in Figure 1.

²Other papers in the literature have considered leisure activities. In particular, Kopecky (2011) and Vandenbroucke (2009) consider leisure activities that combine leisure time with goods to explain the reduction in working hours. More recently, Boppart and Ngai (2017) and Bridgman (2016) consider leisure activities that combine capital and leisure time to explain the rising inequality in leisure. None of these papers relates the rise of leisure time with the increase in the service sector.

In this paper, we also provide a joint explanation of the increase in both the service sector and leisure time. In contrast to the aforementioned papers, the mechanism relating these two patterns is based on the recreational nature of leisure. More precisely, our main assumption is that individuals consume recreational services during leisure time.³ Therefore, the consumption of these services increases with leisure, which introduces a mechanism that directly relates leisure time with the service sector.

To quantify the impact of this mechanism on structural change, we measure the fraction of the value added of the service sector explained by the consumption of recreational services. The details of the procedure followed to obtain this fraction are in Appendix B, and the results are displayed in Figure 1. This figure shows that this fraction has increased from 5.2% in 1965 to 8.6% in 2015. This increase has a sizeable effect on sectoral composition, as it accounts for 19% of the observed increase in the service sector share of total value added.

Our purpose is to analyze the effect that recreational activities have on both the sectoral composition and the labor supply. We measure both effects using a multisector exogenous growth model. In the supply side of this model, we distinguish between two sector-specific technologies that produce goods and services. These technologies are differentiated only by the exogenous growth rate of total factor productivity (TFP). In the demand side, we assume that households obtain utility from consuming goods, non-recreational services, and recreational activities. Following Ngai and Pissarides (2007), the utility function is a constant elasticity of substitution (CES) function. Therefore, the only new feature of this model is the introduction of recreational activities. These activities are defined as another CES function relating the amount of time devoted to leisure and the consumption of recreational services. Hence, the utility function considered in this paper is a non-homothetic version of the nested CES function introduced by Sato (1967).

Technological progress drives structural change through three different mechanisms: substitution, income, and recreational mechanisms. First, the substitution mechanism is due to the assumption of different growth rates of sectoral TFP. Consistent with empirical evidence, we will assume that the goods sector experiences the largest TFP growth rate, which causes the increase in the relative price of services in units of goods. As shown by Ngai and Pissarides (2007), this relative price increase contributes to explain the rise of the service sector when the elasticity of substitution of consumption goods is smaller than one.

Second, the income mechanism is due to the introduction of a minimum consumption requirement on the consumption of goods. Preferences are then non-homothetic and the income elasticity of the demand for goods is smaller than one. As a consequence, the employment share of the service sector increases as income grows with technological progress. Thus, the income mechanism also contributes to explain the rise of the service sector.

Third, the recreational mechanism is the new mechanism introduced in this paper. Both leisure time and the fraction of the value-added of the service sector explained by the consumption of recreational services increase when the elasticity of substitution of

³Table B.2 in Appendix B offers an exhaustive classification of industries that provide recreational services. These services include sport, exercise, socialize, travel, hobbies, Tv, radio, entertainment, among others.

recreational activities is smaller than one but larger than the elasticity of substitution of consumption goods.⁴ Thus, when leisure time and recreational services are complements, the recreational mechanism also contributes to explain the rise of the service sector.

We simulate three different models that are calibrated to match the patterns of the US economy in the period 1965-2015. The first one is our benchmark model, where individuals obtain utility from recreational activities. In the second model, we do not consider these activities, and we instead assume that individuals obtain utility directly from leisure. Finally, the third model is a standard multisector growth model without leisure. Therefore, the three mechanisms of structural change are operative only in the benchmark model. We show that the interaction among the three mechanisms accounts for almost all the observed increase of the share of employment allocated in the service sector, of leisure time and of the fraction of the value added of the service sector explained by the consumption of recreational services. In the other two models, the recreational mechanism is not operative and, hence, the increase of the service sector is explained only by the other two mechanisms. We compare the performance of these three models to conclude that the introduction of recreational activities improves substantially the performance of the model in explaining the increase of the service sector.

The introduction of recreational activities modifies the labor supply as it increases the substitutability between leisure time and consumption expenditures. We show that the reduction in employment due to a labor income tax increase is much larger in the benchmark model with recreational activities than in the model where individuals derive utility directly from leisure. This is explained by the fact that the effect of taxes on employment depends on the substitutability between leisure and consumption expenditures and this substitutability is larger when recreational activities are introduced.

The previous result is related to Rogerson (2008), who shows that the larger labor income taxes in Europe in comparison to the US make home production larger. In his analysis, this explains that European economies exhibit both a lower level of employment and a smaller fraction of working time employed in the service sector. In contrast, in our analysis, the larger taxes make recreational activities be more time intensive in European economies. This also explains that both the level of employment and the employment share in services are smaller when taxes are larger. Hence, our paper offers a complementary explanation of the differences between Europe and US regarding both sectoral composition and uses of time.

The rest of the paper is organized as follows. Section 2 introduces the model and Section 3 characterizes the equilibrium. Section 4 solves the model numerically and obtains the main results. Section 5 studies the effect of labor income taxes on employment. Finally, Section 6 includes some concluding remarks.

⁴We distinguish between the elasticity of substitution of recreational activities (the elasticity between leisure time and recreational services) and the elasticity of substitution of consumption goods (the elasticity among recreational activities, the consumption of goods and non-recreational services).

2. The model

We consider a two-sector exogenous growth model, where we distinguish between the service and the goods sectors. The former only produces a consumption good that can be devoted to either recreational or non-recreational activities, whereas the latter produces both a consumption and an investment good.

2.1. Firms

Each sector i produces by using the following constant returns to scale Cobb-Douglas technology:

$$Y_i = A_i (s_i K)^\alpha (u_i L)^{1-\alpha}, \quad i = s, g, \quad (2.1)$$

where Y_i is the amount produced in sector i , $\alpha \in (0, 1)$ is the capital-output elasticity, s_i is the share of total capital K devoted to sector i , u_i is the share of total employment L employed in sector i , A_i measures total factor productivity (TFP) in sector i , and the subindexes s and g amount for the services and goods sectors, respectively. Obviously, the capital and employment shares satisfy $s_g + s_s = 1$ and $u_g + u_s = 1$. We assume that TFP grows in each sector at a constant growth rate γ_i . Consistent with empirical evidence, we also assume that $\gamma_g > \gamma_s$.

Each individual has a time endowment of measure one that can devote to either leisure activities or work in the market. Let l be the amount of time an individual devotes to work, $1-l$ the amount of time devoted to leisure activities and N the constant number of individuals. Then, total employment in the economy satisfies $L = lN$. It follows that (2.1) can be rewritten in per capita terms as

$$y_i = A_i (s_i k)^\alpha (u_i l)^{1-\alpha}, \quad i = s, g, \quad (2.2)$$

where $y_i = Y_i/N$ and $k = K/N$.

Perfect competition and perfect factors' mobility imply that each factor is paid according to its marginal productivity and that marginal productivities equalize across sectors, implying that the firms' optimization conditions are

$$r = \alpha p_i A_i (s_i k)^{\alpha-1} (u_i l)^{1-\alpha} - \delta, \quad (2.3)$$

and

$$w = (1 - \alpha) p_i A_i (s_i k)^\alpha (u_i l)^{-\alpha}, \quad (2.4)$$

where r is the rental price of capital, w is the wage per unit of employment, p_i is the relative price and $\delta \in (0, 1)$ is the depreciation rate of capital. We assume that the commodity produced in the goods sector is the numeraire and, hence, $p_g = 1$. From using (2.3) and (2.4), we obtain $s_i = u_i$ and

$$p_s = \frac{A_g}{A_s}. \quad (2.5)$$

Given the assumed ranking of TFP growth rates, the relative price of services, p_s , increases.

2.2. Individuals

The economy is populated by infinitely lived individuals characterized by the utility function $u = \int_0^\infty e^{-\rho t} \ln C dt$, where $\rho > 0$ is the subjective discount rate and C is the following composite consumption good:

$$C = \left[\eta_g (c_g - \bar{c})^{\frac{\varepsilon-1}{\varepsilon}} + \eta_s (x c_s)^{\frac{\varepsilon-1}{\varepsilon}} + \eta_l c_l^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where c_g is the consumption of goods, c_s is the consumption of services, c_l are recreational activities, $x \in [0, 1]$ is the fraction of services devoted to non-recreational activities, $\varepsilon > 0$ is the elasticity of substitution among the different consumption goods, \bar{c} is a minimum consumption requirement and $\eta_i > 0$ measures the weight of the different consumption goods in the utility function. We assume that $\eta_g + \eta_s + \eta_l = 1$. We also assume that recreational activities depend on both leisure time and the amount consumed of recreational services, according to the following function:

$$c_l = \left\{ \beta [(1-x) c_s]^{\frac{\sigma-1}{\sigma}} + (1-\beta) (1-l-\bar{o})^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}, \quad (2.6)$$

where $\sigma > 0$ is the elasticity of substitution between recreational services and leisure, \bar{o} is a minimum requirement of leisure and $\beta \in [0, 1]$ measures the weight of recreational services in recreational activities.⁵

Individuals decide on leisure, the value of consumption expenditures, the sectoral composition of these expenditures and the fraction of services devoted to recreational activities to maximize the utility function subject to the budget constraint $wl + rk = E + \dot{k}$, where $E = c_g + p_s c_s$ is total consumption expenditures. The solution of this maximization problem is characterized by the following consumers' optimization conditions:

$$\frac{c_g}{E} = \frac{1}{\kappa_1} + \frac{\bar{c}}{E} \left(\frac{\kappa_1 - 1}{\kappa_1} \right), \quad (2.7)$$

$$\frac{p_s c_s}{E} = \left(1 - \frac{\bar{c}}{E} \right) \left(\frac{\kappa_1 - 1}{\kappa_1} \right), \quad (2.8)$$

$$x = \frac{\kappa_4}{1 + \kappa_4}, \quad (2.9)$$

$$1 - l = \bar{o} + \left(\frac{w \eta_g}{\eta_l (1-\beta) \kappa_2^{\frac{\varepsilon-\sigma}{\varepsilon\sigma}}} \right)^{-\varepsilon} \left(\frac{E - \bar{c}}{\kappa_1} \right), \quad (2.10)$$

and

$$\frac{\dot{E}}{E - \bar{c}} = r - \rho - \frac{\dot{\kappa}_7}{\kappa_7}, \quad (2.11)$$

where $\{\kappa_i\}_{i=1}^7$ are functions of both the price and the wage obtained in Appendix C. Equations (2.7) and (2.8) characterize the sectoral composition of consumption expenditures, while (2.9) determines the fraction of services devoted to non-recreational

⁵The parameter \bar{o} is introduced to disentangle σ from the elasticity of substitution of the labor supply with respect to the wage.

activities and (2.10) is the labor supply. Finally, (2.11) is the Euler condition driving the intertemporal trade-off between consuming today and in the future.⁶

3. Equilibrium

In this section, we define the equilibrium and obtain the long run values of employment, of the employment share in services, and of the fraction of services devoted to recreational activities. The first step is to obtain the employment share in services. To this end, we define per capita GDP as $Q = p_s y_s + y_g$ and, using (2.2) and (2.5), we obtain $Q = A_g k^\alpha l^{1-\alpha}$. We next use the market clearing condition in the service sector, $y_s = c_s$, and (2.5) and (2.8) to obtain

$$u_s = \left(\frac{\kappa_1 - 1}{\kappa_1} \right) \left(\frac{E - \bar{c}}{Q} \right). \quad (3.1)$$

Equations (2.9), (2.10) and (3.1) show that the sectoral composition and leisure time depend on the relative price, the wage and the time path of E and k .⁷ We can now define a dynamic equilibrium of this economy as a path of $\{k, E, u_s, l, x, p_s, w\}_{t=0}^\infty$ that, given initial conditions $k(0)$, $A_g(0)$ and $A_s(0)$, satisfies the consumers' optimization conditions, the firms' optimization conditions, the market clearing conditions and $A_i = A_i(0) e^{-\gamma_i t}$, $i = s, g$.

The assumption of permanent bias in technological progress implies that both the relative price and the wage diverge to infinite (see 2.4 and 2.5). As a consequence, the long run equilibrium can only be attained asymptotically when the variables characterizing the sectoral composition and leisure converge to a corner solution where, depending on the value of ε and σ , they take either its minimum or its maximum possible value.⁸ Given that these long-run values arise because technological progress is permanently biased towards a given sector, they inform about the direction of structural change while the process of biased technological progress is maintained. The following propositions obtain the long run values of these variables.

Proposition 3.1. *The long run value of employment, l^* , satisfies: $l^* = 0$ if $\sigma < 1$ and $\varepsilon < 1$, and $l^* = 1 - \bar{o}$ otherwise.*

Since the wage increases with technological progress, using (2.10), it can be shown that employment increases and converges to its maximum value when individuals can substitute leisure for other consumption goods. This happens when either $\sigma > 1$ or $\varepsilon > 1$. Therefore, we can only explain the increase in leisure time shown in Figure 1 when $\varepsilon < 1$ and $\sigma < 1$. In what follows, we show that these values of the elasticities of substitution are also consistent with the other observed patterns of structural change.

⁶Equation (2.11) shows that the growth rate of consumption expenditures depends on the growth rate of κ_7 and, therefore, it depends on the growth rate of prices. Alonso-Carrera, Caballé, and Raurich (2015) discuss when the growth of prices affects the Euler condition in multisector growth models.

⁷We obtain the system of differential equations governing the time path of the variables in Appendix D. This appendix also contains the proof of Propositions 3.1, 3.2 and 3.3.

⁸The long-run equilibrium is an asymptotic balanced growth path along which the interest rate, the ratio of capital to GDP and the variables characterizing the sectoral composition remain constant.

Proposition 3.2. *The long run values of the sectoral composition of employment, u_s^* and u_g^* , satisfy: $u_s^* = 0$ and $u_g^* = 1$ if $\varepsilon > 1$, and $u_s^* = q^*$ and $u_g^* = 1 - q^*$ if $\varepsilon < 1$, where $1 - q^*$ is the long run savings rate.*

The result in the pervious proposition follows from using (3.1) and it was already obtained in Ngai and Pissarides (2007). As these authors explain, when the price of services increases, the employment share in this sector increases only if goods and services are complements. Therefore, the observed increase in the price of services and the increase in the employment share can only be jointly explained when $\varepsilon < 1$.

Proposition 3.3. *The long run value of the fraction of services devoted to non-recreational activities, x^* , satisfies: $x^* = 1$ if $\sigma < \min\{1, \varepsilon\}$, $x^* = 0$ if $\sigma \in (\varepsilon, 1)$, and $x^* = 1 / \left[\beta \left(\frac{1-\varepsilon}{1-\sigma} \right)^\sigma \left(\frac{\eta_l}{\eta_s} \right)^\varepsilon + 1 \right]$ if $\sigma > 1$.*

The result in Proposition 3.3 follows from using (2.9). Along the transition, both leisure and the consumption of services, c_s , increase when $\varepsilon < 1$ and $\sigma < 1$. However, the increase in the consumption of services is substantially larger and faster than the increase in leisure time. As a consequence, when leisure and recreational services are strong complements, $\sigma < \min\{1, \varepsilon\}$, the definition of recreational activities in (2.6) implies that the fraction of services devoted to recreational activities declines and converges to zero. It follows that this fraction increases only when leisure and recreational services are not strong complements, which occurs when $\sigma \in (\varepsilon, 1)$. Note that this is the empirically relevant case, as it is consistent with the evidence in Figure 1. Finally, leisure vanishes when $\sigma > 1$. Since leisure and recreational services are gross substitutes in this case, individuals still consume recreational services in the long run and, hence, $x^* < 1$.

We conclude that the equilibrium path implied by this model is compatible with the observed patterns of structural change when (i) there is complementarity among the different consumption goods ($\varepsilon < 1$) and between leisure and recreational services ($\sigma < 1$) and (ii) when the complementarity between leisure and services is weaker than the complementarity among the different consumption goods ($\sigma > \varepsilon$). The first condition is already obtained in Ngai and Pissarides (2007). The second condition is a contribution of this paper, which is necessary to explain the process of structural change between recreational and non-recreational services displayed in Figure 1. These constraints on the value of the elasticities of substitution are considered in the numerical analysis of the following section.

4. Structural change

We next study the contribution of the recreational mechanism to explain the observed patterns of structural change. To this end, we calibrate three different models. Economy I is our benchmark economy with recreational activities. In Economy II, we assume that $\beta = 0$, implying that $x = 1$ and, hence, there are no recreational services. Individuals derive utility directly from leisure. Finally, in Economy III we assume that $\eta_l = 0$, which implies that $x = 1$ and $l = 1$. This economy corresponds to a classical structural change model without leisure.

We distinguish between two groups of parameters. The first group is displayed in Table A.1 and consists of parameters that have a common value in the three economies. These parameters are $\gamma_g = 1.87\%$ and $\gamma_s = 1.18\%$ that are set to match the GDP growth rate and the growth rate of prices, $A_g(0) = 1$ and $A_s(0) = 1.4$ that are set to obtain the initial relative price of services in units of goods, and $\alpha = 0.348$ that is set to match the average value of the labor income share. Table A.2 reports the rest of parameters. These parameters are jointly set in each model to attain the following targets: the values of the share of recreational services and of employment in 1965 and 2015, the value of the employment share in services in 1965, the long run values of the ratios of investment to capital and of capital to GDP and to minimize the root mean square error of the model's prediction with respect to investment to capital ratio for the period 1965-2015. Finally, note that in this calibration we do not consider the employment share in the service sector in 2015 as a target.

We assume that capital per efficiency unit of labor is at its long-run value from the initial period and, therefore, the transition displayed in Figures 2 and 3 is driven only by the exogenous technological progress governing the three aforementioned mechanisms of structural change. In Economy I, displayed in Figure 2, the interaction among the three mechanisms accounts for the increase in leisure time, the increase in the share of recreational services and almost all the increase in the employment share of the service sector. Figure 3 displays the other two economies. Economy II does not include the recreational mechanism. Therefore, it does not explain the increase in recreational services, but it still accounts for the increase in leisure and the increase in the employment share of the service sector. However, the performance in explaining the rise of the service sector is worse than in Economy I. Finally, in Economy III there is no leisure. Therefore, this model only explains the changes in the sectoral composition of employment. Again, the performance is worse than in Economy I.

Table A.3 compares the performance of the three economies in explaining the increase of the service sector by using three different accuracy measures: total variation explained, root mean square error and Akaike information criteria. From this comparison, we can see that the performance of Economy I is much better, whereas the differences in the performance of Economies II and III are negligible. We can then conclude that leisure contributes to explain the rise of services only through the increase in recreational activities.

5. Fiscal policy

In this section, we study the effect of labor income taxes on both employment and GDP. Duernecker and Herrendorf (2015), Prescott (2004) and Rogerson (2008) have already shown that the labor supply decreases when the labor income tax increases. In fact, the effect of labor income taxes crucially depends on the substitution between leisure and the consumption of goods. As recreational activities modify this substitution, the effect of taxes on employment is modified when these activities are considered. To study this differential impact of labor income taxes, we compare the effect of a permanent tax increase in Economies I and II. We follow Prescott (2004) and we study the consequences of increasing the effective labor income tax from the US average level, 40%, to the

French average level, 59%. For the sake of simplicity, we assume that government revenues returns to individuals as a lump-sum subsidy.

We calibrate again Economies I and II so that they match the level of employment and the fraction of recreational services both in 1965 and in 2015 and the employment share in services in 1965 when taxes are at the US level. Table A.4 provides the new values of the parameters.

Figure 4 shows the effects of a permanent tax increase introduced in 1965. In Economy II, where individuals directly derive utility from leisure, the tax increase rises leisure both initially and during the transition. The increase in leisure causes the initial reduction of GDP. This lower GDP reduces capital accumulation which, in turn, reduces even further employment and GDP during the transition. Figure 4 displays, in Panels (c) and (d), the employment and GDP loss due to the tax increase. Both employment and GDP loss increase during the transition from around 2% in 1965 to 8% in 2015, as a consequence of the reduction in capital accumulation.

Table A.5 shows that in Economy I, where individuals derive utility from recreational activities, the effect on employment and GDP of the tax increase is substantially larger than in Economy II. Initially, employment and GDP decrease over 4%. This substantial initial reduction of GDP causes a larger reduction in capital accumulation which, in turn, implies a more significant GDP loss during the transition. In 2015, the employment loss is almost 10%, while the GDP loss is over 9%. It follows that the effect of taxes on both employment and GDP is substantially larger when we take into account that individuals derive utility from leisure through the consumption of recreational activities. These activities introduce the possibility that individuals can substitute leisure time for expenditure in services. As a consequence, after the tax increase, recreational activities become more time intensive, which facilitates the increase in leisure and the reduction in working time. This mechanism explains the larger impact that a tax increase has when we consider recreational activities.

6. Concluding remarks

This paper explains two important patterns of structural change; first, the large shift in employment and production from the goods to the service sector, and, second, the sustained increase in leisure. We contribute to the literature on structural change by relating these two patterns. We argue that during leisure time we consume recreational services. The observed increase in leisure time then implies an increase in the consumption of these services, which introduces a mechanism explaining structural change in the sectoral composition of employment.

We construct a multi-sector exogenous growth model with sectoral biased technological change to measure the effect on structural change of this mechanism. The new feature of the model is the introduction of recreational activities, which depend on both leisure time and the consumption of recreational services. We calibrate the model and we show that it accounts for the increase in leisure time, the increase in recreational services and the changes in the sectoral composition of employment. We also show that the performance of the model in explaining the rise of the service sector worsens when recreational activities are not considered.

There are large differences in the amount of time devoted to work between the US and European economies. Prescott (2004) and Rogerson (2008) have convincingly argued that large part of these differences can be explained by the differences in the labor income taxes. The effect of taxes on employment depends on the substitution between leisure and consumption of goods. Since recreational activities increase this substitution, the reduction in employment due to a tax increase is substantially larger when recreational activities are considered. Therefore, recreational activities contribute to explain the relation between cross-country labor income tax differences and working time differences.

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A. Tables and Figures

Table A.1: Baseline Calibration for Economies I, II and III

Parameters	Values	Targets	Data
α	0.3480	Labor income share	0.652
γ_g	0.0187	GDP growth rate	0.028
γ_s	0.0118	Growth rate of relative price	0.007
A_s	1.4002	Relative price of services	0.714
A_g	1	Normalized	-

We report data of average labor income share from Penn World Table version 9.0. and the data on the GDP growth rate from Table 1.1.5 in BEA for the period 1965-2015. We compute relative price of services (level and growth rate) using value added data by industry from BEA based on the procedures presented in Herrendorf et al. (2014). Based on these calibrated parameters, the simulated average GDP growth rate in the period 1965-2015 in Economies I, II and III is 0.0258, 0.0252, and 0.029, respectively.

Table A.2: Joint Calibration of Economies I, II and III

Parameters	Targets	Data	Economy I	Economy II	Economy III
			Values	Values	Values
β	Recreational consumption (1965)	0.052	0.2966	0	-
σ	Recreational consumption (2015)	0.086	0.7130	-	-
\bar{o}	Total employment (2015)	0.457	0.4120	0.4183	-
$\bar{\eta}_l$	Total employment (1965)	0.539	10.819	0.6079	0
$\bar{\eta}_s$	Employment in services (1965)	0.546	157.06	9.3059	9.9417
ρ	Long-run value of K/Q	2.680	0.0781	0.0824	0.0824
δ	Long-run value of I/K	0.054	0.0270	0.0227	0.0227
\bar{c}	Minimize RMSE of I/K	-	0.3140	0.2500	0.2500
ε	Minimize RMSE of I/K	-	0.0100	0.1200	0.0100

We calibrate jointly the parameters ($\beta, \sigma, \bar{o}, \bar{\eta}_l, \bar{\eta}_s, \rho, \delta, \bar{c}, \varepsilon$) along with parameters in Table A.1 to match the following targets: we set the values of β, \bar{o}, σ and $\bar{\eta}_l = (\eta_l/\eta_g)^\varepsilon$ to explain 100% of the total variation of employment and recreational services shares in the period 1965-2015; $\bar{\eta}_s = (\eta_s/\eta_g)^\varepsilon$ to match the employment share in services in 1965. We set δ and ρ to match the long-run values of the investment-capital and the capital-output ratios and we set \bar{c} and ε to minimize the root-mean-square errors (RMSE) of the model's predictions with respect to the investment-capital ratio for the period 1965-2015. Long-run value of investment-capital ratio is the average value for the period 2008 to 2015, whereas the long-run value of capita-output ratio is the average value for the period 1965 to 2015. Both values are obtained from the Bureau of Economic Analysis. Based on the calibrated parameters, the average investment-capital ratio in the period 2008-2015 in Economies I, II and III is 0.0528, 0.0479 and 0.0517. The average capital-output ratio in the period 1965-2015 in Economies I, II and III is 2.54, 2.55 and 2.54, respectively.

Table A.3: Performance of Economies I, II and III

Accuracy Measures	Economy I	Economy II	Economy III
Total variation explained	0.9650	0.7894	0.8019
Root mean squared error	0.0069	0.0162	0.0149
Akaike information criterion	-363.35	-275.62	-284.52

Table A.3 reports accuracy measures for the three models to explain the time path of the employment share in services from 1965 to 2015. Total variation explained measures the percentage of the total change between 1965 and 2015 explained by the model. Both Root Mean Squared Errors and the Akaike Information Criterion are obtained by regressing actual employment share in services on simulated employment share and without a constant.

Table A.4: Calibration with taxes: Economies I and II

Parameters	Targets	Data	Economy I	Economy II
			Values	Values
β	Recreational consumption (1965)	0.052	0.4043	0
σ	Recreational consumption (2015)	0.086	0.8085	-
\bar{o}	Total employment (2015)	0.457	0.4030	0.4258
$\bar{\eta}_t$	Total employment (1965)	0.539	5.4412	0.3064
$\bar{\eta}_s$	Employment in services (1965)	0.546	99.784	9.9342

Table A.4 shows the calibrated parameters values for economies I and II with a labor income tax equal to 40%. The values of δ , ρ , \bar{c} and ε remain as in Table A.2.

Table A.5: Tax increase: changes in employment and GDP

year	Economy I		Economy II	
	Employment	GDP	Employment	GDP
1965	4.273%	4.273%	2.741%	2.741%
2015	9.736%	9.258%	8.684%	8.071%

Table A.5 shows the changes in absolute value of employment and GDP when income tax increases from 40% to 59%.

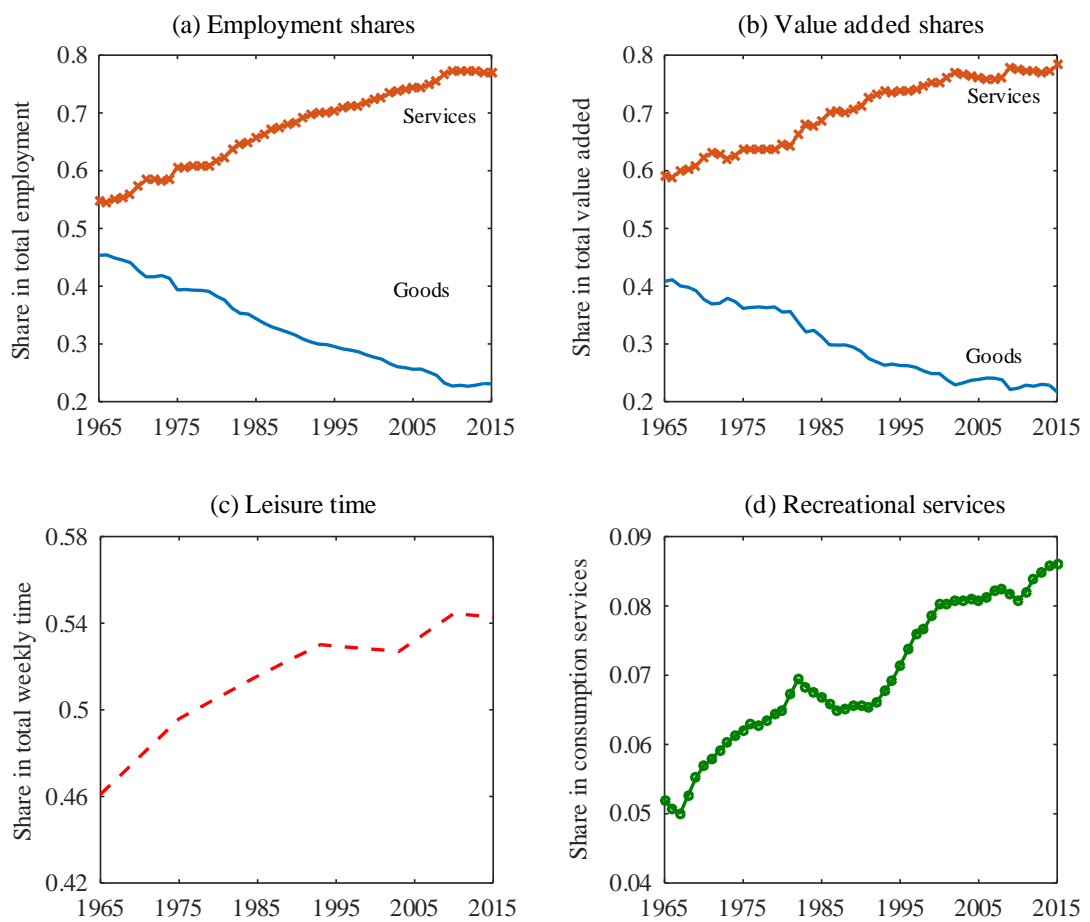


Figure 1. Patterns of Structural Change of the US economy

Source: employment and value-added shares are obtained from Timmer et al (2015) and World Development Indicators. In Appendix B we explain the construction of leisure time and consumption of recreational services.

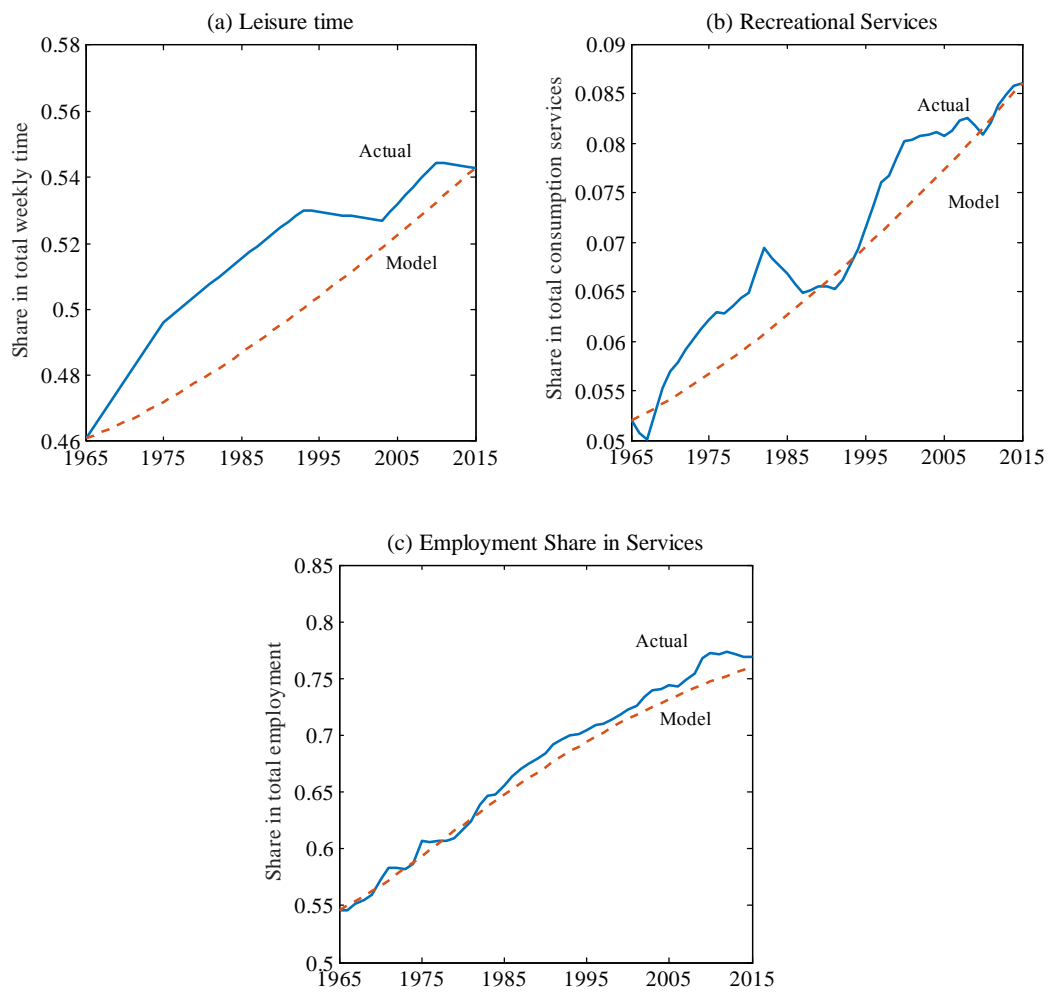


Figure 2. Numerical simulation of Economy I

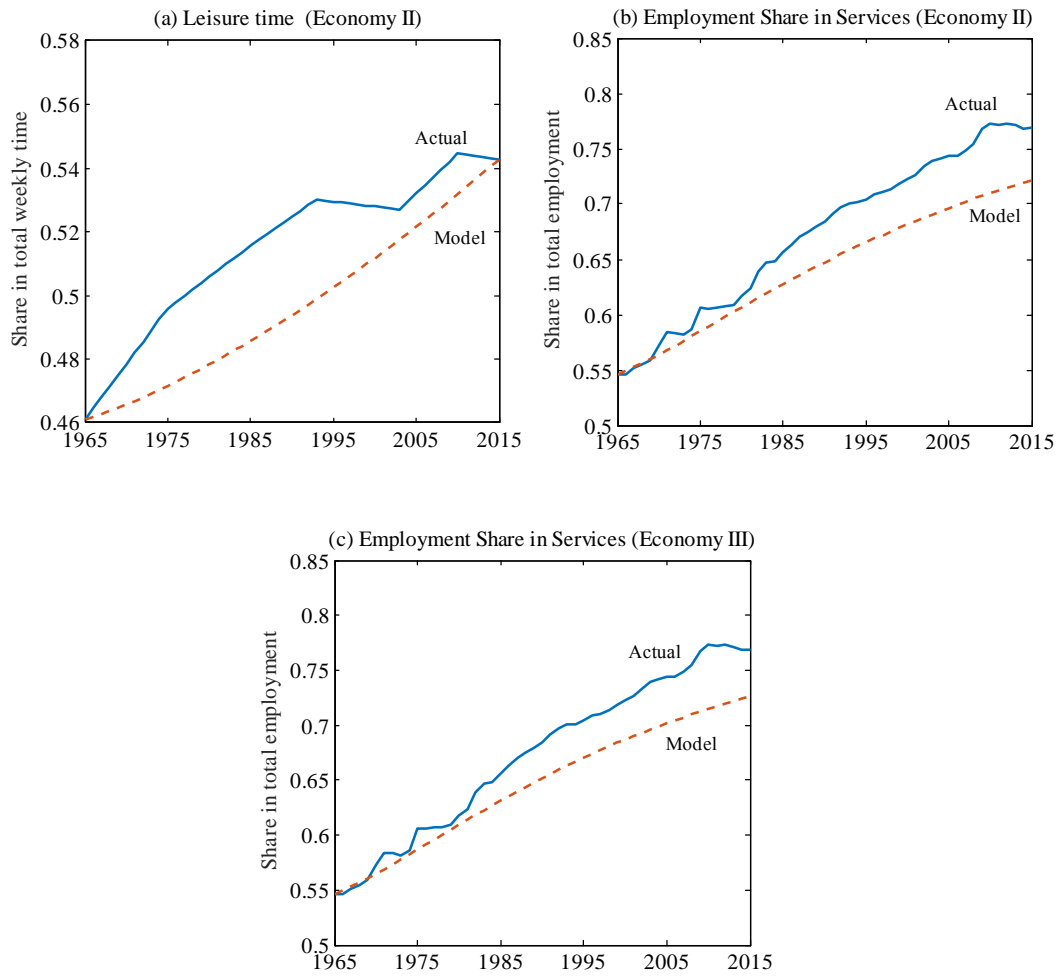


Figure 3. Numerical simulation of Economies II and III

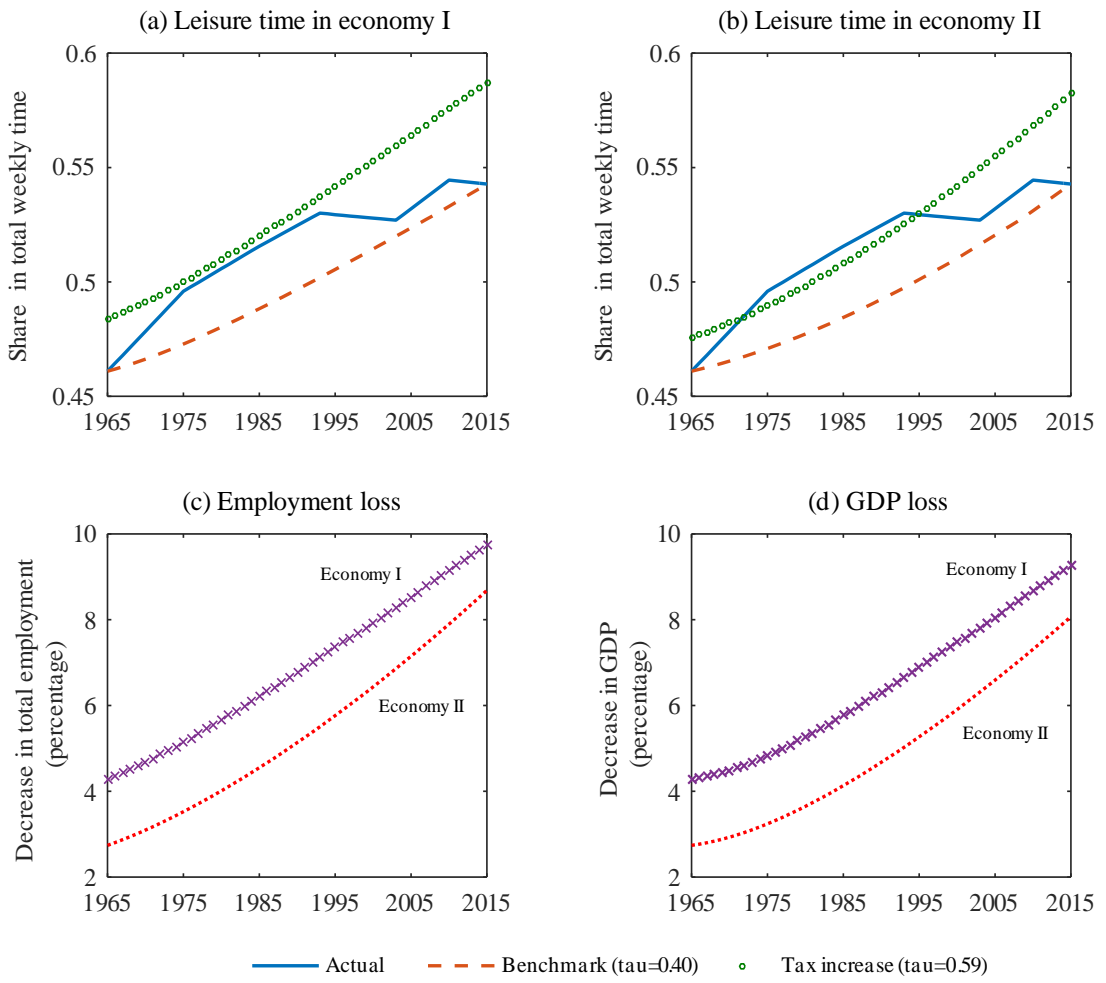


Figure 4. Effect of taxes

B. Leisure time and recreational services

B.1. Leisure time

We construct the uses of time data as in Aguiar and Hurst (2007), who use micro-level data from the American Time Use Survey (ATUS). First, they define time devoted to work as average hours devoted to work in the main job (including time spent working at home), other jobs, plus other work-related activities such as commuting to/from work, meals/breaks at work, searching for a job and applying for unemployment benefits. Second, they define four different measures of leisure based on the type of activities realized during non-working time. The data in our paper refers to their measure *leisure 1*. This measure amounts to the average weekly hours devoted to sports, exercise, socialize, travel, reading, hobbies, TV, radio, entertainment, volunteering, pet care and gardening. We follow the same methodology than Aguiar and Hurst (2007) to compute the average hours per week spent in total market work and leisure, adjusted for changing demographics in the period 1965 to 2015. Table B.1 displays the working and leisure hours for the years in which survey data is available.⁹

Table B.1: Average hours per week devoted to work and leisure

	1965	1975	1985	1993	2003	2010	2015
Working hours	35.96	33.77	32.62	33.29	31.78	29.98	30.12
Leisure hours	30.78	33.22	34.75	37.45	35.33	35.74	35.76

The ATUS reports other uses of time such as time spent on personal care, time devoted to home production and time spend on childcare. The total time devoted to these activities has been roughly constant and equal to 100 hours a week during the period 1965-2015. The remaining time is devoted to either leisure or working in the market. The fraction of the remaining time devoted to leisure is displayed in Figure 1, Panel (c).

B.2. Recreational Services

We obtain the value added generated by the consumption of recreational services using the IO tables published by the Bureau of Economic Analysis (BEA) for the period 1963-2015.¹⁰ We follow the methodology of Herrendorf et al. (2013), who compute

⁹It is important to clarify that the data in the table slightly differs from the data in Aguiar and Hurst (2007). This is a consequence of using a different time period to control for demographics.

¹⁰The Bureau of Economic Analysis (BEA) publishes IO tables for the years 1963, 1967, 1972 1977, 1982, 1987, 1992, 1998. After 1998, IO tables are published annually from 1999 to 2015. To download IO tables prior to 1977, see http://www.bea.gov/industry/io_benchmark.htm. For the years after 1977, IO tables are available in http://www.bea.gov/industry/io_histsic.htm.

the value added in the agricultural, manufacturing and service sectors generated by the final consumption expenditure. We extend this methodology to obtain the value added of recreational and non-recreational services. The former are identified as those services demanded by households to fulfill recreational activities, that are performed by individuals during their leisure time. These activities are defined according to Aguiar and Hurst (2007) first measure of leisure, leisure 1. We then assume that these recreational activities are provided by the following industries in the IO tables: amusement, motion pictures and other recreational services; radio and TV broadcasting; communication; and hotels and lodging places. These industries cover all the activities that according to Aguiar and Hurst (2007) are realized during leisure time except volunteering, pet care, gardening and reading for which we cannot identify and industry in the service sector. Table B.2 provides detailed information of these industries and the codes that identify them in the IO table for each year.

We use this analysis to compute the fraction of the value added of the service sector generated by the consumption of recreational services, which is displayed in Figure 1, Panel (d). At this point, it is important to mention that we do not include industries such as restaurants and transport services, because we cannot claim households consume them only for recreational purposes. Therefore, the value of this fraction reported in Panel d of Figure 1 is underestimated. As a consequence, we interpret the 19% of the increase in the service sector explained by recreational services, that we report in the paper, as a lower bound.

Table B.2: Selected industries in IO tables that provide recreational services

Year	Amusement, Motion pictures and other recreational services	Radio & TV broadcasting	Communication	Hotels and lodging places
1963 & 1967	Motion pictures (7601), Amusement and other recreation services (7602), Business travel, and entertainment (8100).	6700	6600	7201
1972 & 1977	Motion pictures (7601), Amusement and other recreation services (7602).	6700	6600	7201
1982 & 1987	Motion pictures (760100), Theatrical producers, orchestras, and entertainers (760201), Bowling alleys, billiard and pool establishments (760202), Professional sports clubs and promoters (760203), Racing, including track operation (760204), Membership sports and recreation clubs (760205), Other amusement and recreation services (760206).	670000	660000	720100
1993	Motion picture services and theaters (760101), Video tape rental (760102), Theatrical producers (760201), Bowling centers (760202), Professional sports clubs and promoters (760203), Racing, including track operation (760204), Physical fitness facilities and membership sports and recreation clubs (760205).	670000		Hotels (720101), Other lodging places (720102).
2002	Motion pictures and sound recordings (5120), Performing arts, spectator sports, and museums (71A0), Amusements, gambling, and recreation (7130).	Radio and television broadcasting (5151), Cable networks and program distribution (5152).	Internet publishing and broadcasting (5161), Telecommunications (5170), Data processing services (5180), Other information services (5190).	Accommodation (7210).
2007	Motion picture and video industries (512100), Performing arts companies (711100), Spectator sports (711200), Promoters of performing arts and sports and agents for public figures (711A00), Independent artists, writers, and performers (711500), Museums, historical sites, zoos, and parks (712000), Amusement parks and arcades (713100), Gambling industries (except casino hotels) (713200), Other amusement and recreation industries (713900), Sound recording industries (512200).	Radio and television broadcasting (515100), Cable and other subscription programming (515200).	Wired telecommunications carriers (517110), Wireless telecommunications carriers (except satellite) (517210), Satellite, telecommunications resellers, and all other telecommunications (517A00), News syndicates, libraries, archives and all other information services (5191A0), Internet publishing and broadcasting and Web search portals (519130).	Accommodation (721000).
2015	Motion picture and sound recording industries (512), Performing arts, spectator sports, museums, and related activities (711AS), Amusements, gambling, and recreation industries (713).	Broadcasting and telecommunications (513).	Data processing, internet publishing, and other information services (514).	Accommodation (721).

C. Solution of the consumers' problem

The Hamiltonian present value associated to the consumers' maximization problem is

$$\mathcal{H} = \ln C + \lambda (wl + rk - c_g - p_s c_s).$$

The first order conditions with respect to x , c_g , c_s , l and k are, respectively,

$$\frac{x^{-\frac{1}{\varepsilon}}}{(1-x)^{-\frac{1}{\sigma}}} c_s^{\frac{\varepsilon-\sigma}{\varepsilon\sigma}} = \left(\frac{\eta_l \beta}{\eta_s} \right) c_l^{\frac{\varepsilon-\sigma}{\varepsilon\sigma}}, \quad (\text{C.1})$$

$$C^{\frac{1-\varepsilon}{\varepsilon}} \eta_g (c_g - \bar{c})^{-\frac{1}{\varepsilon}} = \lambda, \quad (\text{C.2})$$

$$C^{\frac{1-\varepsilon}{\varepsilon}} \eta_s (x c_s)^{-\frac{1}{\varepsilon}} = \lambda p_s, \quad (\text{C.3})$$

$$C^{\frac{1-\varepsilon}{\varepsilon}} \eta_l c_l^{\frac{\varepsilon-\sigma}{\varepsilon\sigma}} (1-\beta) (1-l-\bar{o})^{-\frac{1}{\sigma}} = \lambda w, \quad (\text{C.4})$$

and

$$\dot{\lambda} = -(r - \rho) \lambda. \quad (\text{C.5})$$

We proceed to obtain c_l , c_s , c_g , l , and x as functions of prices, wages and total consumption expenditures, E , where $E = c_g + p_s c_s$. To this end, we combine (C.2) and (C.3) to get (2.7) and (2.8) in the main text, where the function κ_1 in these equations is

$$\kappa_1 = 1 + p_s \left(p_s \frac{\eta_g}{\eta_s} \right)^{-\varepsilon} \frac{1}{x}. \quad (\text{C.6})$$

We next use (C.1), (C.3) and (C.4) to obtain

$$(1-x) c_s = \left(\frac{(1-\beta) p_s}{w\beta} \right)^{-\sigma} (1-l-\bar{o}). \quad (\text{C.7})$$

We substitute (C.7) in (2.6) to get

$$c_l = \kappa_2 (1-l-\bar{o}), \quad (\text{C.8})$$

where

$$\kappa_2 = \left[\beta \left(\frac{(1-\beta) p_s}{w\beta} \right)^{1-\sigma} + 1 - \beta \right]^{\frac{\sigma}{\sigma-1}}. \quad (\text{C.9})$$

κ_2 can be rewritten as

$$\kappa_2 = \kappa_3 \left(\frac{w}{1-\beta} \right)^{\sigma}, \quad (\text{C.10})$$

where

$$\kappa_3 = \left(\beta^{\sigma} p_s^{1-\sigma} + (1-\beta)^{\sigma} w^{1-\sigma} \right)^{\frac{\sigma}{\sigma-1}}. \quad (\text{C.11})$$

From combining (C.2), (C.4) and (C.8), we get (2.10) in the main text. We combine (C.1), (C.8), (2.8) and (2.10) to get (2.9) in the main text. The expression of the function κ_4 in equation (2.9) is

$$\kappa_4 = \left(\frac{\eta_l \beta}{\eta_s} \right)^{-\sigma} \left(\frac{\eta_s w}{p_s \eta_l (1-\beta)} \right)^{\varepsilon-\sigma} \kappa_2^{\frac{\sigma-\varepsilon}{\sigma}}. \quad (\text{C.12})$$

In what follows, we derive the expression of the Euler condition. To this end, we first use (C.8) and (2.10) to reach

$$c_l = \kappa_2^{\frac{\varepsilon}{\sigma}} \left(\frac{w\eta_g}{\eta_l(1-\beta)} \right)^{-\varepsilon} (c_g - \bar{c}). \quad (\text{C.13})$$

We next substitute (2.8) and (C.13) in the definition of C to obtain

$$\left(\frac{C}{c_g - \bar{c}} \right)^{\frac{\varepsilon-1}{\varepsilon}} \frac{1}{\eta_g} = \kappa_6,$$

where

$$\kappa_6 = 1 + \bar{\eta}_s p_s^{1-\varepsilon} + \bar{\eta}_l \kappa_5^{1-\varepsilon}, \quad (\text{C.14})$$

$$\kappa_5 = \kappa_2^{-\frac{1}{\sigma}} \left(\frac{w}{1-\beta} \right), \quad (\text{C.15})$$

$\bar{\eta}_s = (\eta_s/\eta_g)^\varepsilon$, and $\bar{\eta}_l = (\eta_l/\eta_g)^\varepsilon$. We rewrite (C.2) and substitute the previous relations to reach

$$\lambda = \frac{1}{\kappa_6 (c_g - \bar{c})}. \quad (\text{C.16})$$

From using (2.7), we obtain

$$\frac{1}{\lambda} = \kappa_7 (E - \bar{c}), \quad (\text{C.17})$$

where

$$\kappa_7 = \frac{\kappa_6}{1 + \bar{\eta}_s p_s^{1-\varepsilon} \frac{1}{x}}. \quad (\text{C.18})$$

Finally, from combining (C.5) and (C.17), the Euler condition (2.11) in the main text is obtained.

Note that equations (2.7)-(2.11) in the main text depend on $\{\kappa_i\}_{i=1}^7$. From using (C.6), (C.10), (C.11), (C.12), (C.14), (C.15) and (C.18) it follows that $\{\kappa_i\}_{i=1}^7$ are functions only of the relative price and the wage.

D. Supplementary Appendix

D.1. System of differential equations

In this appendix, we derive the system of differential equations governing the time path of the variables in the equilibrium. To this end, we define the following transformed variables that will take a constant value in the long run equilibrium: $z = k/lA_g^{\frac{1}{1-\alpha}}$ is the capital stock per efficiency unit of labor, $q = E/Q$ is the consumption expenditure per unit of GDP and $\bar{v} = \bar{c}/Q$ is the minimum consumption requirement per unit of GDP.

Using these transformed variables and (2.3), we obtain the interest rate as

$$r = \alpha z^{\alpha-1} - \delta, \quad (\text{D.1})$$

and using (2.4) we obtain

$$w = (1 - \alpha) A_g^{\frac{1}{1-\alpha}} z^\alpha. \quad (\text{D.2})$$

We define per capita GDP as $Q = p_s y_s + y_g$ and using (2.2) and (2.4) we obtain $Q = A_g^{\frac{1}{1-\alpha}} z^\alpha l$. Note that the resource constraint of this economy can be written as $\dot{k} = Q(1 - q) - \delta k$. Note also that (3.1) can be rewritten as

$$u_s = \left(\frac{\kappa_1 - 1}{\kappa_1} \right) (q - \bar{v}), \quad (\text{D.3})$$

Finally, we use (2.10) to obtain the amount of time devoted to work

$$l = 1 - \bar{o} - \left(\frac{w\eta_g}{\eta_l(1-\beta)\kappa_2^{\frac{\varepsilon-\sigma}{\varepsilon}}} \right)^{-\varepsilon} \left(\frac{(q - \bar{v}) A_g^{\frac{1}{1-\alpha}} z^\alpha l}{\kappa_1} \right). \quad (\text{D.4})$$

Equations (2.9), (D.3) and (D.4) show that the sectoral composition of the economy and the amount of time devoted to work depend on the relative price, the wage and the time path of the following three variables: z , q and \bar{v} . We can then define a dynamic equilibrium of this economy as a path of $\{z, q, \bar{v}, u_s, l, x, p_s, w\}_{t=0}^\infty$ that, given initial conditions $z(0)$, $\bar{v}(0)$, $A_g(0)$ and $A_s(0)$, solves the system of differential equations governing the time path of z , q , \bar{v} and satisfies (2.5), (2.9), (D.2), (D.3), (D.4), and $A_i = A_i(0) e^{-\gamma_i t}$, $i = s, g$.

In what follows, we derive the system of differential equations governing the time path of z , q and \bar{v} . The first step is to obtain the expression of $\dot{\kappa}_7/\kappa_7$. We first combine (C.12) and (C.18) to get

$$\kappa_7 = \frac{\kappa_6}{1 + \bar{\eta}_s p_s^{1-\varepsilon} + \beta^\sigma p_s^{1-\sigma} \omega_0}, \quad (\text{D.5})$$

where

$$\omega_0 = \frac{\bar{\eta}_s p_s^{\sigma-\varepsilon}}{\beta^\sigma \kappa_4},$$

and from using (C.12) it follows that

$$\omega_0 = \bar{\eta}_l \kappa_3^{-\left(\frac{\sigma-\varepsilon}{\sigma}\right)}. \quad (\text{D.6})$$

We log-differentiate with respect to time (D.5) to obtain

$$\begin{aligned} \frac{\dot{\kappa}_7}{\kappa_7} &= \frac{\dot{\kappa}_6}{\kappa_6} - \frac{\left[(1-\varepsilon)\bar{\eta}_s p_s^{1-\varepsilon} + (1-\sigma)p_s^{1-\sigma}\beta^\sigma\omega_0 \right] (\gamma_g - \gamma_s)}{1 + \bar{\eta}_s p_s^{1-\varepsilon} + \beta^\sigma p_s^{1-\sigma}\omega_0} \\ &\quad - \left(\frac{\beta^\sigma p_s^{1-\sigma}\omega_0}{1 + \bar{\eta}_s p_s^{1-\varepsilon} + \beta^\sigma p_s^{1-\sigma}\omega_0} \right) \frac{\dot{\omega}_0}{\omega_0}. \end{aligned} \quad (\text{D.7})$$

From using (D.6), we get

$$\frac{\dot{\omega}_0}{\omega_0} = - \left(\frac{\sigma - \varepsilon}{\sigma} \right) \frac{\dot{\kappa}_3}{\kappa_3}. \quad (\text{D.8})$$

We next use (C.5) and (C.16) to reach

$$\frac{\dot{c}_g}{c_g - \bar{c}} = r - \rho - \frac{\dot{\kappa}_6}{\kappa_6}. \quad (\text{D.9})$$

The growth rate of wages is obtained from (D.2) and it is

$$\frac{\dot{w}}{w} = \frac{\gamma_g}{1 - \alpha} + \alpha \frac{\dot{z}}{z},$$

and we use (C.11) and (2.4) to obtain

$$\frac{\dot{\kappa}_3}{\kappa_3} = \omega_1 + \omega_2 \frac{\dot{z}}{z}, \quad (\text{D.10})$$

where

$$\omega_1 = -\sigma \left(\frac{\beta^\sigma p_s^{1-\sigma} (\gamma_g - \gamma_s) + (1-\beta)^\sigma w^{1-\sigma} \left(\frac{\gamma_g}{1-\alpha} \right)}{\beta^\sigma p_s^{1-\sigma} + (1-\beta)^\sigma w^{1-\sigma}} \right), \quad (\text{D.11})$$

and

$$\omega_2 = - \frac{\sigma (1-\beta)^\sigma w^{1-\sigma} \alpha}{\beta^\sigma p_s^{1-\sigma} + (1-\beta)^\sigma w^{1-\sigma}}. \quad (\text{D.12})$$

From using (C.10), (C.15) and (D.9), we get

$$\frac{\dot{\kappa}_5}{\kappa_5} = - \frac{\omega_1}{\sigma} - \frac{\omega_2}{\sigma} \frac{\dot{z}}{z}. \quad (\text{D.13})$$

From using (C.14), we obtain

$$\frac{\dot{\kappa}_6}{\kappa_6} = \frac{(1-\varepsilon)\bar{\eta}_s p_s^{1-\varepsilon} (\gamma_g - \gamma_s) + (1-\varepsilon)\bar{\eta}_l \kappa_5^{1-\varepsilon} \frac{\dot{\kappa}_5}{\kappa_5}}{\kappa_6},$$

and we use (D.13) to obtain

$$\frac{\dot{\kappa}_6}{\kappa_6} = \omega_4 + \omega_5 \frac{\dot{z}}{z}, \quad (\text{D.14})$$

where

$$\omega_4 = \frac{(1-\varepsilon)\bar{\eta}_s p_s^{1-\varepsilon} (\gamma_g - \gamma_s) - (1-\varepsilon)\bar{\eta}_l \kappa_5^{1-\varepsilon} \frac{\omega_1}{\sigma}}{\kappa_6}, \quad (\text{D.15})$$

$$\omega_5 = -\frac{(1-\varepsilon)\bar{\eta}_l\kappa_5^{1-\varepsilon}\omega_2}{\kappa_6}. \quad (\text{D.16})$$

We next use (D.10) to rewrite (D.8) as

$$\frac{\dot{\omega}_0}{\omega_0} = \theta_1 + \theta_2 \frac{\dot{z}}{z}, \quad (\text{D.17})$$

where

$$\theta_1 = -\left(\frac{\sigma-\varepsilon}{\sigma}\right)\omega_1, \quad (\text{D.18})$$

and

$$\theta_2 = -\left(\frac{\sigma-\varepsilon}{\sigma}\right)\omega_2. \quad (\text{D.19})$$

We substitute (D.17) and (D.14) in (D.7) to obtain

$$\frac{\dot{\kappa}_7}{\kappa_7} = \omega_7 + \omega_8 \frac{\dot{z}}{z}, \quad (\text{D.20})$$

where

$$\begin{aligned} \omega_7 = \omega_4 - & \frac{[(1-\varepsilon)\bar{\eta}_s p_s^{1-\varepsilon} + (1-\sigma)\beta^\sigma p_s^{1-\sigma}\omega_0](\gamma_g - \gamma_s)}{1 + \bar{\eta}_s p_s^{1-\varepsilon} + \beta^\sigma p_s^{1-\sigma}\omega_0} \\ & - \frac{\beta^\sigma p_s^{1-\sigma}\omega_0\theta_1}{1 + \bar{\eta}_s p_s^{1-\varepsilon} + \beta^\sigma p_s^{1-\sigma}\omega_0}, \end{aligned} \quad (\text{D.21})$$

and

$$\omega_8 = \omega_5 - \frac{\beta^\sigma p_s^{1-\sigma}\omega_0\theta_2}{1 + \bar{\eta}_s p_s^{1-\varepsilon} + \beta^\sigma p_s^{1-\sigma}\omega_0}. \quad (\text{D.22})$$

The second step is to obtain the growth rate of employment. We first combine (2.10) and (C.10) to get

$$1 - l - \bar{o} = \bar{\eta}_l \kappa_3^{\frac{\varepsilon-\sigma}{\sigma}} \left(\frac{w}{1-\beta}\right)^{-\sigma} \left(\frac{E-\bar{c}}{\kappa_1}\right). \quad (\text{D.23})$$

We use (C.6) and (C.18) to get

$$\kappa_1 = \frac{\kappa_6}{\kappa_7}. \quad (\text{D.24})$$

We combine (D.23) and (D.24) to obtain

$$1 - l - \bar{o} = \bar{\eta}_l \kappa_3^{\frac{\varepsilon-\sigma}{\sigma}} \left(\frac{w}{1-\beta}\right)^{-\sigma} \frac{\kappa_7}{\kappa_6} (E - \bar{c}), \quad (\text{D.25})$$

and we log-differentiate this equation

$$-\frac{\dot{l}}{1-l-\bar{o}} = \left(\frac{\varepsilon-\sigma}{\sigma}\right) \frac{\dot{\kappa}_3}{\kappa_3} - \sigma \frac{\dot{w}}{w} + \frac{\dot{E}}{E-\bar{c}} + \frac{\dot{\kappa}_7}{\kappa_7} - \frac{\dot{\kappa}_6}{\kappa_6}. \quad (\text{D.26})$$

We substitute the growth rate of wages, (2.11), (D.1), (D.10) and (D.14) to rewrite (D.26) as follows

$$\frac{\dot{l}}{l} = -\left(\frac{1-l-\bar{o}}{l}\right) \left(\omega_{10} + \omega_{11} \frac{\dot{z}}{z}\right), \quad (\text{D.27})$$

where

$$\omega_{10} = \left(\frac{\varepsilon - \sigma}{\sigma} \right) \omega_1 - \sigma \left(\frac{\gamma_g}{1 - \alpha} \right) + \alpha z^{\alpha-1} - \delta - \rho - \omega_4, \quad (\text{D.28})$$

and

$$\omega_{11} = \left(\frac{\varepsilon - \sigma}{\sigma} \right) \omega_2 - \sigma \alpha - \omega_5. \quad (\text{D.29})$$

Finally, we proceed to obtain the system of differential equations governing the time path of z and q . We first use the resource constraint to obtain

$$\frac{\dot{k}}{k} = (1 - q) z^{\alpha-1} - \delta. \quad (\text{D.30})$$

We combine (2.11) and (D.1) to get

$$\frac{\dot{E}}{E} = \left(\frac{E - \bar{c}}{E} \right) \left(\alpha z^{\alpha-1} - \delta - \rho - \frac{\dot{\kappa}_7}{\kappa_7} \right). \quad (\text{D.31})$$

From log-differentiating the definition of z and using (D.30), we obtain the dynamic equation for z

$$\frac{\dot{z}}{z} = (1 - q) z^{\alpha-1} - \delta - \frac{\gamma_g}{1 - \alpha} - \frac{\dot{l}}{l},$$

which, using (D.27), can be rewritten as

$$\frac{\dot{z}}{z} = \frac{\left[(1 - q) z^{\alpha-1} - \delta - \frac{\gamma_g}{1 - \alpha} \right] l + (1 - l - \bar{\delta}) \omega_{10}}{l - (1 - l - \bar{\delta}) \omega_{11}}. \quad (\text{D.32})$$

From using (D.27) and (D.32), we get

$$\dot{l} = -l(1 - l - \bar{\delta}) \left(\frac{\omega_{10} + \left[(1 - q) z^{\alpha-1} - \delta - \frac{\gamma_g}{1 - \alpha} \right] \omega_{11}}{l - (1 - l - \bar{\delta}) \omega_{11}} \right). \quad (\text{D.33})$$

From log-differentiating the definition of q and using (D.20), (D.31), (D.32) and (D.33) we reach

$$\begin{aligned} \frac{\dot{q}}{q} = & \left[\left(1 - \frac{\bar{v}}{q} \right) \omega_8 + \alpha \right] \left(\delta + \frac{\gamma_g}{1 - \alpha} - (1 - q) z^{\alpha-1} \right) \\ & + \left(1 - \frac{\bar{v}}{q} \right) (\alpha z^{\alpha-1} - \delta - \rho - \omega_7) - \frac{\gamma_g}{1 - \alpha} \\ & + (1 - l - \bar{\delta}) \left[(1 - \alpha) - \left(1 - \frac{\bar{v}}{q} \right) \omega_8 \right] \left(\frac{\omega_{10} + \left[(1 - q) z^{\alpha-1} - \delta - \frac{\gamma_g}{1 - \alpha} \right] \omega_{11}}{l - (1 - l - \bar{\delta}) \omega_{11}} \right), \end{aligned} \quad (\text{D.34})$$

Finally, we log-differentiate \bar{v} and we use (D.32) and (D.33) to obtain

$$\frac{\dot{\bar{v}}}{\bar{v}} = -\gamma_g - \alpha (1 - q) z^{\alpha-1} + \alpha \delta + (1 - \alpha) (1 - l - \bar{\delta}) \left(\frac{\omega_{10} + \left[(1 - q) z^{\alpha-1} - \delta - \frac{\gamma_g}{1 - \alpha} \right] \omega_{11}}{l - (1 - l - \bar{\delta}) \omega_{11}} \right). \quad (\text{D.35})$$

Note that (D.32), (D.34) and (D.35) form a system of three differential equations governing the time path of z , q and \bar{v} .

D.2. Balanced Growth Path

In order to obtain the BGP of this economy we follow a four steps procedure. First, we compute the long run value of prices. Second, we obtain the long run values of the auxiliary variables $\{\kappa_i\}_{i=1}^7$ and $\{\omega_i\}_{i=1}^{11}$. Third, we compute the long run values of employment and of the transformed variables, z and q , and, finally, we obtain the long run sectoral composition of the economy.¹¹

First, as $\gamma_g > \gamma_s$, equations (2.5) and (D.2) imply that $w^* = \infty$ and $p_s^* = \infty$.¹² Taking this into account, we obtain the long run values of the different auxiliary variables. We first use (C.10), (C.11) and (C.15) to obtain

$$\kappa_5 = \kappa_3^{-\frac{1}{\sigma}} = \left(\beta^\sigma + \left(\frac{w}{p_s} \right)^{1-\sigma} (1-\beta)^\sigma \right)^{\frac{1}{1-\sigma}} p_s.$$

Note first that w/p_s diverges to infinite. To see this, note that the growth rate of this term in the long run is

$$\frac{\dot{w}}{w} - \frac{\dot{p}_s}{p_s} = \frac{\gamma_m}{1-\alpha} - (\gamma_m - \gamma_s) > 0.$$

Then, it follows that $\kappa_5^* = \infty$ and $\kappa_3^* = 0$.

We next use (C.14) to obtain

$$\kappa_6^* = \begin{cases} 1 & \text{when } \varepsilon > 1 \\ \infty & \text{when } \varepsilon < 1 \end{cases}.$$

From using (C.9), we get

$$\kappa_2^* = \begin{cases} (1-\beta)^{\frac{\sigma}{\sigma-1}} & \text{when } \sigma < 1 \\ \infty & \text{when } \sigma > 1 \end{cases}.$$

In order to obtain the long run value of κ_4 , we use (C.10) and (C.11) to rewrite (C.12) as

$$\kappa_4 = \beta^{-\sigma} \left(\frac{\eta_s}{\eta_l} \right)^\varepsilon \left[\beta^\sigma + \left(\frac{w}{p_s} \right)^{1-\sigma} (1-\beta)^\sigma \right]^{\frac{\sigma-\varepsilon}{\sigma-1}}.$$

Then, we get that

$$\kappa_4^* = \begin{cases} \left(\frac{\eta_s}{\eta_l} \right)^\varepsilon \beta^{\left(\frac{1-\varepsilon}{\sigma-1} \right) \sigma} & \text{if } \sigma > 1 \\ \infty & \text{if } \varepsilon > \sigma \text{ and } \sigma < 1 \\ 0 & \text{if } 1 > \sigma > \varepsilon \end{cases}.$$

From using (2.9), we obtain the long run value of x^* in Proposition 3.3. And, from using (C.6), we obtain that

$$\kappa_1^* = \begin{cases} 1 & \text{when } \varepsilon > 1 \\ \infty & \text{when } \varepsilon < 1 \end{cases}.$$

¹¹In order to save space, we do not consider the cases with $\sigma = 1$ or $\varepsilon = 1$.

¹²A star in a variable indicates the long run value of the variable.

From (D.11) and (D.12) we get

$$\omega_1^* = \begin{cases} -\frac{\sigma\gamma_g}{1-\alpha} & \text{if } \sigma < 1 \\ -\sigma(\gamma_g - \gamma_s) & \text{if } \sigma > 1 \end{cases},$$

and

$$\omega_2^* = \begin{cases} -\alpha\sigma & \text{if } \sigma < 1 \\ 0 & \text{if } \sigma > 1 \end{cases}.$$

We use the long run values of κ_5 and κ_6 and equations (D.15) and (D.16) to obtain

$$w_4^* = \begin{cases} (1-\varepsilon)\frac{\gamma_g}{1-\alpha} & \text{if } \sigma < 1 \text{ and } \varepsilon < 1 \\ (1-\varepsilon)(\gamma_g - \gamma_s) & \text{if } \sigma > 1 \text{ and } \varepsilon < 1 \\ 0 & \text{if } \varepsilon > 1 \end{cases},$$

and

$$\omega_5^* = \begin{cases} (1-\varepsilon)\alpha & \text{if } \sigma < 1 \text{ and } \varepsilon < 1 \\ 0 & \text{if } \sigma > 1 \text{ or } \varepsilon > 1 \end{cases}.$$

From (D.18) and (D.19), we also obtain

$$\theta_1^* = -\left(\frac{\sigma - \varepsilon}{\sigma}\right)\omega_1^*,$$

and

$$\theta_2^* = -\left(\frac{\sigma - \varepsilon}{\sigma}\right)\omega_2^*.$$

From using (D.6), we get

$$\omega_0^* = \begin{cases} \infty & \text{if } \sigma > \varepsilon \\ 0 & \text{if } \sigma < \varepsilon \end{cases}. \quad (\text{D.36})$$

Using (D.6) and (C.11), we rewrite ω_0 as follows

$$\begin{aligned} \omega_0 &= \frac{\bar{\eta}_l}{w^{\varepsilon-\sigma}} \left(\beta^\sigma \left(\frac{p_s}{w}\right)^{1-\sigma} + (1-\beta)^\sigma \right)^{\frac{\varepsilon-\sigma}{\sigma-1}} \\ &= \bar{\eta}_l p_s^{\sigma-\varepsilon} \left(\beta^\sigma + \left(\frac{p_s}{w}\right)^{\sigma-1} (1-\beta)^\sigma \right)^{\frac{\varepsilon-\sigma}{\sigma-1}}. \end{aligned} \quad (\text{D.37})$$

This expression is used together with (D.21), (D.22) to obtain

$$w_7^* = \begin{cases} (1-\sigma)\left(\frac{\alpha}{1-\alpha}\gamma_g + \gamma_s\right) & \text{if } \varepsilon < \sigma \text{ and } \sigma < 1 \\ (1-\varepsilon)\left(\frac{\alpha}{1-\alpha}\gamma_g + \gamma_s\right) & \text{if } \varepsilon > \sigma, \sigma < 1 \text{ and } \varepsilon < 1 \\ 0 & \text{if } \varepsilon > 1 \text{ or } \sigma > 1 \end{cases},$$

and

$$\omega_8^* = \begin{cases} (1-\sigma)\alpha & \text{if } \sigma < 1 \text{ and } \varepsilon < \sigma \\ (1-\varepsilon)\alpha & \text{if } 1 > \varepsilon > \sigma \\ 0 & \text{if } \varepsilon > 1 \text{ or } \sigma > 1 \end{cases}.$$

We use (D.28) and (D.29) to get

$$\omega_{10}^* = \begin{cases} \alpha z^{\alpha-1} - \delta - \rho - \frac{\gamma_g}{1-\alpha} & \text{if } \varepsilon < 1 \text{ and } \sigma < 1 \\ (1-\sigma)\gamma_s + \alpha z^{\alpha-1} - \delta - \rho - \frac{\gamma_g}{1-\alpha} & \text{if } \varepsilon < 1 \text{ and } \sigma > 1 \\ -\varepsilon \left(\frac{\gamma_g}{1-\alpha} \right) + \alpha z^{\alpha-1} - \delta - \rho & \text{if } \varepsilon > 1 \text{ and } \sigma < 1 \\ -(\gamma_g - \gamma_s)(\varepsilon - \sigma) - \sigma \left(\frac{\gamma_g}{1-\alpha} \right) + \alpha z^{\alpha-1} - \delta - \rho & \text{if } \varepsilon > 1 \text{ and } \sigma > 1 \end{cases},$$

and

$$\omega_{11}^* = \begin{cases} -\alpha & \text{if } \varepsilon < 1 \text{ and } \sigma < 1 \\ -\sigma\alpha & \text{if } \varepsilon < 1 \text{ and } \sigma > 1 \\ -\alpha\varepsilon & \text{if } \varepsilon > 1 \text{ and } \sigma < 1 \\ -\sigma\alpha & \text{if } \varepsilon > 1 \text{ and } \sigma > 1 \end{cases}.$$

We proceed to get the long run values of employment and of the transformed variables. We first use (D.2), (D.25) and the definitions of q and Q to reach

$$1 - l - \bar{o} = \kappa_3^{\frac{\varepsilon-\sigma}{\sigma}} \left(\frac{\bar{\eta}_l w^{1-\sigma} (1-\beta)^\sigma \kappa_7}{(1-\alpha) \kappa_6} \right) (q - \bar{v}) l$$

and, using (D.5), we obtain

$$1 - l - \bar{o} = \kappa_3^{\frac{\varepsilon-\sigma}{\sigma}} \left(\frac{w^{1-\sigma}}{1 + \bar{\eta}_s p_s^{1-\varepsilon} + \beta^\sigma p_s^{1-\sigma} \omega_0} \right) \frac{\bar{\eta}_l (1-\beta)^\sigma (q - \bar{v}) l}{1-\alpha}.$$

From using (C.11) and (D.37), we get

$$\frac{1-l-\bar{o}}{l} = \left(\frac{\left[\beta^\sigma \left(\frac{p_s}{w} \right)^{1-\sigma} + (1-\beta)^\sigma \right]^{\frac{\varepsilon-\sigma}{\sigma-1}} \bar{\eta}_l (1-\beta)^\sigma (q - \bar{v})}{\frac{1}{w^{1-\varepsilon}} + \bar{\eta}_s \left(\frac{p_s}{w} \right)^{1-\varepsilon} + \beta^\sigma \left(\frac{p_s}{w} \right)^{1-\sigma} \bar{\eta}_l \left[\left(\frac{p_s}{w} \right)^{1-\sigma} \beta^\sigma + (1-\beta)^\sigma \right]^{\frac{\varepsilon-\sigma}{\sigma-1}}} \right)^{\frac{1}{1-\alpha}},$$

which can be rewritten as

$$\frac{1-l-\bar{o}}{l} = \left(\frac{\left(\frac{p_s}{w} \right)^{\sigma-1} \left[\beta^\sigma + \left(\frac{p_s}{w} \right)^{\sigma-1} (1-\beta)^\sigma \right]^{\frac{\varepsilon-\sigma}{\sigma-1}}}{\frac{1}{p_s^{1-\varepsilon}} + \bar{\eta}_s + \beta^\sigma \bar{\eta}_l \left[\beta^\sigma + \left(\frac{p_s}{w} \right)^{\sigma-1} (1-\beta)^\sigma \right]^{\frac{\varepsilon-\sigma}{\sigma-1}}} \right) \frac{\bar{\eta}_l (1-\beta)^\sigma (q - \bar{v})}{1-\alpha}.$$

Using the last two expressions, it can be shown that $l^* = 0$ if $\sigma < 1$ and $\varepsilon < 1$. Otherwise, $l^* = 1 - \bar{o}$. These are the long run values obtained in Proposition 3.1.

In order to obtain the long run values of the transformed variables, we use the system of differential equations (D.32), (D.34) and (D.35). We only consider the case $\sigma < 1$ and $\varepsilon < 1$, as it is the only case implying a declining path of employment consistent with empirical evidence. We use (D.32) and (D.34) to obtain the following long run values:

$$z^* = \left(\frac{(1-\alpha)(\delta + \rho) + \gamma_g}{\alpha(1-\alpha)} \right)^{\frac{1}{\alpha-1}},$$

and

$$q^* = \begin{cases} 1 - \alpha \left(\frac{\delta + \frac{\gamma_g}{1-\alpha} - (1-\sigma) \left(\frac{\alpha\gamma_g}{1-\alpha} + \gamma_s \right)}{\delta + \rho + \frac{\gamma_g}{1-\alpha}} \right) & \text{if } 1 > \sigma > \varepsilon \\ 1 - \alpha \left(\frac{\delta + \frac{\gamma_g}{1-\alpha} - (1-\varepsilon) \left(\frac{\alpha\gamma_g}{1-\alpha} + \gamma_s \right)}{\delta + \rho + \frac{\gamma_g}{1-\alpha}} \right) & \text{if } \sigma < \varepsilon < 1 \end{cases}.$$

Using (D.35), it follows that in the long run $\dot{\bar{v}} < 0$, which implies that $\bar{v}^* = 0$. The expression of the long run employment shares in Proposition 3.2 are obtained using (3.1) and the long run values of \bar{v} and κ_1 .