Revisiting real exchange rate volatility: Non-traded goods and cointegrated TFP shocks

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Abstract: International real business cycle (IRBC) models predict a real exchange rate volatility that is much lower than the levels observed in the data. In this paper, we build a two-country IRBC model with both a traded and a non-traded goods sector, and calibrate it to UK-euro area (EA) data. We provide evidence on the existence of a cointegrating relationship between UK and EA traded sector total factor productivity (TFP) by estimating a vector error correction model (VECM). To account for this relationship, we incorporate non-stationary technology shocks in the traded sectors in our model, and show that then the model is able to match the observed volatility of the UK-EA real exchange rate. Our analysis points out that both the presence of non-traded sectors and non-stationary technology shocks are necessary to account for the observed volatility in the real exchange rate.

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1 Introduction

Empirical evidence shows that international relative prices display large movements over the business cycle. This can be seen in Figure 1, in which we plot volatility in terms of annual demeaned UK real effective exchange rate and real GDP growth rate. It is evident from the figure that the real effective exchange rate series exhibits much higher volatility than the real GDP series. Even in periods of higher GDP volatility, the real exchange rate (RER) displays large swings.

Figure 1: Observed volatility in terms of annual growth rates
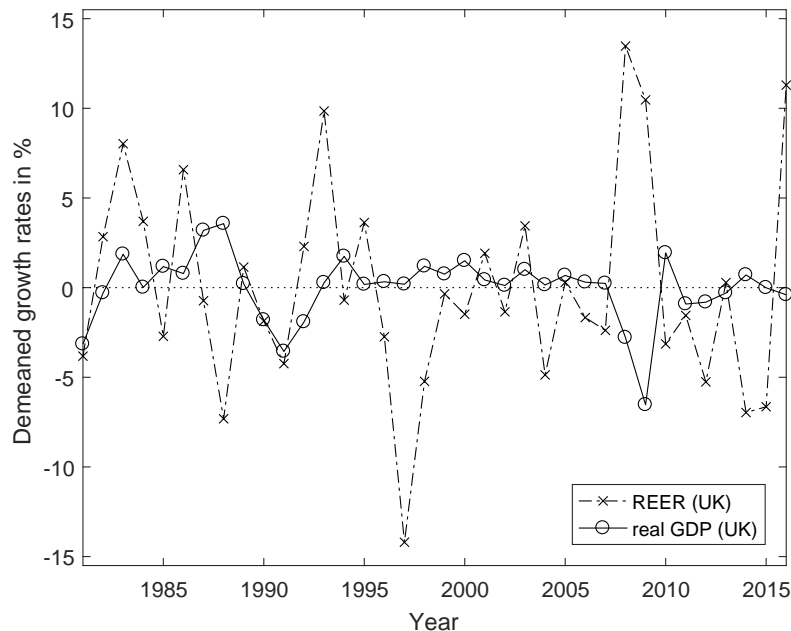
Note: The figure shows annual demeaned growth rates of the UK real effective exchange rate (REER) and real GDP for the period from 1982 to 2016. An increase in the REER corresponds to a depreciation.

Accounting for the high volatility of the RER has become a well-known puzzle in the international macro literature as standard open economy general equilibrium models produce a RER volatility that is much lower than the levels observed in the data. In this paper, we analyse the real world importance of non-traded goods and cointegrated technology shocks to address the volatility of the sterling pound-euro real exchange rate. We present empirical evidence on the cointegrating relationship between UK and euro area (EA) traded sector

TFP using data from the EU-KLEMS database[1]. We show that both series adjust symmetrically towards their common trend path and that the speed of adjustment is relatively slow. This has important implications for the model, because the slow adjustment introduces large wealth effects, which translate into higher RER volatility[2].

We also introduce a role for non-traded goods in our theoretical framework given the significant importance of non-traded goods[3]. In fact, when we look at input-output tables, the share of non-traded goods in total consumption is greater than the traded goods consumption (69% in the UK in 2014)[4]. The large weight of non-traded goods in total consumption implies a high weight of the non-traded goods prices in the CPI. This then means that movements in the prices of non-traded goods will increase the variability of the RER.

We build a two-country, two-sector general equilibrium model with non-traded goods and incomplete international financial markets and assess the model performance in comparison with the UK and EA data. We introduce stationary non-traded sector productivity shocks and cointegrated traded sector productivity shocks and calibrate them directly from the data. The main theoretical contribution of this paper is to introduce non-stationary shocks in a multi-sector general equilibrium framework that is consistent with balanced growth. The existence of a balanced growth path is conditional on the existence of a common trend across non-stationary traded sector productivities and on the assumption of Cobb-Douglas aggregation between non-traded and traded goods consumption. Our model closely follows the model presented by Rabanal et al. (2011), who introduce cointegrated technology shocks in a standard two-country model — with only tradable goods — and show that a model with cointegrated productivity innovations delivers higher RER volatility when compared with a

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[1] In our estimation of technology processes we use the TFP data directly from the EU-KLEMS database rather than relying on the Solow residuals.

[2] We acknowledge that during the sample period we focus on, the UK economy was hit by several other type of shocks (e.g. ERM crisis, 2008 financial crisis, Brexit) which caused fluctuations in the RER. However, even between those episodes, the volatility of the RER was evidently higher than the volatility of real GDP. We investigate whether these fluctuations can be explained by non-stationary TFP improvements that are motivated by direct empirical evidence.

[3] Also empirically, Betts and Kehoe (2008) demonstrate that despite the significant role of traded goods price movements, changes in the relative price of non-traded goods can account for about one third of RER volatility.

[4] We use the Input-Output Tables from the World Input-Output Database, 2016 Release (see, Timmer et al. (2015). We assume that agriculture, mining, manufacturing and the financial intermediation are traded and the remaining are non-traded sectors.

stationary model[5,6]. We extend their model by introducing a non-traded goods sector which then has important implications for the RER fluctuations through the variation in non-traded good prices. Movements in non-traded good prices arise through two channels: First, the existence of non-traded sector productivity shocks cause changes in the relative price of non-traded goods to traded goods and hence in the RER. Second, since labour is mobile across sectors, a productivity improvement in one sector not only affects the prices of the sector where the shock originates but also affects the other sector through fluctuations in wages (Balassa-Samuelson effect).

We find that the model is able to match the RER volatility once it is augmented by non-traded goods and non-stationary technology shocks even with a trade elasticity greater than one[7]. We also show that each of these two channels increases the model generated RER volatility and that the presence of both of these channels are necessary to match the observed RER volatility quantitatively. The volatility of RER relative to output in data is 6.75 and it is equal to 6.23 in the model. Our model also performs reasonably well in matching the correlation of the RER with relative consumption and GDP.

The explanation of this improvement is related to the wealth channels in our model. The non-stationary traded sector shock in our model is cointegrated across countries, implying that, in the long run, traded sectors of the two countries carry the same trend. Generally, global shocks reduce the volatility of international variables as they cannot be insured away by countries. However, our estimations from the VECM deliver a very low speed of adjustment to the common trend, generating significant wealth effects. When a country experiences an improvement in its traded sector productivity, the impact will be very persistent in the country where the shock originates, and the other country's adjustment to that shock will be very slow. For instance, if the home economy faces a persistent productivity improvement in its traded sector, as a result of consumption smoothing motives, the demand for home produced goods will increase by more than the production causing an increase in its price. The larger the persistence the bigger the difference between output and demand will be. Therefore, the slow arrival of the productivity improvement causes larger fluctuations in prices. This wealth effect is also amplified by the theoretical set-up we

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[5]They estimate a VECM model for the US and the rest of the world data.

[6]Similarly to Rabanal et al. (2011), Mandelman et al. (2011) introduce cointegrated investment specific technology shocks to a standard international real business cycle (IRBC) model.

[7]The lower the trade elasticity, the higher the terms of trade volatility will be. With high degrees of home bias, this terms of trade volatility will increase the RER volatility, too.

present. The existence of high non-traded goods consumption and incomplete international asset market structure further increase the obtained RER volatility. As a consequence of the high weight of non-traded goods in the consumption basket, changes in the productivity cause large variations in the relative price of non-traded goods across countries. As our model follows the paper by Rabanal et al. (2011) it is worth noting the difference with their framework. The volatility of RER in their framework increases as a consequence of three important channels: persistence of the non-stationary shock, high degrees of home bias and low trade elasticity. Unlike our setting, they calibrate the trade elasticity to a value lower than one, as they argue this is necessary to match the RER volatility in their model. This is not the case in our paper because our VECM estimates imply higher persistence than the estimates of Rabanal et al. (2011) and the non-traded goods have a large share in aggregate consumption. These features then help match the observed RER volatility without having to calibrate the trade elasticity to a value that is less than one, given that the micro-estimates of trade elasticity is found to be higher than one[8].

Our paper relates to the international business cycle literature that analyses the RER dynamics. Many papers in the literature focus on the role of non-traded goods to study puzzles in international macro as we do — for example, Stockman and Tesar (1995) or Rabanal and Tuesta (2013). Importantly, Dotsey and Duarte (2008) argue that the presence of non-traded goods helps to increase the model generated RER volatility. In their sample the RER volatility is around 3 times as large as the output volatility. They show that a model that incorporates non-traded goods produces a RER volatility that is 1.5 times as volatile as output and that once the non-traded goods sector is eliminated, this value reduces to 1.16. Our findings confirm the importance of non-traded goods but we emphasize that merely incorporating non-traded goods is not sufficient to address RER volatility. There are other papers that stress the role of other channels such as the exchange rate pass through or the international asset market structure — see Chari et al. (2002), Heathcote and Perri (2002), Rabanal and Tuesta (2010), amongst others. Although these papers study RER volatility, they focus on the US exchange rate dynamics. Benigno and Thoenissen (2003) on the other hand, examine UK-EA RER dynamics as we do. Unlike our paper, they investigate the transmission of productivity shocks to the RER and its components — rather than its variance — within a rich theoretical framework that incorporates non-traded goods and nominal rigidities[9].

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[8]See, for instance Imbs and Mejean (2015).

[9]More recent literature on the exchange rate fluctuations focuses on different aspects.

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The remainder of the paper is organised as follows: In the next section we present the model. In Section 3 we lay out the estimation of the productivity shocks and provide evidence on the cointegrating relationship. In Section 4 we describe the parameterization of the remaining parameters. In Section 5 we discuss the performance of our model by comparing data and model moments and in Section 6 we provide provide possible explanations for our findings by performing sensitivity checks for the key parameters of the model. Finally, in Section 8 we conclude. There is a detailed technical appendix in which we show the de-trending of the model and the log-linearised system of equations.

2 The Model

In this section, we present a two-country, two-sector IRBC model with traded and non-traded goods. The two countries, home and foreign, are assumed to differ in population size, n and $1 - n$, and consist of identical, infinitely lived households. Households can consume non-traded goods, domestically produced traded goods and imported goods. We assume that international financial markets are incomplete in the sense that households can trade non-state-contingent claims. The formulation of technological shocks differs in our model from a standard two-sector IRBC model. We assume that productivity innovations in traded sectors have permanent effects while the innovations in non-traded sectors are purely transitory. For the traded sector, as in Mandelman et al. (2011) and Rabanal et al. (2011), we consider permanent technology shocks that are co-integrated across countries. We will denote the foreign country variables with an asterisk (*).

2.1 Households

The preferences over intertemporal decisions are identical across countries, thus we only present the utility maximisation problem of the representative home country household. The representative household, i, receives utility from consumption, C_t^i, and disutility from producing goods, L_t^i. We assume that the utility function is separable in these two arguments and is given by:

For instance, Heyerdahl-Larsen (2014) emphasise the role of deep habits in consumption and consumption home bias in accounting for the RER volatility, while Farhi and Gabaix (2016) show that a model that incorporates the possibility of rare but extreme disasters can address the excess volatility of exchange rates.

$$U_t^i = E_t \sum_{t=0}^{\infty} \beta^t \left[log\left(C_t^i\right) - \frac{(L_t^i)^{1+\eta}}{1+\eta} \right], \quad 0 < \beta < 1 \tag{1}$$

where E_t denotes the expectations operator at time t, β is the discount factor and the parameter η is the inverse of the Frisch elasticity of labour supply. Given the presence of permanent shocks in the model, we ensure a balanced growth path by assuming log consumption utility[10].

The international asset markets are assumed to be incomplete. Following Benigno (2001), we assume that only the foreign issued bonds can be traded internationally although households in the home country can hold domestically issued bonds as well. We assume that households in the home country have to pay a cost in order to engage in a foreign asset market transaction. This cost, $\Theta(.)$, ensures the stationary distribution of wealth across countries[11] (see, Schmitt-Grohe and Uribe, 2003). Households finance their expenditure through the holdings of these bonds in addition to the labour income and dividend payments from the ownership of shares of domestic firms. Households maximise the utility, Equation (1), subject to the following real budget constraint (measured in the units of CPI):

$$C_t^i + \frac{B_{H,t}^i}{(1+r_t)} + \frac{Q_t B_{F,t}^i}{(1+r_t^*)\Theta(Q_t B_{F,t})} \leq B_{H,t-1}^i + Q_t B_{F,t-1}^i + w_t^i L_t^i + \Pi_t^i \tag{2}$$

where $B_{H,t}^i$ and $B_{F,t}^i$ are household i's holdings of the home and foreign currency denominated real risk-free bonds. The real interest rate on these bonds at time t are r_t and r_t^* respectively. Q_t is the real exchange rate expressed as $Q_t = \frac{S_t P_t^*}{P_t}$ and S_t is the nominal exchange rate defined as the home currency price of buying one unit of foreign currency. w_t^i is the real wage and Π_t^i is the real profit income.

This maximisation yields the following equilibrium conditions:

$$\frac{C_{t+1}}{C_t} = \beta\left(1+r_t\right) \tag{3}$$

$$w_t = L_t^\eta C_t \tag{4}$$

$$1 = \beta(1+r_t^*)\Theta(Q_t B_{F,t}) E_t \left[\left(\frac{C_t}{C_{t+1}} \right) \left(\frac{Q_{t+1}}{Q_t} \right) \right] \tag{5}$$

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[10]See, King et al. (1988) for a discussion about the necessary restrictions on preferences for the existence of a balanced growth path.

[11]$\Theta(.)$ is a differentiable decreasing function in the neighbourhood of the steady state level of net foreign assets ($B_{F,t} = 0$) and at the steady state net foreign asset level, the cost function is equal to 1 ($\Theta(0) = 1$). These restrictions ensure a well-defined steady state. See Benigno (2001) for details.

2.2 Final Goods Sectors

Final goods consumption consists of non-traded, $(C_{N,t})$, and traded goods, $(C_{T,t})$. We assume that the consumption index has a Cobb-Douglas functional form. Admittedly, Cobb-Douglas aggregation is much more restrictive than a CES. However, since in the model set-up there is a permanent and a stationary shock, an elasticity of substitution between traded and non-traded goods that is different from one would result in a nonstationary distribution of sector sizes[12]. The aggregate consumption can be expressed in the following way in the home and foreign country respectively:

$$C_t = \frac{C_{T,t}^{\alpha} C_{N,t}^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}} \tag{6}$$

$$C_t^* = \frac{(C_{T,t}^*)^{\alpha^*} (C_{N,t}^*)^{1-\alpha^*}}{(\alpha^*)^{\alpha^*} (1-\alpha^*)^{1-\alpha^*}} \tag{7}$$

where α and α^* are the expenditure share of traded goods in total consumption in the home and foreign country respectively. Consumption of traded goods is a CES aggregate of domestically produced goods and imported goods:

$$C_{T,t} = \left(\nu^{\frac{1}{\theta}} (C_{H,t})^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}} (C_{F,t})^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{8}$$

$$C_{T,t}^* = \left((\nu^*)^{\frac{1}{\theta}} (C_{F,t}^*)^{\frac{\theta-1}{\theta}} + (1-\nu^*)^{\frac{1}{\theta}} (C_{H,t}^*)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{9}$$

where θ is the elasticity of substitution between home and foreign produced goods and ν, ν^* is the weight of domestically produced goods. When ν and ν^* are greater than 0.5, households put a higher weight on domestically produced goods, implying a 'home bias' in preferences. The parameter that determines the share of imported goods in the traded consumption basket is proportional to the size of the importing country and the degree of openness, μ: $1-\nu = (1-n)\mu$ and $1-\nu^* = n\mu$.[13]

Final goods producers maximise the aggregate and traded consumption subject to nominal expenditure. This yields the following optimal demand functions:

$$C_{N,t} = (1-\alpha) \left(\widehat{P_{N,t}} \right)^{-1} C_t, \qquad C_{N,t}^* = (1-\alpha^*) \left(\widehat{P_{N,t}^*} \right)^{-1} C_t^* \tag{10}$$

$$C_{T,t} = \alpha \left(\widehat{P_{T,t}} \right)^{-1} C_t, \qquad C_{T,t}^* = \alpha^* \left(\widehat{P_{T,t}^*} \right)^{-1} C_t^* \tag{11}$$

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[12]Yet assuming that the elasticity of substitution between traded and non-traded goods as one, is not far from some of its calibrations in the literature. For instance, Corsetti et al. (2008) calibrate the elasticity of substitution between traded and non-traded goods to 0.74.

[13]See, De Paoli (2009) for a similar preference structure.

and

$$C_{H,t} = \nu \left(\frac{\widehat{P_{H,t}}}{\widehat{P_{T,t}}} \right)^{-\theta} C_{T,t} \ , \qquad C_{F,t} = (1 - \nu) \left(\frac{\widehat{P_{F,t}}}{\widehat{P_{T,t}}} \right)^{-\theta} C_{T,t} \qquad (12)$$

$$C^*_{F,t} = \nu^* \left(\frac{\widehat{P^*_{F,t}}}{\widehat{P^*_{T,t}}} \right)^{-\theta} C^*_{T,t} \ , \qquad C^*_{H,t} = (1 - \nu^*) \left(\frac{\widehat{P^*_{H,t}}}{\widehat{P^*_{T,t}}} \right)^{-\theta} C^*_{T,t} \qquad (13)$$

We measure all prices relative to the CPI of the corresponding country: $\frac{P_{j,t}}{P_t} = \widehat{P_{j,t}}$ where $j = N, T, H, F$ and $\frac{P^*_{j,t}}{P^*_t} = \widehat{P^*_{j,t}}$ where $j^* = N^*, T^*, H^*, F^*$.

The corresponding price indices are:

$$1 = (\widehat{P_{T,t}})^\alpha \, (\widehat{P_{N,t}})^{1-\alpha} \qquad (14)$$

$$1 = (\widehat{P^*_{T,t}})^{\alpha^*} \, (\widehat{P^*_{N,t}})^{1-\alpha^*} \qquad (15)$$

$$\widehat{P_{T,t}} = \left(\nu \widehat{P_{H,t}}^{1-\theta} + (1 - \nu) \widehat{P_{F,t}}^{1-\theta} \right)^{\frac{1}{1-\theta}} \qquad (16)$$

$$\widehat{P^*_{T,t}} = \left(\nu^* \widehat{P^*_{F,t}}^{1-\theta} + (1 - \nu^*) \widehat{P^*_{H,t}}^{1-\theta} \right)^{\frac{1}{1-\theta}} \qquad (17)$$

We assume that the law of one price (LoOP) holds in the sense that prices are set in the currency of the producer: $\widehat{P_{F,t}} = Q_t \widehat{P^*_{F,t}}$ and $\widehat{P^*_{H,t}} = \widehat{P_{H,t}}/Q_t$.

2.3 Intermediate Goods Sectors

Firms in the intermediate goods sectors produce non-traded and traded goods using labour as the production factor. Non-traded intermediate goods producers sell their goods to the domestic final good producers to be consumed only by domestic households, while traded intermediate goods producers sell their goods to the domestic final goods producers to be consumed by home and foreign households. Production in each industry has a constant returns to scale functional form:

$$Y_{j,t} = A_{j,t} L_{j,t} \qquad (18)$$

where $j = H, F^*, N, N^*$. $Y_{j,t}$ is the output, $A_{j,t}$ is the exogenous technology shock, $L_{j,t}$ is the total labour employed in the respective sector and country.

The technology in non-traded sectors has the following stochastic processes:

$$ln(A_{N,t}) = \rho_{a^N} \, ln(A_{N,t-1}) + \varepsilon_{a^N,t} \tag{19}$$

$$ln(A_{N,t}^*) = \rho_{a^{N*}} \, ln(A_{N,t-1}^*) + \varepsilon_{a^{N*},t} \tag{20}$$

where $0 \leq \rho_{a^N} < 1$, $0 \leq \rho_{a^{N*}} < 1$ and $\varepsilon_{a^N,t} \sim N(0,\sigma_{a^N}^2)$, $\varepsilon_{a^{N*},t} \sim N(0,\sigma_{a^{N*}}^2)$.

Technology in the traded sectors, on the other hand, is assumed to be non-stationary. We explain the functional form of the traded sectors later when we estimate the TFP processes.

2.4 Market Clearing and the Current Account

We close the model with market clearing conditions. The goods market clearing conditions are:

$$Y_{N,t} = C_{N,t} , \qquad Y_{N,t}^* = C_{N,t}^* \tag{21}$$

$$Y_{H,t} = C_{H,t} + \frac{1-n}{n} \, \frac{Q_t \, \widehat{P_{H,t}^*}}{\widehat{P_{H,t}}} \, C_{H,t}^* , \qquad Y_{F,t}^* = C_{F,t}^* + \frac{n}{1-n} \, \frac{\widehat{P_{F,t}}}{Q_t \, \widehat{P_{F,t}^*}} \, C_{F,t} \tag{22}$$

Labour is mobile across sectors but not across countries:

$$L_t = L_{N,t} + L_{H,t} , \qquad L_t^* = L_{N,t}^* + L_{F,t}^* \tag{23}$$

We measure the total output in terms of CPI since we choose CPI as the numeraire:

$$Y_t = C_t + \frac{1-n}{n} \, \widehat{P_{H,t}} C_{H,t}^* - Q_t \widehat{P_{F,t}^*} \, C_{F,t} \tag{24}$$

$$Y_t^* = C_t^* + \frac{n}{1-n} \, \widehat{P_{F,t}^*} C_{F,t} - \frac{\widehat{P_{H,t}}}{Q_t} C_{H,t}^* \tag{25}$$

Finally, the current account dynamics of the home economy can be written as:

$$\frac{Q_t B_{F,t}}{(1+r_t^*)\Theta(Q_t B_{F,t})} - Q_t B_{F,t-1} = \frac{n}{1-n} \, \widehat{P_{H,t}} \, C_{H,t}^* - Q_t \widehat{P_{F,t}^*} C_{F,t} \tag{26}$$

Notice that the right hand side of the current account equation is equal to the trade balance of the home economy. We measure it as a ratio of GDP:

$$\frac{TB_t}{Y_t} = \frac{\dfrac{n}{1-n} \, \widehat{P_{H,t}} \, C_{H,t}^* - Q_t \widehat{P_{F,t}^*} \, C_{F,t}}{Y_t} \tag{27}$$

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3 Estimation of Productivity Shocks

In this section, we describe the estimation of TFP processes in each sector and country that we use to calibrate our model. We calibrate the model to the UK and EA (denoted by an asterisk (*)) data and assume that the UK is the home country.

We compute the sectoral TFP series using the data from the EU-KLEMS database. The data for this calculation is at annual frequency and covers the period from 1982 to 2007. We consider Austria, Spain, Belgium, France, Finland, Germany, Italy and Netherlands as an approximate for the EA. We first take the TFP index data and calculate the TFP growth rates. By computing the value added share of sectors, we construct TFP growth series for the traded and non-traded sectors. We assume that agriculture, mining, manufacturing and financial intermediation are traded and the remaining[14] are non-traded sectors.

The following analysis is based on the assumption that (log) TFP processes of traded sectors are co-integrated in such a way that they follow the same stochastic trend. As mentioned earlier, Mandelman et al. (2011) and Rabanal et al. (2011) find such a behaviour for TFP processes derived from the Solow-residual between the US and the rest of the world. However, the series derived from the EU-KLEMS data, which are plotted in Figure (2), also suggest that traded TFP sectors of the UK and EA follow a strong positive common trend. At the same time, the TFP processes of non-traded sectors remain roughly at the same level. This can be explained using a classic textbook example: Today, the hairdressers still cut hair using the same methods as 30 years ago.

In order to test for a cointegrating relationship between the traded sector (log) TFP processes, we estimate an unrestricted VAR model with a constant and time-trend for both variables. For this model, the Schwarz criterion (SC) suggests a lag order of 1. Afterwards, we test for a cointegrating relation between both series using the Johansen (1991) test. Table (1) displays the cointegration rank test results for the trace and max-eigenvalue statistics. According to the corresponding p-values, the statistics are clearly in favour of one cointegrating relationship between the two variables.

In accordance with Mandelman et al. (2011) and Rabanal et al. (2011), we thus estimate an unrestricted VECM with the specification

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[14]Namely, these sectors are electricity, gas and water; construction; wholesale and retail trade; hotels and restaurants; transport and storage; real estate, renting and business activities and finally community, social and personal services.

Figure 2: Traded and non-traded TFP
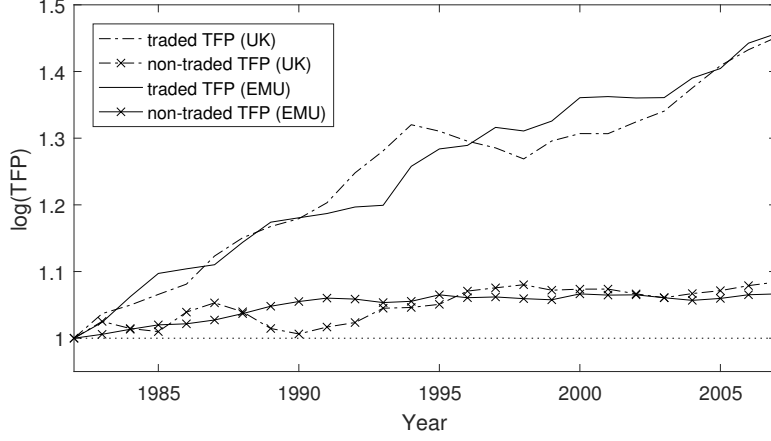
Table 1: Johansen cointegration test

Hypothesized No. of CE(s)	Eigenvalue	trace Statistic	p-value	Max-eigenvalue	p-Value
None	0.704175	34.55006	0.0032	30.44965	0.0008
At most 1	0.151272	4.100408	0.7275	4.100408	0.7275

Note: The table shows trace and max-eigenvalue statistics of the Johansen test under the assumption of a constant and trend in the cointegrating vector. MacKinnon et al. (1999) p-values.

$$\begin{pmatrix} \Delta \log A(s^t) \\ \Delta \log A^*(s^t) \end{pmatrix} = \begin{pmatrix} c \\ c^* \end{pmatrix} + \begin{pmatrix} \kappa \\ \kappa^* \end{pmatrix} [\log A(s^{t-1}) - \gamma \log A^*(s^{t-1}) - \log \zeta] + \begin{pmatrix} \epsilon(s^t) \\ \epsilon^*(s^t) \end{pmatrix}, \quad (28)$$

where $A(s^t)$ and $A^*(s^t)$ denote the home and foreign traded sector TFP processes, respectively. c and c^* represent constant terms. The coefficients representing the speed of adjustment in the cointegrating vector are denoted by κ and κ^*. Without loss of generality, the cointegrating vector is defined as $(1, -\gamma)$. ζ denotes a constant term in the cointegrating relationship. The error terms are $\epsilon(s^t) \sim N(0, \sigma^\epsilon)$ and $\epsilon^*(s^t) \sim N(0, \sigma^{\epsilon,*})$.

In order to test for symmetry across coefficients driving the traded sector

TFP processes, we test the restrictions $\gamma = 1$ and $\kappa = -\kappa^*$ sequentially.[15] The first restriction ($\gamma = 1$) implies that the log difference between both traded sector TFP processes is stationary. Hence, they follow the same trend. The second restriction ($\kappa = -\kappa^*$) tests whether the speed of adjustment towards the common trend is equal across countries. Table (2) presents the results of the likelihood ratio tests for different specifications. Neither the assumption that the cointegrating vector is $(1, -1)$, nor the assumption that $\kappa = -\kappa^*$ is rejected by the data. Consequently, the data does not reject the assumption of the common balanced growth path between both regions.

Table 2: Likelihood ratio test

Restriction	Likelihood value	Degrees of freedom	p-value
none	138.5795	–	–
$\gamma = 1$	138.4412	1	0.5989
$\gamma = 1$, $\kappa = -\kappa^*$	136.6648	2	0.1474

Hence, we estimate the VECM model and impose the symmetry restrictions which the IRBC literature suggests (Mandelman et al. (2011) and Rabanal et al. (2011)). Table (3) shows the coefficient estimates as well as the corresponding t-statistics. All coefficients are statistically significant. The coefficient $\kappa = 0.12$ implies that the (log) traded sector TFP series adjust by approx. 12% towards their common trend within one year. The corresponding cointegration relationship is plotted in Figure (3). We also test whether $\epsilon(s^t)$ and $\epsilon^*(s^t)$ are uncorrelated. The t-statistic of 1.30 suggests that the correlation is not statistically different from zero. Therefore, we abstract from potential cross-correlation in the model.

The country-specific processes of (log) non-traded sector TFP are modeled as univariate AR(1) processes.[16] The estimated autoregressive coefficients are $\rho_a^N = 0.85$ for the UK and $\rho_a^{N*} = 0.87$ for the EA.

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[15]For a detailed discussion of tests with regard to symmetry across countries in a cointegrated VAR environment we refer the reader to Krolzig and Heinlein (2013).

[16]We also considered a VAR(1)-process, but the diagonal entries of the loading matrix (spillovers) were not statistically different from zero. Results are available upon request.

Table 3: VECM estimates

Coefficient	Value	t-statistic
κ	0.123528	2.04672
c	0.018055	5.05613
c^*	0.018279	6.38803
ζ	0.025942	–
σ	0.017539	–
σ^*	0.016176	–

Figure 3: Cointegration relationship
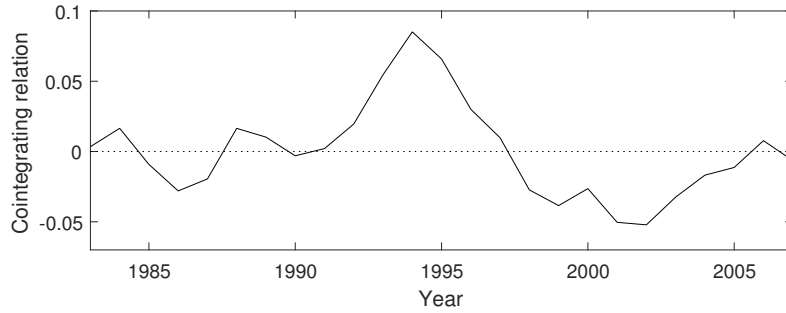
Note: The figure shows the (stationary) cointegrating relation between the UK and EA traded sector TFP series.

4 Parameterization

Calibration of the remaining parameters is shown in Table 4. Since we work with annual data, we calibrate the discount factor, β, to 0.96 which implies an interest rate of 4% per annum. We assume that the inverse Frisch elasticity of the labour supply is equal to 2 in accordance with the DSGE literature. We calibrate the country size to match the population share of the two countries[17]. Following Benigno (2001), we set the value of the cost of intermediation in the foreign asset markets to 0.001. The elasticity of substitution between the home and foreign produced traded goods is assumed to be equal to 1.5 (see Backus et al. (1993) or Chari et al. (2002)).

To calculate the share of traded goods in total consumption basket, α, and

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[17]We obtain the population data from EUROSTAT. We use the total population of age 16-65.

Table 4: Parameter Values

Parameter	Description	Value
β	discount factor	0.96
η	inverse Frisch elasticity of labour supply	2
n	relative country size	0.15
δ	cost of intermediation	0.001
θ	trade elasticity	1.5
$\alpha = \alpha^*$	share of traded goods in total consumption	0.34
$\mu = \mu^*$	degree of openness	0.28

the share of home produced goods in traded consumption basket, ν, we use the Input-Output Tables from the World Input-Output Database, 2013 Release (see Timmer et al. (2015)). We use the consumption shares of 2011 as this is the latest available data for the EA[18]. We make the same sectoral assumption as for the estimation of sectoral TFP's. For the traded sector, we take the sum of final expenditure on agriculture, mining, manufacturing and financial intermediation. We consider both the domestic and import demand for these sectors. For the rest of the sectors, we calculate the expenditure on non-traded goods by only considering the expenditure on domestically produced final goods. We find that the share of non-traded goods is equal to 0.64 in the UK and 0.66 in the EA. To ensure that the share is the same across the countries, we fix this parameter to 0.66. To calculate the degree of openness, we use the share of import expenditure in total traded goods expenditure. We find that the $\mu = 0.29$ and $\mu^* = 0.25$. The size adjusted shares in the data are then: $1 - \nu = 0.25$, $1 - \nu^* = 0.03$. We set the degree of openness to 0.28 so that we match the size adjusted data shares as closely as possible: $1 - \nu = 0.238$, $1 - \nu^* = 0.042$. This calibration implies that UK is much more open to trade compared with the EA.

We will conduct a sensitivity analysis for the value of the trade elasticity and the consumption share of non-traded goods since these parameters are important for the RER dynamics.

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[18]In order to calculate the EA consumption shares, we use the euro zone data from the Regional Input-Output Tables available from the World Input-Output Database. For the UK, the latest available data is from 2014 (2016 Release), but to be consistent between the regions we use 2011 data for the UK as well.

5 Quantitative Properties

In this section, we analyse the performance of our model in terms of matching the second order moments of the data with a special focus on the RER volatility. As our model is non-stationary, we de-trend the non-stationary variables and work with a stationarised model. The de-trending of the model can be found in Appendix A. We log-linearise the de-trended model around the steady state. The log-linearised system of equations are listed in Appendix B.

We report the quantitative properties of the data and model in Table 5. We compute the moments of the data by assuming that the UK is the home country and the EA is the foreign country. The data covers the period from 1982 to 2007 as in our estimations of productivity processes. We use per capita household final consumption expenditures, per capita GDP, bilateral RER and trade balance of goods[19] to calculate the statistics. Details on the data sources can be found in Appendix C. We not only present the moments obtained from the benchmark non-stationary model but also from a model that is only driven by stationary technology shocks (3rd column), from a model without a non-traded sector (4th column) and finally from a model where the elasticity of substitution between home and foreign traded goods is set to 0.85 (last column) for comparison purposes. To obtain the moments generated by a model driven by stationary shocks[20] and by a model without non-traded goods[21], we calibrate the standard deviation of home and foreign traded sector TFPs such that we match the GDP volatility — of home and foreign country — delivered by the non-stationary model. We do not attempt to match the output volatility for the exercise where we change the trade elasticity. Given our interest in the business cycle fluctuations we HP-filter the consumption and the GDP data[22]. We keep the RER and the trade balance to GDP ratio in levels since these variables are stationary in the theoretical framework. To map the model generated moments with the data, we simulate our model and add back the stochastic trends of

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[19] The bilateral trade balance data is only available in nominal terms. This does not cause a problem in terms of the mapping between the data and model because once we divide the nominal trade balance to nominal UK GDP what we obtain is observationally equivalent to Equation (27).

[20] We calibrate the non-traded sector TFP shocks as in our benchmark calibration since they are already stationary. We choose 0.88 as the AR(1) parameter of traded sector TFP shocks. This value implies a significant persistence in accordance with their calibration in the IRBC literature. The rest of the parameters are equivalent to those presented in Table 4.

[21] We set the share of traded goods, α, to 0.999 in order to exclude non-traded goods from the model.

[22] We use 100 for the smoothing parameter of the HP-filter as suggested by Backus et al. (1992).

the trended variables to the simulated data. As we did for the actual data, we
HP-filter those variables[23].

<center>Table 5: Selected Second Moments</center>

	Data	Benchmark ($\theta = 1.5$)	Stationary	No NT	Trade Elast. ($\theta = 0.85$)
Std.dev.s (σ)					
C	1.3	0.91	0.94	0.76	0.89
TB/Y(%)	0.79	1.54	0.48	1.67	1.50
RER	6.75	6.23	2.83	1.32	15.56
Autocorrelations					
Y	0.74	0.48	0.40	0.59	0.48
C	0.73	0.31	0.40	0.21	0.35
TB/Y	0.63	0.86	0.94	0.82	0.90
RER	0.81	0.98	0.94	0.98	0.99
Cross-Correlations					
Y-Y*	0.33	-0.087	0.16	-0.229	-0.09
C-C*	0.43	0.08	0.24	0.1	0.06
Y-C	0.95	0.77	0.99	0.61	0.85
TB/Y-Y	-0.60	0.26	0.20	0.44	0.21
RER-Y	-0.11	0.04	0.28	0.075	-0.008
RER-(C/C*)	0.064	0.094	0.33	-0.04	0.019

Note: Standard deviations of all the variables are reported relative to the standard deviation
of the UK GDP except for the trade balance to GDP ratio. As the trade balance is already
measured as a ratio of GDP, we report its standard deviation directly. EA variables are shown
with an asterisk. All the data is in logs except for the TB/Y. The computed data statistics are
based on HP-filtered annual data for the period 1982-2007 with the exception of the real exchange
rate and the trade balance. These variables are kept in levels since they are stationary in the model.

The benchmark model with co-integrated shocks performs significantly well
in accounting for the volatility of the RER. In our sample the RER is 6.75 times
as volatile as the UK GDP. This volatility is 6.23 in our model, which is very close
to the data. On the other hand, when the permanent shocks or the non-traded
goods sectors are eliminated from the theoretical set-up, models are not able to
generate sufficient volatility — only 2.83 times as large as the GDP volatility
in the stationary model and 1.32 times as large as the GDP volatility in the

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[23]We simulated the model for 2000 periods and we discarded the first 1000 periods.

model without the non-traded sector. In our benchmark model, there are several channels that generate wealth effects and help to increase the volatility of the RER. The combined effect of the slow speed of adjustment in the co-integrated process, the incomplete asset market setting and the high share of non-traded goods in the aggregate consumption basket raises the RER variation. Although the trade elasticity is calibrated to a value higher than 1 ($\theta = 1.5$), which reduces the terms of trade volatility and thus the RER volatility too, our benchmark model matches the observed RER volatility almost perfectly. In fact, once the trade elasticity is lowered to 0.85, the model over-predicts the volatility; the RER is 15 times more volatile than the GDP. We conduct a robustness analysis in the next section on the value of elasticity of substitution between home and foreign produced goods in order to obtain a deeper interpretation of the results.

The improvement in our model's ability to account for the RER volatility is related to the inclusion of both co-integrated TFP shocks and non-traded sectors. The non-stationary traded sector shock in our model is cointegrated across countries implying that, in the long run, traded sectors of the two countries carry the same trend. Generally, global shocks reduce the volatility of international variables as they cannot be insured away by countries. However, since the estimated convergence parameter is very low, when one country experiences a TFP improvement in its traded sector, the other country's traded sector technology process will adjust to that at a very slow speed. This shock thus generates significant wealth effects. The improvement in the model's ability to address the high volatility of the RER has already been emphasised by Rabanal et al. (2011). A model that incorporates co-integrated productivity shocks performs better than a standard IRBC in terms of RER volatility. We discuss the differences in our findings with Rabanal et al. (2011) in the next section where we interpret our results more in detail.

Standard international RBC models fail to account for the negative correlation between the RER and the relative consumption. The lack of international risk sharing is labelled as the consumption-real exchange rate anomaly by Chari et al. (2002) and is also known as the Backus-Smith puzzle (see Backus and Smith (1993)). In our sample, the sign of this correlation is positive but it is very close to zero. Our model does significantly well in matching this correlation. The combination of wealth channels in the theoretical framework, which is amplified by the low adjustment parameter of the co-integrated shock, breaks the link between the RER and the relative consumption. The benchmark model also outperforms the stationary model and the model without non-traded goods in terms of addressing the correlation of the RER with output. In the data,

the RER is counter-cyclical but the value is very close to zero. Although our model does not deliver the correct sign, it generates a correlation that is very close to zero as it is in the data. Once we lower the trade elasticity to 0.85, the model predicts a negative correlation between the RER and output; however, this value is almost equal to zero (-0.008).

Overall, our model performs relatively well in accounting for the volatility of the RER and its correlation with relative consumption and GDP as discussed. However, it fails to match the data in other dimensions. Having a non-stationary shock helps increasing the volatility of the RER, but it comes at the cost of excess volatility in trade balance. While the benchmark model and the model without non-traded goods over-predict the variation in the trade balance, the stationary model under-predicts it. The wealth effects arising from the shock structure increase the volatility of the trade balance. The relative consumption volatility in our sample is above one (1.3). In the theoretical set-up, including a non-traded sector raises the volatility of consumption relative to output, yet our model delivers a consumption volatility that is lower than one (0.91). In terms of matching the persistence of variables, all models fail. They generate too much persistence in international variables and too little in real variables. Finally, even though including a non-stationary shock to the model improves the model performance in several dimensions, our model still generates unrealistic cross-country consumption and output correlations. The reason is that the estimated slow speed of adjustment to the traded sector non-stationary shock reduces the cross-country correlations significantly despite the fact that this shock is common across countries. In our framework, we avoid including ad hoc shock correlations in order to generate higher co-movement.

6   Interpretation of Key Results

6.1   The Role of the Non-traded Goods Sector

Our analysis shows that, once the non-traded sector is excluded from the theoretical framework, the model fails to generate sufficient RER volatility. In order to provide a better understanding of the importance of non-traded goods, we simulate the model by varying the share of traded goods (α) from 0.1 to 0.9 with 0.1 intervals. Figure 4 shows the RER volatility obtained from each simulation once we vary the share of non-traded goods ($1 - \alpha$). It is evident from the figure that the higher the share of non-traded goods (the lower the share of traded goods) in the total consumption basket, the higher the volatility

of the RER. The presence of cointegrated TFP shocks with high persistence is not sufficient to generate the correct RER volatility. However, combining these shocks with non-traded goods consumption that is directly calibrated from the data improves the model performance significantly.

Figure 4: Standard deviation of RER with respect to the changes in the share of traded goods
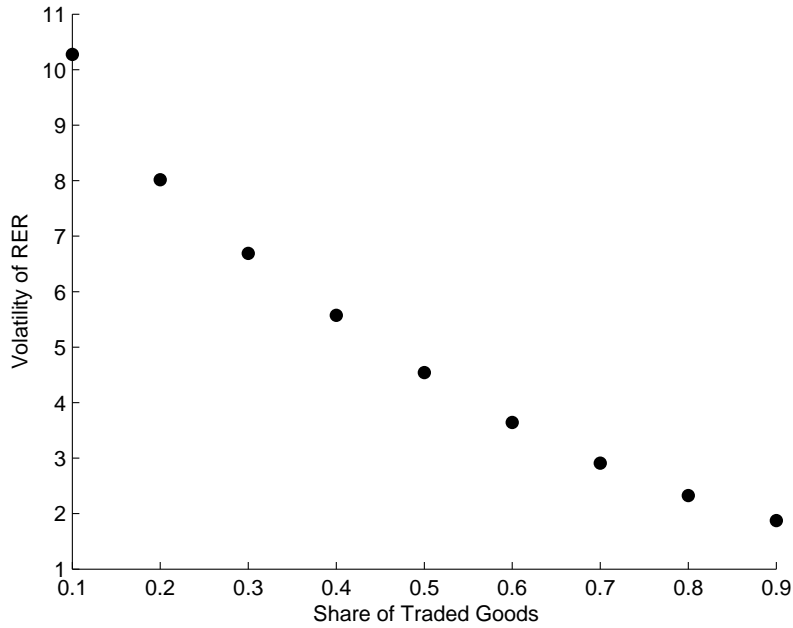

The larger the share of non-traded goods in the total consumption basket, the larger the weight of the non-traded good prices in the CPI. This implies that movements in the prices of non-traded goods will increase the RER variability. The variation in non-traded good prices arises through two channels: First, the existence of non-traded sector productivity shocks cause variations in the relative price of non-traded goods to traded goods and hence in the RER. Second, since labour is mobile across sectors, a productivity improvement in one sector does not affect only the prices of the sector in which the shock originates (Balassa-Samuelson effect). A TFP shock that originates in the traded sector may result in large fluctuations in non-traded good prices when the consumption share of non-traded goods is sufficiently large. This thus raises the volatility of the RER. In fact, when we shut down the non-traded sector TFP shocks in our benchmark model, the volatility of the RER increases to 7.49. This is because the productivity of the traded sector increases without an increase in the pro-

ductivity of the non-traded sector. This generates larger variations in the RER through the Balassa-Samuelson effect[24].

6.2 The Role of Trade Elasticity

The volatility of terms of trade is related to the value of the elasticity of substitution between home and foreign produced goods (θ). The higher the home bias in preferences, the higher the correlation of the RER with the terms of trade. An elasticity that is smaller increases the terms of trade volatility and in the presence of home bias this also causes large movements in the RER too. We check the importance of the value of the trade elasticity for the RER volatility by simulating the model for a range of values from 0.8 to 2 with 0.1 intervals. We plot the corresponding RER volatility to each simulation in Figure 5.

Figure 5: Standard deviation of RER with respect to the changes in the elasticity of substitution between home and foreign produced traded goods
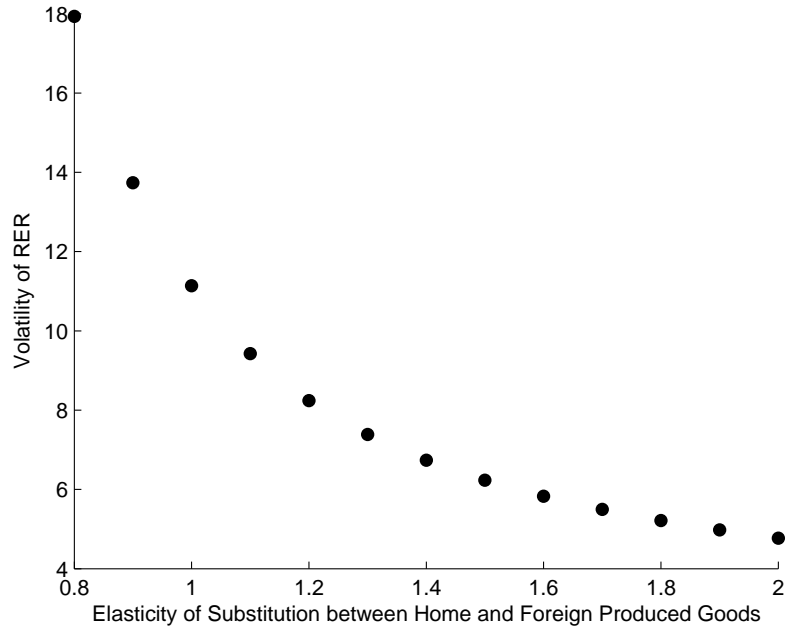
In our model, the high consumption share of non-traded goods combined

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[24]In our model, a productivity improvement in the home traded sector causes wages to increase by more than the TFP improvement and hence generates an increase in the price of home produced traded goods (terms of trade improve, i.e. the relative price of imports to exports falls). The decreasing effect of terms of trade on the RER is amplified by an increase in the non-traded good prices implying an increase in the RER volatility.

with the slow speed of adjustment to the common trend is sufficient to generate the observed RER volatility. As discussed before, Figure 5 shows that an elasticity lower than one causes the model to over-predict the RER variability. Even with an elasticity equal to 2, the model generates an RER volatility that is very close to 5. This value is significantly large compared with the findings in the literature. For instance, Rabanal et al. (2011) can account for the volatility of the RER through a slow convergence to the cointegrating relationship (as in our case) only when combined with an elasticity that is lower than one. Their model requires high degrees of home bias along with a trade elasticity that is lower than one to be able to match the RER volatility. This is not the case in our model due to two reasons: First, our estimates of VECM imply higher persistence than the estimates of Rabanal et al. (2011) and second, the non-traded goods have a large share in aggregate consumption. These then help match the observed RER volatility.

7 Conclusion

The inability of international RBC models to match the real exchange rate volatility has become a well-known puzzle in the international macro literature. The real exchange rate is much more volatile in the data compared to what we obtain from our models. In this paper, we analyse the importance of non-traded goods and cointegrated TFP shocks in accounting for the UK-EA real exchange rate volatility. The analysis is motivated by two key empirical facts: First, non-traded goods have a large share in the total consumption basket and second, the UK and EA traded sector productivities carry the same trend in the long run. We provide direct evidence on the cointegrating relationship by estimating a VECM.

We show that incorporating non-stationary technology shocks along with non-traded goods sectors increases the model generated real exchange rate volatility substantially. Our analysis points out that none of these channels are sufficient enough to account for the observed volatility without the other. These channels also help to match the correlation of the real exchange rate with relative consumption and output. The improvement in our model's performance is a consequence of the wealth effects that arise from the high share of non-traded goods in the consumption basket and the estimated slow speed of adjustment to the common trend.

Appendices

Appendix A   Stationarised Model

In this Appendix we present the de-trended system of equations since we work with a stationary model. We normalise trended variables with the corresponding trend[25]: $\widetilde{Y_{H,t}} = \frac{Y_{H,t}}{A_{H,t}}$, $\widetilde{Y_{F,t}^*} = \frac{Y_{F,t}^*}{A_{F,t}^*}$, $\widetilde{\widehat{P_{N,t}}} = \frac{\widehat{P_{N,t}}}{A_{H,t}^\alpha}$, $\widetilde{\widehat{P_{H,t}}} = \frac{\widehat{P_{H,t}}}{A_{H,t}^{\alpha-1}}$, $\widetilde{\widehat{P_{T,t}}} = \frac{\widehat{P_{T,t}}}{A_{H,t}^{\alpha-1}}$
$\widetilde{\widehat{P_{N,t}^*}} = \frac{\widehat{P_{N,t}^*}}{(A_{F,t}^*)^\alpha}$, $\widetilde{\widehat{P_{F,t}^*}} = \frac{\widehat{P_{F,t}^*}}{(A_{F,t}^*)^{\alpha-1}}$, $\widetilde{\widehat{P_{T,t}^*}} = \frac{\widehat{P_{T,t}^*}}{(A_{F,t}^*)^{\alpha-1}}$, $\widetilde{C_t} = \frac{C_t}{A_{H,t}^\alpha}$, $\widetilde{C_t^*} = \frac{C_t^*}{(A_{F,t}^*)^\alpha}$,
$\widetilde{C_{T,t}} = \frac{C_{T,t}}{A_{H,t}}$, $\widetilde{C_{T,t}^*} = \frac{C_{T,t}^*}{A_{F,t}^*}$, $\widetilde{w_t} = \frac{w_t}{A_{H,t}^\alpha}$, $\widetilde{w_t^*} = \frac{w_t^*}{(A_{F,t}^*)^\alpha}$, $\widetilde{C_{H,t}} = \frac{C_{H,t}}{A_{H,t}}$, $\widetilde{C_{F,t}} = \frac{C_{F,t}}{A_{F,t}^*}$,
$\widetilde{C_{H,t}^*} = \frac{C_{H,t}^*}{A_{H,t}}$, $\widetilde{C_{F,t}^*} = \frac{C_{F,t}^*}{A_{F,t}^*}$, $\widetilde{Y_t} = \frac{Y_t}{A_{H,t}^\alpha}$, $\widetilde{Y_t^*} = \frac{Y_t^*}{(A_{F,t}^*)^\alpha}$.

Then the de-trended system of equations for the home economy are:

$$\left(\frac{\widetilde{C_{t+1}}}{\widetilde{C_t}}\right)\left(\frac{A_{H,t+1}}{A_{H,t}}\right)^\alpha = \beta\left(1 + r_t\right) \tag{A.1}$$

$$\widetilde{w_t} = L_t^\eta \widetilde{C_t} \tag{A.2}$$

$$1 = \beta(1 + r_t^*)\Theta(Q_t B_{F,t})E_t\left[\left(\frac{\widetilde{C_{t+1}}}{\widetilde{C_t}}\right)\left(\frac{A_{H,t+1}}{A_{H,t}}\right)^\alpha\left(\frac{Q_{t+1}}{Q_t}\right)\right] \tag{A.3}$$

$$\widetilde{Y_{H,t}} = L_{H,t} \tag{A.4}$$

$$Y_{N,t} = A_{N,t}L_{N,t} \tag{A.5}$$

$$\widetilde{w_t} = A_{N,t}\widetilde{\widehat{P_{N,t}}} = \widetilde{\widehat{P_{H,t}}} \tag{A.6}$$

$$\widetilde{C_t} = \frac{\widetilde{C_{T,t}}^\alpha C_{N,t}^{1-\alpha}}{\alpha^\alpha\left(1-\alpha\right)^{1-\alpha}} \tag{A.7}$$

$$\widetilde{C_{T,t}} = \left(\nu^{\frac{1}{\theta}}(\widetilde{C_{H,t}})^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}}(\widetilde{C_{F,t}})^{\frac{\theta-1}{\theta}}\left(\frac{A_{F,t}^*}{A_{H,t}}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \tag{A.8}$$

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[25]Since we assume symmetry between the preference parameters of the home and foreign countries in our calibration, here we impose $\alpha^* = \alpha$. This assumption ensures a stationary system of equations along the balanced growth path.

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$$1 = (\widetilde{\widetilde{P_{T,t}}})^{\alpha} (\widetilde{\widetilde{P_{N,t}}})^{1-\alpha} \tag{A.9}$$

$$\widetilde{\widetilde{P_{T,t}}} = \left(\nu \widetilde{\widetilde{P_{H,t}}}^{1-\theta} + (1-\nu)\widetilde{\widetilde{P_{F,t}}}^{1-\theta} \left(\frac{A_{F,t}^*}{A_{H,t}} \right)^{(\alpha-1)(1-\theta)} \right)^{\frac{1}{1-\theta}} \tag{A.10}$$

$$C_{N,t} = (1-\alpha) \left(\widetilde{\widetilde{P_{N,t}}} \right)^{-1} \widetilde{C_t} \tag{A.11}$$

$$C_{T,t} = \alpha \left(\widehat{P_{T,t}} \right)^{-1} \widetilde{C_t} \tag{A.12}$$

$$\widetilde{C_{H,t}} = \nu \left(\frac{\widetilde{\widetilde{P_{H,t}}}}{\widetilde{\widetilde{P_{T,t}}}} \right)^{-\theta} \widetilde{C_{T,t}} \tag{A.13}$$

$$\widetilde{C_{F,t}} = (1-\nu) \left[\left(\frac{Q_t \widetilde{\widetilde{P_{F,t}^*}}}{\widetilde{\widetilde{P_{T,t}}}} \right) \left(\frac{A_{F,t}^*}{A_{H,t}} \right)^{\alpha-1} \right]^{-\theta} \widetilde{C_{T,t}} \left(\frac{A_{F,t}^*}{A_{H,t}} \right) \tag{A.14}$$

$$Y_{N,t} = C_{N,t} \tag{A.15}$$

$$\widetilde{Y_{H,t}} = \widetilde{C_{H,t}} + \widetilde{C_{H,t}^*} \tag{A.16}$$

$$\widetilde{Y_t} = \widetilde{C_t} + \widetilde{\widetilde{P_{H,t}}}\widetilde{C_{H,t}^*} - Q_t \widetilde{\widetilde{P_{F,t}^*}} \widetilde{C_{F,t}} \left(\frac{A_{F,t}^*}{A_{H,t}} \right)^{\alpha} \tag{A.17}$$

$$\frac{Q_t B_{F,t}}{(1+r_t^*)\Theta(Q_t B_{F,t})} - Q_t B_{F,t-1} = \widetilde{\widetilde{P_{H,t}}}\widetilde{C_{H,t}^*} - Q_t \widetilde{\widetilde{P_{F,t}^*}} \widetilde{C_{F,t}} \left(\frac{A_{F,t}^*}{A_{H,t}} \right)^{\alpha} \tag{A.18}$$

$$L_t = L_{N,t} + L_{H,t} \tag{A.19}$$

The normalised equilibrium conditions for the foreign country are as follows:

$$\left(\frac{\widetilde{C_{t+1}^*}}{\widetilde{C_t^*}} \right) \left(\frac{A_{F,t+1}^*}{A_{F,t}^*} \right)^{\alpha} = \beta \left(1 + r_t \right) \tag{A.20}$$

$$\widetilde{w_t^*} = L_t^\eta \, \widetilde{C_t^*} \tag{A.21}$$

$$\widetilde{Y_{F,t}^*} = L_{F,t}^* \tag{A.22}$$

$$Y_{N,t}^* = A_{N,t}^* L_{N,t}^* \tag{A.23}$$

$$\widetilde{w_t^*} = A_{N,t}^* \widetilde{P_{N,t}^*} = \widetilde{P_{F,t}^*} \tag{A.24}$$

$$\widetilde{C_t^*} = \frac{(\widetilde{C_{T,t}^*})^\alpha (C_{N,t}^*)^{1-\alpha}}{\alpha^\alpha \, (1-\alpha)^{1-\alpha}} \tag{A.25}$$

$$\widetilde{C_{T,t}^*} = \left(\nu^{*\frac{1}{\theta}} (\widetilde{C_{F,t}^*})^{\frac{\theta-1}{\theta}} + (1-\nu^*)^{\frac{1}{\theta}} (\widetilde{C_{H,t}^*})^{\frac{\theta-1}{\theta}} \left(\frac{A_{H,t}}{A_{F,t}^*} \right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{A.26}$$

$$1 = (\widetilde{\widetilde{P_{T,t}^*}})^\alpha \, (\widetilde{\widetilde{P_{N,t}^*}})^{1-\alpha} \tag{A.27}$$

$$\widetilde{\widetilde{P_{T,t}^*}} = \left(\nu^* \widetilde{\widetilde{P_{F,t}^*}}^{1-\theta} + (1-\nu^*) \widetilde{\widetilde{P_{H,t}^*}}^{1-\theta} \left(\frac{A_{H,t}}{A_{F,t}^*} \right)^{(\alpha-1)(1-\theta)} \right)^{\frac{1}{1-\theta}} \tag{A.28}$$

$$C_{N,t}^* = (1-\alpha) \left(\widetilde{\widetilde{P_{N,t}^*}} \right)^{-1} \widetilde{C_t^*} \tag{A.29}$$

$$C_{T,t}^* = \alpha \left(\widehat{P_{T,t}^*} \right)^{-1} \widetilde{C_t^*} \tag{A.30}$$

$$\widetilde{C_{F,t}^*} = \nu^* \left(\frac{\widetilde{\widetilde{P_{F,t}}}^*}{\widetilde{\widetilde{P_{T,t}^*}}} \right)^{-\theta} \widetilde{C_{T,t}^*} \tag{A.31}$$

$$\widetilde{C_{H,t}^*} = (1-\nu^*) \left[\left(\frac{\widetilde{\widetilde{P_{H,t}}}}{Q_t \widetilde{\widetilde{P_{T,t}^*}}} \right) \left(\frac{A_{H,t}}{A_{F,t}^*} \right)^{\alpha-1} \right]^{-\theta} \widetilde{C_{T,t}^*} \left(\frac{A_{H,t}}{A_{F,t}^*} \right) \tag{A.32}$$

$$Y_{N,t}^* = C_{N,t}^* \tag{A.33}$$

$$\widetilde{Y_{F,t}^*} = \widetilde{C_{F,t}^*} + \widetilde{C_{F,t}} \tag{A.34}$$

$$\widetilde{Y_t^*} = \widetilde{C_t^*} + \widetilde{\widetilde{P_{F,t}^*}}\widetilde{C_{F,t}} - \frac{\widetilde{\widetilde{P_{H,t}}}}{Q_t}\widetilde{C_{H,t}^*}\left(\frac{A_{H,t}}{A_{F,t}^*}\right)^\alpha \tag{A.35}$$

$$L_t^* = L_{N,t}^* + L_{F,t}^* \tag{A.36}$$

Appendix B   Log-Linearised Model

In this section, we present the log-linearised system of equations that we use to make our analysis.

$$\tilde{c}_t = \tilde{c_{t+1}} - r_t + \alpha\Delta a_{H,t+1} \tag{B.1}$$

$$\tilde{c}_t^* = \tilde{c_{t+1}^*} - r_t^* + \alpha\Delta a_{F,t+1}^* \tag{B.2}$$

$$\tilde{w}_t + \eta l_t + \tilde{c}_t \tag{B.3}$$

$$\tilde{w}_t^* + \eta l_t^* + \tilde{c}_t^* \tag{B.4}$$

$$q_{t+1} - q_t = r_t - r_t^* + \delta b_t \tag{B.5}$$

$$y_{\tilde{H},t} = l_{H,t} \tag{B.6}$$

$$y_{N,t} = a_{N,t} + l_{N,t} \tag{B.7}$$

$$y_{\tilde{F},t}^{*} = l_{F,t}^{*} \tag{B.8}$$

$$y_{N,t}^{*} = a_{N,t}^{*} + l_{N,t}^{*} \tag{B.9}$$

$$\tilde{w}_t = a_{N,t} + p_{\tilde{N},t} = p_{\tilde{\hat{H}},t} \tag{B.10}$$

$$\tilde{w}_t^{*} = a_{N,t}^{*} + p_{\tilde{N},t}^{*} = p_{\tilde{F},t}^{*} \tag{B.11}$$

$$0 = \alpha\, p_{\tilde{\hat{T}},t} + (1-\alpha)\, p_{\tilde{\hat{N}},t} \tag{B.12}$$

$$0 = \alpha\, p_{\tilde{\hat{T}},t}^{*} + (1-\alpha)\, p_{\tilde{\hat{N}},t}^{*} \tag{B.13}$$

$$p_{\tilde{\hat{T}},t} = \nu\, p_{\tilde{\hat{H}},t} + (1-\nu)\,(p_{\tilde{\hat{F}},t} + (1-\alpha)\, d_t) \tag{B.14}$$

$$p_{\tilde{\hat{T}},t}^{*} = \nu^{*}\, p_{\tilde{\hat{F}},t}^{*} + (1-\nu^{*})\,(p_{\tilde{\hat{H}},t}^{*} - (1-\alpha)\, d_t) \tag{B.15}$$

where $p_{\tilde{\hat{H}},t}^{*} = p_{\tilde{\hat{H}},t} - q_t$ and $p_{\tilde{\hat{F}},t} = q_t + p_{\tilde{\hat{F}},t}^{*}$.

$$c_{N,t} = -p_{\tilde{\hat{N}}.t} + \tilde{c}_t \tag{B.16}$$

$$c_{\tilde{T},t} = -p_{\tilde{\hat{T}}.t} + \tilde{c}_t \tag{B.17}$$

$$c_{N,t}^{*} = -p_{\tilde{\hat{N}}.t}^{*} + \tilde{c}_t^{*} \tag{B.18}$$

$$c_{\tilde{T},t}^{*} = -p_{\tilde{\hat{T}}.t}^{*} + \tilde{c}_t^{*} \tag{B.19}$$

$$c_{\tilde{H},t} = -\theta\,(p\tilde{\hat{}}_{H,t} - p\tilde{}_{T,t}) + c_{\tilde{T},t} \tag{B.20}$$

$$c_{\tilde{F},t} = -\theta\,(p\tilde{}_{F,t} - p\tilde{}_{T,t} + (1-\alpha)\,d_t) + c_{\tilde{T},t} - d_t \tag{B.21}$$

$$c^*_{\tilde{F},t} = -\theta\,(p\tilde{\hat{}}^*_{F,t} - p\tilde{}^*_{T,t}) + \tilde{c}^*_{T,t} \tag{B.22}$$

$$c^{\tilde{}}_{H,t} = -\theta\,(p\tilde{}^*_{H,t} - p\tilde{\hat{}}^*_{T,t} - (1-\alpha)\,d_t) + \tilde{c}^*_{T,t} + d_t \tag{B.23}$$

$$y_{N,t} = c_{N,t} \tag{B.24}$$

$$y^*_{N,t} = c^*_{N,t} \tag{B.25}$$

$$y_{\tilde{H},t} = \frac{\overline{C_H}}{\overline{Y_H}} c_{\tilde{H},t} + \frac{1-n}{n}\,\frac{\overline{C^*_H}}{\overline{Y_H}} c^{\tilde{}}_{H,t} \tag{B.26}$$

$$y^{\tilde{}}_{F,t} = \frac{\overline{C^*_F}}{\overline{Y^*_F}} c^{\tilde{}}_{F,t} + \frac{n}{1-n}\,\frac{\overline{C_F}}{\overline{Y^*_F}} c_{\tilde{F},t} \tag{B.27}$$

$$l_t = \frac{\overline{L_H}}{\overline{L}} l_{H,t} + \frac{\overline{L_N}}{\overline{L}} l_{N,t} \tag{B.28}$$

$$l^*_t = \frac{\overline{L^*_F}}{\overline{L^*}} l^*_{F,t} + \frac{\overline{L^*_N}}{\overline{L^*}} l^*_{N,t} \tag{B.29}$$

$$\tilde{y}_t = \frac{1}{\overline{Y}}\left(\overline{C}\tilde{c}_t + \frac{1-n}{n}\,\overline{C^*_H}(p\tilde{\hat{}}_{H.t} + c^{\tilde{}}_{H,t}) - \overline{C_F}\,(q_t + p\tilde{\hat{}}^*_{F.t} + c_{\tilde{F},t} - \alpha\,d_t)\right) \tag{B.30}$$

$$\tilde{y}^*_t = \frac{1}{Y^*}\left(\overline{C^*}\tilde{c}^*_t + \frac{n}{1-n}\,\overline{C_F}(p\tilde{\hat{}}^*_{F.t} + c_{\tilde{F},t}) - \overline{C^*_H}(-q_t + p\tilde{}_{H.t} + c^{\tilde{}}_{H,t} + \alpha\,d_t)\right) \tag{B.31}$$

$$\beta\,b_t - b_{t-1} = \frac{1-n}{n}\,\overline{C^*_H}(p\tilde{\hat{}}_{H.t} + c^{\tilde{}}_{H,t}) - \overline{C_F}(q_t + p\tilde{\hat{}}^*_{F.t} + c_{\tilde{F},t} - \alpha\,d_t) \tag{B.32}$$

where the over-bars denote the steady state values. In the steady state, we assume that all relative prices and the real wages are equal to one. Given the

zero net initial asset position: $\overline{Y} = \overline{Y^*} = \overline{C} = \overline{C^*} = \overline{L} = \overline{L^*} = 1$. We then solve for the rest of the steady state relationships using the symmetry between the preference structure of home and foreign countries[26]: $\overline{C_T} = \overline{C_T^*} = \alpha$, $\overline{C_F} = (1-\nu)\,\alpha$, $\overline{C_H^*} = (1-\nu^*)\,\alpha$, $\overline{C_H} = \nu\,\alpha$, $\overline{C_F^*} = \nu^*\,\alpha$, $\overline{C_N} = \overline{C_N^*} = 1-\alpha$, $\overline{Y_H} = \overline{Y_F^*} = \alpha$, $\overline{L_N} = \overline{L_N^*} = 1-\alpha$, $\overline{L_H} = \overline{L_F^*} = \alpha$.

$$a_{N,t} = \rho_{a^N} a_{N,t-1} + \varepsilon_{a^N,t} \tag{B.33}$$

$$a_{N,t}^* = \rho_{a^N}^* a_{N,t-1}^* + \varepsilon_{a^N,t}^* \tag{B.34}$$

$$d_t = d_{t-1} + \Delta a_{H,t} - \Delta a_{F,t}^* \tag{B.35}$$

$$\Delta a_{H,t} = -\kappa\, d_{t-1} + \varepsilon_{a^H,t} \tag{B.36}$$

$$\Delta a_{F,t}^* = \kappa\, d_{t-1} + \varepsilon_{a^F,t}^* \tag{B.37}$$

where $\Delta a_{H,t} = a_{H,t} - a_{H,t-1}$ and $\Delta a_{F,t}^* = a_{F,t}^* - a_{F,t-1}^*$.

Appendix C Data

We obtain growth rates for UK and EA annual real GDP as well as real consumption from the World Bank WDI database and transform them into indexes. The logarithms of these indexes are then detrended using the HP-filter ($\lambda = 100$).

Annual nominal exchange rates (annual averages) for the euro and the pound sterling vis-à-vis the US dollar are obtained from the BIS database and converted to the GBP/EUR rate. In order to compute the bilateral real exchange rate, we employ (annual) CPI indexes for the UK and EA from Deutsche Bundesbank sources. For our analysis the logarithm of the real exchange rate is applied.

Data for the bilateral trade balance stems from the IMF Directions of Trade Statistics database. We compute the difference between annual UK exports to the EA (in million USD) and UK imports from the EA (in million USD) and multiply by the GBP/USD rate. Afterwards, we normalise the trade balance by nominal UK GDP, obtained from the Office for National Statistics.

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[26]In steady state: $\dfrac{1}{1+\bar{r}} = \beta$

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