Partial transitions in the nonmesonic decay of hypernuclei

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Partial transitions between initial and final states of definite relative angular momentum are studied for the nonmesonic weak decay of ${}^{12}_{\Lambda}$C. Compact expressions for the matrix elements in momentum space are given for nucleons in any orbital. It is shown that, in contrast to nuclear matter, the $S \rightarrow S$ and $S \rightarrow P$ transitions contribute significantly to the total rate. Implications are discussed for the ratio of the neutron- to proton-induced rates and of the parity-violating to parity-conserving transitions. Furthermore, transitions coming from initial relative ΛN P states are shown to be small.

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Motivated by new experimental results from Brookhaven [1] and KEK [2] we have recently presented calculations [3] for the nonmesonic weak decay of ${}^{12}_{\Lambda}$C. This decay channel, $\Lambda N \rightarrow NN$, which due to Pauli suppression is the dominant weak decay mechanism for all but the lightest hypernuclei, may help to shed some light on weak quark-quark interactions in the presence of the strong force. Our calculations were based on a relativistic nuclear model in which the details of finite nuclear structure are treated as well as possible. Final-state interactions of the ejected nucleons with the residual nuclear state were incorporated via a relativistic optical potential. The nucleon and lambda bound-state wave functions were solutions of the Dirac equation with large potentials. Short-range correlations were included with a realistic ΛN-correlation function. Appropriate spectroscopic factors, corrected for the c.m. motion, took the shell structure of ^{11}C into account. The transformation into the relative ΛN two-body frame was avoided which allowed a straightforward inclusion of higher nucleon orbitals. Minimizing the model dependency with these ingredients we found that the pion exchange part of the $\Lambda N \rightarrow NN$ amplitude is insufficient to reproduce experimental data. This is in contrast to nuclear matter results [4–6] which found reasonable agreement with the measurements after extrapolating in some way to ${}^{12}_{\Lambda}$C.

The purpose of this study is to compare the contributions of partial transitions between nuclear matter and finite nucleus calculations. By dividing the $\Lambda N \rightarrow NN$ operator into a central, tensor, and parity-violating part we obtain the strengths of the partial transitions for nucleons in the 1s and 1p shells. For s-shell nucleons this corresponds to the relative $S \rightarrow S$, $S \rightarrow D$, and $S \rightarrow P$ transitions, while for p-shell nucleons many more transitions are possible as shown in Table I.

In order to allow a more qualitative discussion in this Brief Report we perform our calculations in a nonrelativistic framework and neglect final-state distortions. Both approximations combined change the total rate by about 30% [3].

Limiting ourselves to pion exchange we can write the nonrelativistic reduction of the $\Lambda N \rightarrow NN$ transition operator as [7]

$$t_\pi(\mathbf{q}) = g_w \frac{g_{\pi NN}}{2M_N} \left[1 + \frac{\lambda}{M_N + M_\Lambda} \boldsymbol{\sigma}_1 \cdot \mathbf{q} \right] \frac{\mathcal{F}^2(\mathbf{q}^2)}{\mathbf{q}^2 + m_\pi^2} \boldsymbol{\sigma}_2 \cdot \mathbf{q} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tag{1}$$

where λ and g_w are coupling constants determining the strengths of the parity-conserving (PC) and parity-violating (PV) interactions. The form factor is denoted by $\mathcal{F}(\mathbf{q}^2)$ which depends on the momentum transfer $\mathbf{q}$. The parity-conserving amplitude is analogous to the NN one pion exchange potential and can be divided into a central and a tensor part using

$$\boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{q} = \tfrac{1}{3} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 q^2 + S_{12}(\hat{\mathbf{q}}) q^2 , \tag{2}$$

where

$$S_{12}(\hat{\mathbf{q}}) = \boldsymbol{\sigma}_1 \cdot \hat{\mathbf{q}} \boldsymbol{\sigma}_2 \cdot \hat{\mathbf{q}} - \tfrac{1}{3} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 .$$

Without performing a transformation into the two-body relative momentum frame we evaluate matrix elements for the three distinct operators between final asymptotic nucleon states of momentum $\mathbf{k}_1$ and $\mathbf{k}_2$ along with their spin projections m_{s_1} and m_{s_2}, and initial bound lambda and nucleon states. The lambda is assumed to be in a $1s_{1/2}$ state while the nucleon can be in any orbital defined by $\{nlj\}$.

Evaluating the matrix elements in momentum space we obtain for the central part (suppressing isospin for simplicity)

TABLE I. Partial transitions present for s- and p-shell nucleons and different parts of the operator.

	Central	Tensor	Parity violating
s shell	$S \rightarrow S$	$S \rightarrow D$	$S \rightarrow P$
p shell	$S \rightarrow S$	$S \rightarrow D$	$S \rightarrow P$
	$P \rightarrow P$	$P \rightarrow P$	$P \rightarrow S$
		$P \rightarrow F$	$P \rightarrow D$

$$\langle \mathbf{k}_1 m_{s_1}, \mathbf{k}_2 m_{s_2} | t_{\rm cent}(\mathbf{q}) | \Lambda(1s_{1/2}, m_\Lambda) N(nlj, m_N) \rangle$$

$$= \frac{\lambda g_w g_{\pi NN}}{M_N(M_\Lambda + M_N)} \hat{j} (-1)^{l-m_{s_1}-m_{s_2}-m_N-1/2} \sum_{M,m_l} (-1)^M \begin{bmatrix} l & \frac{1}{2} & j \\ m_l & m_s & -m_N \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ -m_{s_1} & M & m_\Lambda \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ -m_{s_2} & -M & m_s \end{bmatrix}$$
$$\times \int d^3p \frac{\mathcal{F}^2(\mathbf{q}^2) q^2 Y_0^0(\hat{q})}{\mathbf{q}^2 + m_\pi^2} \varphi_{1s}(p_\Lambda) \varphi_{nlj}(p) Y_l^{m_l}(\hat{p}) \qquad (3)$$

where $\mathbf{p}_\Lambda = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{p}$ is the lambda momentum and $\mathbf{q} = \mathbf{p}_\Lambda - \mathbf{k}_1$. The integration over $\mathbf{p}$, which is the Fermi momentum of the bound nucleon, is performed numerically. Similarly, we obtain the matrix elements for the tensor part

$$t_{fi}^{\rm tens}(\mathbf{q}) = \sqrt{6} \frac{g_w g_{\pi NN} \lambda}{M_N(M_\Lambda + M_N)} \hat{j} (-1)^{l-m_{s_1}-m_{s_2}-m_N-1/2}$$
$$\times \sum_{\substack{m_a m_b \\ M m_l}} \begin{bmatrix} l & \frac{1}{2} & j \\ m_l & m_s & -m_N \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ m_a & m_b & -M \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ -m_{s_1} & m_a & m_\Lambda \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ -m_{s_2} & m_b & m_s \end{bmatrix}$$
$$\times \int d^3p \frac{\mathcal{F}^2(\mathbf{q}^2) q^2 Y_2^{-M}(\hat{q})}{\mathbf{q}^2 + m_\pi^2} \varphi_{1s}(p_\Lambda) \varphi_{nlj}(p) Y_l^{m_l}(\hat{p}) \qquad (4)$$

and the parity-violating contribution

$$t_{fi}^{\rm PV}(\mathbf{q}) = \frac{g_w g_\pi NN}{\sqrt{2} M_N} \hat{j} (-1)^{1+l-m_N-m_{s_2}} \sum_{M m_l} (-1)^M \begin{bmatrix} l & \frac{1}{2} & j \\ m_l & m_s & -m_N \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ -m_{s_2} & M & m_s \end{bmatrix}$$
$$\times \int d^3p \frac{\mathcal{F}^2(\mathbf{q}^2) q Y_l^{-M}(\hat{q})}{\mathbf{q}^2 + m_\pi^2} \varphi_{1s}(p_\Lambda) \varphi_{nlj}(p) Y_l^{m_l}(\hat{p}) . \qquad (5)$$

We have employed harmonic oscillator radial wave functions $\varphi(p)$ for simplicity using the range parameters of Ref. [3] ($b_N = 1.64$ fm, $b_\Lambda = 1.87$ fm). Including short-range correlations in momentum space is not trivial since it requires replacing the integrals $\int d^3p$ in Eqs. (3)–(5) by $\int d^3p\, d^3p' f(\mathbf{p},\mathbf{p}')$ where the correlation function is of the form $f(\mathbf{p},\mathbf{p}') = \delta(\mathbf{p}-\mathbf{p}') - g(|\mathbf{p}-\mathbf{p}'|)$. We have instead used a monopole form factor with a very soft cutoff of $\Lambda_\pi = 600$ MeV since this reproduces the combined correlation and form factor reduction of the rates, as we have verified with our r-space calculation [3].

The results of our calculations are presented in Table II for the central, tensor, and parity-violating parts of the operator separately: The rate $\Gamma_{\rm cent}$ is about 1/3 of $\Gamma_{\rm tens}$ and thus contributes significantly to the total nonmesonic rate. This is in contrast to nuclear matter [4] where the ratio of $\Gamma_{\rm cent}$ to $\Gamma_{\rm tens}$ is less than 1/30. Note that comparisons for the central part between nuclear matter and finite nucleus calculations should only be made after form factors have been included, otherwise the nuclear matter integrals diverge. The PV part of the interaction is also larger in finite nuclei yielding about 30% of the total rate. Therefore, our results $\Gamma_n/\Gamma_p \approx 0.2$ and $\Gamma_{\rm PV}/\Gamma_{\rm PC} \approx 0.5$ are much larger than in nuclear matter where most calculations give numbers of less than 0.1 for both ratios. Note also that the p-shell rates add incoherently to the s-shell rates. This is due to the different parity of the two orbits; in heavier nuclei contributions from orbits with the same parity will add up coherently.

Another result evident from Table II is the important contribution of p-shell nucleons. The magnitude of the initial relative ΛN P-state and S-state contributions can be calculated separately by transforming to relative and center-of-mass variables, using an average parameter $b = (b_N + b_\Lambda)/2$ for the harmonic oscillator wave functions. In order to facilitate the discussion we rewrite the spatial part of the two-body element in coordinate space

TABLE II. Total and partial rates for the nonmesonic decay of ${}^{12}_{\Lambda}$C in units of the free Λ decay rate.

$\Gamma_{nm}/\Gamma_\Lambda$	$\Gamma_{\rm cent}$	$\Gamma_{\rm tens}$	$\Gamma_{\rm PV}$	$\Gamma_{\rm tot}$	Γ_n/Γ_p	$\Gamma_{\rm PV}/\Gamma_{\rm PC}$
s shell	0.024	0.070	0.045	0.139	0.208	0.481
p shell	0.039	0.105	0.083	0.228	0.188	0.571
Total	0.062	0.175	0.128	0.366	0.196	0.536

$$t_{fi}^{p\,\mathrm{shell}}=\frac{1}{(2\pi)^3}\int d^3r_1 d^3r_2 e^{-i\mathbf{k}_1\cdot\mathbf{r}_1}e^{-i\mathbf{k}_2\cdot\mathbf{r}_2}V_\pi(\mathbf{r})\varphi_\Lambda^{1s}(\mathbf{r}_1,b)\varphi_N^{1p}(\mathbf{r}_2,b)$$

$$=\frac{1}{(2\pi)^{3/2}\sqrt{2}}\left[\tilde{\varphi}_P(\mathbf{K},b/\sqrt{2})\int d^3r\, e^{-i\mathbf{k}\cdot\mathbf{r}}V_\pi(\mathbf{r})\varphi_S(\mathbf{r},\sqrt{2}b)-\tilde{\varphi}_S(\mathbf{K},b/\sqrt{2})\int d^3r\, e^{-i\mathbf{k}\cdot\mathbf{r}}V_\pi(\mathbf{r})\varphi_P(\mathbf{r},\sqrt{2}b)\right], \quad (6)$$

where $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, $\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2$, $\mathbf{k}=(\mathbf{k}_1-\mathbf{k}_2)/2$, and $\mathbf{K}=\mathbf{k}_1+\mathbf{k}_2$. The relative wave function is denoted by $\varphi_L(\mathbf{r},\sqrt{2}b)$, while

$$\tilde{\varphi}_L(\mathbf{K},b/\sqrt{2})=\int\frac{d^3R}{(2\pi)^{3/2}}e^{i\mathbf{K}\cdot\mathbf{R}}\varphi_L(\mathbf{R},b/\sqrt{2}) \quad (7)$$

is the c.m. wave function in momentum space. Note that the wave functions have been written as functions of two arguments, the coordinate ($\mathbf{r}$ or $\mathbf{k}$) and the range parameter (b), in order to clearly illustrate their change due to the transformation to the c.m. system. For s-shell nucleons, where one has an S wave in both the relative and center-of-mass motions, the transition amplitude gives a maximum contribution at the back-to-back kinematics $\mathbf{k}_1=-\mathbf{k}_2$ [8] yielding $\mathbf{K}=0$. For p-shell nucleons one would expect the contribution from the initial relative ΛN S-state to be suppressed compared to that of the relative P-state, since $\tilde{\varphi}_P(\mathbf{K}=0,b/\sqrt{2})=0$. This assumption has been used in Ref. [9]. Surprisingly, after integration over all other kinematics for which $\mathbf{k}_2\neq\mathbf{k}_1$, this relative $L=0$ term contributes about 90% to the p-shell rate. This is correcting our result of Ref. [3] where we had erroneously stated that the initial relative ΛN S-state is suppressed and contributes about 30%. Figure 1 depicts the p-shell double differential decay rates $d^2\Gamma_p/d\Omega_p\,dk_1$, at various angles θ_{12} between the two exiting nucleons, for the relative S- and P-state contributions separately. For $|\mathbf{k}_1|=|\mathbf{k}_2|=378$ MeV/c and $\theta_{12}=180°$ the relative S-state part is zero while the relative P-state rate is at its maximum. However, as soon as $\mathbf{k}_1|\neq|\mathbf{k}_2|$ and also for $\theta_{12}\neq 180°$ the S-state rate dominates. Therefore, after integration over the whole phase space, most of the decay rate of p-shell nucleons comes from the relative ΛN S-state.

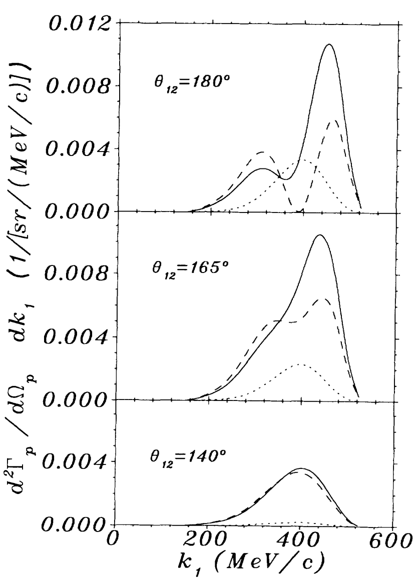

FIG. 1. Double differential decay rate as a function of the proton momentum for the ΛN relative P state (dotted line), relative S state (dashed line), and the total p-shell contribution (solid line).

Finally, we list in Table III the ratio $R^{np}=\Gamma_n/\Gamma_p$ for various parts of the $\Lambda N\to NN$ transition operator. As pointed out before [4–8], the ratio R^{np}_{tens} of the s-shell is zero since only the spin triplet $S\to D$ transition is permitted in the tensor contribution. Antisymmetry in the final two-nucleon state requires the isospin to be $T=0$ and, therefore, the transition $\Lambda n\to nn$ is excluded. However, for p-shell nucleons the presence of the relative ΛN P-state allows tensor transitions to $T=1$ final states leading to a R^{np}_{tens} which is small but nonzero. For the central part of the operator, R^{np}_{cent} is the same for both s- and p-shell nucleons and is close to 1/2. This comes from the fact that the neutron-induced decay rate has to be multiplied with a statistical factor of 1/2 since it contains two identical particles in the final state. Therefore, Table III demonstrates that the non-negligible contributions of $S\to S$ (central) and $S\to P$ (parity-violating) transitions lead to an enhanced ratio R^{np}. However, within the pion exchange model this ratio cannot exceed ~ 0.5, whereas the experiments [1] suggest a ratio closer to one albeit with large error bars.

In conclusion, we have shown that in a finite nucleus calculation the $S\to S$ and $S\to P$ transitions contribute significantly to the total decay rate. This is in contrast to nuclear matter where almost all the strength came from the dominant $S\to D$ transition. Our ratios Γ_n/Γ_p and $\Gamma_{\mathrm{PV}}/\Gamma_{\mathrm{PC}}$—even though still small—are therefore much larger than the nuclear matter results. It remains to be seen how the inclusion of heavier mesons—especially the ρ meson [10]—affects these ratios in a finite nucleus computation. Furthermore, we found the magnitude of the relative ΛN P-state to be small compared to the S state. It is not clear if this conclusion will hold for heavier hypernuclei when nucleons in $1d,2d,2p$, and higher orbits will contribute to the total rate; then it is advantageous to

TABLE III. The ratio Γ_n/Γ_p for various pieces of the operator.

Γ_n/Γ_p	Central	Tensor	PC	PV
s shell	0.53	0	0.10	0.53
p shell	0.53	0.03	0.12	0.31

avoid separating the $\Lambda N \rightarrow NN$ two-body matrix elements into relative and c.m. contributions. Studying the A dependence of the nonmesonic decay will tell us when saturation is achieved, thus permitting a direct comparison between nuclear matter and finite nucleus results.

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