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Double folding with a density-dependent effective interaction and its analytical approximation

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The real part of the optical potential for heavy ion elastic scattering is obtained by double folding of the nuclear densities with a density-dependent nucleon-nucleon effective interaction which was successful in describing the binding, size, and nucleon separation energies in spherical nuclei. A simple analytical form is found to differ from the resulting potential considerably less than 1% all through the important region. This analytical potential is used so that only few points of the folding need to be computed. With an imaginary part of the Woods-Saxon type, this potential predicts the elastic scattering angular distribution in very good agreement with experimental data, and little renormalization (unity in most cases) is needed.

[NUCLEAR REACTIONS Optical model for nucleus-nucleus, double folding model, nucleon-nucleon effective interaction.]

I. INTRODUCTION

Folding potentials are an appealing option for the construction of the optical potential to be used in the calculation of heavy ion elastic scattering: the simple folding of the (target) nuclear density with a nucleon-nucleus (projectile) optical potential[1,2] or the double folding of an effective nucleon-nucleon interaction into the nuclear density distributions of the two colliding nuclei.[3-6] This approach seems to be more physical than the free fitting to data of, for example, Woods-Saxon parameters with ambiguous meaning. Unfortunately the frequent need for a renormalizing factor weakens the basic character of this approach.[7]

Recently Satchler and Love have done extensive work in the application of the folding method to the calculation of the elastic scattering cross section between heavy ions.[5] The real part of the potential is obtained by double folding of the nuclear densities with a realistic effective interaction which has been derived[8] from G-matrix elements based on the Reid nucleon-nucleon potential and consists of a δ term plus two Yukawians with ranges 0.25 (repulsive) and 0.4 fm (attractive). An imaginary part of the Woods-Saxon (WS) type is fitted to the experimental data. The normalizing factor needed for the real part is in general close to 1, except with a ^{6}Li projectile, which requires a much smaller factor, typically 0.60. The same effective interaction, used in double folding for $^{10}B + ^{40}Ca$, requires[6] a normalizing factor as high as 1.38.

We thought it worthwhile to test the double folding model with a density-dependent effective interaction which would take into account the overlap of the two density distributions. The interaction is described in Sec. II. It was built for the purpose of computing the general properties of spherical nuclei in a globally self-consistent shell model.[9] In Sec. III we give some details on the method chosen for the description of the nuclear matter distributions.

The six-dimensional folding integral is a tedious task[10] and the density dependence of the interaction adds some complexity. This will be compensated for by the use of an analytical potential which requires few folding points and approximates the folded potential very closely in the range of distances that determine the elastic scattering. This is shown in Sec. IV. These analytical potentials, together with a WS imaginary part, have been used for computing the elastic scattering of almost twenty pairs of heavy ions, and the agreement with experimental data is reported in Sec. V. Distances are given in fm and energies in MeV all through this work.

II. THE EFFECTIVE INTERACTION

The following effective nucleon-nucleon interaction will be used in our folding

$$V_{\text{eff}}(\vec{r},\vec{r}') = -U_0(1 - s_{ij}\alpha)[1 - c\rho(\vec{r})^{1/3}\rho(\vec{r}')^{1/3}] \times \exp\left[-\left(\frac{\vec{r}-\vec{r}'}{a}\right)^2\right], \quad (1)$$

where $\vec{r}$ and $\vec{r}'$ are the positions of the two nucleons, and $s_{ij} = +1$ (-1) for the interaction between like (unlike) nucleons. The use of a stronger effective force between unlike nucleons compensates for the lack of complete antisymmetrization.

Interaction (1) was created by two of us (F.J.V. and G.M.) for the specific purpose of using it in a shell model with global self-consistency[9]: The central potential is produced by folding the nuclear density with interaction (1), where nuclear density is the sum of the squared wave functions obtained by solving the single-particle Schrödinger equation with such a simple folding potential. Also included in the Schrödinger equations is the Coulomb potential for protons and a spin-orbit contribution of the Thomas type, proportional to the gradient of the central potential, $-(\kappa/r)(dV/dr)\vec{l}\cdot\vec{s}$, with a strength $\kappa = 0.36$ fm^2 fitted to the experimental split of the $l \pm \frac{1}{2}$ levels.

We summarize here some information on our effective interaction (1). A density-independent interaction was excluded because (a) it systematically fails to reproduce at the same time the size and the energy of nuclei in Hartree-Fock calculations[11]; (b) it cannot produce a self-consistent density distribution that is sufficiently uniform[12]; and (c) the saturation conditions investigated by Calogero *et al.*[13] for density-independent forces of the Wigner type require that their Fourier transform be positive for any non-negative value of the momentum p. In particular, for $p = 0$, this means that the volume integral of the interaction is positive, and its convolution with a density distribution gives a repulsive potential unsuitable for shell model computations.

A linear dependence in $\rho^{2/3}$ was chosen, as favored by Bethe[14] and Moszkowski.[15] Usually the density is taken in the center of the two interacting nucleons; this prescription overestimates the influence of the center of the nucleus and leads to insufficient binding.[16] We take the geometric mean of the densities at the sites of the two nucleons.

Our interaction has four parameters: the strength U_0a^3 and the density coefficient c can be related to the equilibrium density and binding per nucleon of symmetrical nuclear matter, and hence to the corresponding parameters of one of the existing mass formulas, as described in Ref. 9; also the asymmetry coefficient α can be related to the symmetry coefficient of the mass formula via computation of asymmetric nuclear matter. A first determination of U_0a^3, c, and α was thus deduced from Seeger's 1970 mass formula.[17] A parallel link of the interaction range a to the surface energy of a mass formula can be established by calculating semi-infinite nuclear matter in the Thomas-Fermi approximation. But this approximation in its simpler form is not reliable in the surface where the density and the potential vary rapidly,[12] and such a link was not even attempted; rather the value of a was left as the only free parameter to be fitted to the energy and size of the five magic nuclei ^{16}O, ^{40}Ca, ^{48}Ca, ^{90}Zr, and ^{208}Pb. A final readjustment of less than 5% was made in three of the parameters to end up with the following values:

$$\begin{aligned} &U_0a^3 = 174.4 \text{ MeV fm}^3, \\ &\alpha = 0.49, \\ &c = 1.73 \text{ fm}^2, \\ &a = 0.87 \text{ fm}. \end{aligned} \tag{2}$$

Interaction (1), with the value of the parameters given in (2) and the spin-orbit term mentioned above, was used in the shell model described in Ref. 9 and was found to predict the binding energies and rms charge radii of the five mentioned magic nuclei with an error less than 1% on the average. It also predicts remarkably well the separation energy of the less bound nucleons. The use of this interaction has been extended successfully to nine groups of even isotopes and isotones with the introduction of a pairing interaction.[18] Further (unpublished) computations of over fifty nuclides gave the correct binding energy within less than 0.5%.

III. NUCLEAR DENSITY DISTRIBUTIONS

The considerations of the previous sections give us confidence that the use of interaction (1) will give reasonable results in computing the nucleus-nucleus interaction by double folding with the nuclear densities. But we are not committed to the use of density distributions genetically related to the same effective interaction; any good density distribution will do it. Actually for extrinsic reasons our choice has been different.

Nuclear density distributions were obtained as the sum of squared single-particle wave functions ψ_i weighted with occupation numbers n_i,

$$\rho(r) = \sum n_i |\psi_i|^2, \tag{3}$$

according to the method developed by Malaguti *et al.*[19-22] Eigenfunctions are first evaluated for a WS potential

$$-V_1 f_1(r) + V_2\left(\frac{\hbar}{mc}\right)^2 \frac{df_2}{dr}\,\vec{l}\cdot\vec{s} + V_C(r), \tag{4}$$

where

$$f_i(r) = \{1 + \exp[(r - R_i)/a_i]\}^{-1}, \quad R_i = r_i A^{1/3}$$

and V_C is the Coulomb repulsion for protons only. The Perey nonlocality correction factor is then applied[23] to the wave functions with a nonlocality parameter $\beta = 0.87$ and the final set is orthogon-

alized by the Gram-Schmidt procedure. Expression (3) is the nucleon point density distribution used for our folding. For comparison with experimental rms charge radii the separated proton and neutron densities are further folded into their own charge distributions and added to a total charge density.

Integer occupation numbers n_i have been assumed. Ideally fractional numbers near the Fermi surface are determined from the empirical spectroscopic factors or from nuclear structure computations. Actually the latter are not reliable enough and the former are not available in many cases. Similarly the potential parameters can be fitted to reproduce the experimental separation energies and charge rms radii in particular cases.

As another alternative we have preferred to take from Ref. 21 a common set of parameters that fit a wide range of data from ^{12}C to ^{208}Pb. These are

$$V_1 = 55.7 \pm 39.3\frac{N-Z}{A} + 0.51(B-15)\delta(B-15)\,, \tag{5}$$

with $\delta(x)$ the step function, B the binding energy of the level, and the +(−) sign holds for protons (neutrons).

$$r_1 = 1.236 \qquad a_1 = 0.52$$
$$V_2 = 7 \qquad r_2 = 1.1 \qquad a_2 = 0.65$$

With this common set of parameters the rms charge radii of the nuclides used in this work either coincide with the experimental values[24] within a standard deviation (for half of the cases) or differ from them not more than 5%. The same general fit has been successful in detecting nuclear structure effects in the critical radii for the elastic scattering of alpha particles by several groups of isotopes[25] or of heavy ions in general[26] as well as in the fusion cross section.[27]

IV. THE FOLDING AND ITS ANALYTICAL APPROXIMATION

In the double folding integral

$$V(r) = \int dr_1 \int dr_2 \rho_1(r_1)\rho_2(r_2)\, V_{\text{eff}}\,, \tag{6}$$

the interaction (1) is written so that the density dependence takes into account the contribution of the two nuclei

$$V_{\text{eff}} = -V_0\left(1 - \frac{N_1-Z_1}{A_1}\frac{N_2-Z_2}{A_2}\alpha\right) \times \{1 - c[\rho_1(r_1) + \rho_2(|\vec{r}_2 - \vec{r}_{12}|)]^{1/3} \times [\rho_1(|\vec{r}_1 + \vec{r}_{12}|) + \rho_2(r_2)]^{1/3}\} \exp\left[-\left(\frac{r_{12}}{a}\right)^2\right].$$

Figure 1 illustrates the meaning of the vectors $\vec{r}_k$, if the $\rho_i(r)$ describe the nuclear density distributions relative to the center of nucleus i and are taken as spherically symmetric functions. The coefficient of α implies that the proton and neutron distributions are assumed to be proportional to the total density for each nuclide. This approximation we hope will be sufficient in computing the contribution to the energy of the system coming from the interaction between the A_1 nucleons of the projectile and the A_2 nucleons of the target when their centers are at a distance r such that the extreme tails of the two nuclides overlap slightly.

The interactions between the $A_1(A_2)$ nucleons of the projectile (target), being density dependent, will be modified by the invasion of the target (projectile) density tail. Nevertheless, we expect this effect to be negligible at the internuclei distances which are relevant for elastic scattering, not less than the changes that will take place in the distributions $\rho_i(r)$ themselves and are systematically ignored in the usual foldings. Actually the explicit computation of this effect in some points yielded a modification of no more than a few percent of the folding potential.

Satchler and Love[5] have reported that the tails of their folded potentials do not decrease like a simple exponential, but are well approximated by a form factor $s^n \exp(-s/a_R)$ for $s > 1$, where

$$s = r - C_1 - C_2 \tag{7}$$

is the separation between the surfaces of the nuclei, which can be localized at the half density distance[28]

$$C_i = R_i - b^2/R_i \quad (i = 1, 2)$$

with

$$b = 1 \text{ for our purpose}$$

and

$$R_i = (1.13 + 0.0002A_i)A_i^{1/3}\,.$$

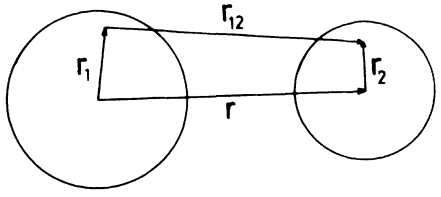

FIG. 1. Notation for the coordinates in the folding.

TABLE I. Values of the parameters in Eq. (8) that fit the folding Eq. (6) in the region $s>1$. Column 6 gives the constant depth of the potential from the origin up to the value $r = C_1 + C_2 + a_R n$ of column 5. Energies are in MeV, distances in fm.

Nuclides	V_0	n	a_R	$C_1 + C_2 + a_R n$	$-V(0)$
$^{11}B + ^{209}Bi$	162.7	0.986	0.531	9.45	32.1
$^{12}C + ^{62}Ni$	155.7	0.967	0.538	7.03	31.5
$^{12}C + ^{74}Ge$	170.9	1.081	0.519	7.37	31.0
$^{12}C + ^{100}Mo$	170.3	1.064	0.529	7.92	31.9
$^{12}C + ^{208}Pb$	190.7	1.043	0.524	9.55	35.8
$^{13}C + ^{207}Pb$	218.3	1.397	0.470	9.73	30.0
$^{16}O + ^{28}Si$	157.0	1.077	0.493	6.19	27.0
$^{16}O + ^{59}Co$	159.2	0.951	0.518	7.22	31.3
$^{16}O + ^{60}Ni$	160.7	1.059	0.504	7.29	28.7
$^{16}O + ^{72}Ge$	166.5	1.044	0.508	7.59	30.3
$^{16}O + ^{74}Ge$	174.1	1.182	0.495	7.69	28.3
$^{16}O + ^{76}Ge$	170.6	1.107	0.504	7.71	29.5
$^{16}O + ^{208}Pb$	198.4	1.011	0.510	9.82	37.0
$^{18}O + ^{72}Ge$	155.9	0.886	0.544	7.67	33.7
$^{18}O + ^{74}Ge$	161.5	0.910	0.541	7.73	34.1
$^{18}O + ^{76}Ge$	157.6	0.880	0.547	7.76	34.4
$^{40}Ar + ^{209}Bi$	207.8	1.054	0.509	10.98	37.6

Moreover, it is well established that the elastic scattering of heavy ions in the range of energies we are interested in (5 to 10 MeV/nucleon) is determined by the value of the potential at distances greater than $C_1 + C_2 + 1$. Hence we have decided to substitute for the exact folded potential a function which approximates to the tail in the form suggested by Satchler and Love and which is constant from the point with zero derivative ($s = a_R n$) up to the origin.

Our analytical approximation to the folding potential is therefore

$$V(r) = \begin{cases} -V_0 (a_R n)^n e^{-n} & \text{if } s \leq a_R n, \\ -V_0 s^n \exp(-s/a_R) & \text{if } s \geq a_R n. \end{cases} \quad (8)$$

The parameters V_0, n, and a_R, are fixed by least square approximation to the folded values (6) computed at six to eight points uniformly spaced in the interval $1.5 < s < 3.8$. Table I gives the values of the parameters which approximate the folding for each one of the pairs we use in this work. In all the points used for the fits the difference between the folded potential and the analytical approximation is smaller than 1%, and the rms relative difference (folded to analytical approximation) for the fitting points of each pair is, on the average of all the pairs, 0.26%. The approximation is quite satisfactory with the advantage of having an analytical expression for the potential. Figure 2 shows the folding potential compared with the approximation (8) in the case $^{16}O + ^{28}Si$.

A pleasing feature of this approximation is that the values each parameter takes for different pairs are fairly uniform. The average values and rms deviations are $V_0 = 176 \pm 14\%$, $n = 1.045 \pm 11\%$, $a_R = 0.516 \pm 4\%$, and $-V(0) = 32.5 \pm 11\%$.

V. PREDICTIONS AND COMPARISON WITH DATA

The elastic scattering cross section for the pairs and at the energies reported in Table II was computed with the optical potential

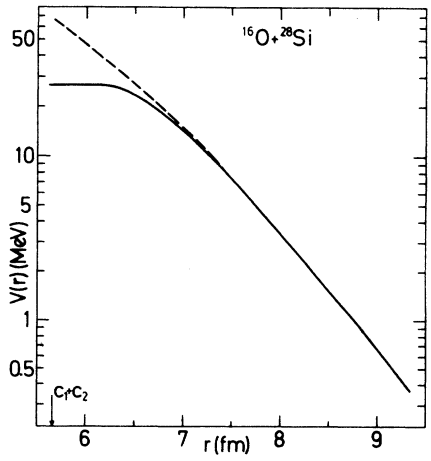

FIG. 2. Folded potential, Eq. (6), (dashed line) and its analytical approximation, Eq. (8), (solid line) for $^{16}O + ^{28}Si$.

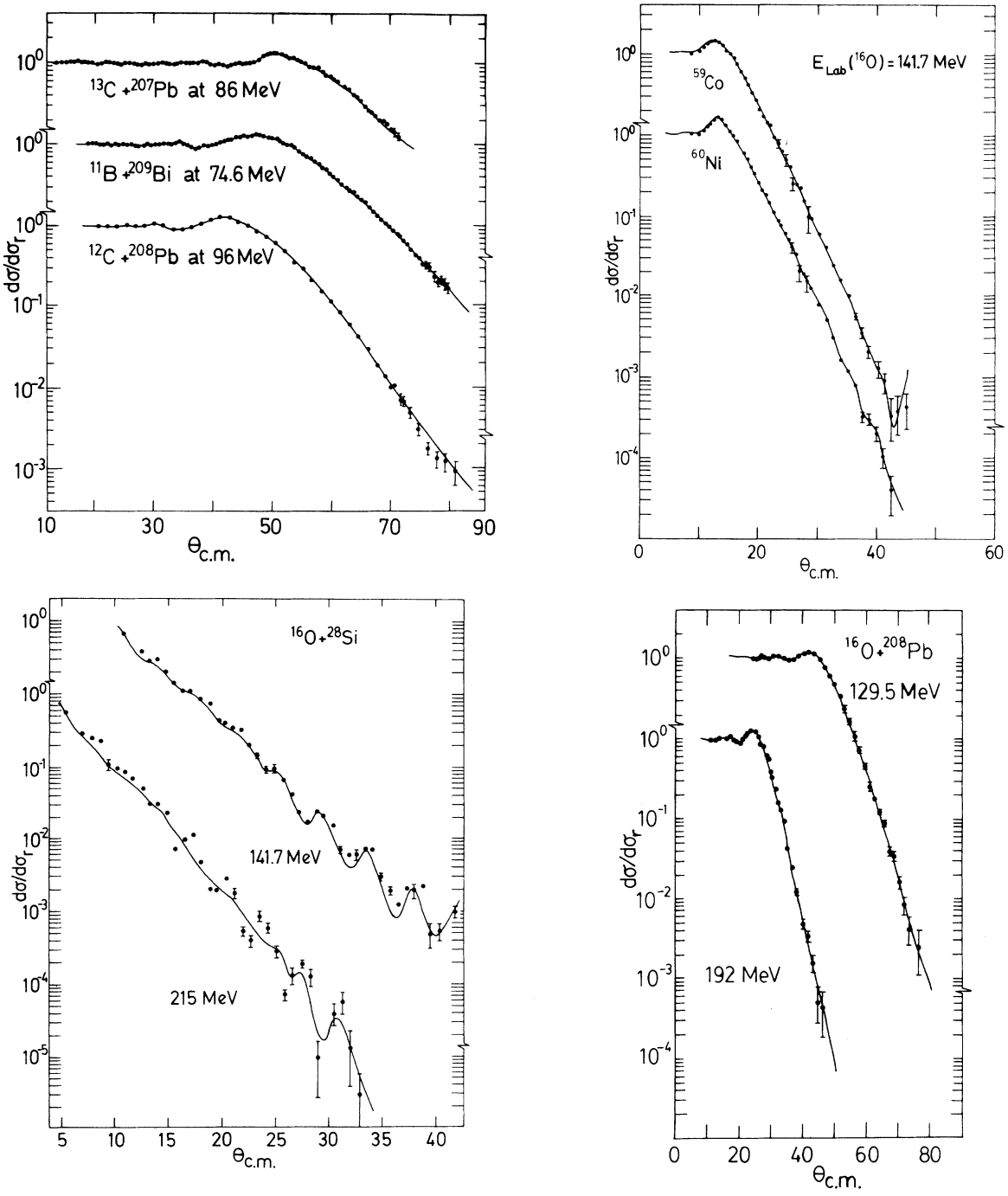

FIG. 3. Angular distributions predicted by our folding (solid line) compared with experimental data.

$$\mathfrak{N}V(r) - iW_0\left[1+\exp\frac{r-r_I(A_1^{1/3}+A_2^{1/3})}{a_I}\right]^{-1} + V_C(r)\,, \tag{9}$$

where $V(r)$ is given in Eq. (8) and the values of the parameters in Table I; V_C is the Coulomb potential produced by a sphere of radius $1.35(A_1^{1/3}+A_2^{1/3})$. The imaginary radius parameter r_I was initially fixed at 1.25 and the depth W_0 and diffuseness a_I were fitted to the experimental data with the optical model code GENOA.

In most cases this search led to a good fit with

TABLE II. Imaginary potential parameters (columns 5–7) and renormalizing factor $\mathfrak{N}$ needed for the fit of data. Energies are in MeV and distances in fm.

Nuclides	Data E_{Lab}	Ref.	Potential $\mathfrak{N}$	W_0	r_I	a_I	χ^2/N
$^{11}B + ^{209}Bi$	74.6	29	1.00	32.66	1.25	0.508	1.37
$^{12}C + ^{62}Ni$	48	30	0.61	12.92	1.25	0.491	2.11
$^{12}C + ^{74}Ge$	42	30	1.00	37.83	1.25	0.443	0.42
$^{12}C + ^{100}Mo$	48	30	1.00	48.80	1.25	0.444	1.06
$^{12}C + ^{208}Pb$	96	29	0.92	52.20	1.25	0.414	4.70
$^{13}C + ^{207}Pb$	86.1	29	1.00	32.87	1.25	0.514	1.10
$^{16}O + ^{28}Si$	141.5	29	0.83	31.78	1.072	1.097	6.36
	215	31	0.73	80.6	1.072	1.101	27.2
$^{16}O + ^{59}Co$	141.7	29	0.93	58.27	1.085	0.783	2.33
$^{16}O + ^{60}Ni$	141.7	29	0.96	52.75	1.071	0.814	2.54
$^{16}O + ^{72}Ge$	56	30	1.00	8.98	1.25	0.750	1.99
$^{16}O + ^{74}Ge$	56	30	1.00	6.32	1.25	0.795	0.70
$^{16}O + ^{76}Ge$	56	30	1.00	43.64	1.25	0.455	0.50
$^{16}O + ^{208}Pb$	129.5	29	1.00	27.15	1.25	0.676	0.55
	192	29	1.00	26.25	1.25	0.618	1.16
$^{18}O + ^{72}Ge$	56	30	1.00	20.48	1.25	0.634	0.28
$^{18}O + ^{74}Ge$	56	30	1.00	21.09	1.25	0.637	0.38
$^{18}O + ^{76}Ge$	56	30	1.00	15.70	1.25	0.674	0.50
$^{40}Ar + ^{209}Bi$	286	32	1.00	188.0	1.25	0.373	4.17

$\chi^2/N \lesssim 1$, comparable to the optimum WS fit (see Table II of Ref. 33), and $\mathfrak{N} = 1$. In other cases a smaller imaginary radius was preferred and a renormalizing factor $\mathfrak{N} \neq 1$ improved the quality of the fit. Table II gives the final values of the parameters and the χ^2/N for the reactions considered in this work. Figures 3(a) to 3(d) show some of the angular distributions predicted by our folding against the experimental data used for the fit of the imaginary potential (and eventually of the $\mathfrak{N}$).

The agreement is quite satisfactory, particularly if we recall that no free parameter was available for the construction of the real potential (with the exception of $\mathfrak{N}$, when different from 1) and that a bad real potential cannot be healed by any imaginary partner to avoid catastrophically high values of χ^2. The effective interaction itself was created for another purpose and deduced from an existing mass formula with little freedom.

We believe this result confirms the powerful ability of such a simple density dependence as included in Eq. (1) to describe the overall features of the average nucleon-nucleon interaction in a wide range of circumstances.

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