

Two nucleon induced  $\Lambda$  decay and the neutron to proton induced ratio

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We reanalyze the decay mode of  $\Lambda$  hypernuclei induced by two nucleons modifying previous numerical results and the interpretation of the process. The repercussions of this channel in the ratio of neutron to proton induced  $\Lambda$  decay is studied in detail in connection with the present experimental data. This leads to ratios that are in greater contradiction with usual one pion exchange models than those deduced before.

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## I. INTRODUCTION

The decay of  $\Lambda$  hypernuclei has traditionally been assumed to proceed via two channels: the mesonic decay,  $\Lambda \rightarrow N\pi$ , and the nonmesonic decay,  $\Lambda N \rightarrow NN$  [1]. The mesonic decay is largely suppressed by Pauli blocking, although the renormalization of the pion properties in the medium [2], or alternatively the consideration of proper distorted waves [3], weakens the effect of Pauli blocking considerably and leads to much larger mesonic widths than those obtained assuming free pion waves. The nonmesonic decay channel is the dominant one for heavy hypernuclei. It can be related to the mesonic decay via the usual meson exchange current mechanism. The  $\Lambda$  decays into a nucleon and a virtual pion, and the pion is absorbed by a second nucleon. This picture can alternatively be described by a  $\Lambda N \rightarrow NN$  transition mediated by pion exchange [see Fig. 1(b)]. However, this picture leads to a ratio of neutron to proton induced rates several times smaller [1,4,5] than the experimental one, which is close to one [6]. This is a severe problem at the present time, and many efforts have been devoted to the understanding of this large discrepancy by introducing the exchange of other mesons in the  $\Lambda N \rightarrow NN$  transition [5,7,8].

A very interesting finding concerning the  $\Lambda$  decay in nuclei was presented in Ref. [9]. The authors there found

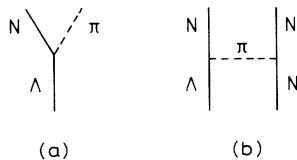


FIG. 1. Illustration of the  $\Lambda$  decay mechanisms in nuclei: mesonic channel (a) and nonmesonic channel (b).

a new decay channel of sizable strength associated with the pionic branch in the nucleus that shows up as a two nucleon induced  $\Lambda$  decay leading to the emission of three nucleons,  $\Lambda NN \rightarrow NNN$  (up to final state interaction of the nucleons). The relatively large strength of this channel (about one half of the free  $\Lambda$  decay width), and the repercussion that it has in the proper interpretation of the neutron to proton induced  $\Lambda$  nonmesonic decay [9], calls for a detailed and accurate analysis of this channel and this is the purpose of the present work. There are several reasons which make the reanalysis advisable in view of the approximations done in the exploratory work of Ref. [9].

(i) A crucial input in Ref. [9] was the pion self-energy associated to  $2p2h$  (two-particle-two-hole) excitation,  $\Pi_{2p2h}$ . The information in Ref. [9] was taken partly from theoretical calculations and partly empirically from experimental data. The value of  $\text{Im} \Pi_{2p2h}$  was taken from the work of Ref. [10], conducted in connection with the study of  $(e, e')$  reactions. The real part,  $\text{Re} \Pi_{2p2h}$ , was taken from fits to pionic atoms (Ref. 23 of [9] from a private communication of L. Tauscher). However, new developments in the evaluation of  $\text{Im} \Pi_{2p2h}$  have been done in Ref. [11] improving the previous results of Ref. [10]. On the other hand, the empirical values of  $\text{Re} \Pi_{2p2h}$  are tied to the assumptions made for the lowest order pion self-energy and for the value of the Lorentz-Lorenz parameter. The empirical value used in Ref. [9] does not match the lowest order pion self-energy implicitly assumed in their model which is given by  $ph$  and  $\Delta h$  excitations, as we shall see. In order to minimize uncertainties, we have taken  $\Pi_{2p2h}$  from an empirical analysis of pionic atoms consistent with our input for the lowest order self-energy.

(ii) Even then, the phase space in the  $\Lambda$  decay forces one to move away from the situation in pionic atoms to values of  $(q^0, \mathbf{q})$  different of  $(\mu, \mathbf{0})$ , where  $\mu$  is the pion mass. In Ref. [9]  $\Pi_{2p2h}$  was taken as a constant, but

$\text{Im } \Pi_{2p2h}$  was deliberately set to zero in the region where  $\text{Im } \Pi_{1p1h} \neq 0$ . We shall see that this approximation is not too accurate and that there is still an appreciable amount of strength of  $\text{Im } \Pi_{2p2h}$  when  $\text{Im } \Pi_{1p1h} \neq 0$ . On the other hand, the approximation of a constant  $\text{Im } \Pi_{2p2h}$  is improved here by carefully taking into account the phase space available for 2p2h excitation as a function of  $(q^0, \mathbf{q})$ .

(iii) The calculations of Ref. [9] were done in infinite nuclear matter where the pionic decay is forbidden. We have done the calculations in finite nuclei using the local density approximation. However, performing the calculations in finite nuclei clearly shows that this nonmesonic channel is connected with the mesonic decay channel and particular care has to be taken to separate the modified pionic decay into a genuine pion emission channel and the  $3N$  emission channel.

(iv) The study in finite nuclei is rewarding because one can clearly see the origin of the new channel and is thus forced to reinterpret its meaning. In Ref. [9] this new channel was associated with the decay into "a collective nuclear state which is a coherent superposition of pionic and  $\Delta h$  states." This interpretation is misleading because we show that, even in the presence of the coherent superposition of pion and  $\Delta h$  states, if the coupling to pion absorption is ignored there is no new decay channel apart from the conventional mesonic and nonmesonic. The new decay channel is exclusively tied to the two nucleon absorption of virtual pions. Furthermore, we can also show that the elimination of the  $\Delta h$  components reduces the width of this new decay channel by less than 10%. A more intuitive picture of the new channel can be visualized as follows. The strength of a free pion is given by a delta function  $\delta(q^0 - \mathbf{q}^2 - \mu^2)$ . The strength of a renormalized pion in a nuclear medium is given by  $\text{Im } D(q) = \text{Im} (q^0^2 - \mathbf{q}^2 - \mu^2 - \Pi)^{-1}$  which leads to an approximate Breit-Wigner distribution if one has a finite value for  $\text{Im } \Pi$ . For pions not far away from the on-shell region at pion threshold,  $\text{Im } \Pi$  is basically given by  $\text{Im } \Pi_{2p2h}$  from pion absorption, and  $\text{Im } \Pi_{\Delta h}$  is negligible. Now, due to the broadening of the pion strength, part of it is located in the region of momenta which is not Pauli blocked and hence the decay is possible. Thus, a more appropriate qualification of the new channel would be "the cheating of Pauli blocking by the finite width of the pions due to pion absorption."

(v) A reanalysis of the neutron to proton induced  $\Lambda$  nonmesonic decay in light of the findings in this new channel shows that the discrepancy of the experimental data with the predictions of one pion exchange worsens, which is the opposite conclusion to the one suggested in Ref. [9]. Our detailed analysis clearly shows that even if the  $3N$  emission channel is a small fraction of the total (about 1/3 of the free decay width), its consideration is important to derive a proper ratio of the neutron to proton induced  $\Lambda$  decay from the experimental data. Thus, the experimental study of this channel becomes necessary to extract an accurate value for this important ratio.

In Sec. II, the method of calculation and the ingredients of our model are presented. Results for the two

body induced decay width in nuclear matter and in finite hypernuclei are shown and discussed in Sec. III. A reanalysis of the neutron to proton induced ratio in relation to the present experimental data is done in Sec. IV. Finally, our conclusions are given in Sec. V.

## II. METHOD

The  $\Lambda$  decay in nuclear matter was studied in Ref. [2] using the language of propagators which provides a unified picture of the different decay modes. The starting point is the  $\Lambda \rightarrow \pi N$  Lagrangian, accounting for this weak process, which is given by

$$\mathcal{L}_{\pi\Lambda N} = G\mu^2 \bar{\psi}_N (A - B\gamma_5) \boldsymbol{\tau} \boldsymbol{\phi}_\pi \psi_\Lambda + \text{H.c.}, \quad (1)$$

where  $G$  is the weak coupling constant,

$$(G\mu^2)^2 / 8\pi = 1.945 \times 10^{-15}, \quad (2)$$

$\mu$  is the pion mass, and  $A$  and  $B$  are given by

$$A = 1.06, \quad B = 7.10. \quad (3)$$

The constants  $A$  and  $B$  in Eq. (1) determine the parity violating and parity conserving transition amplitudes, respectively. In Eq. (1) the  $\Lambda$  is assumed to behave as the state  $|1/2, -1/2\rangle$  of an isospin doublet with  $T = 1/2$ . This implements the  $\Delta T = 1/2$  rule by means of which the  $\Lambda \rightarrow \pi^- p$  decay rate is twice as large as the  $\Lambda \rightarrow \pi^0 n$  one.

A practical way to evaluate the  $\Lambda$  width in nuclear matter and introduce the medium corrections is to start from the  $\Lambda$  self-energy,  $\Sigma$ , represented by the diagram of Fig. 2(a), and then use the relationship

$$\Gamma = -2\text{Im } \Sigma. \quad (4)$$

The self-energy is readily evaluated as

$$-i\Sigma(k) = 3(G\mu^2)^2 \int \frac{d^4q}{(2\pi)^4} G(k-q)D(q) \times \left( S^2 + \left( \frac{P}{\mu} \right)^2 \mathbf{q}^2 \right), \quad (5)$$

where  $G$  and  $D$  are the nucleon and pion propagators respectively,  $S = A$  and  $P/\mu = B/2M$ , with  $M$  the nucleon mass. By using the free nucleon and pion propagators one

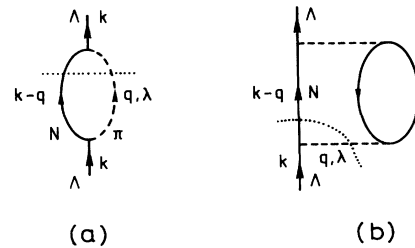


FIG. 2. Free (a) and lowest order (b)  $\Lambda$  self-energy graph. The dotted cuts represent the mesonic decay channel.

immediately obtains the free  $\Lambda$  width [2]

$$\Gamma_\Lambda = 3(G\mu^2)^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega(\mathbf{q})} 2\pi\delta(E_\Lambda - \omega(\mathbf{q}) - E_N(\mathbf{k} - \mathbf{q})) \left( S^2 + \left( \frac{P}{\mu} \right)^2 \mathbf{q}^2 \right). \quad (6)$$

In a Fermi sea of nucleons, both the nucleon and pion propagators are changed:

$$G(p) = \frac{1 - n(\mathbf{p})}{p^0 - E(\mathbf{p}) - V_N + i\epsilon} + \frac{n(\mathbf{p})}{p^0 - E(\mathbf{p}) - V_N - i\epsilon}, \quad (7)$$

$$D(q) = \frac{1}{q^0{}^2 - \mathbf{q}^2 - \mu^2 - \Pi(q^0, \mathbf{q})}, \quad (8)$$

where  $V_N$  is the nucleon potential for which we take  $V_N = -50 \rho/\rho_0$  MeV,  $\Pi(q^0, \mathbf{q})$  is the pion self-energy in the nuclear medium, and  $n(\mathbf{p})$ , the nucleon occupation number, is  $n(\mathbf{p}) = 1$  for  $|\mathbf{p}| \leq k_F$ ,  $n(\mathbf{p}) = 0$  for  $|\mathbf{p}| > k_F$ , with  $k_F$  the Fermi momentum. As shown in Refs. [1,2], the analytical structure of the integrand in Eq. (5) allows one to evaluate the integral over  $q^0$  by performing a Wick rotation, with the result

$$\Gamma(k) = -6(G\mu^2)^2 \int \frac{d^3q}{(2\pi)^3} [1 - n(\mathbf{k} - \mathbf{q})] \theta(k^0 - E(\mathbf{k} - \mathbf{q}) - V_N) \times \left( S^2 + \left( \frac{P}{\mu} \right)^2 \mathbf{q}^2 \right) \text{Im} \frac{1}{q^0{}^2 - \mathbf{q}^2 - \mu^2 - \Pi(q^0, \mathbf{q})} \Big|_{q^0 = k^0 - E(\mathbf{k} - \mathbf{q}) - V_N}. \quad (9)$$

In our calculation, the effect of form factors and short range correlations is also considered. We take a monopole form factor

$$F(q) = \frac{\Lambda^2 - \mu^2}{\Lambda^2 - q^2} \quad (10)$$

at both the weak and strong pion vertices with  $\Lambda = 1200$  MeV as is needed in the empirical study of the  $NN$  interaction. The pion interaction between two nucleons is replaced by the particle-hole interaction developed in Refs. [12,13] which includes  $\pi$  and  $\rho$  exchange modulated by the effect of nuclear short range correlations

$$V_{\text{ph}} = \{V_L(q)\hat{q}_i\hat{q}_j + V_T(q)(\delta_{ij} - \hat{q}_i\hat{q}_j)\}\sigma_i\sigma_j\boldsymbol{\tau}\boldsymbol{\tau}. \quad (11)$$

Detailed expressions for  $V_L$  and  $V_T$  can be found in Eq. (20) of Ref. [2], where they are called  $B$  and  $A$ , respectively. One can write  $V_L(q) = (f^2/\mu^2) [\mathbf{q}^2 F^2(q)D_0(q) + g'(q)]$  in terms of the more familiar Landau-Migdal parameter  $g'$ , where  $D_0$  is the free pion propagator. The  $\Lambda N$  correlations are included in a similar way by modifying the pion exchanged between the weak and strong vertices. Taking all these considerations into account, the decay width is [2]

$$\Gamma(k) = -6(G\mu^2)^2 \int \frac{d^3q}{(2\pi)^3} [1 - n(\mathbf{k} - \mathbf{q})] \times \theta(k^0 - E(\mathbf{k} - \mathbf{q}) - V_N) \times \text{Im} \alpha(q) \Big|_{q^0 = k^0 - E(\mathbf{k} - \mathbf{q}) - V_N} \quad (12)$$

with

$$\alpha(q) = \left( S^2 + \left( \frac{P}{\mu} \right)^2 \mathbf{q}^2 \right) F^2(q)D_0(q) + \frac{\tilde{S}^2(q)\bar{\Pi}^*(q)}{1 - V_L(q)\bar{\Pi}^*(q)} + \frac{\tilde{P}_L^2(q)\bar{\Pi}^*(q)}{1 - V_L(q)\bar{\Pi}^*(q)} + 2 \frac{\tilde{P}_T^2(q)\bar{\Pi}^*(q)}{1 - V_T(q)\bar{\Pi}^*(q)}. \quad (13)$$

The explicit expressions for  $\tilde{S}$ ,  $\tilde{P}_L$ , and  $\tilde{P}_T$  are defined in Eqs. (20), (23), and (24) of Ref. [2], where they are denoted by  $C'$ ,  $B'$ , and  $A'$ , respectively. The function  $\bar{\Pi}^*$  is related to the irreducible pion self-energy  $\Pi^*$  through

$$\Pi^*(q) = \frac{f^2}{\mu^2} \mathbf{q}^2 F^2(q)\bar{\Pi}^*(q). \quad (14)$$

By irreducible we mean that  $\Pi^*$  does not contain pieces iterated with  $V_{\text{ph}}$ , since they are accounted for automatically in the summed series of the second, third, and fourth terms of Eq. (13). In the model of Ref. [2] the pion self-energy was obtained from ph and  $\Delta h$  excitation and, therefore,  $\bar{\Pi}^* = \bar{\Pi}_{1\text{p}1\text{h}}^* + \bar{\Pi}_{\Delta h}^* = U_N + U_\Delta$ , where  $U_N$  and  $U_\Delta$  are the ordinary Lindhard functions for ph and  $\Delta h$  excitation [14] with the normalization given in the appendix of Ref. [1]. To account for the new 3N emission channel, the pion self-energy must include the coupling to 2p2h excitation and therefore

$$\bar{\Pi}^* = \bar{\Pi}_{1\text{p}1\text{h}}^* + \bar{\Pi}_{\Delta h}^* + \bar{\Pi}_{2\text{p}2\text{h}}^*, \quad (15)$$

where the derivation of  $\bar{\Pi}_{2\text{p}2\text{h}}^*$  is specified at the end of this section.

Although the first term in  $\alpha(q)$  shows an apparent

singularity at the pole of the free pion propagator, one can easily see, by summing explicitly all the longitudinal terms in Eq. (13), that the strength moves to the position of the renormalized pion pole and, since  $\text{Im } \Pi \neq 0$  there, it shows up as a pronounced peak at the energy that satisfies the dispersion relation

$$q^0{}^2 - \mathbf{q}^2 - \mu^2 - \text{Re } \Pi(q^0, \mathbf{q}) = 0, \quad (16)$$

where  $\Pi(q^0, \mathbf{q})$  is the pion proper self-energy given by

$$\Pi(q^0, \mathbf{q}) = \frac{\Pi^*(q^0, \mathbf{q})}{1 - \frac{f^2}{\mu^2} g'(q) \bar{\Pi}^*(q^0, \mathbf{q})}. \quad (17)$$

The widths in finite nuclei are obtained via the local density approximation

$$\Gamma(\mathbf{k}) = \int d^3r |\psi_\Lambda(\mathbf{r})|^2 \Gamma(\mathbf{k}, \rho(\mathbf{r})), \quad (18)$$

where  $\psi_\Lambda$  is the  $\Lambda$  wave function for which we have taken the  $1s_{1/2}$  solution of a harmonic oscillator with parameter  $\hbar\omega = [45A^{-1/2} - 25A^{-2/3}] \text{ MeV}$ . The experimental  $\Lambda$  binding energy in each hypernucleus [15] is used. For the nuclear density  $\rho(\mathbf{r})$  we have taken a Fermi distribution with skin parameter  $a = 0.52 \text{ fm}$  and radius  $R = 1.1A^{1/3} \text{ fm}$ . A further average over the momentum distribution of the  $\Lambda$  wave function

$$\Gamma = \int d^3k \bar{\rho}(\mathbf{k}) \Gamma(\mathbf{k}) \quad (19)$$

is also done as in Ref. [2].

The language of propagators which we have used here provides a unified picture of the  $\Lambda$  nuclear decay. This can be seen diagrammatically by expanding the pion propagator as depicted in Fig. 3. The imaginary part of a self-energy diagram is obtained when the set of intermediate states are placed simultaneously on shell in the intermediate integration. The horizontal line in Fig. 3 gives a source corresponding to placing on shell a nucleon and the ph of the pion self-energy. This corresponds to a channel where there are no pions and only nucleons in the final state. The physical process which has occurred is  $\Lambda N \rightarrow NN$  and this is normally referred to as

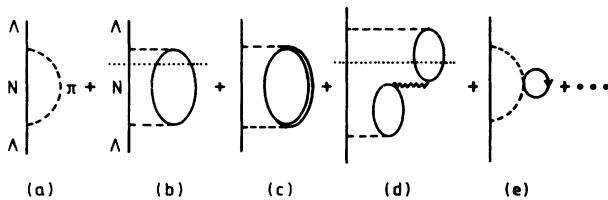


FIG. 3.  $\Lambda$  self-energy diagrams included in Eq. (12). (a) Free self-energy graph. (b) and (c) Insertion of  $p$ -wave pion self-energy at lowest order. (d) Generic RPA graph from the expansion of the pion propagator in powers of the pion self-energy. (e)  $s$ -wave pion self-energy at lowest order. The cuts represent the nonmesonic decay channel.

the nonmesonic channel. Technically, it is obtained by substituting in Eqs. (12) and (13)

$$\text{Im} \frac{\bar{\Pi}^*(q)}{1 - V_{L,T}(q) \bar{\Pi}^*(q)} \rightarrow \frac{\text{Im } \bar{\Pi}_{1p1h}^*(q)}{|1 - V_{L,T}(q) \bar{\Pi}^*(q)|^2}, \quad (20)$$

where  $\text{Im } \bar{\Pi}_{1p1h}^*$  is obtained from the diagrams in which the pion couples to an intermediate ph state. Since there is no overlap between  $\text{Im } \Pi_{1p1h}$  and the pion pole in the propagator, the separation between this channel and the mesonic one is clean.

The mesonic channel would correspond to a different cut, the one where the  $N$  and the  $\pi$  are placed on shell. This is shown diagrammatically in Fig. 2. The terms in Fig. 2(b), and further iterations contained in Eq. (12), lead to a renormalization of the pion in the medium which increases the mesonic width by about two orders of magnitude in heavy nuclei, as shown in Ref. [2]. These results were later corroborated by more detailed calculations in finite nuclei using the wave function approach [3,16,17]. The mesonic width is calculated by omitting  $\text{Im } \Pi$  in Eq. (9), hence

$$\text{Im } D(q) \rightarrow -\pi \delta(q^0{}^2 - \mathbf{q}^2 - \mu^2 - \text{Re } \Pi). \quad (21)$$

This procedure to calculate the mesonic channel was found to be in agreement [2] with the method of wave functions and matrix elements of Ref. [18]. As discussed in Refs. [1,2] the reason why the mesonic width is so drastically changed lies in the attractive character of the pion self-energy which leads to a larger pion momentum for the same pion energy and thus, to a larger nucleon momentum by momentum conservation. Thus, the nucleon has more chances to have a momentum bigger than the Fermi momentum, therefore increasing the mesonic width.

The two nucleon induced  $\Lambda$  decay mode appears when one considers the absorption of the virtual pion by  $2p2h$  states. The physical idea is the following: a real pion in a nuclear medium has a large width because of the coupling to  $2p2h$  components which lead to pion absorption. This means that the strength of the pion is spread over a wide region, unlike a free pion which has all its strength accumulated at one point (one energy for a certain value of  $\mathbf{q}$ ). The decay leading to the emission of one pion is drastically reduced in nuclei because of Pauli blocking. However, one end of the pion distribution in the nucleus could be saved from Pauli blocking, because we can have a smaller energy for the pion and correspondingly more energy for the nucleon, and thus this part of the nuclear pion spectrum could participate in the  $\Lambda$  decay. Technically we could say that the strength of a free pion, which is accumulated in a  $\delta$  function, becomes a Breit-Wigner distribution and part of the tail will correspond to a Pauli unblocked situation. Since the width of the Breit-Wigner distribution at pion energies close to threshold is mostly due to pion absorption through  $2p2h$  emission, the new mode would be observed as three particle emission from  $\Lambda NN \rightarrow NNN$  as depicted in Fig. 4. These ideas are more clearly shown in Fig. 5, where the integrand of Eq. (12), without the Pauli blocking factor

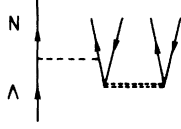


FIG. 4. Schematic representation of the  $\Lambda$  decay coupling to 2p2h components through virtual (close to real) pion absorption.

( $1 - n$ ), is depicted as a function of  $|\mathbf{q}|$  for a  $\Lambda$  at rest decaying in nuclear matter at density  $\rho = 0.69\rho_0$ . For a given  $|\mathbf{q}|$  the pion energy is  $q^0 = M_\Lambda + V_\Lambda - E(\mathbf{q}) - V_N$ , where  $M_\Lambda$  is the  $\Lambda$  mass. Taking  $V_\Lambda = -30\rho/\rho_0$  MeV, we obtain  $V_\Lambda - V_N = 14$  MeV. The full line is the contribution coming from the full  $\text{Im } \alpha$  [Eq. (13)] and the dashed line corresponds to the piece related to 1p1h excitation [Eq. (20)]. The pionic branch would correspond to a delta function located at the energy of the pion pole [Eq. (16)], which is where the integrand peaks. Since the  $\Lambda$  momentum is zero, Pauli blocking forces  $|\mathbf{q}|$  to be larger than  $k_F$ , which is indicated by an arrow in the figure. Consequently, at this density, the mesonic decay would be forbidden by Pauli blocking. However, due to the coupling to 2p2h excitations, the delta function has broadened and there is a contribution coming from a pion of energy smaller than the pion mass being absorbed by two nucleons. This corresponds, as mentioned above, to the two body induced channel  $\Lambda NN \rightarrow NNN$ . The figure also shows that in nuclear matter the contribution of this channel can be obtained in a similar way as in Eq. (20), now using  $\text{Im } \Pi_{2p2h}^*$ , since the mesonic decay is forbidden by Pauli blocking, and the separation between this channel and the pionic branch is also clean. This is the procedure followed in Ref. [9]. However, in finite nuclei, where the local density can be small, there can be an overlap between the mesonic and the 2p2h channels and one must be cautious in separating both contributions. The problem is that whereas the widths from mesonic and 2p2h channels are well defined in finite

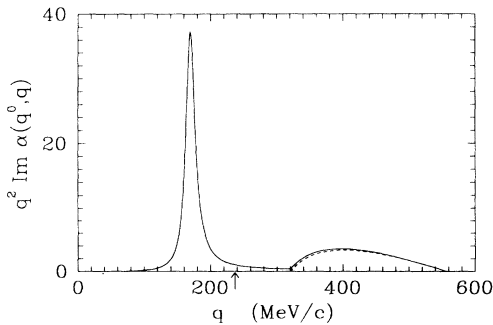


FIG. 5. Illustration of the integrand of Eq. (12) as a function of  $|\mathbf{q}|$  in arbitrary units, for a  $\Lambda$  at rest decaying in nuclear matter at a density  $\rho = 0.69\rho_0$ . The pion energy is  $q^0 = M_\Lambda + V_\Lambda - E(|\mathbf{q}|) - V_N$ . The full line denotes the total contribution of the pion propagator implicit in Eq. (13) and the dashed line is the contribution from coupling to a 1p1h excitation. The arrow shows the location of  $k_F$ .

nuclei, in nuclear matter only their sum has a nontrivial limit. Strictly speaking, in an infinite system all the width comes from nonmesonic channels, because all the pions are eventually absorbed. Thus to separate both channels within the local density approximation requires some prescription. This can be done by first calculating the contribution of the pion pole as described above and associating it to pion emission. Then the contribution of the two body induced channel is obtained by subtracting from the total decay width [obtained from Eqs. (12) and (13)] the pionic width and the 1p1h width [obtained from Eqs. (12) and (20)]. This prescription is equivalent to the assumption that the lambda pionic decay width is little affected by the absorption of the pions, which is indeed the case for the low energy pions involved.

The 2p2h piece  $\bar{\Pi}_{2p2h}^*$  is obtained in the following way. From the work of Ref. [19] in pionic atoms, reanalyzed in Ref. [20] to account for different neutron and proton radii, one finds

$$\Pi_{2p2h}(q_0 = \mu, \mathbf{q} \approx 0, \rho) = -4\pi\mathbf{q}^2 \left(1 + \frac{1}{2}\epsilon\right)^{-1} C_0\rho^2 \quad (22)$$

with

$$\epsilon = \frac{\mu}{M}, \quad C_0 = (0.073 + i0.068)\mu^{-6}. \quad (23)$$

However, this result must be corrected for two reasons. First, in Ref. [19] the second order piece is fitted without the explicit Lorentz-Lorenz correction factor, and it corresponds to a proper pion self-energy piece. However, our approach automatically generates the Lorentz-Lorenz correction, and this must be taken into account. In other words, we need  $\Pi_{2p2h}^*$  which, by virtue of Eq. (17), is given by

$$\Pi_{2p2h} = \frac{\Pi_{2p2h}^*}{1 - \frac{f^2}{\mu^2} g' \bar{\Pi}^*}. \quad (24)$$

By using the same input for  $\bar{\Pi}^*$  found in Ref. [19] (mostly from the  $\alpha_1$  term of the optical potential), the value  $g' \simeq 0.6$ , and the effective density  $\rho_{\text{eff}} = 0.75\rho_0$  for the  $p$ -wave part of the potential [21] we find

$$\Pi_{2p2h}^*(q_0 = \mu, \mathbf{q} \approx 0, \rho) = -4\pi\mathbf{q}^2 C_0^* \rho^2 \quad (25)$$

with

$$C_0^* = (0.105 + i0.096)\mu^{-6}. \quad (26)$$

Note that we have obtained a value of  $\text{Im } C_0^*$  which is about one-half of the one used in Ref. [9], while  $\text{Re } C_0^*$  is about four times smaller. We also note that the lowest order potential in Ref. [19], modified explicitly by the Lorentz-Lorenz correction, is equivalent to the one used here, which is denoted by  $\Pi_{1p1h} + \Pi_{\Delta h}$ .

The second point to consider is the extension of  $\Pi_{2p2h}^*(q^0, \mathbf{q})$  to new kinematical regions away from pionic atoms. Since the imaginary part comes from the physical excitation of 2p2h states, it will be approximately

proportional to the available phase space of these excitations at  $(q^0, \mathbf{q})$  and density  $\rho$ . This factor is responsible for the  $\rho^2$  dependence in Eq. (25). The other factor comes from the two body absorption matrix element squared, which we will take proportional to  $\mathbf{q}^2$  as in pionic atoms. To simplify we neglect possible off-shell corrections. By fixing the proportionality constant to reproduce pionic

atoms, i.e., at  $(\mu, \mathbf{0})$  and  $\rho_{\text{eff}}$ , we obtain

$$\text{Im } \Pi_{2p2h}^*(q^0, \mathbf{q}, \rho) = -4\pi \mathbf{q}^2 \text{Im } C_0^* \rho_{\text{eff}}^2 \frac{PH(q^0, \mathbf{q}, \rho)}{PH(\mu, 0, \rho_{\text{eff}})}. \quad (27)$$

Following the procedure of Ref. [22] and neglecting the momentum dependence of the interaction we find

$$PH(q^0, \mathbf{q}, \rho) \propto \int \frac{d^4 k}{(2\pi)^4} \text{Im } U_N\left(\frac{q}{2} + k, \rho\right) \text{Im } U_N\left(\frac{q}{2} - k, \rho\right) \theta\left(\frac{q^0}{2} + k^0\right) \theta\left(\frac{q^0}{2} - k^0\right). \quad (28)$$

In the low density approximation [22]

$$\text{Im } U_N(k, \rho) \theta(k^0) = -\pi \rho \delta(k^0 - T(\mathbf{k})) P_F(|\mathbf{k}|, \rho) \quad (29)$$

where  $T(\mathbf{k})$  is the nucleon kinetic energy and

$$P_F(|\mathbf{k}|, \rho) = 1 - \theta\left(2 - \frac{|\mathbf{k}|}{k_F}\right) \left[1 - \frac{3|\mathbf{k}|}{4k_F} + \frac{1}{16} \left(\frac{|\mathbf{k}|}{k_F}\right)^3\right], \quad (30)$$

with  $\rho = 2k_F^3/3\pi^2$ . Therefore, Eq. (27) finally reads

$$\text{Im } \Pi_{2p2h}^*(q^0, \mathbf{q}, \rho) = -4\pi \mathbf{q}^2 \text{Im } C_0^* \rho^2 \frac{\int_0^1 d(\cos \widehat{\mathbf{q}\mathbf{k}}) P_F(|\mathbf{q}/2 - \mathbf{k}|, \rho) P_F(|\mathbf{q}/2 + \mathbf{k}|, \rho)}{\sqrt{4M\mu} P_F^2(\sqrt{M\mu}, \rho_{\text{eff}})}. \quad (31)$$

where  $|\mathbf{k}| = \sqrt{Mq^0 - \mathbf{q}^2}/4$ . The real part,  $\text{Re } \Pi_{2p2h}^*$ , is taken constant since the virtual 2p2h excitations are rather insensitive to the values of  $q^0$  and  $\mathbf{q}$ .

The pion self-energy also includes an  $s$ -wave contribution. As shown in Ref. [2] this affects the mesonic decay because the pion momentum for this channel is small, but the 1p1h channel was unchanged. Therefore we add the  $s$ -wave piece  $-4\pi(1 + \epsilon)b_0\rho$ , with  $b_0 = -0.0285 \mu^{-1}$  taken from the parametrization of Ref. [21], to the proper self-energy of Eq. (17), already containing 1p1h and 2p2h excitations. This is basically equivalent to the results of Ref. [19], which includes  $b_0\rho$  and  $B_0\rho^2$  terms, once the effective density,  $\rho_{\text{eff}} = 0.5\rho_0$ , which is the appropriate one for  $s$  waves [21], is used.

We have also included some minor corrections stemming from the use of the relativistic invariant vertex, instead of the nonrelativistic ones. This affects essentially the  $\gamma_5 \Lambda N \pi$  vertex, which is the only one we modify. The relativistic corrections are easy to implement by comparing the sum over spins of the matrix elements squared with the vertices  $\gamma_5$  or  $\sigma \mathbf{q}/2M$ . To implement this relativistic correction one simply needs to make the following replacement in the parity conserving part of the weak vertex

$$\sigma \mathbf{q} \rightarrow \sigma \mathbf{q} \sqrt{\frac{-q^2 + (M_\Lambda - M)^2}{\mathbf{q}^2}}, \quad (32)$$

where  $q^2 = q^{0^2} - \mathbf{q}^2$ .

### III. RESULTS AND DISCUSSION

We will first discuss how our improvements modify the results for the two nucleon induced  $\Lambda$  decay width,  $\Gamma_{2p2h}$ , obtained in Ref. [9]. The calculations are done for a  $\Lambda$  at rest decaying in nuclear matter at a density  $\rho = 0.69\rho_0$ . Our results are shown in Table I where the  $S$ -wave, the  $P$ -wave longitudinal, and the  $P$ -wave transverse contributions, corresponding to the different terms in Eq. (13), are given separately. In the first column we have taken the same input as in the last column of Table I in Ref. [9] and we reproduce their results up to a few percent. The next column shows our results with the same input as in Ref. [9] but replacing only their value of  $C_0$  by our  $C_0^*$  given in Eq. (26). We observe that the results are reduced by a factor of 3 due to our smaller absorptive piece  $\text{Im } \Pi_{2p2h}$  and to our less attractive  $\text{Re } \Pi_{2p2h}$ , which moves the position of the pion pole to a smaller pion momentum. In the results of the next column, our value of  $C_0^*$  is used but we do not enforce  $\text{Im } \Pi_{2p2h} = 0$  when  $\text{Im } \Pi_{1p1h} \neq 0$  as done in Ref. [9]. Relaxing this assumption increases the results by about a factor of 2. The next column shows the results when instead of taking  $\Pi_{2p2h}$  to be constant, as done for the previous columns, one takes into account the phase space modifications as discussed in Eq. (31). We find a decrease of about 30% from the results using a constant  $\Pi_{2p2h}$ . Finally, the last column shows the results using an invariant relativistic  $\gamma_5$  coupling in the parity conserving weak term. Since

TABLE I. Two nucleon induced decay rate (in units of the free  $\Lambda$  width).

	Ref. [9]	$C_0^*$	$\text{Im } \Pi_{2p2h} \neq 0$	Phase space	$q_{\text{inv}}^2$
S	0.427	0.119	0.180	0.148	0.148
P-L	0.081	0.023	0.028	0.026	0.030
P-T	0.035	0.023	0.122	0.065	0.076
$\Gamma_{2p2h}$	0.542	0.165	0.329	0.238	0.254

this correction only affects the  $p$ -wave piece, which is not the most important one here, the changes in the width are rather small, less than 10%.

From the former discussion, it is clear that one must use a realistic input for  $\Pi_{2p2h}$  as a function of  $(q^0, \mathbf{q})$  and that one should consider the strength of the new channel over the whole phase space available (and not only when  $\text{Im } \Pi_{1p1h} = 0$ ).

It is worthwhile to comment on the interpretation of this new channel given in Ref. [9]. As quoted there, this channel is associated with the decay into a collective nuclear state which is a coherent superposition of pionic and  $\Delta h$  states. We find this interpretation to be inappropriate. What makes the new channel possible is the coupling of the pion to the absorption channels and the information that one obtains from this process is about pion absorption and not about the coupling to  $\Delta h$  components. To show this we eliminate the  $\Delta h$  components by setting  $\Pi_{\Delta h} = 0$  and the results for  $\Gamma_{2p2h}$  are only reduced by 10%.

Our results for finite hypernuclei are shown in Table II, where the different contributions to the total width (mesonic, one-body induced, and two-body induced) are given separately in units of the free  $\Lambda$  decay width. Here, the same input as in Ref. [2] is used and we take  $g'_\Lambda \sim 0.52$  [ $g'$  of the  $\Lambda N$  effective interaction of Eqs. (23) and (24) of Ref. [2]]. The results for  $\Gamma_{2p2h}$  are rather insensitive to this correlation parameter, which is also the case for the mesonic channel [2]. We might worry that in some nuclei the results for the mesonic channel in Table II do not agree with the more accurate ones obtained in Ref. [17]. The main reason is the consideration of the  $Q$  values for the nuclear transitions done in Ref. [17], which is omitted here. In our calculation, this would affect both the mesonic and the total width, and therefore it would be inappropriate to subtract the better values of  $\Gamma_m$  found in Ref. [17] from the total rate calculated here, in order to obtain the  $2p2h$  contribution. The two

TABLE II. Mesonic, one body, and two body induced decay rates (in units of the free  $\Lambda$  width).

	$\Gamma_m$	$\Gamma_{1p1h}$	$\Gamma_{2p2h}$
$^{12}_\Lambda\text{C}$	0.31	1.45	0.27
$^{16}_\Lambda\text{O}$	0.24	1.54	0.29
$^{20}_\Lambda\text{Ne}$	0.14	1.60	0.32
$^{40}_\Lambda\text{Ca}$	0.03	1.76	0.32
$^{56}_\Lambda\text{Fe}$	0.01	1.82	0.32
$^{89}_\Lambda\text{Y}$	—	1.88	0.31
$^{100}_\Lambda\text{Ru}$	—	1.89	0.31
$^{208}_\Lambda\text{Pb}$	—	1.93	0.30

calculations (mesonic and total) must be done within the same approximation. Neglecting the  $Q$  values might introduce some uncertainties in the results for the lightest nuclei ( $^{12}_\Lambda\text{C}$ ,  $^{16}_\Lambda\text{O}$ ) shown in Table II. However, for heavier hypernuclei, where the mesonic channel is small, the values of  $\Gamma_{2p2h}$  are more stable and relatively unaffected by an energy shift as one can see in Fig. 5, where it is shown that the width  $\Gamma_{2p2h}$  comes from a wide range of momenta (or equivalently energies) and a shift of a few MeV would not appreciably alter the results. Alternatively, one can think that the  $3N$  emission channel is much less affected by Pauli blocking and shell effects than the mesonic one. Indeed, simple kinematical considerations neglecting binding effects indicate that the most favorable final state occurs when the three nucleons equally share the initial excess energy  $M_\Lambda - M = 176$  MeV, and therefore exit with a kinetic energy of about 60 MeV. This implies a nucleon momentum of about 340 MeV, which is well above the Fermi momentum. Therefore, a small energy shift in the available energy for the  $3N$  final state will not affect the results appreciably.

From Table II we find that  $\Gamma_{2p2h}/\Gamma_\Lambda$  is about 0.3 and remains rather stable as a function of the mass number. Up to now the experiments for  $\Lambda$  decay have focused on two channels, the mesonic and the nonmesonic. In view of the former results and the fact that the  $2p2h$  channel has a larger strength than the mesonic one from hypernuclei like  $^{16}_\Lambda\text{O}$  on, it would be very interesting to conduct experimental searches for this channel as well.

#### IV. REANALYSIS OF THE NEUTRON TO PROTON INDUCED RATIO

Even if  $\Gamma_{2p2h}$  is at most 20% of the nonmesonic width, the existence of this channel has important repercussions with regard to the number of neutrons and protons emitted in the  $\Lambda$  decay process, information which is used to determine the ratio of neutron to proton induced  $\Lambda$  decay  $\Gamma_n/\Gamma_p$ . It is clear that in view of the new results one cannot associate all  $n$  or  $p$  emerging from the experiment to the primary  $\Lambda n \rightarrow nn$  or  $\Lambda p \rightarrow np$  reactions and hence a reanalysis of the experimental data is needed.

One of the interesting consequences of the existence of the two body induced channel, noted in Ref. [9], is that it increases the number of neutrons emitted in the  $\Lambda$  decay, and therefore it should not be ignored in the problem of the ratio of neutron to proton induced  $\Lambda$  decay. The idea is that, since pions are mainly absorbed in correlated neutron-proton pairs [13], the two nucleon induced process proceeds mainly through the reaction  $\Lambda np \rightarrow nnp$  and would provide twice as many neutrons as protons. On the other hand, assuming the pion exchange mechanism to be the dominant one, the one body process is dominated by  $\Lambda p \rightarrow np$ . It is clear then that some of the observed neutrons and protons should be attributed as coming from the two body induced channel  $\Lambda np \rightarrow nnp$ , and it looks like this should affect the neutron induced rate more strongly than the proton induced one.

In this section we show that the consideration of the

two body induced channel in the analysis leads, in general, to the opposite conclusion. The processes to be considered are the neutron ( $\Lambda n \rightarrow nn$ ), proton ( $\Lambda p \rightarrow np$ ), and two body induced ( $\Lambda np \rightarrow nnp$ ). Therefore, in the absence of final state interactions, one obtains

$$N_n = (2\Gamma_n + \Gamma_p + 2\Gamma_{2p2h}) \frac{N}{\Gamma}, \quad (33)$$

$$N_p = (\Gamma_p + \Gamma_{2p2h}) \frac{N}{\Gamma}, \quad (34)$$

$$\Gamma_{nm} \equiv \Gamma_n + \Gamma_p + \Gamma_{2p2h}, \quad (35)$$

where  $N$  is the total number of hypernuclear decays,  $\Gamma$  the total decay rate derived from a lifetime measurement, and  $N_n$ ,  $N_p$  the number of detected neutrons and protons, respectively, after corrections for acceptance and efficiency of the detectors. Note that the nonmesonic decay rate,  $\Gamma_{nm}$ , is the sum of the one body induced rates,  $\Gamma_n$  and  $\Gamma_p$ , and the two body induced one,  $\Gamma_{2p2h}$ . Since the two body induced channel was not considered in Ref. [6], the corresponding analysis would be obtained from the relations

$$N_n = (2\Gamma_n^{\text{exp}} + \Gamma_p^{\text{exp}}) \frac{N}{\Gamma}, \quad (36)$$

$$N_p = \Gamma_p^{\text{exp}} \frac{N}{\Gamma}, \quad (37)$$

$$\Gamma_{nm}^{\text{exp}} \equiv \Gamma_n^{\text{exp}} + \Gamma_p^{\text{exp}}. \quad (38)$$

Comparing Eqs. (33) and (34) with Eqs. (36) and (37), and using the definition of Eq. (38) one can derive the expression

$$\frac{\Gamma_n}{\Gamma_p} = \left( \frac{\Gamma_n^{\text{exp}}}{\Gamma_p^{\text{exp}}} \right) \frac{1 - \frac{1}{2} \left( \frac{\Gamma_p^{\text{exp}}}{\Gamma_n^{\text{exp}}} + 1 \right) \frac{\Gamma_{2p2h}}{\Gamma_{nm}^{\text{exp}}}}{1 - \left( \frac{\Gamma_n^{\text{exp}}}{\Gamma_p^{\text{exp}}} + 1 \right) \frac{\Gamma_{2p2h}}{\Gamma_{nm}^{\text{exp}}}}, \quad (39)$$

written in terms of the rate  $\Gamma_{nm}^{\text{exp}}$  and the ratio  $\Gamma_n^{\text{exp}}/\Gamma_p^{\text{exp}}$  derived in Ref. [6]. It can clearly be seen from Eq. (39) that if the ratio  $\Gamma_n^{\text{exp}}/\Gamma_p^{\text{exp}}$  is larger than 0.5, as it is the case of almost all the experimental results (see Table I in Ref. [6]), the presence of  $\Gamma_{2p2h}$  reduces the denominator more than the numerator. Consequently, the re-analyzed neutron to proton ratio increases when the two body process is considered, contrary to the naive expectation. Care must be taken not to use Eqs. (33)–(39) in the case of  ${}^5_\Lambda\text{He}$  since there the neutrons accounted for in the experiment are those for which no other charged particle is simultaneously emitted [6].

As an example, let us take the case of  ${}^{12}_\Lambda\text{C}$  from the experiment of Ref. [6]. The veto on the charged particles was not applied in the  ${}^{12}_\Lambda\text{C}$  experiment [23] and, therefore, the former equations hold. In order to minimize the effect of final state interaction corrections, we consider the data

obtained after a cutoff of 40 MeV for the kinetic energy of the emitted particles is applied. Ignoring the two body induced mechanism, as implicitly assumed in Ref. [6], one obtains  $\Gamma_n^{\text{exp}}/\Gamma_p^{\text{exp}} = \frac{1}{2} (N_n/N_p - 1) = 0.75$  corresponding to the values  $N_n = 2530 \pm 1050$  and  $N_p = 1012 \pm 130$ . Final state interaction corrections included via an intranuclear cascade calculation modify this ratio up to a value  $\Gamma_n^{\text{exp}}/\Gamma_p^{\text{exp}} = 1.04$  [23]. Taking our theoretical estimate of  $\Gamma_{2p2h} = 0.27\Gamma_\Lambda$  and the value  $\Gamma_{nm}^{\text{exp}} = 1.14\Gamma_\Lambda$  quoted in Ref. [6], Eq. (39) gives  $\Gamma_n/\Gamma_p = 1.54$ , which represents a 50% increase with respect to the value without the two body induced decay process. The obtained ratio of 1.54 is in strong disagreement with the values provided by theoretical models based on the one pion exchange mechanism [1,4,5], which typically range between 0.1 and 0.2.

It is worth noting that the ratio 1.54 has big error bars, coming, among other sources, from the experimental uncertainties in the values of  $N_p$  and  $N_n$ . Therefore, improved experiments with reduced uncertainties are necessary in order to make the corrections implemented by the two body induced channel more meaningful. Nevertheless, the former example has clearly shown that the consideration of the  $2p2h$  induced decay channel becomes imperative in order to extract a proper result for  $\Gamma_n/\Gamma_p$ . On the other hand, the observation of this channel would provide very valuable information on the absorption of off shell pions, complementary to the one from the absorption of real pions, which should broaden our microscopic understanding of the pion absorption mechanisms in nuclei.

## V. CONCLUSIONS

In this paper we have made a detailed and accurate analysis of the two body induced decay mode in  $\Lambda$  hypernuclei which was qualitatively explored in Ref. [9]. This new channel appears because the pions in the medium acquire a width due to their coupling to the  $2N$  absorption process. The original pionic strength of free pions, which accumulates at one point of the spectrum, now spreads out as a consequence of the pionic absorption width and part of this strength overcomes the Pauli blocking, which is responsible for the drastic reduction of the pion emission channel. This  $2p2h$  decay channel shows up as a three nucleon emission process  $\Lambda NN \rightarrow NNN$ , leading mostly to  $nnp$  triplets. The decay width for this channel,  $\Gamma_{2p2h}$ , is found to be remarkably constant as a function of the atomic number and of order one-third of the free  $\Lambda$  width. As a consequence this channel overcomes the mesonic one from hypernuclei like  ${}^{16}_\Lambda\text{O}$  on. In view of that, the conventional wisdom about  $\Lambda$  decay into the mesonic and the nonmesonic channel should rather be changed and one should better refer to the mesonic, one nucleon induced and two nucleon induced  $\Lambda$  decays.

We have shown that the consideration of this new channel is essential for the extraction of the ratio of neutron to proton induced  $\Lambda$  decay from the experimental information. From existing data on neutron and proton emission in the decay of  ${}^{12}_\Lambda\text{C}$ , we have obtained that  $\Gamma_n/\Gamma_p$



increases by about 50% with respect to previous determinations which ignored the  $2p2h$  decay channel, posing additional problems to the theoretical interpretation of this ratio. The experimental study of this channel is thus imperative for further progress in this field. Also, as shown in our study, the measurement of this decay channel would provide valuable information on the mechanisms of pion absorption for off shell pions, complementary to the one obtained from absorption of real pions.

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- [1] E. Oset, P. Fernández de Córdoba, L.L. Salcedo, and R. Brockmann, *Phys. Rep.* **89**, 79 (1990).
  - [2] E. Oset and L.L. Salcedo, *Nucl. Phys.* **A443**, 704 (1985).
  - [3] K. Itonaga, T. Motoba, and H. Bandō, *Z. Phys. A* **330**, 209 (1988).
  - [4] K. Tacheuchi, H. Takaki, and H. Bandō, *Prog. Theor. Phys.* **73**, 841 (1985).
  - [5] J. Dubach, *Nucl. Phys.* **A450**, 71c (1986); J. Dubach, in *Weak and Electromagnetic Interactions in Nuclei, Heidelberg, 1986*, edited by H.V. Klapdor (Springer-Verlag, Berlin, 1986).
  - [6] J.J. Szymanski *et al.*, *Phys. Rev. C* **43**, 849 (1991).
  - [7] A. Ramos, E. van Meijgaard, C. Bennhold, and B.K. Jennings, *Nucl. Phys.* **A544**, 703 (1992); A. Ramos and C. Bennhold, in *Proceedings of the International Symposium on Spin-Isospin Responses and Weak Processes in Hadrons and Nuclei, Osaka, Japan, 1994*, edited by H. Ejiri, Y. Mizuno, and T. Suzuki [*Nucl. Phys. A* (in press)].
  - [8] K. Itonaga, T. Ueda, and T. Motoba, in *Proceedings of the International Symposium on Spin-Isospin Responses and Weak Processes in Hadrons and Nuclei* [7].
  - [9] W.M. Alberico, A. De Pace, M. Ericson, and A. Molinari, *Phys. Lett. B* **256**, 134 (1991).
  - [10] W.M. Alberico, M. Ericson, and A. Molinari, *Ann. Phys. (N.Y.)* **154**, 356 (1984).
  - [11] J. Nieves, E. Oset, and C. García-Recio, *Nucl. Phys.* **A554**, 509 (1993).
  - [12] E. Oset and W. Weise, *Nucl. Phys.* **A319**, 477 (1979).
  - [13] E. Oset, H. Toki, and W. Weise, *Phys. Rep.* **83**, 281 (1982).
  - [14] A.L. Fetter and J.D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).
  - [15] H. Bandō, T. Motoba, and J. Žofka, *Int. J. Mod. Phys. A* **5**, 4021 (1990).
  - [16] T. Motoba, K. Itonaga, and H. Bandō, *Nucl. Phys.* **A489**, 683 (1988).
  - [17] J. Nieves and E. Oset, *Phys. Rev. C* **47**, 1478 (1993).
  - [18] H. Bandō and H. Takaki, *Phys. Lett.* **150B**, 409 (1985).
  - [19] O. Meirav, E. Friedman, R.R. Johnson, R. Olszewski, and P. Weber, *Phys. Rev. C* **40**, 843 (1989).
  - [20] C. García-Recio, J. Nieves, and E. Oset, *Nucl. Phys.* **A547**, 473 (1992).
  - [21] R. Seki and K. Masutani, *Phys. Rev. C* **27**, 2799 (1983).
  - [22] C. García-Recio, E. Oset, and L.L. Salcedo, *Phys. Rev. C* **37**, 194 (1988).
  - [23] G. Franklin, private communication.