Comment on "Influence of bulk properties on the surface structure of finite nuclei"

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(Received 24 May 1993)

We comment on a recent paper by Uma Maheswari *et al.* in which it is claimed that quantal calculations of the half-infinite nuclear matter, in contrast to semiclassical approximations, exhibit an unusually strong dependence of the 90%–10% surface thickness of the density profile on the Fermi momentum k_F at saturation. This conclusion was carried over to the surface incompressibility. On the contrary we find essential agreement between semiclassical and quantal results and very weak dependence on k_F of the quantities in question.

PACS number(s): 21.10.–k, 21.65.+f

In a recent paper by Uma Maheswari *et al.* [1] the dependences of the surface thickness of the density profile of half-infinite nuclear matter on the nuclear binding energy per nucleon (a_v), the nuclear matter incompressibility (K_∞), and the Fermi momentum at saturation (k_F) were investigated in a schematic Skyrme force (Zamick [2]) model. Both quantal and semiclassical calculations were performed. The semiclassical investigations were done in the framework of the density functional theory (DFT) [3] up to second and fourth order in $\hbar$. The surface energy was minimized using as trial density a Fermi function to some power. Also in the quantal calculation the variation was not unrestricted. Rather the wave functions of the half-infinite Woods-Saxon potential (which are known analytically [4]) were taken as trial wave functions and the dependence on the surface parameter of the potential fixed variationally. We think that both the restricted variations (semiclassical and quantal) should be accurate enough to address the aforementioned problems.

For the dependence of the 90%–10% surface thickness (t) on a_v , semiclassical and quantal results show very similar trends though their absolute values can differ locally by as much as 30% (see Figs. 6 and 7 of [1]). The same situation prevails when t is studied as a function of K_∞ . However, in the calculation reported in Ref. [1] t as a function of k_F shows strongly diverging results in the semiclassical and quantal cases. Indeed the quantal result for $a_{\text{eff}} = t/4.4$ is shown to sharply decrease when k_F is only slightly increased whereas the semiclassical values stay practically constant. This result is then carried over to a strong difference between the surface incompressibility coefficient K_S in the semiclassical and quantal cases. A decrease of K_S by more than 100%, when varying k_F from 1.3 fm^{-1} to 1.35 fm^{-1} , is reported. The semiclassical results again stay roughly constant.

This result called for verification, since it has important consequences on the extraction of K_∞ from experimentally measured breathing mode energies [5]. We also were intrigued by the apparent failure of the semiclassical method for this problem which would constitute a very rare case.

We therefore repeated the calculations in the quan-

tal and DFT cases with an unrestricted variation, i.e., fully self-consistently. The procedures employed are reported in Refs. [6,7]. Our results for the quantal and semiclassical cases are given in Tables I and II, respectively. Strictly the same parameters as those employed by Uma Maheswari *et al.* to build up their Fig. 8 and Table I [1] have been used. In order to calculate K_S in both quantal and semiclassical cases we have followed the prescription given in Ref. [8]. It should also be mentioned that for the $\hbar^2$ -order calculations of Table II we took $\beta = 1/36$ for the coefficient of the Weizsäcker term in the kinetic-energy density functional [3].

From the comparison of Tables I and II we see that, in contradiction to the findings of Ref. [1], semiclassical and quantal values are in quite good agreement in what concerns the very weak dependence of a_{eff} and K_S on k_F . The absolute values still differ but such inaccuracies of DFT have been reported elsewhere [9]. (The unusually big relative error in a_S from DFT is partly due to the small value of a_S and partly due to the lack of gradient terms in the force used.) It therefore follows from our studies that the usually employed semiclassical technique for the determination of K_∞ from monopole resonances should be reliable.

The difference of the present quantal calculation with the one of [1] concerning $a_{\text{eff}}(k_F)$ calls for some remarks as to its origin. We for instance very carefully checked that this is not due to numerical inaccuracy on our side. In order to demonstrate this explicitly we evalu-

TABLE I. Quantal results for surface energy (a_S), 90%–10% surface thickness divided by 4.4 (a_{eff}), and surface incompressibility K_S coefficient are shown for different values of k_F .

$k_F \text{ (fm}^{-1}\text{)}$	$a_S \text{ (MeV)}$	$a_{\text{eff}} \text{ (fm)}$	$K_S \text{ (MeV)}$
1.300	10.73	0.340	–158.40
1.325	10.89	0.336	–157.50
1.350	11.00	0.333	–156.20
1.375	11.07	0.329	–154.70
1.400	11.11	0.326	–153.20

TABLE II. Same as in Table I but for the results from density functional theory. The values in the first and second columns of a_s , a_{eff} and K_s refer to $\hbar^2$ and $\hbar^4$ order calculations respectively. The effective surface coefficient a_{eff} is related to the 90%–10% surface thickness by $a_{\text{eff}} = t/4.4$.

k_F (fm $^{-1}$)	a_s (MeV)		a_{eff} (fm)		K_s (MeV)	
	$\hbar^2$	$\hbar^4$	$\hbar^2$	$\hbar^4$	$\hbar^2$	$\hbar^4$
1.300	7.49	8.22	0.201	0.269	−90.0	−155.0
1.325	7.64	8.36	0.201	0.267	−91.6	−156.7
1.350	7.79	8.50	0.201	0.266	−93.3	−158.3
1.375	7.93	8.64	0.201	0.264	−94.9	−160.0
1.400	8.08	8.78	0.201	0.263	−96.5	−161.6

TABLE III. Convergence behavior of the density profile as a function of the number of iterations in the semiclassical and quantal cases. For the definition of χ_ρ^2 , see text.

Semiclassical calculations		Hartree-Fock calculations	
χ_ρ^2	Number of iterations	χ_ρ^2	Number of iterations
0.42×10^{-1}	10	0.51×10^{-3}	10
0.46×10^{-2}	1000	0.56×10^{-5}	15
0.81×10^{-5}	2000	0.13×10^{-6}	20
0.30×10^{-8}	3000	0.43×10^{-8}	25

ated for both semiclassical and Hartree-Fock approaches the quantity χ_ρ^2 defined as follows

$$\chi_\rho^2 = \sum_{i=1}^N \left[\frac{\rho^{(n+1)}(r_i) - \rho^{(n)}(r_i)}{\rho^{(n)}(r_i)} \right]^2,$$

where $\rho^{(n)}$ is the particle density after n iterations. The sum is running over all the points where the density is calculated. For a typical case we get the results shown in Table III. From this table we see that the density profiles

are indeed very well converged. We therefore suppose that the conflicting quantal results of [1] are due either to the restricted variation or to a numerical error.

This work was supported in part by DGICYT (Spain) under Grant PB 89-0332 and by the IN2P3-CICYT exchange program. One of us (M.C.) is very grateful to the CIRIT (Catalonia) for the financial support during his stay at the ISN (Grenoble). The Supercomputing Center of Catalonia (CESCA) is gratefully acknowledged for providing computing time on an IBM 3090/600J.

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