World-line condition and the noninteraction theorem

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A $6N$-dimensional alternative formulation is proposed for constrained Hamiltonian systems. In this context the noninteraction theorem is derived from the world-line conditions. A model of two interacting particles is exhibited where physical coordinates are canonical.

I. INTRODUCTION

The theory of action at a distance in relativistic dynamics[1] was very enhanced with Dirac's work,[2] where a program to construct this kind of theory was established. Dirac's program consists of obtaining a realization of the Poincaré group $\mathscr{P}$ by ten functions which are the infinitesimal generators of $\mathscr{P}$ through the Poisson brackets. The subgroups of $\mathscr{P}$ with generators maintaining their free form were chosen and three possibilities were studied in Dirac's paper: the instant form, the front form, and the point form. The noninteraction theorems (NIT's)[3,4] involve the Dirac program adding new difficulties because canonical positions and the correct world-line behavior are allowed only in the free-particle case.

These difficulties are now solved in several ways. In the predictive-relativistic-mechanics[5] formalism, which works in the Dirac instant form, by removing the canonical condition for the position coordinates, the difficulties were transferred to finding the symplectic form. In the constrained Hamiltonian formalism[6–8] another point of view was developed. It works in a $(6N+1)$-dimensional extended phase space and the evolution of the system is given by a new generator which is independent of the Poincaré generators, and the positions are rarely canonical.

The main aim of this work is to study the relations existing between this generator and those of the Poincaré group, and also to clarify in what form the noninteraction theorem is circumvented in the constrained Hamiltonian formalism. We also exhibit a model with canonical position coordinates that describes interacting particles.

II. CONSTRAINED HAMILTONIAN SYSTEMS

We shall consider this formalism as it was used in Refs. 6 and 7. This approach starts from $T^*M_4^N$ endowed with the symplectic form

$$\Omega=\sum_{a=1}^{N} dq_a^{\mu}\wedge dp_{a\mu} , \tag{2.1}$$

where (q_a^{μ},p_b^{ν}) are a set of $8N$ adapted coordinates for the symplectic form Ω and with appropriate transformation properties under the action of the Poincaré group $\mathscr{P}$; i.e., q_a^{μ} behave like positions and p_b^{ν} behave like momenta.

The generating functions of the Poincaré group associated to the Poisson brackets are

$$P_{\mu}=\sum_{a=1}^{N} p_{a\mu},\ \ J_{\mu\nu}=\sum_{a=1}^{N}(q_{a\mu}p_{a\nu}-q_{a\nu}p_{a\mu}) . \tag{2.2}$$

To obtain the right number of variables, i.e., $6N$ for the dynamical problem of N point particles without spin, a $6N$ submanifold Σ_0 is defined. The pullback of Ω gives us a closed form ω_0 on Σ_0 and it can be obtained by defining Σ_0 through $2N$ functions:

$$K_a(q,p)=0,\ \ \chi_a(q,p)=0,\ \ a=1,\ldots,N , \tag{2.3}$$

verifying the relations

$$\{K_a,K_b\}=0,\ \ \{K_a,\chi_b\}=S^{-1}{}_{ab} ,$$

where $S^{-1}{}_{ab}$ is an invertible matrix.

The Poisson brackets associated to ω_0 are the Dirac brackets and can be expressed in terms of the constraints (2.3) by the classical expression[8]

$$\begin{aligned}\{f,g\}^*=&\{f,g\}+\{f,K_a\}S_{ba}\{\chi_b,g\}\\ &-\{f,\chi_a\}S_{ab}\{K_b,g\}\\ &-\{f,K_a\}S_{ba}\{\chi_b,\chi_c\}S_{cd}\{K_d,g\} .\end{aligned} \tag{2.4}$$

In order to obtain a realization of the Poincaré group in Σ_0 in general we cannot use the infinitesimal generators given by

$$\vec{\mathbf{P}}_{\mu}=\{P_{\mu},\ldots\},\ \ \vec{\mathbf{J}}_{\mu\nu}=\{J_{\mu\nu},\ldots\} \tag{2.5}$$

because they are not tangent fields to Σ_0 except for the very special case in which χ_a and K_b are Poincaré invariants (and the Poincaré generators can be expressed as in the free case). The standard option requires all the K_a's to be Poincaré invariant while it requires some among the χ_a not to be. Then there is an elegant procedure to obtain a realization of $\mathscr{P}$ on Σ_0;[9] it consists of looking for the combinations of the fields $\vec{\mathbf{K}}_a=\{K_a,\ldots\}$ which are tangent to $T^*M_4^N$ (but not to Σ_0 because $\{\chi_a,\chi_b\}$ is an invertible matrix) with the $\vec{\Lambda}_I$ fields of the $T^*M_4^N$ realization of the Poincaré group, getting in this way for each $\vec{\Lambda}_I$ a unique field $\vec{\Lambda}_I^*$ tangent to Σ_0,

$$\vec{\Lambda}_I^* = \vec{\Lambda}_I - \sum_{a,b=1}^{N} \{\Lambda_I,\chi_b\} S_{ba} \vec{K}_a , \tag{2.6}$$

where S_{ba} is the inverse matrix of $\{K_a,\chi_b\}$.

Moreover, since the K_a are Poincaré invariant, the Lie algebra of these fields (2.6) agrees with the Lie algebra of the Poincaré group.

We have got a symplectic $6N$ submanifold (phase space) with a realization of the Poincaré group, $\vec{\Lambda}_I^*$, that allows us to describe the dynamics of an N-particle system.

Let z be the $6N$ coordinates of Σ_0; the time evolution of any dynamical variable $f(z)$ can be obtained through the action of $\vec{P}_0^*$, the generator for time translations on Σ_0:

$$\frac{df(z)}{dt} = \vec{P}_0^* f = \{P_0, f\}^* . \tag{2.7}$$

The parameter t must be handled here with care: it is not a common physical time (the phase-space points do not correspond to simultaneous configurations of particles). We have, however, kept the symbol t since this parameter is the one associated to P_0.

Nevertheless, the constrained Hamiltonian formalisms[6–8] are a description of the physical system in a $(6N+1)$-dimensional submanifold, i.e., the extended phase space containing Σ_0 and the parameter τ to describe the evolution of the physical variables.

It can be inserted in the description through the χ_a constraints that now depend on τ. Generally the τ dependence is chosen to be[9]

$$\chi_1(q,p,\tau) = h(q,p) - \tau \tag{2.8}$$

and the remaining $\chi_A(q,p)$, $A=2,\ldots,N$, are τ independent.

Now for each value of τ we have a $6N$-dimensional space Σ_τ and their union for every τ gives us a $(6N+1)$-dimensional extended phase space Σ that can be expressed through the equations $K_a=0$, $a=1,\ldots,N$, and $\chi_A=0$, $A=2,\ldots,N$. In Σ we have the same Poincaré realization (2.6); in fact, each $\vec{\Lambda}_I^*$ field leaves Σ_τ invariant. Furthermore, there is only a combination of the $\vec{K}_a$ fields that is tangent to Σ verifying the conditions $\vec{H}\chi_a + \partial\chi_a/\partial\tau = 0$,

$$\vec{H} = \sum_{a,b=1}^{N} \frac{\partial \chi_b}{\partial \tau} S_{ba} \vec{K}_a = \sum_{a=1}^{N} S_{1a} \vec{K}_a . \tag{2.9}$$

For the second equality we have taken into account the particular expression of the χ_a constraints, in which $\chi_1 = h - \tau$ and χ_A, $A=2,\ldots,N$, are τ independent. The vector field $\vec{H}$ commutes with the Poincaré generators (2.6) and the application

$$e^{\sigma \vec{H}} : \Sigma_\tau \to \Sigma_{\tau+\sigma} \tag{2.10}$$

commutes with the action of $\mathscr{P}$ in each Σ_τ. An isomorphism can be established among the Σ_τ that preserves the realization of $\mathscr{P}$ in each Σ_τ.[9] In this way a contact structure on Σ is defined, where an Abelian extension of the Poincaré group $\mathscr{P}\otimes\mathscr{A}$ acts. $\mathscr{A}$ is the one-dimensional algebra generated by $\vec{H}$. Then the equation of evolution for any physical variable $f(q,p)$ is given by

$$\frac{df}{d\tau} = \vec{H} f . \tag{2.11}$$

Any of these models must be completed by giving the relation between the (q,p) coordinates on Σ, or z in Σ_0, and the physical positions x_a^μ of the particles in Minkowski space for any inertial observer. In all the models we know the physical positions are identified with the coordinates on Σ.

The N projections

$$\Pi_a : \Sigma \to M_4$$
$$(q,p) \to q_a^\mu \tag{2.12}$$

allow us to build the N world lines of the N-particle system using the solution in Σ of the following equations of motion:

$$\frac{dq_a^\mu}{d\tau} = \vec{H} q_a^\mu , \quad \frac{dp_a^\mu}{d\tau} = \vec{H} p_a^\mu . \tag{2.13}$$

Let $(\phi_a^\mu(\tau;q_0,p_0)\psi_b^\nu(\tau;q_0,p_0))$ be the general solution.

The problem is how the Poincaré standard action on M_4 and the realization (2.6) of the Poincaré group can be made compatible. The M_4 projections of a Σ trajectory of the system and the ones of the Poincaré-transformed trajectory in Σ must be related by the standard transformation of $\mathscr{P}$ in M_4,[6,7] i.e., given a transformation (Λ, A) with parameters $(\epsilon_I; I=1,\ldots,10)$, functions $\tau_a(\tau;q_0,p_0;\epsilon_I)$ must exist such that

$$\phi_a^\mu(\tau_a; G^*(\epsilon_I;q_0,p_0)) = \Lambda^\mu{}_\nu[\phi_a^\nu(\tau;q_0,p_0) - A^\nu] , \tag{2.14}$$

where G^* is the action of the realization of $\mathscr{P}$ in Σ on (q_0,p_0). Their infinitesimal expressions are

$$\frac{(\vec{\Lambda}_I^* - \vec{\Lambda}_I) q_a^i}{(\vec{\Lambda}_I^* - \vec{\Lambda}_I) q_a^0} = \frac{\vec{H} q_a^i}{\vec{H} q_a^0} \quad \begin{cases} i=1,2,3, \\ I=1,\ldots,10 , \\ a=1,\ldots,N . \end{cases} \tag{2.15}$$

These equations are restrictions on the constraints and they are known as the world-line conditions (WLC). If the constraints do not verify (2.15) the M_4 trajectories of the particles do not transform correctly under the Poincaré group. In this way, we have constructed two different dynamical systems: the one with the $\vec{H}$ generator giving the "time evolution" and the other one in Σ with $\vec{P}_0^*$ as evolution operator. How can both systems be related? It can be shown[9] that this relation is the same as the one existing between the $(6N+1)$-dimensional extended phase space and the $6N$-dimensional phase space in classical mechanics. We are going to prove that the M_4 trajectories of the particles, projections of the Σ and Σ_0 ones, coincide by adding the time t to the zero component for every inertial observer to the projections of the trajectories in Σ_0.

The equations of motion in Σ_0 are

$$\frac{dq_a^\mu}{dt}=\vec{\mathrm{P}}{}_0^*q_a^\mu,\quad \frac{dp_a^\mu}{dt}=\vec{\mathrm{P}}{}_0^*p_a^\mu \tag{2.16}$$

and let

$$(f_a^\mu(t;q_0,p_0),g_a^\mu(t;q_0,p_0))$$

be its solution corresponding to the initial conditions $(q_0,p_0)\in\Sigma_0$. Then we can choose N functions $\tau_a(t;q_0,p_0)$ in such a way that

$$\eta_0^\mu t+f_a^\mu(t;q_0,p_0)=\phi_a^\mu(\tau_a;q_0,p_0)\ . \tag{2.17}$$

Their infinitesimal expressions are

$$t(\eta_0^\mu+\vec{\mathrm{P}}{}_0^*q_a^\mu)=\tau_a\vec{\mathrm{H}}q_a^\mu\ . \tag{2.18}$$

The functions τ_a can be eliminated by using the zero component and we obtain

$$\frac{\vec{\mathrm{P}}{}_0^*q_a^i}{\vec{\mathrm{P}}{}_0^*q_a^0-1}=\frac{\vec{\mathrm{H}}q_a^i}{\vec{\mathrm{H}}q_a^0}\quad \begin{cases} i=1,2,3,\\ 1=1,\ldots,N\ .\end{cases} \tag{2.19}$$

These equations coincide with (2.15) for the $\vec{\mathrm{P}}{}_0^*$ generator because $\vec{\mathrm{P}}_0q_a^\mu=\eta_0^\mu$. Then the physical trajectories are the same *provided that the WLC (2.15) holds.* From (2.15) and (2.19) we obtain the WLC in the Σ_0 description:

$$\frac{(\vec{\Lambda}_I-\vec{\Lambda}{}_I^*)q_a^i}{(\vec{\Lambda}_I-\vec{\Lambda}{}_I^*)q_a^0}=\frac{\vec{\mathrm{P}}{}_0^*q_a^i}{\vec{\mathrm{P}}{}_0^*q_a^0-1}\ . \tag{2.20}$$

From this result we can see the "irrelevance" of the eleventh generator $\vec{\mathrm{H}}$ on Σ, since it does not contain any dynamical information which has not been previously introduced by the ten Poincaré generators. The use of $\vec{\mathrm{H}}$ and Σ instead of $\vec{\mathrm{P}}{}_0^*$ on Σ_0 is a question of taste or convenience, but it is empty of any physical content. A similar situation can be found in classical dynamics between the extended phase space and the phase space.

There is, however, a difference between the common $6N+1$ to $6N$ transition in analytical mechanics and the one presented in this paper. The equivalence here has been established paying attention only to world lines (i.e., to the q's and not to the p's at all).

In predictive relativistic mechanics (PRM) we have a similar situation. There are two equivalent formulations,[10] one in a $6N$-dimensional space with $\vec{\mathrm{P}}_0$ generating the evolution and another in an $8N$-dimensional space adding to the Poincaré group an Abelian extension generated by commuting fields. The choice between them is also a question of taste or convenience.

III. THE NONINTERACTION THEOREM

The relativistic models of action at a distance run into difficulties with the noninteraction theorem (NIT),[3,4] and hinder the construction of relativistic theories. We will use the Leutwyler version of the NIT:[4]

(i) We have a $6N$-dimensional symplectic manifold with adapted coordinates.

(ii) In this manifold a realization of the Poincaré group with functions Λ_I through the Poisson brackets acts.

(iii) The equations of evolution are

$$\frac{dq_a^i}{dt}=\{P_0,q_a^i\},\quad \frac{dp_a^i}{dt}=\{P_0,p_a^i\}$$

and the trajectories in M_4 are given by $(t,q_a^i(t))$, where $q_a^i(t)$, $a=1,\ldots,N$ are the solutions of the evolution equations.

(iv) The trajectories in M_4 transform correctly under the Poincaré group, i.e.,

$$\begin{aligned}&\{P_i,q_a^j\}=-\delta_i^j,\quad \{J_i,q_{aj}\}=\epsilon_{ijk}q_{ak}\ ,\\ &\{K_i,q_{aj}\}=-q_{ai}\{P_0,q_a^j\}\ .\end{aligned} \tag{3.1}$$

Then the trajectories of the particles are straight lines, i.e., a canonical formalism with canonical q_a^i and representing the instantaneous physical positions can only describe free particles.

Provided that the constrained Hamiltonian models usually work in a $(6N+1)$-dimensional extended phase space and the evolution operator is not $\vec{\mathrm{P}}_0$ but the eleventh generator $\vec{\mathrm{H}}$, some authors[6] have suggested the existence of this generator as the reason for giving up the NIT. Nevertheless, we have seen in Sec. II the equivalence of the $(6N+1)$-dimensional formulation with the $6N$-dimensional one using $\vec{\mathrm{P}}{}_0^*$ as evolution generator, that is to say, the trajectories of both models coincide. Then we can look at the $6N$-dimensional version and see if the NIT holds.

When we check if this $6N$-dimensional version agrees with hypotheses (i) and (ii) of the NIT, we can easily see that two conditions are not generally accomplished.

(a) The q_a^i coordinates are not generally canonical with respect to the Dirac brackets associated with the symplectic form ω_0 of Σ_0, i.e., $\{q_a^i,q_b^j\}^*\neq 0$.

(b) The q_a^i coordinates are generally not simultaneous; i.e., $(t,q_a^i(t))$ is not the trajectory for the particle in Minkowski space. This is so because the constraints $K_a=0$ and $\chi_b=0$ fix the values for q_a^0 and p_a^0 as functions of q_a^i and p_a^i (it happens to be so in all the models we know, although nobody has imposed this condition explicitly), and from (2.17) we have as the trajectory in M_4 $(t+f_a^0(t;\mathrm{C.I.}),f_a^i(t;\mathrm{C.I.}))$. Then condition (iv) is not accomplished.

Now it can be asked, if only one of these conditions (a) or (b) is given up , whether or not we arrive at the noninteraction theorem; in Ref. 6 the authors proved that simultaneity in the two-particle case leads to no interaction. We are going to prove that this is true also when we have an N-particle system.

Lemma. $\chi_a=q_a^0-\tau \Longrightarrow \{q_a^i,q_b^j\}^*=0$, effectively

$$\{\chi_a,\chi_b\}=\{q_a^0,q_b^0\}=0\ ,$$

$$\{\chi_a,q_b^i\}=\{q_a^0,q_b^i\}=0\ .$$

Then from (2.4) we have

$$\{q_a^i,q_b^j\}^*=\{q_a^i,q_b^j\}=0\ ,$$

i.e., the simultaneity ($q_a^0=\tau$, $\forall a$) leads to canonical positions q_a^i [condition (i)] and furthermore in $\Sigma_0(\tau=0)$ we have $q_a^0=0$, $\forall a=1,\ldots,N$, and the trajectory in M_4 for the particle is $(t,q_a^i(t))$ [condition (iii)]. The WLC guarantees that the trajectory has the right Poincaré behavior[9] [condition (iv)], and the system verifies all the hypotheses of the NIT; then in the constrained Hamiltonian models the simultaneity (in Σ) for the q_a^i leads to noninteraction: the trajectories are straight lines.

Therefore the nonsimultaneity for different particles is an essential condition in the constrained Hamiltonian models to describe interacting-particle systems.

Now we give an example where it can be seen that canonicity for the physical coordinates q_a^i permits interacting-particle systems.

IV. AN INTERACTING MODEL WITH CANONICAL PHYSICAL COORDINATES

From expression (2.4) we can easily find some conditions assuring the canonical property of the q_a^i coordinates in Σ_0. These are

$$\{\chi_a,\chi_b\}=0,\quad \{\chi_a,q_b^i\}=0\ . \tag{4.1}$$

These can be done by choosing the χ_a constraints depending only on the coordinates q_a^μ,τ.

Furthermore the model must verify the WLC to assure the correct transformation of the trajectories by the action of the Poincaré group. An easy way to guarantee the WLC is to take all the χ_a excepting one Poincaré invariant.[9]

Let us introduce the following notation for a two-particle system:

$$X^\mu=\frac{q_1^\mu+q_2^\mu}{2},\quad z^\mu=q_1^\mu-q_2^\mu,\quad P^\mu=p_1^\mu+p_2^\mu\ ,$$
$$y^\mu=\frac{P_1^\mu-P_2^\mu}{2},\quad \Pi^\mu{}_\nu=\eta^\mu{}_\nu-\frac{P^\mu P_\nu}{P^2},\quad \tilde{a}^\mu=\Pi^\mu{}_\nu a^\nu\ . \tag{4.2}$$

(P,X) and (y,z) are canonical conjugate variables and $\tilde{a}^\mu$ is the orthogonal projection of a^μ to the vector P^μ.

The following constraints verifying the above-mentioned conditions give us an interacting model of a two-particle system:

$$K_a=\tfrac{1}{2}(p_a{}^2+m_a{}^2)+V(\tilde{z}),\quad a=1,2$$
$$\chi_1=X^2-\tau,\quad \chi_2=z^2+A\ , \tag{4.3}$$

where V is an arbitrary function and A a constant.

The K_a, $a=1,2$ are chosen in this way to guarantee[11] that

$$\{K_a,K_b\}=0\ .$$

Let us look for the invertibility of the matrix S_{ab}. Straightforward calculation gives us

$$\{K_a,\chi_1\}=\frac{4V^1}{P^2}(X,z)(P,z)-(X,p_a)\ ,$$
$$\{K_a,\chi_z\}=(-1)^a 2(p_a,z)\ , \tag{4.4}$$
$$\mathrm{Det}S^{-1}=\frac{8V^1}{P^2}(X,\tilde{z})(P,z)^2-2[(X,p_1)(p_2,z)+(X,p_2)(p_1,z)]=\Delta\ ,$$

where $(a,b)=a^\mu b_\mu$ is the product of the space M_4.

We can see that $\det S\neq 0$, except perhaps for very special configurations that can be excluded from Σ_0 or Σ.

The evolution field $\vec{\mathrm{H}}$ generating the evolution in Σ from (2.9) will be

$$\vec{\mathrm{H}}=\sum_{a=1}^{2} S_{1a}\vec{\mathrm{K}}_a=\frac{2}{\Delta}[(p_2,z)\vec{\mathrm{K}}_1-(p_1,z)\vec{\mathrm{K}}_2]\ . \tag{4.5}$$

A lengthy and tedious calculation gives us

$$\vec{\mathrm{H}}q_a^\mu=B_1\tilde{z}^\mu+C_1p_a^\mu\ ,$$
$$\vec{\mathrm{H}}(\vec{\mathrm{H}}q_a^\mu)=B_2\tilde{z}^\mu+C_2p_a^\mu+\frac{8V^1}{\Delta^2P^2}(P,z)(y,z)(p_a,z)p_{a'}^\mu\ , \tag{4.6}$$

where B_i,C_i, $i=1,2$ are involved scalar functions of the variables q_a^μ and p_b^ν.

We can see now that $\vec{\mathrm{H}}(\vec{\mathrm{H}}q_a^\mu)$ is not parallel to $\vec{\mathrm{H}}q_a^\mu$ and the trajectories are not straight lines; therefore the particles interact. The constraints (4.3) give us the q_a^0,p_b^0 components but the spatial coordinates are arbitrary.

So the vectors $\tilde{z}^\mu$, p_1^μ, p_2^μ are linearly independent except for some very special configurations. Furthermore, in general $B_1/B_2\neq C_1/C_2$. Therefore from (4.6) we see that $\vec{\mathrm{H}}(\vec{\mathrm{H}}q_a^\mu)$ cannot be parallel to $\vec{\mathrm{H}}q_a^\mu$ except for some isolated points (inflection points) of the trajectory.

Then this model with canonical position coordinates is proved not to be a free-particle model due to the nonsimultaneity of the q_a coordinates.

The model proposed here has been worked out in the $(6N+1)$-dimensional version because most of the models in the literature are presented in this way. Hence, the eleventh generator $\vec{\mathrm{H}}$ has been used to define the "velocities" and "accelerations," i.e., $\vec{\mathrm{H}}q_a^\mu$ and $\vec{\mathrm{H}}(\vec{\mathrm{H}}q_a^\mu)$. However, dealing with the model in the $6N$-dimensional formalism would not represent any additional work. The τ-dependent constraint $\chi_1=X^2-\tau$ would then become $\chi_1=X^2$ and $\vec{\mathrm{P}}_0^*q_a^\mu$ and $\vec{\mathrm{P}}_0^*(\vec{\mathrm{P}}_0^*q_a^\mu)$ would have to be taken as velocities and accelerations, respectively. As in the case we dealt with, the latter vectors would not be parallel, anyhow.

V. CONCLUSION

There are two main conclusions to this paper. One is the existence of an alternative $6N$-dimensional approach to the usual[6,7] constrained Hamiltonian models. In this $6N$-dimensional formulation we can obtain the same world lines (generated by $\vec{\mathrm{P}}_0^*$) as in the $(6N+1)$-dimensional approach. The trajectories generated by $\vec{\mathrm{H}}$ are defined through the N projections Π_a, while the $\vec{\mathrm{P}}_0^*$ ones are obtained adding the time t to the zero component of the solution (2.17). This is the same mechanism used in classical mechanics to obtain the relation between the trajectories in the phase space or in the extended phase space.

The second conclusion is the possibility of the application to this formalism of the known noninteraction theorem[3,4] due to the $6N$ formulation of the model. The known results from other relativistic formulations (predictive relativistic mechanics and constrained systems) seem to make us conclude that the noncanonical property of the positions permits us to circumvent the NIT, but we have shown that the nonsimultaneity is the essential property to give up the noninteraction theorem in the case of constrained Hamiltonian models. The example in Sec. IV shows that the canonical behavior for the physical positions is not enough to forbid interaction.

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