Bubble fluctuations in $\Omega<1$ inflation

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In the context of the open inflationary universe, we calculate the amplitude of quantum fluctuations which deform the bubble shape. These give rise to scalar field fluctuations in the open Friedmann-Robertson-Walker universe which is contained inside the bubble. One can transform to a new gauge in which matter looks perfectly smooth, and then the perturbations behave as tensor modes (gravitational waves of very long wavelength). For $(1-\Omega)\ll 1$, where Ω is the density parameter, the microwave temperature anisotropies produced by these modes are of order $\delta T/T\sim H(R_0\mu l)^{-1/2}(1-\Omega)^{l/2}$. Here, H is the expansion rate during inflation, R_0 is the intrinsic radius of the bubble at the time of nucleation, μ is the bubble wall tension, and l labels the different multipoles $(l>1)$. The gravitational back reaction of the bubble has been ignored. In this approximation, $G\mu R_0\ll 1$, and the new effect can be much larger than the one due to ordinary gravitational waves generated during inflation (unless, of course, Ω gets too close to 1, in which case the new effect disappears). [S0556-2821(96)00918-6]

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I. INTRODUCTION

The possibility of an open inflationary universe, in which the cosmological density parameter Ω is less than 1, has been intensively studied in recent years [1–7]. According to this model, the universe is initially in a de Sitter phase, driven by the potential energy of a scalar field trapped in a false vacuum σ_f (see Fig. 1). A bubble of the new vacuum σ_t nucleates and its interior undergoes a second period of inflation. The homogeneity of our universe is then attributed to the O(3,1) symmetry of the bubble: the interior of the light cone from the nucleation event is isometric to an open Friedmann-Robertson-Walker (FRW) universe [8]. The second period of inflation has to be sufficiently long to generate the observed entropy, but if it is too long then Ω is driven exponentially close to 1. As a result, to obtain $\Omega<1$ the parameters of the scalar field potential have to be fine tunned to some extent [1,3]. Nevertheless, if observations ultimately determine that Ω is smaller than one, then open inflation may be regarded as a ''natural'' scenario [2].

The cosmic microwave background anisotropies produced by a nearly massless scalar field in open inflation were analyzed in [3]. It was shown that a ''supercurvature'' mode [4], which is not normalizable on the open FRW spacelike sections, would give a significant contribution to the low multipoles if $\Omega<0.1$ (see also [6,9]). A more complete study of cosmological perturbations in open inflation requires the quantization of fields in the presence of a bubble. Quantum field theory in a bubble background was pioneered in [10] and further developped in [5–7]. As noted in [2], large contributions to microwave perturbations may result from the quantum fluctuations of the bubble wall itself [11]. The purpose of this paper is to calculate the amplitude of such fluctuations and their effect on the microwave background.

In Sec. II we briefly describe the bubble geometry. In Sec. III we calculate the amplitude of wall fluctuations for bubbles nucleated during inflation. This extends previous work for bubbles in flat space [11] (see also [7]). In Sec. IV we show that the effect of wall perturbations can be described in terms of long wavelength tensor modes (analogous to gravitational waves), and we evaluate their impact on the microwave sky. Finally, in Sec. V we summarize our conclusions and compare them with recent related work [2,12].

II. BUBBLE GEOMETRY

Before calculating the amplitude of bubble wall perturbations, it will be useful to summarize some of the features of the spacetime containing the bubble. A conformal diagram is given in Fig. 2. The nucleation event is marked as N. The bubble wall is represented by the timelike hypersurface w (solid line). In region I, which is the interior of the light cone from N, the line element is given by [8]

$$ds^2=-dt^2+a^2(t)d\Omega_{H^3}, \tag{1}$$

where $d\Omega_{H^3}$ is the metric on the unit spacelike hyperboloid

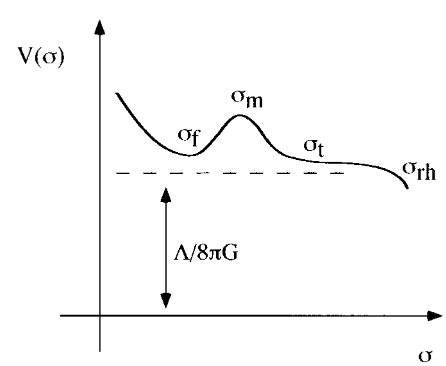

FIG. 1. A scalar field potential $V(\sigma)$ which leads to open inflation. The universe is initially in the false vacuum phase σ_f when a bubble of σ_t nucleates. Then the scalar field slowly rolls down the hill until it reaches $\sigma_{\rm rh}$, at which point the universe reheats. We separate the potential into a large part $\Lambda/8\pi G$ and a small part with the barrier feature, since we want to neglect the self-gravity of the bubble. The top of the barrier is denoted by σ_m.

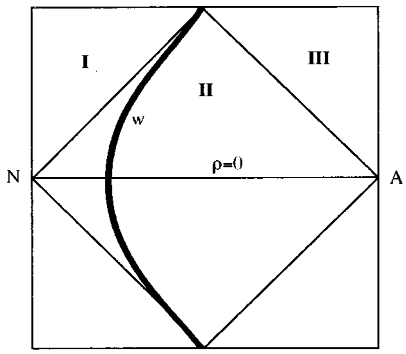

FIG. 2. A conformal diagram of spacetime in the presence of a bubble. The nucleation event is marked as N. Region I corresponds to an open FRW which inflates and eventually becomes our observable universe. The trajectory of the domain wall is marked as w. It lies in region II, which is covered by the chart (4). The spacelike hypersurface $\rho=0$, connecting N with the antipodal point A, is a good Cauchy surface for the entire spacetime. Clearly, no such surfaces exist in region I. Region III, the interior of the light cone from A, is uninteresting for our purposes.

$$d\Omega_{H^3}=dr^2+\sinh^2 r(d\theta^2+\sin^2\theta d\varphi^2). \tag{2}$$

Equation (1) represents the geometry of an open FRW universe. This open universe inflates up to the time when the scalar field reaches the value $\sigma=\sigma_{\rm rh}$ (see Fig. 1), and then reheats. After the usual radiation and matter dominated eras, it eventually becomes our observable universe. At all stages of expansion, the scale factor obeys the Friedmann equation

$$(1-\Omega)\dot{a}^2=1, \tag{3}$$

where Ω is the ratio of the matter density ρ_m to the critical density $\rho_c=(3/8\pi G)(\dot{a}/a)^2$.

The chart (1) covers only the interior of the light cone from N. One can cover the outside by analytically continuing the coordinates t and r to the complex plane. By taking $t=i\tau$ and $r=\rho+i(\pi/2)$, where τ and ρ are real, we have [8]

$$ds^2=+d\tau^2+R^2(\tau)d\Omega_{dS}. \tag{4}$$

Here $R(\tau)=-ia(i\tau)$, and

$$d\Omega_{dS}=-d\rho^2+\cosh^2\rho(d\theta^2+\sin^2\theta d\varphi^2) \tag{5}$$

is the metric of a $(2+1)$-dimensional de Sitter space of unit ''Hubble length.'' Note that τ is now a spacelike coordinate, playing the role of a proper radial distance from N. In this way, the spacetime outside the light cone from N is foliated into $(2+1)$-dimensional de Sitter leaves with constant τ. In each one of these leaves ρ plays the role of time.

The scalar field σ obeys the field equation

$$-\Box\sigma+\frac{dV(\sigma)}{d\sigma}=0, \tag{6}$$

where $\Box$ is the covariant d'Alembertian and $V(\sigma)$ is a potential of the form depicted in Fig. 1. The bubble configuration is a solution of Eq. (6) of the form

$$\sigma=\sigma_0(\tau).$$

It has the shape of a kink that interpolates between the false vacuum σ_f and the true one σ_t [8]. The locus where the field is at the top of the barrier, $\sigma(\tau_w)=\sigma_m$, can be identified with the trajectory of the domain wall (solid timelike line in Fig. 2). There, we have actually drawn an eternal bubble. A real bubble would nucleate at a given moment of time, say $\rho=0$, with intrinsic radius

$$R_0=R(\tau_w). \tag{7}$$

The intrinsic radius would subsequently expand with ρ as $R_0\cosh\rho$. The scalar field in region I can be found by analytically continuing from τ back to t [8].

Both the bubble configuration and the corresponding spacetime metric enjoy an O(3,1) symmetry inherited from the spherical symmetry of the instanton describing the tunneling [8]. This is the group of isometries of the de Sitter leaves in region II and of the open hyperboloids in region I. For the scalar field, the symmetry simply means that σ_0 only depends on τ (or t).

III. SCALAR FIELD FLUCTUATIONS

In this section we calculate the amplitude of small fluctuations for a bubble that nucleates during inflation. This extends previous results for bubbles in flat space [11,10,7]. We shall work in the approximation in which the gravitational back reaction of the bubble can be ignored. In practice, this means that up to the time of reheating, the energy density of matter can be expressed as a large cosmological constant part $\Lambda/8\pi G$, plus a small part $\delta\rho_m$ which contains the barrier feature of the potential energy (see Fig. 1) plus the gradient and kinetic contributions of the scalar field. Then we take the limit

$$8\pi G\,\delta\rho_m\ll\Lambda. \tag{8}$$

The case when gravitational back reaction is included will be discussed elsewhere.

In our approximation, the geometry during inflation is that of de Sitter space. In region II the line element is given by Eq. (4), with

$$R(\tau)=H^{-1}\sin(H\tau)\quad(0<\tau<\pi H^{-1}), \tag{9}$$

where $H=(\Lambda/3)^{1/2}$ is the de Sitter Hubble rate. Note that R vanishes both at $\tau=0$ (the nucleation event N) and at $\tau=\pi H^{-1}$ (the antipodal point A). To study the wall fluctuations, we expand the scalar field as

$$\sigma(\tau,x^i)=\sigma_0(\tau)+\phi(\tau,x^i), \tag{10}$$

where x^i are the coordinates on the $(2+1)$-dimensional de Sitter leaves (5). The small perturbation ϕ is promoted to a quantum operator $\hat{\phi}$, which is then expanded into a sum over modes times the usual creation and anihilation operators as $\hat{\phi}=\sum\phi_{klm}a_{klm}+$ H.c. As noted in [6], the spacelike surface $\rho=0$, connecting the nucleation event N with its antipodal A, is a good Cauchy surface for the entire spacetime (see Fig. 2). Therefore we shall normalize our modes on that hypersurface. Some of them, the so-called supercurvature modes

[3,4], may not be normalizable on the open hyperboloids of the Friedmann-Robertson-Walker chart (2), but this is just because the hyperboloids are not good Cauchy surfaces. As we shall see, the perturbations of the bubble wall are supercurvature.

The equation of motion for small perturbations is

$$[-\Box+m^2(\sigma_0)]\phi_{klm}=0, \tag{11}$$

where $m^2(\sigma_0)=d^2V/d\sigma^2|_{\sigma=\sigma_0(\tau)}$. With the ansatz

$$\phi_{klm}=R^{-1}(\tau)F_k(\tau)Y_{klm}(x^i), \tag{12}$$

this separates into

$$[-{}^{(3)}\Box+k^2]Y_{klm}(x^i), \tag{13}$$

and

$$-\frac{d^2F_k}{d\eta^2}+R^2[m^2(\sigma_0)-2H^2]F_k=(k^2-1)F_k. \tag{14}$$

Here ${}^{(3)}\Box$ stands for the covariant d'Alembertian on the (2+1)-dimensional de Sitter leaves (5), k^2 is a separation constant, and the conformal ''radial'' coordinate η is defined through the relation $R(\tau)d\eta\equiv d\tau$,

$$\cosh\eta\equiv\frac{1}{\sin(H\tau)}=\frac{1}{HR(\tau)}. \tag{15}$$

Equations (13) and (14) have a familiar interpretation [10,11]. The first one tells us that Y_k behave as scalar fields of mass k^2 living in a (2+1)-dimensional unit de Sitter space. The masses k^2 are determined as the eigenvalues of Eq. (14), which is simply a one-dimensional Schrödinger equation with effective potential

$$U_{\mathrm{eff}}=R^2[m^2(\sigma_0)-2H^2]. \tag{16}$$

Note also that the modes ϕ_{klm} must obey the Klein-Gordon normalization condition

$$-i\int\phi_{klm}\overleftrightarrow{\partial}_\mu\phi^*_{k'l'm'}d\Sigma^\mu=\delta_{kk'}\delta_{ll'}\delta_{mm'}, \tag{17}$$

where Σ is the hypersurface $\rho=0$. If we choose the Y_{klm} to be Klein-Gordon normalized on the $(2+1)$-dimensional de Sitter leaves, then Eq. (17) reduces to

$$\int_{-\infty}^{+\infty}F_kF_{k'}d\eta=\delta_{kk'}, \tag{18}$$

which is the usual normalization condition for eigenfunctions of the Schrödinger equation.

The effective potential (16) is schematically represented in Fig. 3. The height of U_{eff} at $\eta=0$ [which corresponds to $R(\tau)=H^{-1}$] is basically given by $(m_f^2H^2-2)$, where m_f is the scalar field mass in the false vacuum. The narrow well on the left corresponds to the location of the bubble wall, where $m^2(\sigma)$ is negative and large in absolute value. The equation of motion (6) for σ_0 written in terms of the conformal coordinate η is

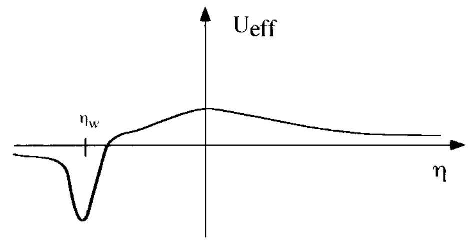

FIG. 3. The effective potential U_{eff} as a function of conformal radius η. The narrow well at η_w corresponds to the location of the bubble wall, where the effective mass $m^2(\sigma)$ is negative.

$$\sigma_0''+2\frac{R'}{R}\sigma_0'-\frac{dV(\sigma_0)}{d\sigma}R^2=0,$$

where primes denote derivatives with respect to η. Taking one more derivative with respect to η it is straightforward to show that

$$F_{-3}\equiv N\sigma_0'(\eta) \tag{19}$$

is a solution of Eq. (14) with eigenvalue $k^2=-3$. This is analogous to what happens for bubbles in flat space [11]. Note that $\sigma_0'=R\dot{\sigma}_0$, where a dot indicates derivative with respect to the ''radial'' variable τ. But in order for the instanton to be smooth, we must have $\dot{\sigma}_0\to0$ both at the nucleation event $(\eta\to-\infty)$ and at the antipodal point$(\eta\to+\infty)$ [8]. From Eq. (15), $R(\eta)$ also vanishes exponentially at $\eta\to\pm\infty$. Therefore it is clear that the mode (19) is normalizable and that its eigenvalue $k^2=-3$ belongs to the spectrum. In addition, since σ_0 is a monotonous function interpolating between true and false vacuum, the mode (19) has no nodes and is the eigenstate of lowest eigenvalue. Although all higher modes will contribute to density perturbations and microwave temperature distorsions [6], for the remainder of this paper we shall focus on the lowest mode. This mode has a clear geometrical interpretation as deformations of the bubble shape, which is the effect we are concentrating on. To linear order we can write the perturbed field as

$$\sigma_0(\tau)+\phi_{-3}(\tau,x^i)\approx\sigma_0(\tau+NY_{-3lm}(x^i)).$$

Therefore, like in the case of flat space [11], the perturbations associated with $k^2=-3$ correspond to deformations that shift the position of the bubble wall in a x^i dependent way, without altering the ''radial'' profile function $\sigma_0(\tau)$.

The normalization constant in Eq. (19) will eventually determine the magnitude of the effect. In order to calculate it we use Eq. (18), in the form

$$N^2\int R(\tau)\dot{\sigma}_0^2d\tau=1. \tag{20}$$

The integral can be numerically evaluated for any particular type of bubble, but its meaning is best illustrated in the thin wall case. In this case $\dot{\sigma}_0$ is very small except in a small region of size comparable to the width of the narrow well in Fig. 3, which is centered around the location of the wall, at

$\tau=\tau_w$. The factor $R(\tau_w)$ can be pulled out of the integral and what remains is simply the wall tension μ. Therefore,

$$N^2=\frac{1}{R_0\mu}.$$

Here, as in Eq. (7), $R_0=R(\tau_w)$ is the radius of the bubble at the moment of nucleation. In the general case, the denominator in the right-hand side is just shorthand for the integral in Eq. (20).

In the thin wall case, the explicit expression for the radius of the bubble at nucleation is given by (see, e.g., [15])

$$R_0=\frac{3\mu}{(9\mu^2H^2+\epsilon^2)^{1/2}}, \tag{21}$$

where ϵ is the jump in energy density between the true and the false vacuum. As mentioned before, we have neglected the bubble's gravitational back reaction. For the approximation to be valid, we need on one hand that $G\epsilon\ll\Lambda$ [see Eq. (8)]. On the other hand, we are neglecting the gravity of the wall. As is well known, the gravitational field of a domain wall is characterized by a Rindler-type horizon distance [16]

$$l_w\equiv\frac{1}{8\pi G\mu}. \tag{22}$$

We need this distance to be much larger than the radius of the bubble at nucleation

$$G\mu R_0\ll 1. \tag{23}$$

So far we have found the modes describing quantum fluctuations outside the light cone from N:

$$\phi_{-3lm}=\frac{\dot\sigma_0(\tau)}{(R_0\mu)^{1/2}}Y_{-3lm}(x^i). \tag{24}$$

The modes Y_{-3lm} are those of a scalar field of tachyonic $(\text{mass})^2=k^2=-3$ [11] living in a (2+1)-dimensional de Sitter space (5). If we want to preserve the O(3,1) symmetry of the bubble solution then we need to choose the Bunch-Davies vacuum in the lower-dimensional timelike $\tau=$const sections. The corresponding normalized modes are [11]

$$Y_{-3lm}=-\left(\frac{\pi\Gamma(l-1)}{4\Gamma(l+3)}\right)^{1/2}\frac{1}{\cosh\rho}R_l^2(\tanh\rho)Y_{lm}(\theta,\varphi), \tag{25}$$

where Y_{lm} are the spherical harmonics and $R_\nu^\lambda(x)\equiv P_\nu^\lambda(x)-(2i/\pi)Q_\nu^\lambda(x)$. Here P and Q are the Legendre functions on the cut $-1<x<1$.

In order to assess the effect of these fluctuations in our Universe today, we must first analytically continue to the interior of the light cone,

$$\phi_{-3lm}=\frac{\dot\sigma(t)}{(R_0\mu)^{1/2}}Y_{-3lm}(r,\theta,\varphi), \tag{26}$$

where now the analytically continued harmonics can be cast, after some algebra [13], into the form

$$Y_{-3lm}=\left(\frac{\Gamma(3+l)\Gamma(l-1)}{2}\right)^{1/2}\frac{P_{3/2}^{-l-1/2}(\cosh r)}{\sqrt{\sinh r}}Y_{lm}(\theta,\varphi). \tag{27}$$

Here we have used Eqs. (8.738.2) and (8.732.5) of Ref. [13]. The Legendre functions can be given in terms of elementary functions. For $l=0,1$, and 2 they are

$$P_{3/2}^{-1/2}=(2\pi\sinh r)^{-1/2}\sinh(2r), \tag{28}$$

$$P_{3/2}^{-3/2}=\frac{1}{\Gamma(5/2)}\left(\frac{\sinh r}{2}\right)^{3/2},$$

$$P_{3/2}^{-5/2}=\left(\frac{2}{\pi}\right)^{1/2}\frac{1}{8(\sinh r)^{5/2}}\left[\tfrac{1}{12}\sinh(4r)-\tfrac{2}{3}\sinh(2r)+r\right],$$

and for higher l they can be obtained through the well known recurrence relations.

Several comments should be made. First of all, for $l=0$ and $l=1$ the normalization factor in Eq. (27) diverges. This is not a problem, since these modes do not contribute to observables. They simply correspond to spacetime translations of the nucleation event [11]. Second, the modes are real on the open chart, and hence their Klein-Gordon norm vanishes there. This is not a problem either, because the open hyperboloids are not good Cauchy surfaces [6]. Finally, the analytic continuation of Eq. (13) with $k^2=-3$ tells us that

$$\Delta Y_{-3lm}=+3Y_{-3lm},$$

and so the eigenvalue of the Laplacian has the ''wrong'' sign. Not surprisingly, the modes diverge exponentially for large r. As we shall see, this divergence does not appear in the physical effect.

IV. FROM SCALAR TO TENSOR MODES

The wall fluctuations induce scalar field fluctuations of the form (26) in the open FRW universe. These will locally advance or retard by an amount

$$\delta t=\frac{\phi_{-3lm}}{\dot\sigma}=(R_0\mu)^{-1/2}Y_{-3lm}(r,\theta,\varphi) \tag{29}$$

the time at which the universe reheats. Deformations of the reheating surface generically induce density fluctuations and perturbations in the microwave background. It turns out that the particular modes (26) do not cause density perturbations [12], but, as we shall see, they do affect the microwave background just like gravitational waves do.

A nice framework to study deformations of the constant scalar field surfaces is that of Ref. [14]. One defines a ''fluid'' velocity

$$u_\mu\equiv\frac{\sigma_{,\mu}}{(-\sigma_{,\mu}\sigma^{,\mu})^{1/2}} \tag{30}$$

orthogonal to the constant field surfaces, and projects its covariant derivative $u_{\mu;\nu}$ onto these surfaces:

$$u_{\mu|\nu}\equiv(\delta_\mu^\rho+u_\mu u^\rho)u_{\rho;\nu}.$$

One can separate $u_{\mu|\nu}$ into a symmetric and an antisymmetric part. The antisymmetric part is called vorticity and it can be shown that it vanishes for a four vector of the form (30). As a consequence, the projected covariant derivative $u_{\mu|\nu}$ coincides with the intrinsic covariant derivative on the surfaces $\sigma=\text{const}$ [14]. The symmetric tensor $K_{\mu\nu}=u_{\mu|\nu}$ is also known as the extrinsic curvature. Its trace $K^{\mu}_{\mu}\equiv\Theta$ is the expansion, and in the unperturbed FRW $\Theta=3(\dot{a}/a)$. A straightforward calculation shows that under perturbations of the form (26) the expansion does not change [11]. The traceless part of $K_{\mu\nu}$ is the shear tensor $\sigma_{\mu\nu}=K_{\mu\nu}-(\Theta/3)(g_{\mu\nu}+u_{\mu}u_{\nu})$. To linear order in perturbations and in the coordinate system (1), we have

$$u_{\mu}=(-1,Y_{,i}), \tag{31}$$

where Y stands for $Y_{klm}(x^i)$, with $x^i=(r,\theta,\varphi)$. Then $\sigma_{00}=\sigma_{0i}=0$ and

$$\sigma_{ij}=Y_{|ij}-Y\gamma_{ij}. \tag{32}$$

Here γ_{ij} is the metric on the unit spacelike hyerboloid (2). In addition to being traceless, the shear tensor for $k^2=-3$ is transverse $\sigma_{ij|}{}^{j}=0$, just like a tensor mode [7].

This immediately suggests going to a new coordinate system which straightens out the constant scalar field surfaces, while still remaining in a synchronous gauge

$$t'=t+\delta t, \quad x^{i\,\prime}=x^i-\gamma^{ij}\frac{\dot{a}}{a}\,\delta t_{|j}.$$

Here δt is given by Eq. (29). In this new gauge $\sigma=\sigma(t')$ is constant on $t'=\text{const}$ surfaces, but the metric reads

$$ds^2=-dt^2+a^2(\gamma_{ij}+h_{ij})dx^i dx^j.$$

Here

$$h_{ij}=-2E\sigma_{ij}, \tag{33}$$

and, during inflation,

$$E=\frac{\dot{a}}{a}(R_0\mu)^{-1/2}. \tag{34}$$

Once in the new transverse and traceless gauge, the matter distribution looks perfectly smooth. It is now legitimate to evolve h_{ij} with the usual equation for tensor perturbations through the entire cosmic evolution [17]

$$\ddot{h}_{ij}+3\frac{\dot{a}}{a}\dot{h}_{ij}-\frac{1}{a^2}(\Delta h_{ij}+2h_{ij})=0. \tag{35}$$

Note that although $\Delta Y=+3Y$, the corresponding tensor mode, derived from Eqs. (33) and (32) satisfies

$$\Delta h_{ij}=-3h_{ij},$$

with the ''correct'' sign for the Laplacian eigenvalue. For $l=0$ and $l=1$, it can be readily checked that $\sigma_{ij}=0$, and so the tensor mode only exists for $l>1$, as expected of gravitational waves [18].

Introducing Eq. (33) in Eq. (35) we can calculate the evolution of the amplitude E throughout the different stages of expansion. In terms of conformal time $(d\eta=a^{-1}dt)$, we have

$$E''+2\frac{a'}{a}E'+E=0.$$

During inflation E is given by Eq. (34) and it tends to a constant:

$$E=\frac{H}{\sqrt{R_0\mu}},$$

where $H=(\Lambda/3)^{1/2}$. In the radiation era $\Omega=1$ to very high accuracy, and it can be checked that as a result E stays constant.

During the matter era $a\sim t^{2/3}\sim\eta^2$, so

$$E''+\frac{4}{\eta}E'+E=0.$$

This can be solved in terms of Bessel functions. For small η

$$E=\frac{H}{\sqrt{R_0\mu}}\left(1-\frac{\eta^2}{8}+\cdots\right).$$

From Eq. (3) we have $\eta=2(1-\Omega)^{1/2}$, and in what follows we shall concentrate in the case $(1-\Omega)\ll 1$. (The general case can be treated numerically along the lines of Refs. [4,3].)

The amplitude of microwave fluctuations due to h_{ij} is given by the well known Sachs-Wolfe formula

$$\frac{\delta T}{T}=\frac{1}{2}\int_0^{r_{\rm ls}}\frac{dh_{rr}}{d\eta}dr,$$

where $r_{\rm ls}\approx 2(1-\Omega)^{1/2}$ is the comoving distance to the surface of last scattering and h'_{rr} is evaluated at $\eta=r_{\rm ls}-r$. Since $r_{\rm ls}$ is assumed to be small, we can use the asymptotic form of the Legendre functions in Eqs. (27) to obtain

$$Y_{-3lm}\approx\Gamma_l 2^{-(l+1)}r^l Y_{lm},$$

where $\Gamma_l=[\Gamma(3+l)\Gamma(l-1)]^{1/2}/\Gamma(l+3/2)$ Using this form in Eqs. (32) and (33) one immediately obtains

$$\frac{\delta T}{T}\approx\frac{H}{\sqrt{R_0\mu}}\frac{\Gamma_l}{8}(1-\Omega)^{l/2}, \tag{36}$$

which for low spatial curvature is dominated by the quadrupole $l=2$.

It should be noted that the tensor mode h_{ij} is pure gauge during inflation (as it should since we are neglecting the bubble's back reaction.) However, the overall configuration taking the scalar field into account is not. The $\sigma=\text{const}$ surfaces have nonvanishing shear. As we evolve h_{ij} past the reheating surface, it gradually ceases to be pure gauge, as can be checked by expressing the evolved mode in the longitudinal gauge [17]. Also, it can be checked that although the scalar modes Y_{-3lm} diverge exponentially at large distances, the corresponding tensor components h_{rr} entering the Sachs-Wolfe formula do not.

V. CONCLUSIONS

We have calculated the amplitude of small fluctuations in the shape of bubbles nucleated during inflation. In the case of thin bubbles, the result takes the simple form (26). Since the de Sitter modes Y_{-3lm} grow like the intrinsic radius of the bubble $r=R_0\cosh\rho$ (ρR_0 is the proper time coordinate on the bubble wall), the relative amplitude of proper local radial wall displacements is given by

$$\frac{\delta r}{r}\sim(R_0^3\mu)^{-1/2},$$

where R_0 is the radius of the bubble at the time of nucleation, given by Eq. (7), and μ is the wall tension. This coincides by order of magnitude with the estimate of Ref. [2] in the case when the size of the bubbles at the time of nucleation is much smaller than the de Sitter horizon H^{-1}.

The propagation of wall perturbations to the interior of the light cone gives rise to shear deformations of the reheating surface, and consequently, of the surface of last scattering. The simplest way to study the cosmological evolution of such perturbations is to use a synchronous coordinate system in which matter looks perfectly smooth. Somewhat surprisingly, the metric fluctuations in this gauge are transverse and traceless, just like usual gravitational waves. This transmutation of scalar into tensorlike modes is only possible because of the peculiar (supercurvature) eigenvalue of the Laplacian for the scalar modes corresponding to wall fluctuations, $k^2=-3$ [7].

For $(\Omega-1)\ll 1$, the anisotropies of the microwave sky produced by these waves are given in Eq. (36). The dominant effect is in the quadrupole, with

$$\frac{\delta T}{T}\sim\frac{H}{\sqrt{R_0\mu}}(1-\Omega).$$

Within the limits of validity of our approximation [see Eq. (23)], and unless Ω is too close to 1, this can be much larger than the distortion produced by usual gravitational waves produced during inflation, which is of order $G^{1/2}H$ (however, see [18,19]). The case with strongly gravitating domain walls may yield a different result [12], and is currently under investigation.

Finally, we would like to compare our results with those of Ref. [12]. There the amplitude of a homogeneous fluctuation is estimated by considering the small change ΔS in the instanton action when the radius of the bubble is changed by an amount δa. This is used to estimate the ''typical deviation'' δa as the one that corresponds to $\Delta S\sim 1$. However, the physical meaning of that prescription is unclear, because the radius of the bubble cannot have a homogeneous fluctuation. In flat space, this would violate energy conservation, and a similar argument can be applied in curved space. The only homogeneous radial fluctuations allowed in the thin wall limit are time translations of the nucleation point.

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