Correlations between black holes formed in cosmic string breaking

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(Received 5 July 1995)

An analysis of cosmic string breaking with the formation of black holes attached to the ends reveals a remarkable feature: the black holes can be correlated or uncorrelated. We find that, as a consequence, the number-of-states enhancement factor in the action governing the formation of uncorrelated black holes is twice the one for a correlated pair. We argue that when an uncorrelated pair forms at the ends of the string, the physics involved is more analogous to thermal nucleation than to particle-antiparticle creation. Also, we analyze the process of intercommuting strings induced by black hole annihilation and merging. Finally, we discuss the consequences for grand unified strings. The process whereby uncorrelated black holes are formed yields a rate which significantly improves over those previously considered, but still not enough to modify string cosmology.

PACS number(s): 04.70.−s, 11.27.+d, 98.80.Cq

I. INTRODUCTION

Recent investigations on pair production of black holes have yielded new evidence for the conjecture that black holes have a number of internal states given by the exponential of one-quarter of the area of the event horizon. Processes where black holes are spontaneously created in pairs show an enhancement of the probability relative to formation of, e.g., monopoles, although we are still far from having a fully satisfactory microscopic explanation of the underlying degrees of freedom.

Black hole pair creation has been investigated in several scenarios: Schwinger-like production in Maxwell fields [1–4], inflation-driven creation in de Sitter space [5] and, very recently, the breaking of a cosmic string with black holes forming at its ends [6–8]. All these are quantum tunneling processes mediated by gravitational instantons. In fact, since these seem to provide a consistent picture of physically reasonable processes we may take it as an indication that the Euclidean approach to semiclassical quantum gravity is a meaningful one and that topology change should be taken into account.

The instantons describing black hole pair production have been mainly based on different variations of the Euclidean section of the C metric [9]. The Lorentzian section of this space describes two black holes uniformly accelerating away from each other. This hyperbolic trajectory turns into a circular orbit when continuing to Euclidean time. One can then proceed in analogy with the calculation of the Schwinger effect in Ref. [10] and construct an instanton for computing the rate for pair production in semiclassical approximation. A nice feature of the general relativistic description of the process is that the C-metric signals by itself the need of a force to accelerate the black holes, in the form of conical singularities along the axis joining the black holes. The different

pair production processes mentioned above correspond to different ways of dealing with these singularities.

In this paper our interest will be focused on situations where the force is provided by a cosmic string. The process by which a string breaks by forming a pair of black holes attached to its ends has also attracted recent attention because of its possible relevance to cosmic string cosmology. Although vortex solutions are not easily handled analytically, recent investigations on Nielsen-Olesen vortices piercing a black hole [11] or ending at it [8] have shown that one can consistently model a thin gravitating physical vortex by a conical deficit in the spacetime geometry. Moreover, it has been argued that it is possible for a topologically stable string to end at a black hole [11,7]. Then the C metric can be interpreted as providing a picture of a pair of black holes formed by the breaking of a cosmic string and which are subsequently being pulled apart by the string tension.

One could also consider a seemingly similar process in which the cosmic string breaks in such a way that the string tension is exactly balanced by the gravitational attraction of the black holes. This black-hole–cosmic-string configuration has been considered in Ref. [12] and very recently in Ref. [8]. However, as we will see below, the similarities between the two processes are rather superficial and, in fact, differences between them will be more significant. We will find a main difference in that the created black holes are correlated in one case and uncorrelated in the other. Also, the mechanisms involved in string breaking are utterly different in each process. We will also discuss how black hole annihilation or merging provides a mechanism for intercommuting strings and, finally, we will analyze the relevance of the different breaking processes in the context of grand unified strings. The idealization will always be made that a thin cosmic string is well approximated by a conical singularity.

II. BREAKING THE STRING

Let us first consider summarily the breaking of a cosmic string as described by the C metric. The leading

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contribution to the instanton action governing the semi-classical rate for string breaking, $\exp(-I)$, was computed in Ref. [6] to be

$$I \simeq \frac{\pi m^2}{\mu}, \qquad (1)$$

where m is the black hole mass and μ the string tension. This result is the same as the one for breaking the string forming monopoles at its ends [14]. To find out whether substitution of monopoles with black holes results in an enhanced rate, we will expand to next-to-leading order the exact result in Ref. [6]. For gravitationally corrected monopoles the action is the same as for creation of extremal black holes, since the topology of the instantons is the same in both cases. We find

$$I_{\mathrm{mon}} = \frac{\pi m^2}{\mu} - \pi m^2 + \cdots. \qquad (2)$$

For nearly extremal black holes (the wormhole configuration) the result is

$$I_{\mathrm{nonext}} = \frac{\pi m^2}{\mu} - 2\pi m^2 + \cdots. \qquad (3)$$

The difference between both is, as expected [2,3], the factor $\pi m^2 = A_{\mathrm{BH}}/4$. Therefore, breaking by nonextremal black holes is enhanced by a factor of $(\exp S_{\mathrm{BH}})$.

Let us now consider the process of static breaking of a cosmic string.[1] The metric describing it belongs to a family of axisymmetric multi-Schwarzschild configurations which are conveniently written in Weyl's canonical coordinates [13]

$$ds^2 = -e^{2\psi}dt^2 + e^{-2\psi}[e^{2\gamma}(d\rho^2 + dz^2) + \rho^2\,d\phi^2], \qquad (4)$$

with

$$\psi = \frac{1}{2}\sum_{i=1}^{N} \ln \frac{r_i^{(+)} + r_i^{(-)} - 2M_i}{r_i^{(+)} + r_i^{(-)} + 2M_i}, \qquad (5)$$

and

$$\gamma = \frac{1}{4}\sum_{i,j=1}^{N} \ln \frac{[r_i^{(+)}r_j^{(-)} + l_i^{(+)}l_j^{(-)} + \rho^2]\,[r_i^{(-)}r_j^{(+)} + l_i^{(-)}l_j^{(+)} + \rho^2]}{[r_i^{(-)}r_j^{(-)} + l_i^{(-)}l_j^{(-)} + \rho^2]\,[r_i^{(+)}r_j^{(+)} + l_i^{(+)}l_j^{(+)} + \rho^2]} + \gamma_0. \qquad (6)$$

Here we have defined

$$r_i^{(\pm)2} = l_i^{(\pm)2} + \rho^2,$$
$$l_i^{(\pm)} = z - z_i \pm M_i. \qquad (7)$$

This metric describes N black holes of masses M_i, with event horizons at $\rho = 0$, $|z - z_i| \leq M_i$. In general, the metric possesses conical singularities at the intervals between black holes, $z_i + M_i < z < z_{i+1} - M_{i+1}$, $\rho = 0$. If ϕ has periodicity 2π, then for a small circumference centered at these points

$$\frac{\mathrm{circumference}}{\mathrm{radius}}\bigg|_{\mathrm{rad}\to 0} = 2\pi e^{-\gamma}|_{\rho=0} = 2\pi \frac{(z_{i+1} - z_i)^2 - (M_i - M_{i+1})^2}{(z_{i+1} - z_i)^2 - (M_i + M_{i+1})^2} e^{-\gamma_0}, \qquad (8)$$

while at the axis outside the black holes ($z < z_1 - M_1$ or $z > z_N + M_N$) we have a conical deficit $2\pi[1 - \exp(-\gamma_0)]$. It is therefore impossible in general to adjust γ_0 to cancel the conical singularities everywhere. The physical reason for this is clear: we need a force to balance the gravitational attraction between the black holes. We can choose γ_0 so that we have a conical deficit running from the outermost black holes to infinity, and smaller (or zero) conical deficits inbetween the black holes. The physical configuration described in this way is that of a thin cosmic string split or "thinned" by a set of black holes.

The most interesting cases correspond to $N = 1, 2$, also considered in Ref. [12]. For $N = 1$ and $\gamma_0 \neq 0$, we have a black hole of gravitational mass M and internal energy [Arnowitt-Deser-Misner (ADM) mass] $Me^{-\gamma_0}$ pierced by a cosmic string of energy density $\mu = (1 - e^{-\gamma_0})/4$. For $N = 2$ and taking $M_1 = M_2 = M$, the choice

$$\gamma_0 = \ln \frac{(\Delta z)^2}{(\Delta z)^2 - 4M^2} \qquad (9)$$

($\Delta z \equiv |z_2 - z_1|$), results in the geometry of a thin cosmic string broken by a pair of static black holes separated a distance

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[1]We will refer to this process as "static breaking"; however, it should be noted that the C metric, although describing accelerating objects, is also static in a way similar to Rindler space.

$$\Delta z = M/\sqrt{\mu}\,. \tag{10}$$

This equation can also be read as the balance between the string tension and the Newtonian gravitational attraction. For the thin string limit to be consistent we must have $M > \mu^{-1/2}$ [11]. Also, for the semiclassical calculations to be reliable we will need $M \gg M_{\rm Pl}$ and small μ. One deduces from here that the black holes must be far from each other, $\Delta z > \mu^{-1}$.

We will focus on this solution, and compute its Euclidean action to obtain the probability for static string breaking. First we continue $t \to i\tau$. To avoid conical singularities at the horizons ($e^\psi = 0$) the Euclidean time must have period $\beta = 8\pi M$. This amounts to requiring the black holes to be in a thermal bath so that they are in thermal equilibrium with their surroundings (this condition also requires equality of the black hole masses). In contrast, thermal equilibrium in the situation described by the C metric is achieved by equating the black hole and acceleration temperatures and, to obtain this, one must introduce a charge to lower the black hole temperature to a value close to extremality.[2]

Following Ref. [16], we compute the Euclidean action as

$$I_E = \beta H - \frac{1}{4}A_{\rm hor}\,. \tag{11}$$

The action must be defined relative to a reference background, which we take to be thermal flat space containing a thin cosmic string of tension μ, and at the same temperature. The Hamiltonian H contains two terms: one is proportional to the Hamiltonian constraint of general relativity and vanishes for exact solutions. The other is a boundary term which is easily computed. We find

$$\beta H = 2\beta M(1 - 4\mu)\,. \tag{12}$$

The area term in Eq. (11) appears because surfaces of constant τ intersect at the horizons. It contains the difference between the area of all the horizons with respect to the reference background. In our case, two black hole horizons of the same area are present, while the reference metric contains none. For each of these horizons we find

$$A_{\rm BH} = \int_0^{2\pi} d\phi \int_{z_2-M}^{z_2+M} dz \sqrt{|g_{zz}g_{\phi\phi}|}\Big|_{\rho=0}$$
$$= 2\beta M \frac{\Delta z}{\Delta z - 2M}\,. \tag{13}$$

Then

$$I_E = \beta M \left(1 - \frac{2\sqrt{\mu}}{1 - 2\sqrt{\mu}} - 8\mu\right) \tag{14}$$

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[2]It has been argued [15] that physical vortices can dress the horizons and relax the periodicity requirements. This would allow to consider, e.g., an uncharged C metric [8]. However, for simplicity we will continue to keep the usual conditions for Euclidean time periodicity. Our conclusions will not be affected in any essential way by this.

and, for small μ, we will keep only the leading term $I_E \simeq \beta M$.

By slicing in half the Euclidean geometry and gluing it to a constant Lorentzian time section we obtain an instanton describing the quantum tunneling from the initial string to a string split by a pair of black holes. The leading semiclassical value for the rate of this process is given by $\exp(-I_E)$.

We want to compare this result to the process where the string breaks by forming particles without horizons at its ends. In this case we cannot consider extremal black holes: the modification of Eqs. (5) and (6) to include magnetic charge, i.e., the axisymmetric multi-Reissner-Nordstrom metric, describes a pair of *equally charged* objects whose repulsion tends to balance the gravitational attraction, doing so completely in the extremal case; conical singularities and therefore cosmic strings can be avoided in the latter case. It is also clear that we cannot use this form of the metric if magnetic charges are to be conserved.

On the other hand, if we wanted the string to break into monopoles, it should be topologically unstable. In any case, string stability will not be an issue for the arguments in this section, which concern the properties of black holes and not those of strings. In the next section we will discuss some aspects of the breaking of real cosmic strings, but for the moment we will consider that the string can break into two neutral massive particles without horizons, separated a distance much greater than their mass. The rate for this process will be given by Eq. (11) without the area terms, i.e., $\sim \exp(-2\beta M)$.

Therefore we find that the probability for string breaking by black holes is enhanced by a factor $\exp(2S_{\rm BH})$, where $S_{\rm BH} = 4\pi M^2$ is the entropy of a single black hole. Instead, the enhancement factor for the C metric is only $\exp(S_{\rm BH})$. This is a clear indication that string breaking by accelerating black holes and by static ones are markedly different processes. Actually, whereas the C metric produces a black-hole–anti-black-hole pair, the static breaking does not. The Euclidean section of the C metric describes the circular trajectory of a single black hole in Euclidean time. When one slices the Euclidean geometry at $\tau = \tau_0$ and at the corresponding antipodal value $\tau = \tau_0 + \beta/2$, the spatial section contains two oppositely charged black holes. In this case we expect the internal state of one of the members of the pair to be correlated with the internal state of the other, and therefore we expect to obtain just one factor $\exp(S_{\rm BH})$. Mathematically, this happens because the Euclidean C metric contains only one black hole "bolt" (the "acceleration bolt" is dealt with by taking the adequate reference metric).

In contrast, the Euclidean multi-black hole metric contains N "bolts," each of which contributes a factor $-A_{\rm BH}/4$ to the action. In this case it seems more proper to regard the formation of two black holes at the ends of the string not as the creation of a particle-antiparticle pair, but rather as a process of *thermal nucleation of black holes in a heat bath*, like the one studied in Ref. [17]. The Euclidean topology of the static two-black hole instanton is different from the one required for pair creation (i.e.,

$S^2 \times S^2 - \{pt\}$ with spatial sections $S^2 \times S^1 - \{pt\}$ for pair creation of nonextremal black holes in a noncompact space). In the instanton for the static process we have two different "bolts" corresponding to different horizons and therefore we do not expect any correlation between the internal states of the black holes. This explains the enhancement factor in this case. Each time a black hole is added to the configuration, the leading semiclassical probability is multiplied by a factor $\sim \exp(-\beta M + S_{BH})$.

This interpretation is consistent with what we have said about the charge of black holes in each process. The C-metric instanton requires the black holes to have opposite charges. The static instanton would yield, if modified to include charge, two black holes with equal charges of the same sign, clearly something not expected in particle-antiparticle creation.

Differences between the two processes not only concern the correlations between the internal states of the black holes. Actually, the action Eq. (14) is completely dominated by the thermal nucleation factors βM. In contrast, the C-metric instanton is dominated by the string action: the leading term in Eq. (1) can be obtained by computing the Nambu-Goto action of the string that is absorbed when the string breaks apart (see Ref. [4] for a similar calculation), and the number-of-states enhancement factor appears only at next-to-leading order. Therefore, in the static process the string does not break because of its tension, but rather because black holes nucleate on it. One could say that in the accelerating process the black hole pair is created by the breaking of the string, whereas in the static situation it is the formation of the black holes that causes the string to break.

What happens to a string swallowed by a black hole? Amusingly, the string does not actually break. For the C-metric process (involving nonextremal black holes) the string passes through the wormhole connecting the black holes [7]. Apparently we may run into trouble in the static process, since we have argued that there is not such a wormhole. However, no real problem is posed by this. For a single Schwarzschild solution there are two causally disconnected asymptotic regions, and the string can pass through the Einstein-Rosen bridge from one region to the other [11]. In the two-black hole configuration that we have been considering, the black holes connect to another asymptotic region, identical to the initial one (it is easily seen that both black holes must connect the same asymptotic regions: since the global geometry is static, the string tension in each asymptotic region must be balanced by the gravitational attraction of the other black hole). Then, the string can disappear and reappear in them without needing to connect the black holes. In this sense, we cannot break the string, but only "hide away" some part of it. If the string disappears down a nonextremal black hole, it must reappear through another one, either in the same asymptotic region or in a different one. In contrast, if the black hole is extremal, the string disappears down its infinite throat. Finally, when a string is created joining a pair of nonextremal black holes as in Ref. [4], we can think of it as a closed string.

Finally, we point out that the static configuration is unstable. The reason is that black holes have negative specific heat, and therefore a small fluctuation would cause them to grow or to evaporate. The balance between string tension and gravitational attraction would then be destroyed, and the black holes would either approach and merge or accelerate away. A small acceleration temperature would then appear, but it is unlikely that it may compensate for the evaporation.

III. BLACK HOLE ANNIHILATION AND INTERCOMMUTING STRINGS

We would like to address now processes of annihilation or merging of black holes created in one of the two ways presented here. Black hole annihilation has been analyzed in the context of pair creation processes in Ref. [2].

The same instanton that describes pair creation can, when reversed in time, describe pair annihilation, yielding an equal rate for the process. It is clear that we can describe in this way the annihilation of two black holes previously created as a particle-antiparticle pair. However, as it has been argued in [2], black hole annihilation between members of different pairs should also be possible, with a probability to happen of order $\exp(-S_{BH})$. This crossed annihilation of two pairs would contain just one black hole loop, instead of the two loops involved in independent creation and annihilation of the two pairs. In general, the action for these processes of creation or annihilation will contain one factor of black hole entropy per each black hole loop.

It is interesting to find that when the black holes are created at the end points of the string, annihilation between members of different pairs results in a process of intersecting and intercommuting strings, analyzed in [14]. Conservation laws require that these processes of crossed creation and annihilation take place only between oppositely charged black holes and with zero total magnetic flux. Using the instanton methods that we have been describing, one can compute the probability that two intersecting strings intercommute by this process. The previous argument implies that intersecting and intercommuting should be suppressed relative to nonintercommuting by a factor $\exp(-S_{BH})$.

Instead, we could consider a process whereby the strings intercommute without annihilating the black holes. In this case, black holes would merge into a bigger one, and we would end with a pair of strings each threading a black hole. The probabilities for intercommuting and nonintercommuting in this case would be of the same order. However, intercommuting by this process would still be very low because of the need to first break the string by quantum tunneling, and it is generally assumed that another kind of mechanism is involved which makes the probability to be of the order of unity.

If we consider now black holes thermally nucleated at the string ends, reversed instantons would correspond to thermal fluctuations in which black holes disappear, not necessarily in pairs. In this case once the strings are broken their ends do not know to which other end they were

previously joined (as long as the string tensions and black hole parameters are the same). We could still consider direct annihilation with a probability of order $\exp(-S_{BH})$ but, again, the dominant process for string intercommuting mediated by black holes would involve merging.

IV. STRINGS AND BLACK HOLES AT GUT SCALE

If a string is present in hot flat space, it is more probable for a black hole to nucleate on it than away from it. The reason is that, for a given temperature, $T = (8\pi M)^{-1}$, the energy required to nucleate a black hole piercing a string is

$$E = M_{\rm int} = M(1 - 4\mu) < M \qquad (15)$$

(this can be calculated easily using the expression for the energy in Ref. [16]; see also Refs. [12,11]). The action in this case is $I = \beta M_{\rm int}/2$, and there is an enhancement over nucleation in flat space given by $\delta I = 2\beta M\mu$.

The temperature at which grand unified strings are expected to form is $T_{\rm GUT} \sim v \sim 10^{16}$ GeV, where v is the Higgs vacuum expectation value corresponding to string formation. The tension of the strings would then be $\mu \sim v^2 \sim 10^{-6} M_{\rm Pl}$. At this temperature, we expect to find nucleation of black holes of mass $M \simeq 1/(8\pi T_{\rm GUT}) \sim 1/(8\pi\sqrt{\mu})$. However, for the thin string limit to hold we should let the temperature lower down at least one order of magnitude. We will consider $M \gtrsim \mu^{-1/2}$. The mass of these black holes would be $M \sim 10^3 M_{\rm Pl}$, still large enough for the semiclassical approximation to be reliable. In this case,

$$I \simeq \frac{1}{2}\beta M \geq O(\mu^{-1}) \sim 10^6 \ , \qquad (16)$$

which yields a negligible nucleation rate. However, the relative ratio for nucleation at a string compared to nucleation in flat space is $\sim \exp(16\pi M^2\mu) \gtrsim 1$. Thus, when strings are present, if black hole nucleation were to occur, it would mainly take place at a string.

Let us now consider the process of string breaking. In Refs. [6,7] the rate for the process mediated by the C-metric instanton has been calculated using Eq. (1); an approximate estimation yields $I > 10^{12}$, but this is only a lower limit, since one expects that the dominant process should involve breaking by black holes smaller than the string thickness. A more detailed examination in Ref. [8] shows that it may be possible to lower the value down to

$I \sim 10^7$.

Now we want to find out whether the breaking induced by thermal black hole nucleation can modify these rates. An issue to clarify previously is whether it is more probable for the string to break than to "thin" by nucleating black holes a distance larger than (10), in which case a residual string tension $\mu_{\rm in}$ would remain inbetween the black holes.

It is not difficult to find that when γ_0 is selected so as to match the required string tensions, the following relation must hold

$$\frac{M}{\Delta z} = \sqrt{\frac{\mu - \mu_{\rm in}}{1 - 4\mu_{\rm in}}} \ . \qquad (17)$$

The Hamiltonian is still given by Eq. (12), but the black hole area is modified. The final result for the action is

$$I = 2\beta M(1 - 4\mu) - \beta M \frac{1 + 2M/\Delta z}{1 - 4\mu}$$
$$= \beta M \left(1 - 2\sqrt{\mu} + \frac{\mu_{\rm in}}{\sqrt{\mu}} + \cdots \right), \qquad (18)$$

where the small μ limit has been taken in the last line. It is evident from here that the action takes its minimum value for $\mu_{\rm in} = 0$.

Then, at finite temperatures and in the presence of cosmic strings, we could expect to find black hole nucleation at the strings in such a way that the strings break. The value of the action governing this process at grand unified theory (GUT) temperatures is $I \gtrsim 10^6$, significantly lower than the naive estimation based on Eq. (1), but still negligible. It is unlikely that the arguments considered in Ref. [8] to improve Eq. (1), which involve the strong magnetic fields present near the string core, would be of use in this situation, since, as we have seen, it is essentially the thermal bath and not the string that causes black hole nucleation. Therefore, even if the mechanism we have been considering could improve over those previously studied, breaking by black holes is not likely to modify in any essential way string cosmology.

ACKNOWLEDGMENTS

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