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Cosmic strings in extra dimensions

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The Einstein equations coupled with a cloud of geometric strings for a five-dimensional Bianchi type-I cosmological model are studied. The cosmological consequences of having strings along the fifth dimension are examined. Particular solutions with dynamical compactifications of the extra dimensions and compatibility with expanding three-dimensional spaces are presented.

One of the most appealing mechanisms for producing density fluctuations that grow fast enough to produce galaxies compatible with present observational data is the motion of open relativistic strings and the oscillation of closed loops.[1,2] The relativistic strings can also be used (a) to describe the large-scale anisotropy[3] of the Universe, (b) to explain the observed two-point correlation function for clusters of galaxies[4] (Abel clusters), and (c) to describe extended objects in general relativity.[5]

The Kaluza-Klein theories present an attractive way to unify all gauge interactions with gravity.[6] One of the current problems facing these theories is to explain why the extra-dimensional space is so small, since otherwise it would be observed. Some authors explain entropy production[7] and inflation[8] as a result of the contraction of the extra space. In addition, different solutions to Kaluza-Klein theory with different symmetries, dimensions, and models of matter have been found.[9]

The purpose of this paper is to study the cosmological implications of having a "cloud of ordered" strings along an extra dimension of spacetime.

A three-dimensional analog of this model can be seen as follows. A single straight cosmic string in ordinary four-dimensional spacetime along the z axis, say, has the same metric[10] as a point particle in three-dimensional spacetime,[11] in which we simply add a dz^2 term (a similar construction holds for N parallel strings[12]). But as noted by Gott, Simon, and Alpert[13] the addition of this extra dimension amounts to the construction of a Kaluza-Klein theory which would unify electromagnetism and gravity in three-dimensional spacetime. Of course the extra dimension has to be compactified and this may be achieved[14] by forcing the spacetime to be periodic in the z direction with a period $2\pi a_z$, where a_z is a constant; this we can do because the metric is independent of the z coordinate. Thus we have a spacetime with three macroscopic dimensions and one compact dimension. The strings are now loops of length $2\pi a_z$, which would look like point particles to macroscopic observers living in a three-dimensional spacetime.

Now, in our five-dimensional model, which is just ordinary Kaluza-Klein theory aimed to unify electromagnetism and gravity, the strings lie along the fifth dimension; as no more extra dimensions are included, this should be taken as a toy model. Since no dependence is assumed on the fifth coordinate for the metric coefficients we can achieve compactification by simply forcing the fifth coordinate to be periodic as in the previous case. Thus the resulting space is a four-dimensional spacetime and an extra compactified dimension.

The model of a cloud of ordered strings, i.e., a "cloud" formed by a distribution of geometric strings lying along the same direction, has already been used in cosmology.[15] The fact that all strings lie along the fifth dimension might be produced by the same process that brought about the compactification of that dimension.

We shall analyze the five-dimensional Einstein equations coupled to a cloud of strings for a Bianchi type-I universe, i.e., for the metric

$$ds^2 = dt^2 - \sum_{i=1}^{4} a_i^2(t)(dx^i)^2 , \tag{1}$$

where $a_i(t)$ are functions of the indicated argument only. The extra compact dimension has a circumference of $2\pi a_4$, as remarked above.

The energy-momentum tensor for the cloud of pure geometric strings is given by[15,16]

$$T^{\mu\nu}=\rho(u^{\mu}u^{\nu}-\chi^{\mu}\chi^{\nu}) , \quad (2)$$

where μ and ν run from 0 to 4. ρ is the cloud of string density. χ^{μ} represents the direction of the string tensions and u^{μ} the four-velocity. u^{μ} and χ^{ν} are restricted by

$$u^{\mu}u_{\mu}=-\chi^{\mu}\chi_{\mu}=1 , \quad (3a)$$

$$\chi^{\mu}u_{\mu}=0 . \quad (3b)$$

Note that (2) differs from the usual chaotic ensemble of strings, which is equivalent to a perfect fluid with $p=-\frac{1}{3}\rho$ equation of state.[1] The energy-momentum tensor (2) represents a cloud of strings with tensions not randomized. For this reason (2) is interpreted as a cloud of ordered strings.[15] The energy-momentum tensor (2) is the electromagnetic analog of the energy-momentum tensor for a flux of ordered radiation.[17]

Let us take the strings as lying along the fifth dimension; thus,

$$u^{\mu}=(1,0,0,0,0) , \quad (4a)$$

$$\chi^{\mu}=(0,0,0,0,a_4^{-1}) . \quad (4b)$$

The Einstein equations

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R=-T_{\mu\nu} \quad (5)$$

for the metric (1) and the energy-momentum tensor (2) reduce to

$$\frac{\dot a_1}{a_1}\left[\frac{\dot a_2}{a_2}+\frac{\dot a_3}{a_3}+\frac{\dot a_4}{a_4}\right]+\frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3}+\frac{\dot a_2}{a_2}\frac{\dot a_4}{a_4}+\frac{\dot a_3}{a_3}\frac{\dot a_4}{a_4}=\rho , \quad (6a)$$

$$\frac{\ddot a_1}{a_1}+\frac{\ddot a_2}{a_2}+\frac{\ddot a_3}{a_3}+\frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2}+\frac{\dot a_1}{a_1}\frac{\dot a_3}{a_3}+\frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3}=\rho , \quad (6b)$$

$$\frac{\ddot a_2}{a_2}+\frac{\ddot a_3}{a_3}+\frac{\ddot a_4}{a_4}+\frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3}+\frac{\dot a_2}{a_2}\frac{\dot a_4}{a_4}+\frac{\dot a_3}{a_3}\frac{\dot a_4}{a_4}=0 , \quad (6c)$$

$$\frac{\ddot a_1}{a_1}+\frac{\ddot a_3}{a_3}+\frac{\ddot a_4}{a_4}+\frac{\dot a_1}{a_1}\frac{\dot a_3}{a_3}+\frac{\dot a_1}{a_1}\frac{\dot a_4}{a_4}+\frac{\dot a_3}{a_3}\frac{\dot a_4}{a_4}=0 , \quad (6d)$$

$$\frac{\ddot a_1}{a_1}+\frac{\ddot a_2}{a_2}+\frac{\ddot a_4}{a_4}+\frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2}+\frac{\dot a_1}{a_1}\frac{\dot a_4}{a_4}+\frac{\dot a_2}{a_2}\frac{\dot a_4}{a_4}=0 , \quad (6e)$$

where the overdots denote derivation with respect to t. Note that the field equations (6a) tells us that ρ is a function of t only. The Bianchi identity in this case gives

$$\dot\rho+\rho\left[\frac{\dot a_1}{a_1}+\frac{\dot a_2}{a_2}+\frac{\dot a_3}{a_3}\right]=0 . \quad (7)$$

The system of equations (6) is an autonomous system of differential equations of the quadratic type[18] in the variables $y_i=\dot a_i/a_i$: i.e.,

$$\dot y_1=y_1^2+\tfrac{1}{3}(2y_2y_3-y_1y_2-y_1y_3)+\tfrac{1}{3}y_4(2y_2+2y_3-y_1) , \quad (8a)$$

$$\dot y_2=-y_2^2+\tfrac{1}{3}(2y_1y_3-y_1y_2-y_2y_3)+\tfrac{1}{3}y_4(2y_1+2y_3-y_2) , \quad (8b)$$

$$\dot y_3=-y_3^2+\tfrac{1}{3}(2y_1y_2-y_1y_3-y_2y_3)+\tfrac{1}{3}y_4(2y_1+2y_2-y_3) , \quad (8c)$$

$$\dot y_4=-y_4^2-\tfrac{1}{3}(y_1y_2+y_2y_3+y_1y_3)-\tfrac{4}{3}y_4(y_1+y_2+y_3) , \quad (8d)$$

$$\rho=y_1y_2+y_1y_3+y_1y_4+y_2y_3+y_2y_4+y_3y_4>0 . \quad (8e)$$

From (8) we find

$$y_2-y_1-\frac{C_{21}}{\bar R^{3}a_4} , \quad (9a)$$

$$y_3-y_2-\frac{C_{32}}{\bar R^{3}a_4} , \quad (9b)$$

$$y_3-y_1=\frac{C_{31}}{\bar R^{3}a_4} , \quad (9c)$$

where C_{21}, C_{32}, and C_{31} are integration constants related by $C_{32}+C_{21}=C_{31}$ and $\bar R^{3}\equiv a_1a_2a_3$. $\bar R$ can be interpreted as the mean radius[19] of the Universe. Equations (9) tell us that when the Universe expands the anisotropy decreases and that when the fifth dimension decreases the Universe anisotropy increases.

From Eq. (7) we get

$$\rho=\frac{M_0}{\bar R^{3}} , \quad (10)$$

where M_0 is a constant. Thus, when the Universe expands the cloud of string density decreases.

From a four-dimensional viewpoint, Eqs. (6a), (6c), (6d), and (6e) can be regarded as the Einstein equations for the usual four-dimensional Bianchi type-I universe coupled to an anisotropic fluid of energy-momentum tensor:

$$T^{ab}=\rho^{*}u^{a}u^{b}+\sum_{i=1}^{3}P_iZ^{a}_{(i)}Z^{b}_{(i)} , \quad (11)$$

where a and b run from 0 to 3. $u^{a}=\delta^{a}_{0}$ and $Z^{a}_{(i)}=\delta^{a}_{i}a_i(t)$. The density and the pressure are given by

$$\rho^{*}=\rho+\frac{\dot a_4}{a_4}\frac{\dot\rho}{\rho} , \quad (12a)$$

$$p_1=\frac{\ddot a_4}{a_4}+\frac{\dot a_4}{a_4}\left[\frac{\dot a_2}{a_2}+\frac{\dot a_3}{a_3}\right] , \quad (12b)$$

$$p_2=\frac{\ddot a_4}{a_4}+\frac{\dot a_4}{a_4}\left[\frac{\dot a_1}{a_1}+\frac{\dot a_3}{a_3}\right] , \quad (12c)$$

$$p_3=\frac{\ddot a_4}{a_4}+\frac{\dot a_4}{a_4}\left[\frac{\dot a_1}{a_1}+\frac{\dot a_2}{a_2}\right] . \quad (12d)$$

Equation (6b) in this case can be interpreted as an equation of state. Note that the four-dimensional "conservation laws" $T^{ab}_{;b}=0$ do not give new constraints. The interpretation of the extra dimensions as a fluid can be found in Refs. 20 and 21.

Let us study the specialization $a \equiv a_1 = a_2 = a_3$, i.e., the isotropic three-space case.

From (8) we find

$$\dot{x} = -x^2 + xy \ , \quad (13a)$$

$$\dot{y} = -y^2 - x^2 - 4xy \ , \quad (13b)$$

$$\rho = 3x(x+y) > 0 \ , \quad (13c)$$

where $x = \dot{a}/a$ and $y = y_4 = \dot{a}_4/a_4$. The system of equations (13) can be reduced [cf. Eqs. (6)] to

$$a_4 = \dot{a} \ , \quad (14)$$

$$\dddot{a}a^2 + 4\ddot{a}\dot{a}a + \dot{a}^3 = 0 \ . \quad (15)$$

The general solution of Eq. (15) can be easily found by noticing that (15) is equivalent to

$$(\ddot{a}a^2 + \dot{a}^2 a)^{\cdot} = 0 \ . \quad (16)$$

Thus

$$\ddot{a}a^2 + \dot{a}^2 a = C_1 \ . \quad (17)$$

C_k, $k=1,2,3$ are integration constants (C_2 and C_3 are introduced for later use). Defining $z = a\dot{a}$ we have that (17) reduces to $zz' = C_1$, where the prime indicates derivation with respect to a. The case $C_1 = 0$ gives $a \sim t^{1/2}$. $C_1 \neq 0$ yields the cubic equation for a:

$$9C_1^4(t+C_3)^2 = 2C_1^3 a^3 - 3C_1^2 C_2 a^2 + C_2^3 \ . \quad (18)$$

Another simple, particular solution is obtained by setting $C_2 = C_3 = 0$ in (18); we get $a \sim t^{2/3}$. In summary we have, for the two above-mentioned particular solutions,

$$a \sim t^{1/2}, \quad a_4 \sim t^{-1/2}, \quad \rho \sim t^{-2} \ , \quad (19)$$

$$a \sim t^{2/3}, \quad a_4 \sim t^{-1/3}, \quad \rho \sim t^{-2} \ . \quad (20)$$

Now since the extra dimension has a circumference of $2\pi a_4$ we have in both cases a dynamical compactification of that dimension. Both solutions represent three-dimensional expanding universes: solution (19) expands like a flat radiative universe, whereas solution (20) expands like a flat matter-dominated universe. Note that since the circumference of the extra dimension is responsible for fixing the value of the electric charge this model leads to a value of the electric charge that changes with time. This is typical of higher-dimensional cosmological models.[14]

The system of Eqs. (8) as well as (13) have only a critical point:

$$(y_1, y_2, y_3, y_4) = (0,0,0,0)$$

and $(x,y) = (0,0)$, respectively. Using standard techniques for quadratic critical points we find for the solutions of the system (13) near the critical point the behavior shown in Fig. 1. The lines that separate the regions II

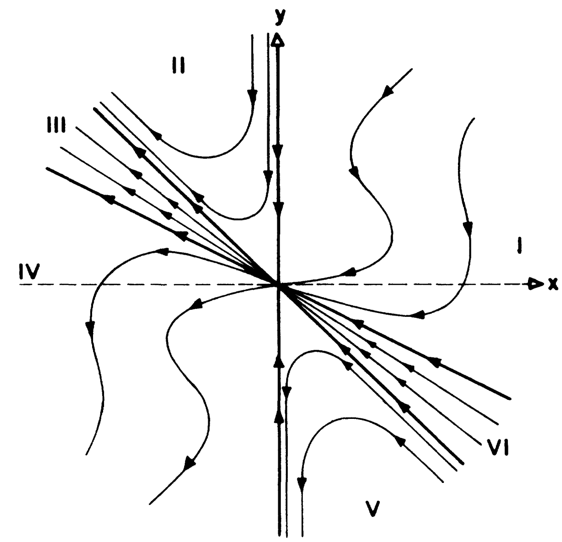

FIG. 1. The phase-plane trajectories are shown for a variety of possible initial values for x and y.

and III (V and VI) and III and IV (VI and I) are $y = -x$ and $y = -0.5x$, respectively. For the evolution in the regions I and VI we have that the critical point is stable. And for the trajectories in III and IV we have that (0,0) is unstable. The constraint $\rho > 0$ limits the evolution to the regions I, III, IV, and VI, i.e., the condition that the energy be positive allows only evolutions with either a stable or unstable critical point. The condition $y > 0$ ($y < 0$) represents expansion (contraction) in the fifth dimension. Thus for both physical regions (I and VI and III and IV) we have subregions wherein compactification of the fifth dimension occurs.

We have also studied numerically the system of Eq. (8) in the general case, for different initial values. We have found a behavior similar to the one found in the isotropic case; i.e., the trajectories in phase space have the same behavior with respect to the critical point as shown in Fig. 1.

It is interesting to note that under the assumption of three-dimensional isotropy the inclusion of strings in the fifth dimension is not only compatible with the dynamical compactification of the extra dimension but that it is also compatible with radiative and matter-dominated universes.

When the compactification of the fifth dimension occurs $\dot{a}_4/a_4$ and $\dot{\rho}/\rho$ are negative and $\rho^* - \rho$ is positive [cf. Eq. (12a)]. Thus from a four-dimensional viewpoint we have that fluctuation in the fifth-dimensional metric coefficient as well as fluctuations in the string density give rise to fluctuations in the four-dimensional density ρ^*. For the particular cases (19) and (20) we have that ρ and $\rho^* - \rho$ decay with the same time scale. This suggests that the contributions coming from the fifth dimension are of the same importance as the ones coming from the strings.

The special cases studied suggest that the inclusion of strings in the extra dimensions can be used as a mechanism to make them disappear at latter epochs of the

Universe. In fact, their presence in the extra space is compatible with the dynamical compactification of such space and the three-dimensional expansion of the visible Universe. Note, however, that such strings are not the normal cosmic strings which are extended in one of the macroscopic directions. They are ordinary cosmic strings in five-dimensional spacetime, but since they are extended along the compactified dimension they look as if they are points in the macroscopic four-dimensional model, in a way somewhat similar to the way ordinary straight strings look in a three-dimensional model. Here, however, the gravitational effects produced on four-dimensional spacetime are similar to those produced by a four-dimensional anisotropic fluid.

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