Reheating in inflationary cosmologies: Geometric coupling of the "inflaton" to quantum fields

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We propose a simple geometrical prescription for coupling a test quantum scalar field to an "inflaton" (classical scalar field) in the presence of gravity. When the inflaton stems from the compactification of a Kaluza-Klein theory, the prescription leaves no arbitrariness and amounts to a dimensional reduction of the Klein-Gordon equation. We discuss the possible relevance of this coupling to "reheating" in inflationary cosmologies.

I. INTRODUCTION

The possible role of scalar fields in cosmology has been extensively studied since the invention of the inflationary scenario (see Ref. 1 for a review). In "old",[2] "new",[3] or "chaotic"[4] inflation, they are identified with various grand-unifying entities such as Higgs fields. Effective scalar fields also appear in theories formulated in higher-dimensional space-times (see, e.g., Ref. 5) as the "radius of the internal space" when the theory is "reduced" to four dimensions.[6] Finally they can describe the extra degrees of freedom of "higher-derivative" theories of gravity (see, e.g., Ref. 7), based on $f(R)$ Lagrangians (R is the scalar curvature), when those are formulated as general relativity (Lagrangian R) coupled to matter.[8]

Among the many issues that inflationary scenarios have to face, one is "reheating," that is, how these scalar fields (or "inflatons") transmute themselves into ordinary radiation at the end of the inflationary stage, so that the standard hot scenario can resume—or start. A number of mechanisms have been proposed. One uses the predicted (temporal) oscillations of the scalar field at the end of inflation to excite the vacuum modes of ordinary matter, thus transferring energy. Present models of the process are however very scant as they amount to introducing a phenomenological friction term in the evolution equation of the inflaton.[9,10] Another, more elaborate, mechanism exploits the fact that the inflaton is self-coupled (its "Klein-Gordon" evolution equation is nonlinear); its fluctuations hence satisfy a (linear) Klein-Gordon equation with a time-dependent mass; when quantized they can be interpreted as scalar particles which are supposed to subsequently decay into ordinary radiation.[11,12] Finally a third reheating mechanism is based on the prediction that ordinary matter can be created when coupled to a time-varying *gravitational* field (see, e.g., Ref. 13). In addition to a few schematic toy models[14] a framework for treating the problem in a self-consistent way was developed in Ref. 15 (taking into account the back reaction on to the scale factor of the stress-energy tensor of the scalar quanta). Clearly a satisfactory solution of the reheating problem should somehow combine the last two mechanisms as well as introduce some kind of irreversibility in order for genuine dissipation and entropy production to occur.[12]

In this paper we shall make only one point. In the last reheating mechanism mentioned (triggered by gravitational particle production) only the coupling to gravity is usually considered: the (scalar) quantum fields obey a covariant Klein-Gordon equation, optionally coupled to the scalar curvature of space-time. Now when an inflaton is present as a source for the gravitational field, the coupling of the quantum fields to this inflaton should also be taken into account (after all, the phenomenological treatment of reheating first mentioned above relies on the existence and efficiency of such a coupling). The problem of course is to describe in mathematical terms how the inflaton couples to the quantum matter fields.[9] What we suggest here is that simple geometrical considerations can serve as a guide to restrict the possible couplings. Indeed a family of classical actions for a test particle coupled to a scalar field has long been known[16,17] and yields, via the standard Bohr correspondence principle, to modified Klein-Gordon equations whose mass terms depend on the scalar field. If moreover that inflaton has itself a geometrical origin, as is the case in higher-dimensional theories "compactified" on to four dimensions, a single modified Klein-Gordon equation can be obtained from a dimensional reduction of either the geodesic or the free Klein-Gordon equations in the higher-dimensional space-time. Taking into account this coupling of matter fields to the inflaton should modify, at least qualitatively, the analysis of particle production in inflationary cosmologies.

In Sec. II we examine the possible actions, geodesics and Hamilton-Jacobi equations for a classical test particle coupled to an "inflaton," and deduce by the Bohr correspondence principle the modified Klein-Gordon equations for quantum scalar fields. In Sec. III we first recall how a higher-dimensional pure gravity (Kaluza-Klein) theory, when reduced to four dimensions, can be identified to general relativity coupled to a scalar field (and to gauge fields, but we shall ignore those). We then show how the geodesics or the free Klein-Gordon equations in the higher-dimensional space-time are "reduced" and how the coupling of quantum scalar fields to the inflaton is thus fixed. In the concluding section, Sec. IV, we sketch the possible relevance of this coupling to the reheating process of the Universe, using the model of Yoon and Brill[18] as an example.

II. THE COUPLING OF A TEST PARTICLE TO A SCALAR FIELD

The action for a classical test particle moving in a gravitational and a scalar field can be taken to be[16,17]

$$S=-\int_a^b m^* ds, \quad m^*=m+\epsilon\chi(\sigma)\ , \tag{1}$$

where $ds=\sqrt{-g_{\mu\nu}dx^\mu dx^\nu}$, m is the mass of the particle, ϵ its "scalar charge," and where $\chi(\sigma)$ is an *a priori* arbitrary function of the "inflaton" σ. (We exclude other couplings, such as $\partial_\mu\sigma\, dx^\mu$, which is a total divergence, or $\partial_\mu\sigma\partial^\mu\sigma ds$, which would imply that the force exerted on the particle depends on the second derivatives of σ.) The variation of this action with respect to infinitesimal deformations of the path between a and b is

$$\delta S=-\int_a^b\left[m^* g_{\mu\nu}\frac{Du^\mu}{ds}+\epsilon\frac{d\chi}{d\sigma}\frac{\partial\sigma}{\partial x^\mu}(\delta^\mu_\nu+u^\mu u^\rho g_{\nu\rho})\right]ds\,\delta x^\nu+[m^* g_{\mu\nu}u^\mu\delta x^\nu]_a^b\ , \tag{2}$$

where D is the covariant derivative and $u^\mu=dx^\mu/ds$. The particle follows the path that extremizes the action, and its equation of motion therefore is

$$u^\rho D_\rho u^\nu=-\frac{\epsilon}{m+\epsilon\chi(\sigma)}(g^{\mu\nu}+u^\mu u^\nu)\partial_\mu\chi(\sigma)\ . \tag{3}$$

The right-hand side (RHS) is the scalar equivalent of the Lorentz force where $m^*=m+\epsilon\chi$ plays the role of a σ- and hence x^μ-dependent inertial mass.[17] On the dynamical trajectories, that is when (3) is satisfied, we have from (2) that $u_\mu=\partial_\mu S/m^*$ which, from the normalization condition $u^\nu u_\nu=-1$, yields the Hamilton-Jacobi equation

$$g^{\mu\nu}\partial_\mu S\partial_\nu S+[m+\epsilon\chi(\sigma)]^2=0\ . \tag{4}$$

We refer to, e.g., Ref. 17, for a deduction from (4) of the classical trajectories.

The standard procedure to describe now a quantum spin-zero particle is to replace in the Hamilton-Jacobi equation (4) $\partial_\nu S$ by the operator $-iD_\nu$ (Bohr's correspondence principle). One thus gets a modified Klein-Gordon equation for the wave function ϕ of the quantum particle:

$$-\Box\phi+m^{*2}\phi=0, \quad m^*=m+\epsilon\chi(\sigma)\ , \tag{5}$$

which when $\epsilon=0$ or in the absence of external field σ reduces to the standard minimally coupled Klein-Gordon equation. We postpone a discussion of this equation until we have chosen the function $\chi(\sigma)$. We only mention here as a side remark that a way to arrive at a nonminimally coupled equation would be to replace m by $\sqrt{m^2+\xi R}$ in the action (1); however, the equation of motion (3) would then read (for $\epsilon=0$)

$$u^\mu D_\mu u^\nu=-\frac{\xi}{2}\frac{g^{\mu\nu}+u^\mu u^\nu}{m^2+\xi R}\partial_\mu R\ , \tag{6}$$

and the corresponding classical particle would not follow a geodesic.

III. FIXING THE COUPLING BY DIMENSIONAL REDUCTION

In the absence of experimental data it is difficult to specify the function χ characterizing in (4) or (5) the coupling of a classical or quantum particle to the scalar field σ. This however can be done when σ arises from the reduction to four dimensions of a theory of gravity in a higher-dimensional space-time.

Let us recall then how pure gravity in a D-dimensional space-time can be reduced by means of a conformal transformation to general relativity coupled to a scalar field.[6,19] The D-dimensional Einstein equations read

$$R_{AB}-\tfrac{1}{2}Rg_{AB}+Cg_{AB}=0, \quad A,B=0,1,\ldots,D-1\ , \tag{7}$$

where R_{AB} is the Ricci tensor, R is the scalar curvature and C is a "bare" cosmological constant. When one is interested not in the decomposition of g_{AB} into a gravitational and gauge fields but in the scalar field such a decomposition gives rise to, solutions of (7) of the following type are sought for (see, e.g., Ref. 5):

$$g_{AB}=\begin{pmatrix}\hat{g}_{\mu\nu}(x^\rho) & 0\\ 0 & e^{2\sigma(x^\mu)}\tilde{g}_{ab}(x^c)\end{pmatrix} \tag{8}$$

with $\mu,\nu=0,1,2,3$, $a,b=4,\ldots,D-1$ and where $\tilde{g}_{ab}$ is the metric of a compact "internal" space whose volume at each space-time point x^μ is measured by the field $\sigma(x^\mu)$. (For an analysis of the off-diagonal terms in terms of gauge fields see, e.g., Refs. 5 and 19.) It was shown in Ref. 6 (and references therein) that in terms of a metric $\bar{g}_{\mu\nu}$, conformally related to $\hat{g}_{\mu\nu}$ by

$$\bar{g}_{\mu\nu}=e^{n\sigma}\hat{g}_{\mu\nu}\ , \tag{9}$$

where $n=D-4$ is the number of "internal" dimensions, the field equations (7) split into two: first a constraint on the geometry of the internal space,

$$\tilde{R}_{ab}=\frac{1}{n}K\tilde{g}_{ab}\ , \tag{10}$$

where K is a constant [when $n=1$, $K=0$ and when $n=2$, (10) is an identity]; and second the equations of general relativity for the metric $\bar{g}_{\mu\nu}$, minimally coupled to the "dilaton" σ,

$$\bar{R}_{\mu\nu}-\tfrac{1}{2}\bar{g}_{\mu\nu}\bar{R}=\frac{n(n+2)}{2}(\partial_\mu\sigma\partial_\nu\sigma-\tfrac{1}{2}\bar{g}_{\mu\nu}\partial_\rho\sigma\bar{\partial}^\rho\sigma)-\bar{g}_{\mu\nu}V(\sigma) \tag{11}$$

where all barred quantities are formed using $\bar{g}_{\mu\nu}$ and where the potential $V(\sigma)$ is given by

$$V(\sigma)=Ce^{-n\sigma}-\frac{K}{2}e^{-(n+2)\sigma}\ . \tag{12}$$

As for the "Klein-Gordon" equation for σ, it follows from (11) and (12) via the Bianchi identity:

$$-\Box\sigma+\frac{2}{n(n+2)}\frac{dV}{d\sigma}=0\ . \tag{13}$$

Let us examine now how a classical particle of "mass" μ, which is free in the higher-dimensional space-time, can be described as a particle in four-dimensional (4D) space-time coupled to a scalar field.[6,19] Its action is

$$S=-\tau\mu\int_A^B dS=-\tau\mu\int_A^B L\,dp\ , \tag{14}$$

where $dS=\sqrt{-\tau g_{AB}dx^A dx^B}$ and $\tau=\pm1$ (as the particle may behave as a tachyon in higher-dimensional space-time); the Lagrangian is $L=dS/dp$ where p parametrizes the path. It is at this stage convenient to distinguish the five-dimensional (5D) from the general case. Indeed when $D=5$ the metric (8) does not depend on x^5; we can then take advantage of the Routh procedure[19,20] and define an equivalent action

$$S'=-\tau\mu\int_A^B\left[L-q\frac{dx^5}{dp}\right]dp\ , \tag{15}$$

where q is a constant of the motion:

$$q=\frac{\partial L}{\partial(dx^5/dp)}=-\tau e^{2\sigma}\frac{dx^5}{dS}\ . \tag{16}$$

Using the decomposition (8) and (9) of the metric and (16) to eliminate dx^5/dp, S' can be rewritten as[19]

$$S'=-\int_a^b d\bar{s}\,\mu e^{-\sigma/2}\sqrt{\tau+q^2e^{-2\sigma}}\ , \tag{17}$$

where $d\bar{s}=\sqrt{-\bar{g}_{\mu\nu}dx^\mu dx^\nu}$. Comparing (17) with (1) we see that the free action of the particle in the 5D space appears, when "projected" on the 4D space-time, as the action of a particle interacting with a scalar field σ. The form of the interaction $\chi(\sigma)$ as well as the relation between (m,ϵ) and (μ,q) are determined by

$$m^*=m+\epsilon\chi(\sigma)=\mu e^{-n\sigma/2}\sqrt{\tau+q^2e^{-2\sigma}} \tag{18}$$

where $n=1$. Once this identification has been made the modified Klein-Gordon equation (5) for a quantum scalar field, with m^* given by (18) follows straightforwardly—see Sec. II. Now when $D>5$ the metric (8) depends on x^a, and an alternative route to arrive at (18) is to start from the geodesic equation in D dimensions deduced from the variation of (14):

$$\frac{DU^A}{dS}=0\ , \tag{19}$$

where $U^A=dx^A/dS$. For $A=a$ and the ansatz (8) for the metric, (19) reads

$$U^\mu\partial_\mu U_a+U^b\tilde{D}_bU_a=0\ , \tag{20}$$

which implies that the quantity

$$q^2=\tilde{g}^{ab}U_aU_b \tag{21}$$

is a constant of the motion. For $A=\mu$ and the forms (8) and (9) for the metric, (19) first becomes

$$\frac{\bar{D}U^\mu}{dS}=nU^\mu U^\nu\partial_\nu\sigma-\frac{n}{2}\bar{g}_{\nu\rho}U^\nu U^\rho\bar{\partial}^\mu\sigma+q^2e^{(n-2)\sigma}\bar{\partial}^\mu\sigma\ . \tag{22}$$

The D-dimensional metric dS must then be expressed in terms of the (4D) one:

$$dS=d\bar{s}\frac{e^{-n\sigma/2}}{\sqrt{q^2e^{-2\sigma}+\tau}} \tag{23}$$

with $d\bar{s}=\sqrt{-\bar{g}_{\mu\nu}dx^\mu dx^\nu}$ [note in passing that inserting (23) into (14) does not yield (17)]. Using (23) the geodesic equation (22) finally reads[19]

$$\bar{u}^\nu\bar{D}_\nu\bar{u}^\mu=\left[\frac{n}{2}+\frac{q^2e^{-2\sigma}}{q^2e^{-2\sigma}+\tau}\right](\bar{u}^\mu\bar{u}^\nu+\bar{g}^{\mu\nu})\partial_\nu\sigma \tag{24}$$

where $\bar{u}^\mu=dx^\mu/d\bar{s}$. The RHS of (24) shows how the internal space acts on the 4D projection of the trajectory of a free particle. When compared to the RHS of (3) it can be interpreted as the force exerted by a scalar field σ. Identifying the two expressions

$$-\frac{\epsilon}{m+\epsilon\chi}\frac{d\chi}{d\sigma}=\frac{n}{2}+\frac{q^2e^{-2\sigma}}{q^2e^{-2\sigma}+\tau} \tag{25}$$

and integrating yields (18) and, hence, (1) and (5).

We saw how a free classical test particle in a higher-dimensional space-time behaves as if coupled to a scalar field when the metric is decomposed according to (8) and we deduced, using Bohr's principle, the corresponding Klein-Gordon equation, (5) and (18). Another way to arrive at the result is to simply decompose the free D-dimensional Klein-Gordon equation. Indeed consider the D-dimensional wave equation:

$$(\Box_D+\tau\mu^2)\Phi(x^A)=0\ , \tag{26}$$

where $\Box_D=g^{AB}D_AD_B$, μ is the "mass" of the field Φ, $\tau=\pm1$. When the metric g_{AB} is imposed to be of the form (8) and (9) the wave function can be decomposed as

$$\Phi(x^A)=\sum_q\phi(x^\mu)\tilde{y}_q(x^c)\ , \tag{27}$$

where $\tilde{y}_q(x^c)$ are the harmonics on the n-sphere (since $\tilde{R}=$const and we assume a compact internal space) and the eigenvalues q are defined as

$$\tilde{\Box}\tilde{y}_q(x^c)=-\mu^2q^2\tilde{y}_q(x^c)\ . \tag{28}$$

Substituting (27) into (26) a standard calculation—spelled out in the Appendix—yields the expected result:

$$-\bar{\Box}\phi+m^{*2}\phi=0 \quad \text{with } m^{*}=\mu e^{-n\sigma/2}\sqrt{\tau+q^2e^{-2\sigma}} \ . \tag{29}$$

IV. REHEATING AND THE COUPLING TO THE DILATON IN INFLATIONARY COSMOLOGIES

As we mentioned in the Introduction, three main different lines of attack to the problem of reheating in inflationary cosmologies have been proposed in the literature. The first one consists in coupling "by hand" the inflaton σ to a radiation fluid (density ρ_r) by means of a friction term Γ. This coupling is supposed to become effective at the end of the inflationary era when σ approaches the minimum of its potential V, so that $V\sim\frac{1}{2}M^2\sigma^2$. The evolution equations for σ, ρ_r, and the scale factor S of the Universe (assumed to have a Robertson-Walker geometry) are thus taken to be[9,10]

$$\begin{aligned} &3(H^2+k/S^2)=\tfrac{1}{2}\dot\sigma^2+\tfrac{1}{2}M\sigma^2+\rho_r \ ,\\ &\dot\rho_r+4H\rho_r-\Gamma\dot\sigma^2=0 \ ,\\ &\ddot\sigma+(3H+\Gamma)\dot\sigma+M\sigma=0 \ , \end{aligned} \tag{30}$$

where the dot is the derivative with respect to the cosmic time t, $H=\dot S/S$, and $k=0,\pm1$. Noting that for $\Gamma=0$, $\rho_r=0$ and large t we have $H\sim 2/3t$ (for $k=0$) so that the inflaton behaves like a pure dust perfect fluid,[21] the problem is sometimes[10] reduced to the conversion of dust into radiation in a dust-dominated Friedmann model. As long as a field theoretical derivation of the Γ term is not given, this mechanism can however hardly be said to have any explanatory power.

In a second line of attack[11,12] the inflaton σ is regarded as the sum of a classical field $\sigma_{\rm cl}$ and a small quantum correction ϕ. The field equations for σ and S,

$$\begin{aligned} &3(H^2+k/S^2)=\tfrac{1}{2}\dot\sigma^2+V(\sigma) \ ,\\ &-\Box\sigma+\frac{dV}{d\sigma}=0 \Longleftrightarrow \ddot\sigma+3H\dot\sigma+\frac{dV}{d\sigma}=0 \ , \end{aligned} \tag{31}$$

are then solved iteratively. The zero order [Eq. (31) where $\sigma=\sigma_{\rm cl}$] determines the evolution of the background; the first order governs the quantum field ϕ:

$$-\Box\phi+\bar m^2\phi=0, \quad \bar m^2(t)=\left.\frac{d^2V(\sigma)}{d\sigma^2}\right|_{\rm cl} \ . \tag{32}$$

For a self-interacting inflaton [$d^2V/d\sigma^2\neq$const—which means $V(\sigma)\neq\frac{1}{2}M\sigma^2$], the time dependence of $\bar m$ implies that quanta of the field ϕ are created.[11] Their back reaction onto the evolution of $\sigma_{\rm cl}$ and S appears at the second iteration:

$$\begin{aligned} &3(H^2+k/S^2)=\tfrac{1}{2}\dot\sigma_{\rm cl}^2+V(\sigma_{\rm cl})+\langle T_{00}(\phi)\rangle \ ,\\ &-\Box\sigma_{\rm cl}+\left.\frac{dV}{d\sigma}\right|_{\rm cl}+\tfrac{1}{2}\langle\phi^2\rangle\left.\frac{d^3V}{d\sigma^3}\right|_{\rm cl}=0 \ , \end{aligned} \tag{33}$$

where $\langle T_{00}(\phi)\rangle$ and $\langle\phi^2\rangle$ should be some suitably renormalized vacuum expectation values (see Ref. 12). The connection between this mechansim and the previous one is however not straightforward. Indeed the back-reaction term in the "Klein-Gordon" equation (33) is a function of $\sigma_{\rm cl}$ and turning it into a friction term of the type $\Gamma\dot\sigma$ involves introducing some kind of time irreversibility.[12]

The third reheating mechanism[14,15] is mathematically very similar to the previous one. The difference is that the quantum field ϕ is not thought of as the fluctuations of the inflaton but represents "ordinary" scalar particles of *constant* mass $\bar m=m$ that are produced by the time-varying *gravitational* field. In fact, the presence of an inflaton is not even necessary in that scenario.[15] Indeed the stress-energy tensor of the gravitationally created quanta [$\langle T_{\mu\nu}(\phi)\rangle$] involves terms that are quadratic in the scalar curvature of space-time and can react on the scale factor to produce the required inflation.[22] In that model then the evolution equations to be solved are

$$\begin{aligned} &-\Box\phi+m^2\phi=0 \ ,\\ &3(H+k/S^2)=\langle T_{00}\rangle \ ,\\ &-6(\dot H+2H^2+k/S^2)=\langle T^{\mu}_{\mu}\rangle \ . \end{aligned} \tag{34}$$

See Ref. 15 for details.

The (at least qualitatively) new reheating mechanism that we propose here is simply to include in the previous scheme a coupling of ϕ to the inflaton σ. According to the geometrical prescription that we defined in the previous sections this amounts to replacing in the Klein-Gordon equation (34) m by $m^*=m+\epsilon\chi(\sigma)$ [Eq. (5)]; when moreover σ arises from the compactification of a higher-dimensional gravity theory we saw that the function χ can be determined to be $m^{*2}=\mu^2\tau e^{-n\sigma}+q^2\mu^2e^{-(n+2)\sigma}$ [Eq. (18)]. In that latter case however this new coupling is very similar to the standard mechanism described by (31) and (32), based on the quantization of a field with the variable mass $\bar m^2=d^2V/d\sigma^2$. Indeed when V is given by (12), its second derivative $\bar m^2$ has the same σ dependence as m^{*2} and can even be identical to m^{*2} if $n^2C=\tau\mu^2$ and $K(n+2)^2=-2q^2\mu^2$. Only in cases when C and τ cannot have the same sign can the two mechanisms really differ.

In order to illustrate the possible role of these couplings we shall briefly examine the cosmological model of Yoon and Brill.[18] Starting from a theory of pure gravity in a $(n+4)$-dimensional space-time, but allowing for the presence of fine-tuned torsional bilinears, these authors arrive at an effective 4D theory of the type (11)–(13) where the potential $V(\sigma)$ is modified as to have a zero minimum:

$$V(\sigma)=\frac{\Lambda}{2}\left[\frac{2}{n+2}+\frac{n}{n+2}e^{-(n+2)\sigma}-e^{-n\sigma}\right], \quad \Lambda>0 \ . \tag{35}$$

We therefore have, in this case,

$$m^{*2}=\mu^2\tau e^{-n\sigma}+q^2\mu^2 e^{-(n+2)\sigma}\ ,$$
$$\overline{m}^{\,2}=-\frac{n^2\Lambda}{2}e^{-n\sigma}+\frac{n(n+2)\Lambda}{2}e^{-(n+2)\sigma}\ . \tag{36}$$

[We assume here that the geodesic equation (19), which is linear in the torsion, is not affected by the nonvanishing of the torsional bilinears.] In a Robertson-Walker background (with $k=0$) the field equations (11), (13), and (35) read

$$3H^2=\frac{1}{2}\frac{n(n+2)}{2}\dot\sigma^2+V(\sigma)\ ,$$
$$\ddot\sigma+3H\dot\sigma+\frac{2}{n(n+2)}\frac{dV}{d\sigma}=0\ . \tag{37}$$

For large negative σ the potential is exponential $[V(\sigma)\propto e^{-(n+2)\sigma}]$, and could in principle lead to power-law inflation.[23,24] The coefficient of σ is however larger than 2 which implies that not only is the horizon problem left unsolved[23] but also that the solution is not an attractor for $k\neq 0$ models.[24] For large positive σ on the other hand, the potential tends exponentially fast to a constant. The "slow-roll" approximation of (37) then gives $H\approx\sqrt{\Lambda(n+2)/3}$, so that the effective masses (36) of the quantum fields ϕ are, during that inflationary period,

$$m^{*2}\approx\mu^2\tau e^{-n\sigma},\quad \overline{m}^{\,2}\approx-\frac{n^2\Lambda}{2}e^{-n\sigma}\ ,$$
$$e^{-n\sigma}\approx\left[\frac{3(n+2)}{n^2\Lambda}\right]^{1/2}\frac{1}{t_0-t}\ . \tag{38}$$

We see that $\overline{m}^{\,2}<0$ whereas $m^{*2}>0$ if $\tau>0$. The two processes are then qualitatively different. Consider now the oscillatory regime $\sigma\sim 0$. Then $V(\sigma)\approx\frac{1}{2}n\Lambda\sigma^2$ and an approximate solution of the system (37) is, for large t,[22]

$$H\approx\frac{2}{3t},\quad \sigma\approx\frac{1}{t}\left[\frac{2}{3n\Lambda}\right]^{1/2}\sin\left[t\left[\frac{2\Lambda}{n+2}\right]^{1/2}\right]\ . \tag{39}$$

The two effective masses (36) then both sinusoidally approach a constant:

$$m^{*2}\approx\mu^2(\tau+q^2)-\mu^2[n\tau+(n+2)q^2]\sigma\ ,$$
$$\overline{m}^{\,2}\approx n\Lambda[1-(n+2)\sigma] \tag{40}$$

with σ given by (39). We leave a detailed study of the self-consistent system (34), with m given by (39) and (40), to further work.

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APPENDIX

Although the calculation leading from (26) to (29) is standard we shall spell it out for completeness. Let us start with the D-dimensional wave equation

$$\Box_D\Phi+(\tau\mu^2+\xi R_D)\Phi=0\ , \tag{A1}$$

where $\Box_D=g^{AB}D_AD_B$, μ is the "mass" of the field Φ, $\tau=\pm 1$, and ξ allows for a nonminimal coupling to the D-dimensional Ricci scalar R_D. When the metric g_{AB} is decomposed as

$$g_{AB}=\begin{bmatrix}\hat g_{\mu\nu}(x^\rho) & 0\\ 0 & e^{2\sigma(x^\mu)}\tilde g_{ab}(x^c)\end{bmatrix}\ , \tag{A2}$$

we have

$$\Box_D\Phi=\hat\Box\Phi+e^{-2\sigma}\tilde\Box\Phi+n\hat\partial^\mu\sigma\partial_\mu\Phi\ , \tag{A3}$$
$$R_D=\hat R+e^{-2\sigma}\tilde R-n[2\hat\Box\sigma+(n+1)\partial_\mu\sigma\hat\partial^\mu\sigma]\ . \tag{A4}$$

If we now perform the conformal transformation,

$$\overline{g}_{\mu\nu}=e^{2\omega}\hat g_{\mu\nu}\ , \tag{A5}$$

where $\omega=\omega(x^\mu)$, and if we set $\Phi=\overline{\Phi}e^{p\omega}$, we have

$$\hat\Box\Phi=e^{(p+2)\omega}[\overline\Box\,\overline\Phi+p\overline\Phi\,\overline\Box\omega+2(p-1)\overline\partial^\mu\omega\partial_\mu\overline\Phi +p(p-2)\overline\Phi\,\overline\partial^\mu\omega\partial_\mu\omega]\ , \tag{A6}$$
$$\hat R=e^{2\omega}[\overline R+6(\overline\Box\omega-\partial_\mu\omega\overline\partial^\mu\omega)]\ . \tag{A7}$$

It is then straightforward to obtain:

$$\begin{aligned}[-\Box_D\Phi+(\tau\mu^2+\xi R_D)\Phi]e^{-(p+2)\omega}=&-\overline\Box\,\overline\Phi+(\mu^2\tau e^{-2\omega}+\xi\overline R)\overline\Phi+e^{-2(\sigma+\omega)}(-\tilde\Box\,\overline\Phi+\xi\tilde R\,\overline\Phi)\\ &-\partial_\mu\overline\Phi[2(p-1)\overline\partial^\mu\omega+n\overline\partial^\mu\sigma]+\overline\Phi[(6\xi-p)\overline\Box\omega-2n\xi\overline\Box\sigma]\\ &-\overline\Phi[6\xi+p(p-2)]\overline\partial^\mu\omega\partial_\mu\omega-n\overline\Phi(p-4\xi)\overline\partial^\mu\sigma\partial_\mu\omega-n(n+1)\xi\overline\Phi\,\overline\partial^\mu\sigma\partial_\mu\sigma\ .\end{aligned} \tag{A8}$$

When $n=0$ we recover that the massless Klein-Gordon equation is conformally invariant if $p=1$ and $\xi=\frac{1}{6}$. In the case considered here where $\omega=n\sigma/2$ the Klein-Gordon equations (A1) and (A8) reduce to

$$\begin{aligned}-\overline\Box\,\overline\Phi+(\mu^2e^{-n\sigma}+\xi\overline R)\overline\Phi+e^{-(n+2)\sigma}(-\tilde\Box\,\overline\Phi+\xi\tilde R\,\overline\Phi)\\ -pn\overline\partial^\mu\sigma\partial_\mu\overline\Phi+n(-p/2+\xi)\overline\Phi\,\overline\Box\sigma-(n/4)[p^2n+2\xi(n+2)]\overline\partial^\nu\sigma\partial_\mu\sigma=0\ .\end{aligned} \tag{A9}$$

We then see that the coupling of Φ to σ will not involve derivatives of σ only if $p=\xi=0$. Decomposing then Φ as

$$\Phi(x^A)=\sum_q \phi(x^\mu)\tilde{y}_q(x^c) , \quad \text{(A10)}$$

where $\tilde{y}_q(x^c)$ are the eigenfunctions of $\tilde{\Box}$ with eigenvalues q defined as

$$\tilde{\Box}\,\tilde{y}_q(x^c)=-\mu^2 q^2\tilde{y}_q(x^c) \quad \text{(A11)}$$

we arrive at the final result:

$$(-\Box_D+\tau\mu^2)\Phi(x^A)=0 \Longleftrightarrow [-\tilde{\Box}+\mu^2 e^{-n\sigma}(\tau+q^2e^{-2\sigma})]\phi(x^\mu)=0 . \quad \text{(A12)}$$

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