Finite-temperature QED effects in atoms

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We argue that low-temperature effects in QED can, if anywhere, only be quantitatively interesting for bound electrons. Unluckily the dominant thermal contribution turns out to be level independent, so that it does not affect the frequency of the transition radiation.

At low temperature, $T << m$, where m is the electron mass, temperature effects in QED are negligible because the dimensionless temperature scale T/m is very small (our units are $\hbar = c = k_B = 1$). Thus the mass shift is proportional to $\alpha(T/m)^2$,[1-5] the anomalous magnetic moment shift is proportional to $\alpha(T/m)^2$,[2,5,6] the speed-of-light shift is proportional to $\alpha^2(T/m)^4$,[7] and the electric-charge shift is proportional to $\alpha\sqrt{m/T}\exp(-m/T)$,[5] α being the fine-structure constant. The only way of having a larger temperature effect at $T << m$ is to have an energy scale smaller than m. The obvious physical system which has such a scale is an atom, where the Rydberg constant which measures the binding energy is much smaller than the electron mass, $\alpha^2 m << m$. One can then hope that low-temperature effects in atomic systems are given by the dimensionless temperature scale $T/(m\alpha^2)$ and therefore much more important and measurable even at $T \sim 10^3$ K, as $\frac{1}{2}\alpha^2 m \sim 10^5$ K. The aim of this note is to show that, in spite of the correctness of the previous statement, the unfortunate existence of a Thomas-Reiche-Kuhn-type sum rule makes this dominant low-temperature effect unmeasurable.

To be specific, let us consider the hydrogen Lamb shift which is a quantum field effect as it is due to virtual photons. Recall (see, e.g., Ref. 8 which we follow along this work) that for s waves the naive free-electron one-loop computation of the energy shift is infrared divergent. This is not surprising and only signals that at large wavelengths $\lambda >> (m\alpha)^{-1}$ a relativistic free-electron description of the electron bound in a region of the order of a Bohr radius $(m\alpha)^{-1}$ is incorrect. This leads to the introduction of a cutoff K, such that $m\alpha^2 << K << m$, which separates the large-momentum contribution, computed within relativistic quantum-field perturbation theory around free electrons, and the low-momentum contribution, computed within second-order nonrelativistic quantum-mechanical perturbation theory around bound electrons. When both contributions are summed the cutoff K cancels and the well-known Lamb-shift formula follows. The Rydberg constant enters this expression obviously coming from the second contribution.

Let us now proceed with the computation of temperature effects. The large-momentum temperature effects will be weighted with m, as the electrons are seen as free by small-wavelength photons. They do not interest us. On the contrary, the low-momentum temperature effects are weighted in principle by $\alpha^2 m$, as large-wavelength photons are sensitive to the whole atom and therefore see a bound electron. The only difference in the computation of the energy shift in second-order nonrelativistic perturbation theory due to the temperature is that our initial state is not a pure one-electron state but is given by a density matrix which describes the electron within an equilibrium photon sea at temperature T. The temperature energy shift is then given by

$$\delta E_n(T) = \frac{2\alpha}{3\pi} \sum_{n' \neq n} \int_0^K \frac{dk\,k}{e^{k/T}-1} \left(\frac{1}{E_n - E_{n'} - k} + \frac{1}{E_n - E_{n'} + k} \right) |\langle n | \vec{P}/m | n' \rangle|^2 \ , \tag{1}$$

where n and n' indicate all the quantum numbers needed for a complete description of the state.

The first term corresponds to an intermediate state with one photon more and the second with one photon less than the initial state. At temperatures below 10^4 K, $T << m\alpha^2$, it is convenient to divide the range of integration into two parts form 0 to T and from T to K. In the first range $E_n - E_{n'} \pm k \simeq E_n - E_{n'}$ and the integration is performed trivially. It is not difficult to see that the second range gives a contribution depressed by $(E_n - E_{n'})/T$ with respect to the first one. Thus the result for $T << m\alpha^2$ is

$$\delta E_n(T) \simeq \frac{2\alpha}{9\pi} \left(\frac{\pi T}{m} \right)^2 \sum_{n' \neq n} \frac{|\langle n | \vec{P} | n' \rangle|^2}{E_n - E_{n'}} \ . \tag{2}$$

This leads to an energy shift proportional to $\alpha T^2/m$. Recalling that the zero-temperature Lamb shift is proportional to $m\alpha^5$ we find that the temperature effects are of the order of $(T/m\alpha^2)^2$ and thus weighted by the Rydberg constant and not the electron mass. This is what we expected and pushes the effect into the domain of present-day experimental capabilities. Unluckily, temperature effects do not want to be measured so easily. It turns out that $\delta E_n(T)$ does not depend on n, and in particular does not depend on angular momentum. This can easily be seen recalling that

$$\vec{P} = im[H, \vec{R}] \ , \tag{3}$$

so that

$$|\langle n | \vec{P} | n' \rangle|^2 = m^2 (E_n - E_{n'})^2 |\langle n | \vec{R} | n' \rangle|^2 \ , \tag{4}$$

which then implies that

$$\sum_{n' = n} \frac{|\langle n | \vec{P} | n' \rangle|^2}{E_n - E_{n'}} = -\tfrac{3}{2} m \ , \tag{5}$$

independently of the value of n. In other words, the dominant low-temperature energy shift is the same for all levels and does not change the frequency of the transition radiation. Only subdominant effects enter into the Lamb shift, but those are not large enough to be measurable in the near future.

In atoms the dominant thermal QED effects are the same for all levels, and thus not seen in transitions.

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[1]J. L. Cambier, J. R. Primack, and M. Sher, Nucl. Phys. **B209**, 372 (1982); **B222**, 517(E) (1983).

[2]G. Peressutti and B. S. Skagerstam, Phys. Lett. **110B**, 406 (1982).

[3]K. Babu Joseph, Phys. Lett. **115B**, 252 (1982).

[4]J. F. Donoghue and B. R. Holstein, Phys. Rev. D **28**, 340 (1983).

[5]A. Palanques-Mestre and R. Tarrach, University of Barcelona Report No. UBFT-FP-3-83 (unpublished).

[6]Y. Fujimoto and J. Hyung Yee, Phys. Lett. **114B**, 359 (1982).

[7]R. Tarrach, Phys. Lett. **133B**, 259 (1983).

[8]C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).