Determination of anomalies in supersymmetric theories

Friedemann Brandt* and Jordi París†

Instituut voor Theoretische Fysica, Katholieke Universiteit Leuven, Celestijnenlaan 200 D, B-3001 Leuven (Heverlee), Belgium

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We develop an efficient technique to compute anomalies in supersymmetric theories by combining the so-called nonlocal regularization method and superspace techniques. To illustrate the method we apply it to a four-dimensional toy model with potentially anomalous $N=1$ supersymmetry and prove explicitly that in this model all the candidate supersymmetry anomalies have vanishing coefficients at the one-loop level. [S0556-2821(97)05904-3]

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I. INTRODUCTION

Supersymmetric quantum field theories have many remarkable properties. In particular, quantum corrections are usually better under control in such theories than in others due to nonrenormalization properties implied by supersymmetry. However, it is not clear from the outset whether the supersymmetry of a classical theory survives as a symmetry of the quantized theory, due to the lack of consistent regularization methods which manifestly preserve supersymmetry in perturbation theory. Nevertheless, supersymmetry ''miraculously'' appears to be preserved in standard supersymmetric theories.

An indirect but powerful and regularization-independent tool to investigate whether or not supersymmetry can be anomalous consists in an analysis of the supersymmetric analogue of the Wess-Zumino consistency condition [1]. Nontrivial solutions to this consistency condition are candidate supersymmetry anomalies whereas the absence of such solutions indicates that supersymmetry is not anomalous.

The consistency condition for supersymmetry anomalies, in combination with the usual Wess-Zumino consistency condition in the case of supersymmetric gauge theories, has been studied already for various $D=4$, $N=1$ globally supersymmetric models (see, e.g., [2,3]) and, recently, also for minimal supergravity [4]. It turns out that whether or not candidate supersymmetry anomalies exist depends decisively on the way supersymmetry is represented on the fields, i.e., on the structure of the supersymmetry multiplets present in the model in question. For standard representations, such as multiplets that can be described in terms of unconstrained or chiral scalar superfields, one finds that candidate anomalies for supersymmetry itself do not exist. However, this does not exclude the existence of supersymmetrized versions of other candidate anomalies such as Adler-Bell-Jackiw (ABJ) chiral anomalies in super Yang-Mills theories. Moreover, there are nonstandard representations of supersymmetry (''non-QDS representations''[1] in the terminology of [3]) which do give rise to candidate anomalies for supersymmetry itself.

When the cohomological analysis alone is not sufficient to exclude candidate anomalies due to the existence of nontrivial solutions to the consistency condition (for supersymmetry or other symmetries), one has to check by an explicit calculation whether or not these candidate anomalies have vanishing coefficients. To that end one needs an appropriate regularization method. One of the main disadvantages of most of the regularization methods designed for supersymmetric theories is the lack of a consistent implementation of the superspace techniques [5,6] — one of the main tools in supersymmetry — at the regularized level [5]. This drawback, somewhat analogous to the dimensional regularization troubles when dealing with chiral theories, becomes then relevant in analyzing the presence of anomalies in the model under consideration. Indeed, naive manipulations in superspace may lead to inconsistencies or ambiguities when computing divergent expressions, making it impossible to detect and calculate (unambiguously) such anomalies. It would thus be desirable to design a method in which superspace computations were unambiguously defined.

In this paper we develop a new efficient technique to investigate anomaly issues in supersymmetric theories. It combines naturally superspace techniques, which facilitate the perturbative calculations in supersymmetric theories considerably, with the so-called nonlocal regularization [7,8], which has already been successfully used to compute one- [9] and higher-loop anomalies [8] in other (nonsupersymmetric) theories. Among others, the method allows one to check whether or not supersymmetry itself is anomalous. We illustrate this by applying the method to a four-dimensional supersymmetric toy model whose supersymmetry is potentially anomalous, as cohomological results indicate [3].

The paper is organized as follows. First we describe our method in Sec. II. To that end we briefly recall the basic concepts of nonlocal regularization, emphasizing its use to determine anomalies, and describe how superspace techniques are naturally implemented in it. In Sec. III we introduce the toy model and present its candidate supersymmetry

*Present address: Dept. ECM, Facultat de Física, Universidad de Barcelona, Diagonal 647, 08028 Barcelona, Spain. Electronic address: brandt@ecm.ub.es

†Present address: Dept. ECM, Facultat de Física, University de Barcelona, Diagonal 647, 08028 Barcelona, Spain. Electronic address: paris@ecm.ub.es.

[1]''A theory is called QDS theory if the D_α-representation decomposes into a sum of (Q) and (D) multiplets and singlets which have only dotted indices'' [3].

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anomalies. In Sec. IV we then apply our method to this toy model and prove the absence of supersymmetry anomalies at the one-loop level. Three appendixes finally collect our conventions.

II. NONLOCAL REGULARIZATION OF SUPERSYMMETRIC THEORIES

There exist many ways in the literature to algebraically compute (one-loop) anomalies. All of them are essentially based in testing the response of the — suitably regulated — partition function of the model under the (infinitesimal version of the) symmetry transformation under study. Departures from unity of the Jacobian arising upon this change which cannot be absorbed by suitable counterterms reflect then the presence of anomalies in the model.

The so-called ''nonlocal regularization'' method, recently introduced in [7,8], fits perfectly well in this philosophy. Indeed, this approach proceeds by constructing from the original action $S(\Phi^A)$ and symmetry transformations $\delta\Phi_A$ of the model a regulated action $S_\Lambda(\Phi^A)$, invariant under a ''regulated'' version of the original symmetry, $\delta_\Lambda\Phi_A$, where Λ stands for a cutoff or regulating parameter. Such invariant action, exponentiated afterwards in the path integral, generates then a modified set of Feynman rules and propagators that yield finite Feynman integrals for finite values of the cutoff at all loop levels and, thus, a finite partition function.

For our purposes, there are two main advantages of this approach relative to other ''standard'' regularization methods. First of all, the nonlocally regularized action $S_\Lambda(\Phi^A)$ can just be seen as a ''smooth'' deformation of the original one such that its main features (dimensionality, field content, symmetries, etc.) remain unaltered. Therefore, when dealing with supersymmetric theories, in particular, superspace computations at a regulated level can be performed in exactly the same way as in the original theory. Second, and on top of that, the invariance of S_Λ under δ_Λ directly relates potential one-loop anomalies to the finite part of the functional trace — now completely regulated — of the Jacobian matrix, namely,[2]

$$\mathcal{A}=\left[(-1)^A\frac{\partial_r(\delta_\Lambda\Phi_A)}{\partial\Phi_A}\right], \tag{2.1}$$

where $(-1)^A\equiv(-1)^{|\Phi_A|}$ stands for the Grassmann parity of the field Φ_A. In view of these facts, nonlocal regularization appears thus as an excellent candidate to implement our program.

In what follows, we briefly summarize the construction of the nonlocal action S_Λ and of its symmetries δ_Λ, as well as the specific form of the anomaly (2.1), along the lines of Refs. [7,8], implementing afterwards the standard superspace techniques in this framework.

A. Basics of nonlocal regularization

Consider a theory defined by a classical action $\mathcal{S}(\Phi^A)$, which admits a sensible perturbative decomposition into free and interacting parts:

$$\mathcal{S}(\Phi)=F(\Phi)+I(\Phi)\quad\text{with}\quad F(\Phi)=\tfrac{1}{2}\Phi^A F_A{}^B\Phi_B. \tag{2.2}$$

Introduce now a field-independent operator $(T^{-1})_A{}^B$ such that a second-order derivative ''regulator'' $R_A{}^B$ arises through the combination

$$R_A{}^B=(T^{-1})_A{}^C F_C{}^B,$$

and construct from this object the so-called smearing operator $\varepsilon_A{}^B$ and shadow kinetic operator $(\mathcal{O}^{-1})_A{}^B$:

$$\varepsilon_A{}^B=\exp\left(\frac{R_A{}^B}{2\Lambda^2}\right), \tag{2.3}$$

$$(\mathcal{O}^{-1})_A{}^B=T_A{}^C\int_0^1\frac{dt}{\Lambda^2}\exp\left(t\frac{R_C{}^B}{\Lambda^2}\right). \tag{2.4}$$

To each original field Φ_A is now associated an auxiliary, or ''shadow,'' field Ψ_A with the same statistics. Both sets of fields are then coupled by means of the auxiliary action

$$\widetilde{\mathcal{S}}(\Phi,\Psi)=F(\hat{\Phi})-A(\Psi)+I(\Phi+\Psi), \tag{2.5}$$

with $A(\Psi)$, the kinetic term for the auxiliary fields, constructed with the help of Eq. (2.4) as

$$A(\Psi)=\tfrac{1}{2}\Psi^A(\mathcal{O}^{-1})_A{}^B\Psi_B,$$

and where the ''smeared'' fields $\hat{\Phi}_A$ appearing in the free part of the auxiliary action (2.5) are defined, using Eq. (2.3), by $\hat{\Phi}_A\equiv(\varepsilon^{-1})_A{}^B\Phi_B$.

The perturbative theory described by Eq. (2.5), when only external Φ lines are considered, is then seen to describe the same theory as the original action (2.2). However, the special form of propagators and couplings in Eq. (2.5) lead the loops formed with shadow propagators to isolate the divergent parts of the original diagrams. As a consequence, dropping out these loop contributions, i.e., the quantum fluctuations of the shadow fields, by hand regularizes the theory. Such an *ad hoc* procedure may, however, be simply implemented by putting the auxiliary fields Ψ classically on shell. The classical shadow field equations of motion,

$$\frac{\partial_r\widetilde{\mathcal{S}}(\Phi,\Psi)}{\partial\Psi_A}=0\Rightarrow\Psi^A=\left(\frac{\partial_r I}{\Phi_B}(\Phi+\Psi)\right)\mathcal{O}_B{}^A, \tag{2.6}$$

should then be solved, in general, in a perturbative fashion and its solution $\Psi_0(\Phi)$ substituted in the auxiliary action (2.5). The result of this process is the nonlocalized action to be used in regularized perturbative computations:

$$\mathcal{S}_\Lambda(\Phi)\equiv\widetilde{\mathcal{S}}(\Phi,\Psi_0(\Phi)). \tag{2.7}$$

[2]de Witt notation is assumed throughout the paper whenever capital indices $A,B,\ldots$ are used. These indices indicate the different fields, their components, *and* the space-time point on which they depend (unless it is explicitly displayed). In this way, a summation over A includes not only discrete summations, but also integration over (super)space-time. The derivatives are left and right functional derivatives.

Moreover, as mentioned above, the nonlocalization procedure just presented has the merit of preserving at the tree level a distorted version of any of the original continuous symmetries of the theory. Indeed, assume the original action (2.2) to be invariant under the infinitesimal transformation

$$\delta\Phi_A = R_A(\Phi).$$

Then, the auxiliary action (2.5) is seen to be invariant under the auxiliary infinitesimal transformations

$$\tilde{\delta}\Phi_A = (\varepsilon^2)_A{}^B R_B(\Phi+\Psi), \quad \tilde{\delta}\Psi_A = (1-\varepsilon^2)_A{}^B R_B(\Phi+\Psi),$$

while the nonlocally regulated action $\mathcal{S}_\Lambda(\Phi)$, Eq. (2.7), becomes invariant under

$$\delta_\Lambda \Phi_A = (\varepsilon^2)_A{}^B R_B[\Phi + \Psi_0(\Phi)],$$

with $\Psi_0(\Phi)$ the solution of Eq. (2.6). In this way, an extensive use of the chain rule allows us to determine a closed form for the anomaly (2.1) in terms of propagators and vertices of the original theory as

$$\mathcal{A} = [(-1)^A (\varepsilon^2)_A{}^B J_B{}^C (\delta_\Lambda)_C{}^A], \quad \text{where } J_A{}^B = \frac{\partial_r R_A}{\partial \Phi_B}, \quad (2.8)$$

and with the regulated identity $(\delta_\Lambda)_A{}^B$ defined by

$$(\delta_\Lambda)_A{}^B \equiv (\delta_A{}^B - \mathcal{O}_A{}^C I_C{}^B)^{-1} = \delta_A{}^B + \sum_{n \geq 1} (\mathcal{O}_A{}^C I_C{}^B)^n,$$

in terms of the functional Hessian of the original interaction in Eq. (2.2)

$$I_A{}^B = \frac{\partial_l \partial_r I}{\partial \Phi^A \partial \Phi_B}. \quad (2.9)$$

The proof of these statements is straightforward and can be found in the original references [7,8], to which we refer the reader for further details.

B. Implementation of superspace techniques

The nonlocal regularization procedure outlined above applies of course to all kinds of perturbative models, including supersymmetric ones. Now, it is well known that in supersymmetric theories perturbative calculations can often be considerably simplified by means of superspace techniques due to the cancellation of terms caused by supersymmetry. It is therefore natural to look for a way to implement these techniques in the nonlocal regularization procedure. An obvious idea is to replace ordinary fields by superfields. However, one faces immediately the following related difficulties: How should one define functional derivatives with respect to arbitrary (constrained) superfields and integrations over their ‘‘superspace coordinates?’’ These two problems appear to make the simple substitution ‘‘fields → superfields’’ impossible except in very special cases where one deals only with particular superfields such as unconstrained or chiral ones. Thus in general we cannot simply take the Φ’s of the previous subsections to be superfields.

Fortunately this is not necessary at all since superspace techniques are of course not restricted to true superfields.[3] In fact, we will show now that they apply also to ‘‘constituents’’ of superfields such as

$$\varphi(x,\overline{\theta}) = a(x) + \overline{\theta}_{\dot\alpha} b^{\dot\alpha}(x) + \tfrac{1}{2}\overline{\theta}^2 c(x), \quad (2.10)$$

provided a,b,c are elementary fields. Namely we can then *define* functional derivatives[4] with respect to φ simply through

$$\frac{\partial}{\partial\varphi(x,\overline{\theta})} = -\frac{\partial}{\partial c(x)} + \overline{\theta}^{\dot\alpha}\frac{\partial}{\partial b^{\dot\alpha}(x)} - \frac{1}{2}\overline{\theta}^2 \frac{\partial}{\partial a(x)}, \quad (2.11)$$

which results in

$$\frac{\partial\varphi(x,\overline{\theta})}{\partial\varphi(x',\overline{\theta}')} = \delta^2(\overline{\theta}-\overline{\theta}')\,\delta^4(x-x') \equiv \delta^6(\overline{z}-\overline{z}').$$

Summation over their indices in de Witt’s condensed notation includes then simply an integration $\int d^6\overline{z} \equiv \int d^4x\, d^2\overline{\theta}$.

Alternatively we can (and will) use instead of φ the quantity

$$\Phi(z) = \exp(-i\theta\partial\overline{\theta})\varphi(x,\overline{\theta}), \quad (2.12)$$

which is antichiral in the sense that

$$\mathcal{D}_\alpha \Phi = 0, \quad (2.13)$$

where the standard covariant derivatives are defined as

$$\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\overline{\theta}^{\dot\alpha}\partial_{\alpha\dot\alpha}, \quad \overline{\mathcal{D}}_{\dot\alpha} = -\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} - i\theta^\alpha\partial_{\alpha\dot\alpha}. \quad (2.14)$$

However, Φ is not in general a superfield (see Appendix B); i.e., Eq. (2.13) does not reflect the transformation properties of Φ. The functional derivative with respect to Φ is then defined by means of Eq. (2.11) according to

$$\frac{\partial}{\partial\Phi(z)} = \exp(-i\theta\partial\overline{\theta})\frac{\partial}{\partial\varphi(x,\overline{\theta})}.$$

This results in

$$\frac{\partial\Phi(z)}{\partial\Phi(z')} = \frac{1}{2}\mathcal{D}^2\delta^8(z-z'), \quad (2.15)$$

due to the identity

$$\exp(-i\theta\partial\overline{\theta} + i\theta'\partial\overline{\theta}')\,\delta^6(\overline{z}-\overline{z}') = \tfrac{1}{2}\mathcal{D}^2\delta^8(z-z').$$

Formula (2.15) can indeed be found in many textbooks on supersymmetry for functional derivatives with respect to antichiral superfields — we just extend it to constituents of superfields satisfying Eq. (2.13). Because of the presence of the antichiral projector $\frac{1}{2}\mathcal{D}^2$ in Eq. (2.15), summation over

[3]See Appendix B for a discussion of the concept of a superfield.

[4]For definiteness all formulas are written for left derivatives in this subsection.

TABLE I. Supersymmetry multiplet of the toy model.

ϕ	χ_β	A	$V_{\beta\dot\beta}$	$\bar\psi_{\dot\beta}$	η_β	F
$D_\alpha\phi$	$\varepsilon_{\beta\alpha}A$	0	$-2i\partial_{\alpha\dot\beta}\chi_\beta+\varepsilon_{\alpha\beta}\bar\psi_{\dot\beta}$	$-2i\partial_{\alpha\dot\beta}A$	$2i\partial_{\alpha\dot\alpha}V_\beta{}^{\dot\alpha}-\varepsilon_{\alpha\beta}F$	$2i\partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha}$
$\bar D_{\dot\alpha}\phi$	$V_{\beta\dot\alpha}$	$\bar\psi_{\dot\alpha}$	$\varepsilon_{\dot\alpha\dot\beta}\eta_\beta$	$\varepsilon_{\dot\alpha\dot\beta}F$	0	0
dim(ϕ)	1/2	1	1	3/2	3/2	2

the indices of these constituents does not involve the integration $\int d^8z$ but again only an integration $\int d^6\bar z$. Analogous formulas hold of course for functional right derivatives and chiral quantities.

We conclude that we can use quantities such as Eqs. (2.10) or (2.12) in nonlocal regularization instead of ordinary fields. This remains true even if it is impossible to combine all the elementary fields in such quantities — the remaining elementary fields may be treated as usual; i.e., one can use quantities (2.10) or (2.12) and ordinary fields simultaneously if necessary. The only thing one has to keep in mind when dealing with such constituents is that operators such as Eqs. (2.14) or the usual generators of supersymmetry transformations,

$$\nabla_\alpha=\frac{\partial}{\partial\theta^\alpha}-i\bar\theta^{\dot\alpha}\partial_{\alpha\dot\alpha},\quad \bar\nabla_{\dot\alpha}=-\frac{\partial}{\partial\bar\theta^{\dot\alpha}}+i\theta^\alpha\partial_{\alpha\dot\alpha},\quad (2.16)$$

do not have the same interpretation acting on constituents of superfields as on superfields themselves: In particular the operators (2.16) do not represent the supersymmetry transformations any longer on all of the constituent fields.

III. MODEL

A. Multiplet and supersymmetry transformations

The four-dimensional toy model we are going to use contains only a supersymmetry multiplet considered in Sec. 7 of [3]. This multiplet consists of complex Weyl spinors χ, ψ, and η, a complex vector field V, and two complex scalar fields A and F. On these fields the abstract supersymmetry algebra

$$[P_a,P_b]=[P_a,Q_\alpha]=[P_a,\bar Q_{\dot\alpha}]=0,$$

$$\{Q_\alpha,Q_\beta\}=\{\bar Q_{\dot\alpha},\bar Q_{\dot\beta}\}=0,\quad \{Q_\alpha,\bar Q_{\dot\alpha}\}=-2i\sigma^a{}_{\alpha\dot\alpha}P_a \qquad (3.1)$$

is represented by $(P_a,Q_\alpha,\bar Q_{\dot\alpha})\equiv(\partial_a,D_\alpha,\bar D_{\dot\alpha})$ according to Table I (using $X_{\alpha\dot\alpha}=\sigma^a{}_{\alpha\dot\alpha}X_a$).

The assignment of the dimensions (dim) to the fields in Table I follows from the choice dim(χ)=1/2, which will be the power-counting dimension of χ, and from the standard convention dim(D_α) = dim($\bar D_{\dot\alpha}$)=1/2, dim(∂_a)=1. Supersymmetry transformations δ_{SUSY} of the fields in Table I are then defined according to the relation

$$\delta_{\text{SUSY}}=\epsilon^\alpha D_\alpha+\bar\epsilon_{\dot\alpha}\bar D^{\dot\alpha}\equiv\epsilon^{\underline\alpha}D_{\underline\alpha}, \qquad (3.2)$$

where the parameters ϵ^α are constant anticommuting spinors.

The supersymmetry multiplet and transformation laws of Table I can also be formulated in superspace (cf. Appendix B) which will be useful within the computation of the anomaly coefficients. However, for the reasons we have just explained, we will apply a somewhat unconventional approach involving not only true superfields but also special constituents of them, which will be introduced and discussed in the following.

The fundamental ("defining") superfield of the multiplet of Table I is

$$G^\alpha=\exp(\theta D+\overline{\theta D})\chi^\alpha=H^\alpha+\theta^\alpha K, \qquad (3.3)$$

with

$$H^\alpha=\exp(-i\theta\partial\bar\theta)h^\alpha,\quad K=\exp(-i\theta\partial\bar\theta)k, \qquad (3.4)$$

$$h^\alpha=\exp(\overline{\theta D})\chi^\alpha=\chi^\alpha+\bar\theta_{\dot\alpha}V^{\dot\alpha\alpha}+\tfrac{1}{2}\bar\theta^2\eta^\alpha, \qquad (3.5)$$

$$k=\exp(\overline{\theta D})A=A+\bar\theta_{\dot\alpha}\bar\psi^{\dot\alpha}+\tfrac{1}{2}\bar\theta^2F, \qquad (3.6)$$

where we used the identity (A1), Table I, and the notation $\theta\partial\bar\theta=\theta^\alpha\partial_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}$, $\theta^2=\theta^\alpha\theta_\alpha$, and $\bar\theta^2=\bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha}$. The split of G^α into the constituents H^α and K will be useful later on, in particular since the latter are "antichiral" in the sense that[5]

$$\mathcal{D}_\alpha H_\beta=\mathcal{D}_\alpha K=0, \qquad (3.7)$$

whereas G^α itself satisfies the "constraint"

$$\mathcal{D}_{(\alpha}G_{\beta)}=0. \qquad (3.8)$$

It is important to realize and keep in mind that H_α is *not* a superfield since it does not satisfy the first identity (B1). Rather, its supersymmetry transformations are given by

$$D_\alpha H_\beta=\nabla_\alpha H_\beta+\varepsilon_{\beta\alpha}K,\quad \bar D_{\dot\alpha}H_\beta=\bar\nabla_{\dot\alpha}H_\beta. \qquad (3.9)$$

In contrast, K is a true superfield and thus satisfies Eq. (B1):

$$K=\tfrac{1}{2}\mathcal{D}_\alpha G^\alpha,\quad D_\alpha K=\nabla_\alpha K,\quad \bar D_{\dot\alpha}K=\bar\nabla_{\dot\alpha}K. \qquad (3.10)$$

We remark that the supersymmetry multiplet of Table I can be truncated (consistently with the supersymmetry algebra) in two ways, by setting to zero either all the fields χ,V,η or all the fields $A,\bar\psi,F$. One would then be left with standard antichiral supersymmetry multiplets given by $(A,\bar\psi,F)$ and (χ,V,η), respectively, corresponding to K and H^α, respectively. Hence, the supersymmetry multiplet of Table I may be regarded as a nontrivial merger of these two multiplets. Alternatively, one can regard it itself as the trun-

[5]Throughout the paper superfields or constituents thereof are called antichiral if they satisfy Eq. (3.7) and (functions of) elementary fields and their derivatives are called antichiral if they fulfill $D_\alpha\phi=0$.

cation of a full complex vector multiplet corresponding to an unconstrained complex scalar superfield.

B. Action

Using the techniques of [3] one can prove that the most general real action for the supersymmetry multiplet of Table I, which is (a) polynomial in the elementary fields and their derivatives, (b) constructible out of field monomials of dimension $\leqslant 4$ (with dimensions as in Table I), (c) Poincaré invariant, and (d) invariant (up to surface terms) under the supersymmetry transformations D_α and $\overline{D}_{\dot\alpha}$ given in Table I, can be written, up to surface terms, in terms of superspace integrals in the form

$$S=\int d^4x(L_1+L_2+L_3+L_4), \tag{3.11}$$

$$L_1=\int d^2\overline{\theta}\{\mu^2K+\text{ c.c.}\}, \tag{3.12}$$

$$L_2=\int d^4\theta\{ia_1G\partial\overline{G}+a_2K\overline{K}$$
$$+(\tfrac{1}{4}a_3G\overline{\mathcal{D}}^2G+\tfrac{1}{2}mGG+\text{ c.c.})\}, \tag{3.13}$$

$$L_3=\int d^4\theta\{(\tfrac{1}{2}b_1GG\overline{K}+\tfrac{1}{2}b_2GGK)+\text{ c.c.}\}, \tag{3.14}$$

$$L_4=\int d^4\theta\tfrac{1}{4}b_3GG\overline{GG}, \tag{3.15}$$

where G^α and K are the superfields given in Eqs. (3.3) and (3.4); μ^2,a_3,m,b_1,b_2 are complex parameters and a_1,a_2,b_3 are real parameters. The action is spelled out explicitly in Appendix C.

Some special features of this general action merit now special consideration. First of all, the terms in Eqs. (3.12)–(3.15) corresponding to the parameters μ^2,m,b_2 give rise to a superpotential $(\mu^2K-mK^2-b_2K^3)$ for the antichiral multiplet $(A,\overline{\psi},F)$ since one has

$$\int d^2\overline{\theta}\mu^2K+\tfrac{1}{2}\int d^4\theta(mGG+b_2GGK)$$
$$\cong\int d^2\overline{\theta}(\mu^2K-mK^2-b_2K^3), \tag{3.16}$$

where $\cong$ denotes equality up to a total derivative. Expression (3.16) together with the kinetic term corresponding to the parameter a_2 constitutes thus nothing but the familiar action of a Wess-Zumino model for the fields A,ψ,F making up the (anti)chiral superfields K, $\overline{K}$. The other terms in the action involve also the fields χ,V,η and in particular couple them to A,ψ,F.

For simplicity we will later not work with the above general action but restrict ourselves to the simpler action

$$\int d^8z(ia_1G\partial\overline{G}+a_2K\overline{K}+\tfrac{1}{2}b_1GG\overline{K}+\tfrac{1}{2}\overline{b_1}\overline{GG}K); \tag{3.17}$$

i.e., we will set to zero the Wess-Zumino superpotential (3.16) as well as the coefficients a_3 and b_3. Furthermore, we will assume

$$a_1\neq 0,\quad a_1+a_2\neq 0, \tag{3.18}$$

since otherwise Eq. (3.17) does not give well-defined propagators for all the fields. $a_1\neq 0$ is imposed since otherwise the kinetic terms of Eq. (3.17) reduce to those of the Wess-Zumino model for A,ψ,F and the remaining fields would not propagate. $a_1+a_2\neq 0$ warrants that Eq. (3.17) have no gauge invariance.

C. Candidate anomalies

By standard arguments, analogous to those used in [1] and applied to the vertex functional (effective action), one concludes from the (classical) supersymmetry algebra (3.1) that at lowest order in $\hbar$ supersymmetry anomalies must satisfy the consistency conditions

$$D_{(\alpha}\Delta_{\beta)}=\overline{D}_{(\dot\alpha}\Delta_{\dot\beta)}=D_\alpha\Delta_{\dot\alpha}+\overline{D}_{\dot\alpha}\Delta_\alpha=0, \tag{3.19}$$

where the contributions Δ_α and $\Delta_{\dot\alpha}$ to such an anomaly are local functionals of the fields. Furthermore, one can assume

$$\Delta_\alpha\neq D_\alpha\Gamma_0,\quad \Delta_{\dot\alpha}\neq\overline{D}_{\dot\alpha}\Gamma_0, \tag{3.20}$$

for any local functional Γ_0 of the fields since otherwise the anomaly can be removed through a local counterterm, at least up to terms of higher order in $\hbar$.

The consistency condition (3.19) and the nontriviality condition (3.20) are most efficiently formulated and analyzed using cohomological techniques. To that end one introduces a ''Becchi-Rouet-Stora-Tyutin'' (BRST) operator s corresponding to the algebra (3.1):

$$s=\xi^\alpha D_\alpha+\overline{\xi}^{\dot\alpha}\overline{D}_{\dot\alpha}+C^a\partial_a+2i\xi\sigma^a\overline{\xi}\frac{\partial}{\partial C^a},$$

where ξ^α are constant commuting supersymmetry ghosts and C^a are constant anticommuting translation ghosts (D_α and $\overline{D}_{\dot\alpha}$ vanish on the ghosts). s is nilpotent and allows us to reformulate Eqs. (3.19) and (3.20) through

$$s\Delta=0,\quad \Delta\neq s\Gamma_0, \tag{3.21}$$

with

$$\Delta=\xi^\alpha\Delta_\alpha+\overline{\xi}^{\dot\alpha}\Delta_{\dot\alpha}.$$

In Eqs. (3.19) and (3.21) it is understoood that the operators (D_α and s, respectively) act on the integrands of the Δ's and Γ_0 and, in general, equalities need to hold only on shell (up to surface terms).

For the model in question two complex solutions of Eq. (3.21) have been given in Sec. 7 of [3]:

$$\Delta_1=\xi^\alpha\int d^4x\overline{D}^2\chi_\alpha=-2\xi^\alpha\int d^4x\,\eta_\alpha,$$
$$\Delta_2=\xi^\alpha\int d^4x\overline{D}^2(\chi_\alpha\overline{\psi}'\overline{\psi}'), \tag{3.22}$$

where $\overline{\psi'}$ is the combination

$$\overline{\psi'_{\dot\alpha}} = \overline{\psi_{\dot\alpha}} + 2i\partial_{\alpha\dot\alpha}\chi^{\alpha}. \tag{3.23}$$

The explicit form of Δ_2 is given in Appendix C. We note that both Δ_1 and Δ_2 give in fact rise to two independent real solutions of Eq. (3.21), given by their real and imaginary parts, respectively.

Using the methods of [3] and extending them to the on-shell problem[6] one can prove that, up to trivial solutions of the form $s\Gamma_0$ and surface terms, the functionals (3.22) and their complex conjugates are indeed the only inequivalent solutions to Eq. (3.21) in our model which have the correct Lorentz transformation properties and are polynomials in all the fields and their derivatives with $\dim(\Delta)\leqslant 4$ [using $\dim(\xi)=-1/2$].

It is evident that both functionals (3.22) indeed solve the first condition (3.21), using the fact that $\overline{\psi'}$ is antichiral, i.e.,

$$D_\alpha \overline{\psi'_{\dot\alpha}} = 0.$$

Furthermore, Δ_1 and Δ_2 are cohomologically nontrivial; i.e., there is no local functional Γ_0 of the fields such that $s\Gamma_0$ equals Δ_1 or Δ_2 on-shell modulo a surface term. This can be verified straightforwardly by an explicit inspection of all the relevant candidates for Γ_0. In fact there are only finitely many such candidates as only functionals need to be considered which have the same dimension as the respective Δ (1 and 4, respectively) and which are Lorentz invariant, thanks to the properties of s.

Without going into details we remark that the presence of candidate supersymmetry anomalies in our model is due to the fact that the representation of the supersymmetry algebra given in Table I of Sec. III A does not have ''QDS structure'' in the terminology of [3], in contrast to more standard representations of supersymmetry. Furthermore, we note that the non-QDS property itself can be traced back to the ''constraint'' (3.8).

Finally we add two comments concerning the consistency condition for supersymmetry anomalies in general and its solutions Δ_1 and Δ_2.

(a) In superspace notation Δ_1 and Δ_2 read

$$\Delta_1 = -\xi^\alpha \int d^8z\,\theta^2 G_\alpha, \quad \Delta_2 = -\xi^\alpha \int d^8z\,\theta^2 G_\alpha \overline{\Psi'}\,\overline{\Psi'}, \tag{3.24}$$

with G_α as in Eq. (3.3) and $\overline{\Psi'}$ being the antichiral superfield whose lowest component field is $\overline{\psi'}$, Eq. (3.23):

$$\overline{\Psi'_{\dot\alpha}} = \exp(\theta D + \overline{\theta D})\overline{\psi'_{\dot\alpha}} = \overline{\mathcal{D}_{\dot\alpha}}K + 2i\partial_{\alpha\dot\alpha}G^{\alpha}.$$

The presence of θ^2 in the integrands in Eq. (3.24) indicates that Δ_1 and Δ_2 cannot be written as superspace integrals $\int d^8z$ (or $\int d^6\bar z$) over true (antichiral) superfields. This shows that in general it would be misleading to formulate the consistency conditions (3.19) and (3.21), respectively, in terms of the operators ∇_α defined in Eq. (2.16) instead of the D_α (recall that the ∇'s represent the supersymmetry transformations only on true superfields).

(b) The dimensions of Δ_1 and Δ_2 indicate that they would play different roles if they would occur in the (anomalous) Jacobian of supersymmetry transformations: Δ_1 has dimension 1 and thus would eventually arise as a *divergent* contribution to that Jacobian, in contrast to Δ_2 which has canonical dimension 4 and is interpreted as a genuine potential anomaly.

[6]This is done efficiently by introducing antifields in the manner of [10].

IV. COMPUTATION OF THE ANOMALY COEFFICIENTS

Let us finally pass to investigate the actual presence of the candidate anomalies (3.22) in our toy model by applying expression (2.8) of the nonlocally regularized form of the anomaly to it. For the sake of simplicity, to illustrate the procedure and results we restrict ourselves to the simple version (3.17) of the general action (3.11).

The structure of the superfield (3.3) and the previous considerations immediately suggest to work with its ''(anti)chiral'' constituents (3.4) and use as a basis to express the matrixlike operators

$$\Phi^A \equiv (\Phi^a, \Phi_{\bar a}) \equiv (H^\alpha, K; \overline{H_{\dot\alpha}}, \overline{K}), \quad \Phi_A \equiv \begin{pmatrix} \Phi_a \\ \Phi^{\bar a} \end{pmatrix} \equiv \begin{pmatrix} H_\alpha \\ K \\ \overline{H^{\dot\alpha}} \\ \overline{K} \end{pmatrix},$$

where latin indices express compactly antichiral (a) and chiral ($\bar a$) components. In terms of these (anti)chiral components, the action (3.17) reads then

$$S = \int d^8z\{ia_1(H^\alpha + \theta^\alpha K)\partial_{\alpha\dot\alpha}(\overline{H^{\dot\alpha}} + \overline{\theta^{\dot\alpha}}\overline{K}) + a_2 K\overline{K} + [\tfrac{1}{2}b_1(H^\alpha H_\alpha + 2H^\alpha\theta_\alpha K + \theta^2K^2)\overline{K} + \text{(c.c.)}]\}. \tag{4.1}$$

As pointed out in Sec. II B (and in many textbooks), the constrained character of these (anti)chiral components requires some reinterpretation of their superspace integration and functional differentiation rules. First of all, the functional derivative rules for (anti)chiral fields (2.15), now reading

$$\frac{\partial\Phi^a}{\partial\Phi^b} = \frac{\partial\Phi_b}{\partial\Phi_a} = \tfrac{1}{2}\mathcal{D}^2\delta^a_b,$$

where δ^a_b encodes, according to the compact notation we are using, a discrete identity as well as the eight-dimensional delta function $\delta^8(z-z')$ in superspace, express nothing but the fact that (anti)chiral fields and operators obtained from functional differentiation with respect to them naturally live in six-dimensional superspace. This fact is conveniently expressed by introducing the projector in the space of antichiral-chiral superfields $(P_q)_A{}^B$:

$$(P_q)_A{}^B = \begin{pmatrix} (P_q)_a{}^b & 0 \\ 0 & (P_q)^{\bar a}{}_{\bar b} \end{pmatrix} = \begin{pmatrix} \tfrac{1}{2}\mathcal{D}^2\delta_a{}^b & 0 \\ 0 & \tfrac{1}{2}\overline{\mathcal{D}^2}\delta^{\bar a}{}_{\bar b} \end{pmatrix},$$

verifying[7]

$$(P_q)_a{}^c(P_q)_c{}^b=\int d^6\overline{z''}\tfrac{1}{2}\mathcal{D}_z^2\delta^8(z-z'')\tfrac{1}{2}\mathcal{D}_{z''}^2\delta^8(z''-z')$$
$$=\tfrac{1}{2}\mathcal{D}_z^2\delta^8(z-z')=(P_q)_a{}^b, \qquad (4.2)$$

and an analogous relation for the chiral sector. ''(Anti)Chiral'' kernels will thus be typically expressed, in compact notation, as

$$M_A{}^B=(P_q)_A{}^C\mathcal{M}_C{}^D(P_q)_D{}^B\equiv(P_q\mathcal{M}P_q)_A{}^B,$$

so that supermatrix multiplication will then yield, according to Eq. (4.2),

$$M_A{}^CN_C{}^B=(P_q\mathcal{M}P_q)_A{}^C(P_q\mathcal{N}P_q)_C{}^B=(P_q\mathcal{M}P_q\mathcal{N}P_q)_A{}^B.$$

The nonlocal regularization of the model (4.1) requires now the identification of the basic quantities involved in the computation, namely, the Jacobian (2.8) of the original transformation, the Hessian of the interaction (2.9), and the regulating objects related to the kinetic operator (2.2). The Jacobian of the original transformation (3.2) adopts in the above basis, according to Eqs. (3.9) and (3.10), the form

$$J_A{}^B=\frac{\partial_r(\delta_{\rm SUSY}\Phi_A)}{\partial\Phi_B}=(\mathcal{J}P_q)_A{}^B=\begin{pmatrix}\tfrac{1}{2}\mathcal{D}^2\mathcal{J}_a{}^b & 0\\ 0 & \tfrac{1}{2}\overline{\mathcal{D}}^2\overline{\mathcal{J}}^{\overline{a}}{}_{\overline{b}}\end{pmatrix}, \qquad (4.3)$$

with its antichiral and chiral sectors given by

$$\mathcal{J}_a{}^b=\begin{pmatrix}\epsilon^{\underline{\alpha}}\nabla_{\underline{\alpha}}\delta_\alpha{}^\beta & \epsilon_\alpha\\ 0 & \epsilon^{\underline{\alpha}}\nabla_{\underline{\alpha}}\end{pmatrix},\quad \overline{\mathcal{J}}^{\overline{a}}{}_{\overline{b}}=\begin{pmatrix}\epsilon^{\underline{\alpha}}\nabla_{\underline{\alpha}}\delta^{\dot\alpha}{}_{\dot\beta} & \overline{\epsilon}^{\dot\alpha}\\ 0 & \epsilon^{\underline{\alpha}}\nabla_{\underline{\alpha}}\end{pmatrix}.$$

In an analogous way, the Hessian of the interaction term in Eq. (4.1) results in $I_A{}^B=(P_q\mathcal{I}P_q)_A{}^B$, with the ''naive'' Hessian $\mathcal{I}_A{}^B$ expressed as

$$\mathcal{I}_A{}^B=\begin{pmatrix}b_1\delta_\alpha{}^\beta\overline{K} & b_1\theta_\alpha\overline{K} & 0 & b_1G_\alpha\\ b_1\theta^\beta\overline{K} & b_1\theta^2\overline{K} & \overline{b_1G_{\dot\beta}} & (b_1G^\alpha\theta_\alpha+\overline{b_1G_{\dot\alpha}}\,\overline{\theta^{\dot\alpha}})\\ 0 & \overline{b_1G^{\alpha}} & \overline{b_1}\delta^{\dot\alpha}{}_{\dot\beta}K & \overline{b_1\theta^\alpha}K\\ b_1G^\beta & (b_1G^\alpha\theta_\alpha+\overline{b_1G_{\dot\alpha}}\,\overline{\theta^{\alpha}}) & \overline{b_1\theta_{\dot\beta}}K & \overline{b_1\theta^2}K\end{pmatrix}.$$

Finally, the kinetic operator is found to be $F_A{}^B=(P_q\mathcal{F}P_q)_A{}^B$, with the ''naive'' kinetic term $\mathcal{F}_A{}^B$ given by

$$\mathcal{F}_A{}^B=\begin{pmatrix}0 & 0 & ia_1\partial_{\alpha\dot\beta} & ia_1\partial_{\alpha\dot\beta}\overline{\theta^\beta}\\ 0 & 0 & ia_1\theta^\alpha\partial_{\alpha\dot\beta} & a_2+ia_1\theta\partial\overline{\theta}\\ ia_1\partial^{\dot\alpha\beta} & ia_1\partial^{\dot\alpha\beta}\theta_\beta & 0 & 0\\ ia_1\overline{\theta_{\dot\alpha}}\partial^{\dot\alpha\beta} & a_2+ia_1\overline{\theta}\partial\theta & 0 & 0\end{pmatrix}.$$

Introducing then as operator T^{-1} the free propagator of the model in superspace up to $(-\Box)^{-1}$, namely, $(T^{-1})_A{}^B=(P_q\mathcal{T}^{-1}P_q)_A{}^B$ with

$$(\mathcal{T}^{-1})_A{}^B=\frac{-1}{4(a_1+a_2)}\begin{pmatrix}0 & 0 & \left(\frac{i(a_1+2a_2)}{2a_1\Box}\partial_{\alpha\dot\beta}-\theta_\alpha\overline{\theta_{\dot\beta}}\right) & \theta_\alpha\\ 0 & 0 & \overline{\theta_{\dot\beta}} & -1\\ \left(\frac{i(a_1+2a_2)}{2a_1\Box}\partial^{\dot\alpha\beta}-\overline{\theta^{\alpha}}\theta^\beta\right) & \overline{\theta^{\alpha}} & 0 & 0\\ \theta^\beta & -1 & 0 & 0\end{pmatrix},$$

a suitable regulator, diagonal and quadratic in space-time derivatives, arises:

$$R_A{}^B=-\Box(P_q)_A{}^B.$$

In this way the corresponding smearing and shadow kinetic operators (2.3) and (2.4), adapted to the chiral case, result in

$$(\varepsilon^2)_A{}^B=\varepsilon^2(P_q)_A{}^B,\quad \mathcal{O}_A{}^B=\hat\sigma(P_q\mathcal{T}^{-1}P_q)_A{}^B,$$

with ε^2 and $\hat\sigma$ defined as

$$\varepsilon^2=\exp(-\Box/\Lambda^2),\quad \hat\sigma=\int_0^1\frac{dt}{\Lambda^2}\exp(-t\Box/\Lambda^2).$$

[7]Recall that matrix multiplication among projectors P must be performed using an integration in the corresponding six-dimensional superspaces, i.e., either $\int d^6\overline{z}$ or $\int d^6z$.

The form of the candidate anomalies (3.22), involving only either products of antichiral fields H^α, K, or of chiral fields $\overline{H^\alpha}$, $\overline{K}$, but no crossed terms, indicates that the evaluation of their coefficients by means of the supertrace (2.8) can now be considerably simplified by considering, for instance, only the antichiral sector, i.e., by neglecting the fields $\overline{H^\alpha}$ and $\overline{K}$, and by further restricting the computation to only linear and trilinear terms in H^α, K, namely, to the first- and third-order interaction terms.[8] The coefficients coming from the chiral sector contributions can then be automatically determined by complex conjugation. Therefore, from now on we are going to concentrate our attention on the terms

$$\tilde{\mathcal{A}}_n=[(-1)^A(\varepsilon^2)_A{}^B J_B{}^C(\mathcal{O}_C{}^D I_D{}^A)^n]_{\rm anti} \quad \text{for } n=1,3, \tag{4.4}$$

where the subscript ''anti'' indicates that all terms involving $\overline{H^\alpha}$ and $\overline{K}$ are neglected.

Our main task shall now consist of determining the diagonal entries of the matrix involved in expression (4.4). First of all, the nth power of the matrix $\mathcal{O}_A{}^C I_C{}^B$ reads, under the above restrictions,

$$(\mathcal{O}_A{}^C I_C{}^B)^n_{\rm anti}=\begin{pmatrix} (\mathcal{O}I_n)_a{}^b & \cdots \\ 0 & (\mathcal{O}I_n)^{\bar a}{}_{\bar b}\end{pmatrix}.$$

Its diagonal blocks — the relevant ones taking into account the block diagonal form of the Jacobian (4.3) — can be easily found by using the commutation relation

$$[\tfrac{1}{2}\mathcal{D}^2,\theta_\alpha]=\mathcal{D}_\alpha, \tag{4.5}$$

resulting in

$$(\mathcal{O}I_n)_a{}^b=\tfrac{1}{2}\mathcal{D}^2\begin{pmatrix}\theta_\alpha(S^\gamma\mathcal{D}_\gamma)^{n-1} & -\theta_\alpha(S^\gamma\mathcal{D}_\gamma)^{n-1}\theta_\beta \\ -(S^\gamma\mathcal{D}_\gamma)^{n-1} & (S^\gamma\mathcal{D}_\gamma)^{n-1}\theta_\beta\end{pmatrix}S^\beta\tfrac{1}{2}\mathcal{D}^2,$$

$$(\mathcal{O}I_n)^{\bar a}{}_{\bar b}=\tfrac{1}{2}\overline{\mathcal{D}^2}\begin{pmatrix}0 & \cdots \\ 0 & (\mathcal{D}_\gamma\mathcal{G}^\gamma)^n\end{pmatrix}\tfrac{1}{2}\overline{\mathcal{D}^2},$$

in terms of the quantities S^α, $\mathcal{G}^\alpha$ defined as

$$S^\alpha=\tfrac{1}{2}\overline{\mathcal{D}^2}\mathcal{G}^\alpha, \quad \mathcal{G}^\alpha=\left(\frac{-b_1}{4(a_1+a_2)}\right)\hat{\sigma}G^\alpha, \tag{4.6}$$

where all the operators are understood to act on everything on their right. Terms indicated by ellipses in the above matrices turn out to be irrelevant for the present computation.

Afterwards, straightforward matrix multiplication yields

$$\text{diag}[(\varepsilon^2)_A{}^B J_B{}^C(\mathcal{O}_C{}^D I_D{}^E)^n]_{\rm anti}=((A_n)_\alpha{}^\beta,A_n;0,C_n),$$

where the expressions for the antichiral sector operators are found to be, upon use of the commutation relation $[\epsilon^{\underline{\beta}}\nabla_{\underline{\beta}},\theta_\alpha]=\epsilon_\alpha$,

$$(A_n)_\alpha{}^\beta=\varepsilon^2\tfrac{1}{2}\mathcal{D}^2[\epsilon^{\underline{\delta}}\nabla_{\underline{\delta}}\theta_\alpha-\epsilon_\alpha](S^\gamma\mathcal{D}_\gamma)^{n-1}S^\beta\tfrac{1}{2}\mathcal{D}^2$$
$$=\varepsilon^2\tfrac{1}{2}\mathcal{D}^2\theta_\alpha\epsilon^{\underline{\delta}}\nabla_{\underline{\delta}}(S^\gamma\mathcal{D}_\gamma)^{n-1}S^\beta\tfrac{1}{2}\mathcal{D}^2,$$

$$A_n=\varepsilon^2\tfrac{1}{2}\mathcal{D}^2\epsilon^{\underline{\delta}}\nabla_{\underline{\delta}}(S^\gamma\mathcal{D}_\gamma)^{n-1}\theta_\beta S^\beta\tfrac{1}{2}\mathcal{D}^2, \tag{4.7}$$

whereas the chiral sector operator is directly given by

$$C_n=\varepsilon^2\epsilon^{\underline{\delta}}\nabla_{\underline{\delta}}\tfrac{1}{2}\overline{\mathcal{D}^2}\mathcal{D}_{\alpha_1}\mathcal{G}^{\alpha_1}\tfrac{1}{2}\overline{\mathcal{D}^2}\cdots\tfrac{1}{2}\overline{\mathcal{D}^2}\mathcal{D}_{\alpha_n}\mathcal{G}^{\alpha_n}\tfrac{1}{2}\overline{\mathcal{D}^2}. \tag{4.8}$$

The general expression of $\tilde{\mathcal{A}}_n$, Eq. (4.4), is thus

$$\tilde{\mathcal{A}}_n=\text{Tr}[-(A_n)_\alpha{}^\alpha+A_n]+\overline{\text{Tr}}[C_n], \tag{4.9}$$

where the extra minus sign comes from taking the discrete trace over the fermionic fields, while the symbols Tr and $\overline{\text{Tr}}$ stand, respectively, for the functional traces in the antichiral and chiral superspaces, namely,

$$\text{Tr}[A]=\int d^6\bar{z}A(z,z')|_{\bar z=\bar z'}, \quad \overline{\text{Tr}}[C]=\int d^6zC(z,z')|_{z=z'}. \tag{4.10}$$

Upon substitution of expressions (4.7) and (4.8), both traces in Eq. (4.9) are then seen to share similar structures. However, there is the fundamental difference that such functional traces are taken in different superspaces, according to Eq. (4.10). Therefore, in order to compare both expressions, some mechanism should be found to relate supertraces of antichiral expressions to those of chiral ones. Fortunately, it is not difficult to verify, as shown in Appendix D, that for chiral operators $\overline{\mathcal{A}}$, namely, those verifying $\overline{\mathcal{D}}_{\dot\alpha}\overline{\mathcal{A}}=0$, the following relation holds:

$$\text{Tr}[\tfrac{1}{2}\mathcal{D}^2\overline{\mathcal{A}}\tfrac{1}{2}\mathcal{D}^2]=\overline{\text{Tr}}[\overline{\mathcal{A}}\tfrac{1}{2}\mathcal{D}^2\tfrac{1}{2}\overline{\mathcal{D}^2}]. \tag{4.11}$$

Using this result as well as the commutation relation (4.5) and the cyclic property of the regulated trace, the antichiral sector contribution $\text{Tr}[-(A_n)_\alpha{}^\alpha+A_n]$ to Eq. (4.9) can be rewritten in chiral form as

$$\text{Tr}[-(A_n)_\alpha{}^\alpha+A_n]=\overline{\text{Tr}}[B_n], \tag{4.12}$$

with the operator B_n given by

$$B_n=\varepsilon^2\epsilon^{\underline{\delta}}\nabla_{\underline{\delta}}\tfrac{1}{2}\overline{\mathcal{D}^2}\mathcal{G}^{\alpha_1}\mathcal{D}_{\alpha_1}\tfrac{1}{2}\overline{\mathcal{D}^2}\cdots\tfrac{1}{2}\overline{\mathcal{D}^2}\mathcal{G}^{\alpha_n}\mathcal{D}_{\alpha_n}\tfrac{1}{2}\overline{\mathcal{D}^2},$$

after substitution of S^α by its explicit expression (4.6). In this way, B_n is seen to ''almost'' coincide with C_n, Eq. (4.8), when reading it from the right to the left.

This similarity may conveniently be exploited by using the property that the traces of an operator and of its transpose coincide. Combining further this fact with the cyclic property of regulated traces, the following relations are seen to hold:

[8]This restriction is indeed sufficient even though candidate anomalies are defined only modulo trivial solutions of the consistency conditions. The reason is that the supersymmetry transformations of Table I are linear and do not mix the fields of the chiral and antichiral sectors.

$$\overline{\mathrm{Tr}}[B_n]=\overline{\mathrm{Tr}}[\varepsilon^2\epsilon^{\dot\delta}\nabla_{\dot\delta}\tfrac{1}{2}\overline{\mathcal{D}^2}\mathcal{G}^{\alpha_1}\mathcal{D}_{\alpha_1}\tfrac{1}{2}\overline{\mathcal{D}^2}\cdots\tfrac{1}{2}\overline{\mathcal{D}^2}\mathcal{G}^{\alpha_n}\mathcal{D}_{\alpha_n}\tfrac{1}{2}\overline{\mathcal{D}^2}]$$
$$=\mathrm{Tr}[\tfrac{1}{2}\overline{\mathcal{D}^2}\mathcal{G}^{\alpha_1}\mathcal{D}_{\alpha_1}\tfrac{1}{2}\overline{\mathcal{D}^2}\cdots\tfrac{1}{2}\overline{\mathcal{D}^2}\mathcal{G}^{\alpha_n}\mathcal{D}_{\alpha_n}\tfrac{1}{2}\overline{\mathcal{D}^2}\epsilon^{\dot\delta}\nabla_{\dot\delta}\varepsilon^2]$$
$$=-(-)^{2n}\overline{\mathrm{Tr}}[\varepsilon^2\epsilon^{\dot\delta}\nabla_{\dot\delta}\tfrac{1}{2}\overline{\mathcal{D}^2}\mathcal{D}_{\alpha_1}\mathcal{G}^{\alpha_1}$$
$$\times\tfrac{1}{2}\ \overline{\mathcal{D}^2}\cdots\tfrac{1}{2}\ \overline{\mathcal{D}^2}\mathcal{D}_{\alpha_n}\mathcal{G}^{\alpha_n}\tfrac{1}{2}\overline{\mathcal{D}^2}]$$
$$=-\overline{\mathrm{Tr}}[C_n],$$

so that the contribution coming from the antichiral sector, $\overline{\mathrm{Tr}}[B_n]$, Eq. (4.12), is seen to exactly cancel that coming from the chiral sector, $\mathrm{Tr}[C_n]$, for all n. The present computation leads thus to the vanishing of $\widetilde{\mathcal{A}}_n$, Eq. (4.9), for all n and, with it, of the potential anomalies of our model. Therefore, we conclude that the latter, potentially present on cohomological grounds, actually do not show up in the model we have analyzed at the one-loop level. We have also checked that this remains valid for supersymmetric actions which differ from Eq. (3.17) and arise from Eq. (3.11) by turning on other (combinations of) coefficients such as a_3, m, or b_3. However, we have not performed the computation for the most general action (3.11), as the main purpose of considering the toy model was the illustration of the method outlined in Sec. II.

V. CONCLUSION

The purpose of this paper is to show that implementation of superspace techniques in the framework of nonlocal regularization constitutes a suitable and efficient tool to analyze anomaly issues. To outline and illustrate the method, we have applied it to a toy model whose supersymmetry, by cohomological arguments, is potentially anomalous, but turns out to be actually nonanomalous at the one-loop level. As a by-product, the result of the computation gives further evidence that the remarkable quantum stability of supersymmetry even extends to models which admit nontrivial solutions of the consistency condition for supersymmetry anomalies. Finally, although not proved, our construction also points to nonlocal regularization as a possible candidate for a supersymmetric invariant regularization method.

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APPENDIX A: CONVENTIONS AND NOTATION

1. Lorentz- (SL(2,C)-) invariant tensors

Minkowski metric, ε tensors

$$\eta_{ab}=\mathrm{diag}(1,-1,-1,-1),\quad \varepsilon^{abcd}=\varepsilon^{[abcd]},\quad \varepsilon^{0123}=1,$$
$$\varepsilon^{\alpha\beta}=-\varepsilon^{\beta\alpha},\quad \varepsilon^{\dot\alpha\dot\beta}=-\varepsilon^{\dot\beta\dot\alpha},\quad \varepsilon^{12}=\varepsilon^{\dot1\dot2}=1,$$
$$\varepsilon_{\alpha\gamma}\varepsilon^{\gamma\beta}=\delta_\alpha{}^\beta=\mathrm{diag}(1,1),\quad \varepsilon_{\dot\alpha\dot\gamma}\varepsilon^{\dot\gamma\dot\beta}=\delta^{\dot\beta}{}_{\dot\alpha}=\mathrm{diag}(1,1).$$

σ matrices $\sigma^a{}_{\alpha\dot\beta}$ (α, row index; $\dot\beta$, column index)

$$\sigma^0=\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad \sigma^1=\begin{pmatrix}0&1\\1&0\end{pmatrix},$$
$$\sigma^2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\quad \sigma^3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}.$$

$\overline{\sigma}$ matrices

$$\overline{\sigma^a}^{\dot\alpha\alpha}=\varepsilon^{\dot\alpha\dot\beta}\varepsilon^{\alpha\beta}\sigma^a{}_{\beta\dot\beta},$$

and $\sigma^{ab},\overline{\sigma^{ab}}$ matrices

$$\sigma^{ab}{}_\alpha{}^\beta=\tfrac{1}{4}(\sigma^a\overline{\sigma^b}-\sigma^b\overline{\sigma^a})_\alpha{}^\beta,\quad \overline{\sigma^{ab}}^{\dot\alpha}{}_{\dot\beta}=\tfrac{1}{4}(\overline{\sigma^a}\sigma^b-\overline{\sigma^b}\sigma^a)^{\dot\alpha}{}_{\dot\beta}.$$

2. Spinors, grading, and complex conjugation

We work with two-component Weyl spinors. Undotted and dotted spinor indices α and $\dot\alpha$ distinguish the $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$ representations of SL(2,**C**) related by complex conjugation. Raising and lowering of spinor indices:

$$\psi_\alpha=\varepsilon_{\alpha\beta}\psi^\beta,\quad \psi^\alpha=\varepsilon^{\alpha\beta}\psi_\beta,\quad \overline{\psi}_{\dot\alpha}=\varepsilon_{\dot\alpha\dot\beta}\overline{\psi^{\beta}},\quad \overline{\psi^{\alpha}}=\varepsilon^{\dot\alpha\dot\beta}\overline{\psi}_{\dot\beta}.$$

Contraction of spinor indices:

$$\psi\chi:=\psi^\alpha\chi_\alpha,\quad \overline{\psi\chi}:=\overline{\psi}_{\dot\alpha}\overline{\chi}^{\dot\alpha}.$$

Lorentz vector indices in spinor notation:

$$V_{\alpha\dot\alpha}=\sigma^a{}_{\alpha\dot\alpha}V_a.$$

The grading (Grassmann parity) $|X|$ of a field or an operator X is determined by the number of its spinor indices and its ghost number (gh):

$$|X^{\dot\alpha_1\cdots\dot\alpha_m}_{\alpha_1\cdots\alpha_n}|=m+n+\mathrm{gh}(X)\quad (\mathrm{mod}2).$$

The grading of the fields ϕ^i determines their statistics,

$$\phi^i\phi^j=(-)^{|\phi^i||\phi^j|}\phi^j\phi^i.$$

Complex conjugation of a field or operator X is denoted by $\overline{X}$. Complex conjugation of products of fields and operators is defined by

$$\overline{XY}=(-)^{|X||Y|}\overline{X}\,\overline{Y}.$$

In particular this implies

$$\overline{\partial/\partial\phi}=(-)^{|\phi|}\partial/\partial\overline{\phi}$$

and thus the minus sign in front of $\partial/\partial\overline{\theta}$ in Eqs. (2.16) and (2.14).

3. Superspace conventions and useful identities

θ^α and $\overline{\theta^\alpha}$ are odd graded, constant, and related by complex conjugation. Superspace integration

$$\int d\theta\theta=\int d\bar{\theta}\bar{\theta}=1,\quad \int d^2\theta=\int d\theta^2 d\theta^1,$$

$$\int d^2\bar{\theta}=\int d\bar{\theta}^1 d\bar{\theta}^2,\quad \int d^4\theta=\int d^2\theta d^2\bar{\theta},$$

$$\int d^6 z=\int d^4x d^2\theta,\quad \int d^6\bar{z}=\int d^4x d^2\bar{\theta},$$

$$\int d^8 z=\int d^4x d^4\theta.$$

δ functions

$$\delta^2(\theta-\theta')=-\tfrac{1}{2}(\theta-\theta')^2,$$

$$\delta^2(\bar{\theta}-\bar{\theta}')=-\tfrac{1}{2}(\bar{\theta}-\bar{\theta}')^2,$$

$$\delta^6(z-z')=\delta^2(\theta-\theta')\delta^4(x-x'),$$

$$\delta^6(\bar{z}-\bar{z}')=\delta^2(\bar{\theta}-\bar{\theta}')\delta^4(x-x'),$$

$$\delta^8(z-z')=\delta^2(\theta-\theta')\delta^2(\bar{\theta}-\bar{\theta}')\delta^4(x-x').$$

Useful identities

$$\exp(\theta D+\overline{\theta D})=\exp(i\theta\partial\bar{\theta})\exp(\theta D)\exp(\overline{\theta D})$$
$$=\exp(-i\theta\partial\bar{\theta})\exp(\overline{\theta D})\exp(\theta D),\quad \text{(A1)}$$

$$\mathcal{D}_{\underline{\alpha}}\exp(\theta D+\overline{\theta D})=\exp(\theta D+\overline{\theta D})D_{\underline{\alpha}}.$$

θ integrations over superfields (B2) thus result in

$$\int d^2\theta\exp(\theta D+\overline{\theta D})f(\phi,\partial\phi,\ldots)$$
$$\cong\tfrac{1}{2}D^2\exp(\overline{\theta D})f(\phi,\partial\phi,\ldots),$$

$$\int d^4\theta\exp(\theta D+\overline{\theta D})f(\phi,\partial\phi,\ldots)$$
$$\cong\tfrac{1}{4}D^2\bar{D}^2 f(\phi,\partial\phi,\ldots),$$

where $\cong$ denotes equality up to a total derivative.

APPENDIX B: SUPERFIELDS AND CONSTITUENTS

In this appendix, we briefly review the construction of superfields out of ordinary fields for given supersymmetry transformations of the latter according to the conventions used in this paper. As usual we implement the supersymmetry transformations on superfields through the operators ∇_α, $\bar{\nabla}_{\dot{\alpha}}$, Eq. (2.16). Then, given a (linear) representation $D_\alpha, D_{\dot{\alpha}}$ of the supersymmetry algebra (3.1) on ordinary fields ϕ^i such as in Table I of Sec. III A, superfields are defined as functions Σ of the θ^α, $\bar{\theta}^{\dot{\alpha}}$, ϕ^i and of the derivatives of the ϕ^i, $\Sigma=\Sigma(\theta,\bar{\theta},\phi,\partial\phi,\ldots)$, satisfying

$$D_\alpha\Sigma=\nabla_\alpha\Sigma,\quad \bar{D}_{\dot{\alpha}}\Sigma=\bar{\nabla}_{\dot{\alpha}}\Sigma, \tag{B1}$$

where D_α and $\bar{D}_{\dot{\alpha}}$ act nontrivially only on the ϕ^i and their derivatives and anticommute with all the θ's and $\bar{\theta}$'s. The operators ∇_α, $\bar{\nabla}_{\dot{\alpha}}$, Eq. (2.16), provide then a representation of the supersymmetry algebra (3.1) with $(P_a, Q_\alpha, \bar{Q}_{\dot{\alpha}})\equiv(-\partial_a, -\nabla_\alpha, -\bar{\nabla}_{\dot{\alpha}})$. Note that $\nabla_\alpha\Sigma$ is *not* a superfield since its $\bar{D}_{\dot{\alpha}}$ transformation is not given by $\bar{\nabla}_{\dot{\alpha}}\nabla_\alpha\Sigma$, but rather by

$$\bar{D}_{\dot{\alpha}}\nabla_\alpha\Sigma=-\nabla_\alpha\bar{D}_{\dot{\alpha}}\Sigma=-\nabla_\alpha\bar{\nabla}_{\dot{\alpha}}\Sigma.$$

Instead, and in contrast to the ∇'s, the standard ''covariant'' derivatives $\mathcal{D}_\alpha$, $\bar{\mathcal{D}}_{\dot{\alpha}}$, Eq. (2.14), map superfields to superfields because they anticommute both with the D's and with the ∇'s.

Having characterized superfields abstractly by Eq. (B1), we can now construct them explicitly: Any superfield, i.e., any solution of Eq. (B1), can be written in the form

$$\Sigma=\exp(\theta D+\overline{\theta D})f(\phi,\partial\phi,\ldots), \tag{B2}$$

where $f(\phi,\partial\phi,\ldots)$ is a function of the (ordinary) fields and their derivatives and we used the summation conventions $\theta D=\theta^\alpha D_\alpha$ and $\overline{\theta D}=\bar{\theta}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}$. The proof of this statement is straightforward using that (i) Eq. (B2) satisfies Eq. (B1) for any $f(\phi,\partial\phi,\ldots)$, as can be easily checked directly, and (ii) any nonvanishing superfield has a nonvanishing θ-independent part which is required by Eq. (B1). The assertion is now proved as follows: Given a nonvanishing solution Σ of Eq. (B1) with θ-independent part $f(\phi,\partial\phi,\ldots)$ we consider $\Sigma'=\Sigma-\exp(\theta D+\overline{\theta D})f(\phi,\partial\phi,\ldots)$. The latter is a superfield due to (i) and must vanish due to (ii) since by construction it has no θ-independent part.

APPENDIX C: LAGRANGIAN AND CANDIDATE ANOMALY IN EXPLICIT FORM

The various parts (3.12)–(3.15) of the general Lagrangian read, explicitly (up to total derivatives),

$$\int d^2\bar{\theta}K\cong -F,$$

$$\int d^4\theta iG\partial\bar{G}\cong i\eta\partial\bar{\eta}-i\psi\partial\bar{\psi}+2\psi\Box\chi+2\bar{\psi}\Box\bar{\chi}-4i\chi\Box\partial\bar{\chi}-4A\Box\bar{A}-4(\partial_a V^a)\partial_b\bar{V}^b+2F_{ab}\bar{F}^{ab}+2iF\partial_a\bar{V}^a-2i\bar{F}\partial_a V^a,$$

$$\int d^4\theta K\bar{K}\cong -4A\Box\bar{A}-2i\bar{\psi}\partial\psi+F\bar{F},$$

$$\int d^4\theta \tfrac{1}{4} G\overline{\mathcal{D}^2}G \cong 2i\eta\partial\overline{\psi} + 4\eta\Box\chi - 4V_a\Box V^a - F^2 - 4iF\partial_a V^a,$$

$$\int d^4\theta GG \cong -2\overline{\psi\psi} + 4FA,$$

$$\int d^4\theta \tfrac{1}{2} GG\overline{K} \cong -\overline{A\psi\psi} - 2\chi\chi\Box\overline{A} + 2i\chi\sigma^a\overline{\psi}\partial_a\overline{A} - A\eta\psi + V^{\alpha\dot\alpha}(\overline{\psi}_{\dot\alpha}\psi_\alpha - 2i\chi_\alpha\partial_{\beta\dot\alpha}\psi^\beta)$$
$$-4iAV^a\partial_a\overline{A} - \overline{F}(V^aV_a - \chi\eta) + F(2A\overline{A} - \chi\psi),$$

$$\int d^4\theta \tfrac{1}{6} GGK \cong -A\overline{\psi\psi} + A^2F,$$

$$\int d^4\theta \tfrac{1}{4} GG\overline{GG} \cong \tfrac{1}{2}A^2\overline{A^2} + V^a\overline{V_a}A\overline{A} + \tfrac{1}{1}2V^aV_a\overline{V^b}\,\overline{V_b} - F(\overline{\chi\overline{V}}\chi - A\overline{\chi\chi}) - \chi\eta\overline{V_a}\,\overline{V^a} + \chi V\overline{\eta\overline{A}} - \chi\sigma^a\overline{\sigma^b}\psi V_a\overline{V_b} - \chi\overline{V\psi A}$$
$$-2\chi\psi A\overline{A} + i(A\chi)\partial(\overline{\chi\overline{A}}) - 2iAV^a\partial_a(\overline{\chi\chi}) - i(V^{\dot\alpha\beta}\chi_\beta)\partial_{\alpha\dot\alpha}(\overline{\chi_{\dot\beta}\overline{V}^{\beta\alpha}}) + \tfrac{1}{2}\chi\eta\overline{\chi\eta} + \tfrac{1}{2}\chi\psi\overline{\chi\overline{\psi}} - \tfrac{1}{2}(\chi\chi)(\psi\psi)$$
$$+i\chi\chi\partial_a(\psi\sigma^a\overline{\chi}) - (\chi\chi)\Box(\overline{\chi\chi}) + \text{c.c.},$$

with $F_{ab} = \partial_a V_b - \partial_b V_a$ and $\Box = \partial_a\partial^a = \frac{1}{2}\partial_{\alpha\dot\alpha}\partial^{\dot\alpha\alpha}$.

The integrand of the candidate anomaly Δ_2 in Eq. (3.22) reads, explicitly,

$$\tfrac{1}{2}\overline{D^2}(\xi\chi\overline{\psi'}\,\overline{\psi'}) = \xi\chi\{2i\varepsilon^{abcd}F_{ab}F_{cd} + 8(\partial_aV^a)\partial_bV^b + 4F_{ab}F^{ab} - 2F^2 - 8iF\partial_aV^a + 4i\overline{\psi'}\partial\eta\} - \xi\eta\overline{\psi'}\,\overline{\psi'} - 2\xi V\overline{\psi'}F$$
$$-4i\xi\sigma^a\overline{\sigma^b}\sigma^c\overline{\psi'}V_a\partial_cV_b.$$

APPENDIX D: PROOF OF RELATION (4.11)

In the perturbative computation of the anomaly coefficients performed in Sec. IV, relation (4.11) has been seen to be crucial in checking their vanishing. In this appendix, we prove that relation.

Consider a generic chiral operator $\overline{\mathcal{A}}$, namely, an object verifying $\mathcal{D}_{\dot\alpha}\overline{\mathcal{A}} = 0$, and a typical trace over this quantity of the form

$$\text{Tr}[\tfrac{1}{2}\mathcal{D}^2\overline{\mathcal{A}}\tfrac{1}{2}\mathcal{D}^2] = \int d^6\overline{z}[\tfrac{1}{2}\mathcal{D}^2\overline{\mathcal{A}}\tfrac{1}{2}\mathcal{D}^2\delta^8(z-z')]|_{\overline{z}=\overline{z'}} = \int d^6\overline{z}d^6\overline{z'}[\tfrac{1}{2}\mathcal{D}^2\overline{\mathcal{A}}\tfrac{1}{2}\mathcal{D}^2\delta^8(z-z')][\tfrac{1}{2}\mathcal{D}^2\delta^8(z'-z)]. \tag{D1}$$

The identity for chiral expressions,

$$\overline{\mathcal{A}}(z) = \int d^6z''\tfrac{1}{2}\overline{\mathcal{D}^2}\delta^8(z-z'')\overline{\mathcal{A}}(z''),$$

allows us to rewrite Eq. (D1) as

$$\int d^6\overline{z}d^6\overline{z'}d^6z''[\tfrac{1}{2}\mathcal{D}^2\tfrac{1}{2}\overline{\mathcal{D}^2}\delta^8(z-z'')][\overline{\mathcal{A}}(z'')\tfrac{1}{2}\mathcal{D}^2\delta^8(z''-z')][\tfrac{1}{2}\mathcal{D}^2\delta^8(z'-z)]$$
$$= \int d^6\overline{z}d^6z''[\overline{\mathcal{A}}(z'')\tfrac{1}{2}\mathcal{D}^2\delta^8(z''-z)][\tfrac{1}{2}\mathcal{D}^2\tfrac{1}{2}\overline{\mathcal{D}^2}\delta^8(z-z'')], \tag{D2}$$

where in writing the second expression use has been made of the property (4.2) for the antichiral projector $\frac{1}{2}\mathcal{D}^2$. By exactly the same arguments, Eq. (D2) can be further rewritten as

$$\int d^6\overline{z}d^6z'd^6z''[\tfrac{1}{2}\overline{\mathcal{D}^2}\delta^8(z''-z')][\overline{\mathcal{A}}(z')\tfrac{1}{2}\mathcal{D}^2\delta^8(z'-z)][\tfrac{1}{2}\mathcal{D}^2\tfrac{1}{2}\overline{\mathcal{D}^2}\delta^8(z-z'')]$$
$$= \int d^6z'd^6z''[\tfrac{1}{2}\overline{\mathcal{D}^2}\delta^8(z''-z')][\overline{\mathcal{A}}(z'')\tfrac{1}{2}\mathcal{D}^2\tfrac{1}{2}\overline{\mathcal{D}^2}\delta^8(z''-z')]$$
$$= \int d^6z[\overline{\mathcal{A}}(z)\tfrac{1}{2}\mathcal{D}^2\tfrac{1}{2}\overline{\mathcal{D}^2}\delta^8(z-z')]|_{z=z'} = \overline{\text{Tr}}[\overline{\mathcal{A}}\tfrac{1}{2}\mathcal{D}^2\tfrac{1}{2}\overline{\mathcal{D}^2}],$$

which finally shows the fulfillment of relation (4.11).

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