Generalized string models and their semiclassical approximation

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We construct an extensive family of Bose string models, all of them classically equivalent to the Nambu and Eguchi models. The new models involve an arbitrary analytical function $f(u)$, with $f(0)=0$, and are based on the Brink-Di Vecchia-Howe and Polyakov string action. The semiclassical approximation of the models is worked out in detail.

I. INTRODUCTION

Various string models are known at present. What in the beginning was a phenomenological device, whose aim consisted in explaining the remarkable regularities of Regge trajectories of hadron resonances,[1] has now become a serious candidate for the string of glue connecting quark-antiquark pairs in, say, an SU(3) gauge theory.[2] An impressive amount of work has been done in this new direction (for recent reviews see Refs. 3—5). In particular, the following Bose string models have been studied:

Nambu-Goto[6] $$I_N=M^2\int_{\mathscr{D}}\sqrt{h}\,d^2z\ , \tag{1}$$

Eguchi[7] $$I_E^{(\nu)}=M^{4\nu}\int_{\mathscr{D}}h^\nu d^2z\ , \tag{2}$$

Brink-Di Vecchia-Howe[8] and Polyakov[9]

$$I_G=\tfrac{1}{2}M^2\int_{\mathscr{D}}\sqrt{g}\,g^{ab}h_{ab}d^2z\ , \tag{3}$$

where in all cases

$$M^{-2}=2\pi\alpha',\ \ h=\det h_{ab},\ \ h_{ab}=\partial_a x^\mu\partial_b x^\mu\ ,\qquad \partial_a=\frac{\partial}{\partial z^a},\ \ a=1,2\ . \tag{4}$$

In the Euclidean formulation (which will be used throughout) $\mathscr{D}$ is a compact orientable two-dimensional manifold with boundary $\partial\mathscr{D}$, while $x:\mathscr{D}\to\mathrm{R}^d$ is an embedding of $\mathscr{D}$ in d-dimensional Euclidean space (so $\mu=1,2,\ldots,d$). Summing up, z^a are local coordinates on $\mathscr{D}$, and $x^\mu(z)$ defines the embedding. g^{ab} is the inverse of a new metric tensor g_{ab} (i.e., $g_{ab}g^{bc}=\delta_a^c$), which has nothing to do with h_{ab}, and is taken as an independent variable. Also, $g=\det g_{ab}$.

When use is made of (1)—(3) as phenomenological models for the Wilson loop $\mathscr{C}$ (a single, noninteracting closed contour in R^d), the image of $\partial\mathscr{D}$ by the embedding is identified with $\mathscr{C}$. On the other hand, we shall only consider the case when the embedding of $\mathscr{D}$ in R^d does not have holes or handles (for a complete study with general topology see Ref. 5). Strings with fermions[10] will also not be considered in this work.

The models (1)—(3) above lead to the classical equations of motion

$$\partial_a(\sqrt{h}\,h^{ab}\partial_b x^\mu)=0\ , \tag{5}$$

$$\partial_a(h^\nu h^{ab}\partial_b x^\mu)=0\ , \tag{6}$$

$$g_{ab}=\lambda^2 h_{ab},\ \ \partial_a(\sqrt{g}\,g^{ab}\partial_b x^\mu)=0\ , \tag{7}$$

respectively. The first equation describes the minimal surface enclosed by the curve $\mathscr{C}$, and so does the second, in the special parametrization where $h=$const. Also, Eq. (7) reduces to (5), g_{ab} being a (positive constant) multiple of h_{ab}. Thus, all these models are classically equivalent (aside from boundary conditions, see Ref. 11). Fradkin and Tseytlin[3] have proven that they are also equivalent in the semiclassical approximation, but that, nevertheless, they are *not* equivalent quantum mechanically. The string propagators in the quantum versions of the models (1)—(3) are given by the Wilson-loop *Ansätze*

$$W_N[\mathscr{C}]=\int_{x|_{\partial\mathscr{D}}=\mathscr{C}}[dx]e^{-I_N[x]}\ , \tag{8}$$

$$W_E^{(\nu)}[\mathscr{C}]=\int_0^\infty da\,e^{-a/2}\int_{x|_{\partial\mathscr{D}}=\mathscr{C}}[dx]e^{-k(\nu)I_E^{(\nu)}[x]}\ ,$$
$$k(\nu)=(2-1/\nu)^{2\nu-1}(1/2\nu),\ \ a=\int_{\mathscr{D}}d^2z\ , \tag{9}$$

$$W_G[\mathscr{C}]=\int_{x|_{\partial\mathscr{D}}=\mathscr{C}}[dx][dg]e^{-I_G[x,g]}\ . \tag{10}$$

The constant $k(\nu)$ in the Eguchi model is needed in order to obtain the area law

$$W_E^{(\nu)}[\mathscr{C}]\sim e^{-M^2/A},\ \ A=\int_{\mathscr{D}}\sqrt{h}\,d^2z \tag{11}$$

in the classical limit. The inequivalence of (8), (9), and (10) shows up when one integrates over the metric g^3.

II. STRING ACTIONS

Let us now come to the main point of this work, i.e. the construction of new families of string actions, all of them sharing with the above actions the property of being completely equivalent with them at the classical level.

Let us start with the *Ansatz*

$$I^{(\nu)}[x,g]=b(\nu)M^{2\nu}\int_{\mathscr{D}}d^2z\,g^{\nu/2}(g^{ab}\partial_a x^\mu\partial_b x^\mu)^\nu\ , \tag{12}$$

ν being an arbitrary positive integer, and where $b(\nu)$ is to be determined in order to satisfy the area law (11). Recall Eq. (4) for notation. Action (12) is classically equivalent to (1)—(3). In fact, the following equations of motion are readily obtained:

$$\partial_a[\nu g^{\nu/2}(g^{cd}h_{cd})^{\nu-1}g^{ab}\partial_b x^\mu]=0\ ,$$
$$\nu g^{\nu/2}(g^{cd}h_{cd})^{\nu-1}[h_{ab}-\tfrac{1}{2}(g^{cd}h_{cd})g_{ab}]=0\ , \tag{13}$$

that is,

$$g_{ab}=\lambda^2 h_{ab},\quad \partial_a(h^{\nu/2}h^{ab}\partial_b x^\mu)=0\ , \tag{14}$$

which [as (5)—(7)] describe the minimal surface enclosed by the contour $\mathscr{C}$, in the special parametrization where $h=\text{const}$.

In order to determine the constant $b(\nu)$, for $\nu>1$, we must evaluate the integral

$$\int_0^\infty da\, e^{-a/2}e^{-I(\nu)}\ . \tag{15}$$

This is done by the method of steepest descent, taking into account (11). From $h=\text{const}$ we see that

$$h^{1/2}=A/a\ , \tag{16}$$

which substituted into Eq. (15) yields

$$\int_0^\infty da\,\exp\left\{-\frac{a}{2}\left[1+2b(\nu)\left[\frac{M^2A}{a}\right]^\nu\right]\right\}. \tag{17}$$

The stationary point $\bar{a}$ is

$$\bar{a}=[2(\nu-1)b(\nu)]^{1/\nu}M^2A\ , \tag{18}$$

and from

$$-\frac{\bar{a}}{2}\left[1+2b(\nu)\left[\frac{M^2A}{\bar{a}}\right]^\nu\right]=-M^2A \tag{19}$$

one gets immediately

$$b(\nu)=\left[\frac{2(\nu-1)}{\nu}\right]^{\nu-1}\frac{1}{\nu}\ ,$$
$$\bar{a}=2\frac{\nu-1}{\nu}M^2A\ . \tag{20}$$

Ansatz (12) can be made much more general in the following way. Let $f(u)$ be an arbitrary analytical function in the variable u, with the only condition that $f(0)=0$. The following *Ansatz* can then be considered as some "combination" of the family (12):

$$I^{(f)}[x,g]=b(f)\int_{\mathscr{D}}d^2z\,f(M^2g^{1/2}g^{ab}\partial_a x^\mu\partial_b x^\mu)\ . \tag{21}$$

Again, $b(f)$ will be determined in order to satisfy (11) in the classical approximation.

Let us first check that (21) is classically equivalent to the known actions (1)—(3). The equations of motion are now

$$\partial_a[f(M^2g^{1/2}g^{cd}h_{cd})g^{1/2}g^{ab}\partial_b x^\mu]=0\ ,$$
$$f(M^2g^{1/2}g^{cd}h_{cd})(h_{ab}-\tfrac{1}{2}g^{cd}h_{cd}g_{ab})=0\ , \tag{22}$$

or, equivalently,

$$g_{ab}=\lambda^2h_{ab}\ ,$$
$$f'(M^2h^{1/2})M^2(\partial_a h^{1/2})h^{1/2}h^{ab}\partial_b x^\mu$$
$$+f(M^2h^{1/2})\partial_a(h^{1/2}h^{ab}\partial_b x^\mu)=0\ , \tag{23}$$

which is also the minimal surface enclosed by $\mathscr{C}$, in a parametrization with $h=\text{const}$. $f'(u)$ is the derivative of f with respect to u.

The constant $b(f)$ is determined as before. Now, the stationary point $\bar{a}$ satisfies the equation

$$\bar{a}=\frac{2M^2Af'(M^2A/\bar{a})}{f'(M^2A/\bar{a})+2f(M^2A/\bar{a})}\ , \tag{24}$$

and making use of Eq. (11), one gets

$$b(f)=[f'(M^2A/\bar{a})]^{-1}\ . \tag{25}$$

As is clear, the family of actions (21) has (12) as a particular subfamily [when one takes $f(u)=u^\nu$] and, on its turn, the action (3) is one of the subfamily (12) (for $\nu=1$). All of them are completely equivalent at the classical level, as we have just seen.

III. SEMICLASSICAL APPROXIMATION FOR THE FIRST *ANSATZ*

The string propagators corresponding to the actions (12) and (21) are, respectively, given by

$$W^{(\nu)}[\mathscr{C}]=\int_0^\infty da\, e^{-a/2}\times\int_{x|_{\partial\mathscr{D}}=\mathscr{C}}[dx][dg]e^{-I^{(\nu)}[x,g]}\ , \tag{26}$$

$$W^{(f)}[\mathscr{C}]=\int_0^\infty da\, e^{-a/2}\times\int_{x|_{\partial\mathscr{D}}=\mathscr{C}}[dx][dg]e^{-I^{(f)}[x,g]}\ . \tag{27}$$

Our purpose is now to investigate if the semiclassical approximations to (26) and (27) are still equivalent to the corresponding ones of Eqs. (8)—(10) (which happen, in fact, to be equivalent among themselves).[3]

We start with the integrand of $I^{(\nu)}[x,g]$,

$$g^{\nu/2}(g^{ab}\partial_a x^\mu\partial_b x^\mu)^\nu\ , \tag{28}$$

and make an expansion near a minimal surface, in both variables x^μ and g_{ab},

$$x^\mu=\varphi^\mu+\epsilon\eta^\mu,\quad g_{ab}=h_{ab}+\epsilon\gamma_{ab}\ , \tag{29}$$

where φ^μ and h_{ab} are the solutions of the classical equations of motion (14) (in particular, $h_{ab}=\partial_a\varphi^\mu\partial_b\varphi^\mu$). One easily obtains

$$g=h+\epsilon hh^{ab}\gamma_{ab}+\epsilon^2\gamma\ , \tag{30}$$

$$g^{\nu/2}=h^{\nu/2}\left[1+\epsilon\frac{\nu}{2}\gamma_a^a-\epsilon^2\frac{\nu}{4}\gamma_{ab}\gamma^{ab}+\epsilon^2\frac{\nu^2}{8}(\gamma_a^a)^2\right]\ , \tag{31}$$

$$g^{ab}=h^{ab}-\epsilon\gamma^{ab}+\epsilon^2\gamma^{ac}\gamma_c^b+\cdots\ . \tag{32}$$

Finally, substituting (29)—(32) into (28), we get—up to order ϵ^2—the following expression:

$$g^{\nu/2}(g^{ab}\partial_a x^\mu \partial_b x^\mu)^\nu = h^{\nu/2} + \epsilon\left[\frac{\nu}{2}h^{\nu/2}\gamma_a^a - \nu h^{\nu/2}\gamma^{ab}h_{ab} + 2\nu h^{\nu/2}h^{ab}\partial_a\varphi^\mu\partial_b\eta^\mu\right]$$
$$+\epsilon^2\left\{\nu h^{\nu/2}h^{ab}\partial_a\eta^\mu\partial_b\eta^\mu + h^{\nu/2}\left[-\frac{\nu}{4}\gamma_{ab}\gamma^{ab} + \frac{\nu^2}{8}(\gamma_a^a)^2\right]\right.$$
$$-2\nu h^{\nu/2}\gamma^{ab}\partial_a\varphi^\mu\partial_b\eta^\mu - \frac{\nu^2}{2}h^{\nu/2}\gamma_a^a\gamma^{bc}h_{bc} + \nu^2 h^{\nu/2}\gamma_a^a h^{bc}\partial_b\varphi^\mu\partial_c\eta^\mu$$
$$+\nu h^{\nu/2}\gamma^{ab}\gamma_{ab} + 2\nu(\nu-1)h^{\nu/2}h^{ab}\partial_a\varphi^\mu\partial_b\eta^\mu h^{cd}\partial_c\varphi^\nu\partial_d\eta^\nu$$
$$\left.-2\nu(\nu-1)h^{\nu/2}\gamma^{ab}h_{ab}h^{cd}\partial_c\varphi^\mu\partial_d\eta^\mu + \tfrac{1}{2}\nu(\nu-1)h^{\nu/2}\gamma^{ab}h_{ab}\gamma^{cd}h_{cd}\right\} + \cdots . \tag{33}$$

This expression can be easily simplified to the form

$$g^{\nu/2}(g^{ab}\partial_a x^\mu\partial_b x^\mu)^\nu = h^{\nu/2}\left\{1 - \epsilon\frac{\nu}{2}\gamma_a^a + \epsilon^2[\nu h^{ab}\partial_a\eta^\mu\partial_b\eta^\mu - 2\nu\bar{\gamma}^{ab}\partial_a\varphi^\mu\partial_b\eta^\mu + \tfrac{3}{4}\nu\bar{\gamma}_{ab}\bar{\gamma}^{ab}\right.$$
$$+\tfrac{1}{8}\nu(\nu-1)(\gamma_a^a)^2 + 2\nu(\nu-1)(h^{ab}\partial_a\varphi^\mu\partial_b\eta^\mu)^2$$
$$\left.-2\nu(\nu-1)\gamma_a^a(h^{bc}\partial_b\varphi^\mu\partial_c\eta^\mu)] + \cdots\right\}, \tag{34}$$

where the classical equations of motion (14) have been used, and

$$\bar{\gamma}_{ab} = \gamma_{ab} - \tfrac{1}{2}h_{ab}\gamma_c^c,\quad \bar{\gamma}_a^a = 0 . \tag{35}$$

The semiclassical approximation to Eq. (26) is given by

$$W_{\rm SC}^{(\nu)}[\mathscr{C}] = \int_0^\infty da\, e^{-a/2}\int_{\eta|_{\partial\mathscr{D}}\sim dc/dt}[d\eta][d\gamma]e^{-I_{\epsilon^2}^{(\nu)}[\eta,\gamma]}, \tag{36}$$

where in $I_{\epsilon^2}^{(\nu)}$ only the terms up to order ϵ^2 in Eq. (34) must be taken into account, and where t is a parametrization of $\partial\mathscr{D}$. Notice that the constants $b(\nu)$ in (20) and $M^{2\nu}$ are included in $I_{\epsilon^2}^{(\nu)}$ [see Eq. (12)].

Upon integration over da and over $[d\gamma]$, we obtain

$$\int_0^\infty da\, e^{-a/2}\int_{\eta|_{\partial\mathscr{D}}\sim dc/dt}[d\gamma]e^{-I_{\epsilon^2}^{(\nu)}[\eta,\gamma]}$$
$$\sim e^{-M^2A\frac{2}{3}}\exp\left\{-\epsilon^2M^2\int_{\mathscr{D}}d^2z\,h^{1/2}[h^{ab}\partial_a\eta^\mu\partial_b\eta^\mu - \tfrac{1}{3}(\partial_a\varphi^\mu\partial_b\eta^\mu + \partial_a\eta^\mu\partial_b\varphi^\mu - h_{ab}h^{cd}\partial_c\varphi^\mu\partial_d\eta^\mu)^2\right.$$
$$\left.+2(\nu-1)(h^{ab}\partial_a\varphi^\mu\partial_b\eta^\mu)^2]\right\} \tag{37}$$

$$= e^{-M^2A\frac{2}{3}}\exp\left\{-\epsilon^2M^2\int_{\mathscr{D}}d^2z\,h^{1/2}\partial_a\eta^\mu[M_{\mu\nu}^{\perp ab} + 2(\nu-1)M_{\mu\nu}^{\parallel ab}]\partial_b\eta^\nu\right\}, \tag{38}$$

where $\gamma_a^a = 0$ has been chosen as the quantum Weyl gauge.[3] The transversal and longitudinal parts are defined as usual (see, e.g., Fradkin and Tseytlin,[3] and Lüscher, Symanzyk, and Weisz[7]):

$$M_{\mu\nu}^{\perp ab} = h^{ab}\delta_{\mu\nu} + \tfrac{2}{3}(\partial^a\varphi_\mu\partial^b\varphi_\nu - \partial^a\varphi_\nu\partial^b\varphi_\mu - h^{ab}h_{cd}\partial^c\varphi_\mu\partial^d\varphi_\nu),$$
$$M_{\mu\nu}^{\parallel ab} = \partial^a\varphi_\mu\partial^b\varphi_\nu . \tag{39}$$

We can write

$$\partial_a\eta^\mu M_{\mu\nu}^{\perp ab}\partial_b\eta^\nu = h^{ab}\partial_a\eta^\mu\partial_b\eta^\mu - \tfrac{1}{3}(\delta_1\bar{h}_{ab})^2 , \tag{40}$$

with

$$\delta_1\bar{h}_{ab} \equiv \partial_a\varphi^\mu\partial_b\eta^\mu + \partial_a\eta^\mu\partial_b\varphi^\mu - h_{ab}h^{cd}\partial_c\varphi^\mu\partial_d\eta^\mu , \tag{41}$$

and $\delta_1\bar{h}_{ab} = 0$ can be taken as a coordinate gauge. Notice the factor $\frac{1}{3}$, which was absent in Ref. 3 but gives, after all, no contribution in this gauge.

Now we are ready to perform the last integration in (36), i.e., over $[d\eta]$. Following Ref. 3, let us write

$$\eta_\mu=\eta_\mu^\perp+\eta_\mu^{\|}, \quad \eta_\mu^{\|}=\xi^a\partial_a\varphi_\mu ,$$
$$[d\eta]=[d\eta^\perp][d\eta^{\|}] . \tag{42}$$

We get for (36) the expression

$$W_{\rm SC}^{(\nu)}[\mathscr{C}]\sim e^{-M^2A\frac{2}{3}}Z^\perp Z^{\|} \tag{43}$$

with

$$Z^\perp=[\det(\triangle_0)_\eta]^{-1/2}(\det\triangle_1)^{1/2} ,$$
$$Z^{\|}=[\det(\triangle_0)_N]^{-1/2}[\det(\triangle_0)_D]^{1/2} , \tag{44}$$

where $\triangle_0=-h^{-1/2}\partial_a(h^{1/2}h^{ab}\partial_b)$. The $\triangle_1$ term is the Faddeev-Popov determinant corresponding to the coordinate gauge

$$\triangle_{1ab}\xi^b=-(\nabla_c\nabla^c+\mathscr{R}/2)\xi_a , \tag{45}$$

∇_c being the covariant derivative of h_{ab}, and R the curvature scalar. The subscripts N and D make reference to the Neumann and Dirichlet boundary conditions for $\triangle_0$, respectively.

Finally, all these determinants can be explicitly calculated for $h_{ab}=e^{2\sigma}\delta_{ab}$, with the result (for more details and notation see Ref. 3 and the last Ref. 7)

$$Z^\perp\sim\exp\left[-\frac{26-d}{24\pi}\int_{\mathscr{D}}(\partial_a\sigma)^2d^2z-\frac{2-d}{12\pi}\int_{\partial\mathscr{D}}K_T\sigma\, dz-\frac{d}{8\pi}\int_{\partial\mathscr{D}}\partial_{\tilde n}\sigma\, dz\right][\det(-\Box)_D]^{(2-d)/2},\quad \Box=e^{-2\sigma}\triangle_0 ,$$
$$Z^{\|}\sim\exp\left[\tfrac{1}{2}\ln A-\frac{1}{4\pi}\int_{\partial\mathscr{D}}\partial_{\tilde n}\sigma\, dz\right] . \tag{46}$$

Summing up, the semiclassical approximation (36) of the string propagator (26) is equivalent to the semiclassical approximations of the known strings (8)—(10).

IV. SEMICLASSICAL APPROXIMATION: THE GENERAL CASE

Let us now turn to the more general *Ansatz* (27). Its semiclassical approximation is given by

$$W_{\rm SC}^{(f)}[\mathscr{C}]=\int_0^\infty da\, e^{-a/2}\int_{\eta|_{\partial\mathscr{D}}\sim dc/dt}[d\eta][d\gamma]e^{-I_{\epsilon^2}^{(f)}[\eta,\gamma]} . \tag{47}$$

If

$$f(u)=\sum_{\nu=1}^\infty a_\nu u^\nu \tag{48}$$

is the series expansion of the analytical function f [$f(0)=0$], the exponent $I_{\epsilon_2}^{(f)}$ has the explicit form

$$\begin{aligned}I_{\epsilon^2}^{(f)}[\eta,\gamma]=b(f)\Bigg\{&\sum_{\nu=1}^\infty a_\nu M^{2\nu}\int_{\mathscr{D}}h^{\nu/2}d^2z-\frac{\epsilon}{2}\sum_{\nu=1}^\infty a_\nu\nu M^{2\nu}\int_{\mathscr{D}}h^{\nu/2}\gamma_a^a d^2z\\ &+\epsilon^2\sum_{\nu=1}^\infty a_\nu M^{2\nu}\int_{\mathscr{D}}d^2z\, h^{\nu/2}[\nu h^{ab}\partial_a\eta^\mu\partial_b\eta^\mu-2\nu\overline{\gamma}^{ab}\partial_a\varphi^\mu\partial_b\eta^\mu+\tfrac{3}{4}\nu\overline{\gamma}^{ab}\overline{\gamma}_{ab}\\ &+\tfrac{1}{8}\nu(\nu-1)(\gamma_a^a)^2+2\nu(\nu-1)(h^{ab}\partial_a\varphi^\mu\partial_b\eta^\mu)^2\\ &-2\nu(\nu-1)\gamma_a^a(h^{bc}\partial_b\varphi^\mu\partial_c\eta^\mu)]\Bigg\} .\end{aligned} \tag{49}$$

The constant $b(f)$ is given by (24), (25). Equation (49) can be written in the more compact form

$$\begin{aligned}I_{\epsilon^2}^{(f)}[\eta,\gamma]=b(f)\Bigg[&\int_{\mathscr{D}}d^2z\, f(M^2h^{1/2})-\frac{\epsilon}{2}M^2\int_{\mathscr{D}}d^2z\, h^{1/2}f'(M^2h^{1/2})\gamma_a^a\\ &+\epsilon^2M^2\int_{\mathscr{D}}d^2z\, h^{1/2}\{f'(M^2h^{1/2})(h^{ab}\partial_a\eta^\mu\partial_b\eta^\mu-2\overline{\gamma}^{ab}\partial_a\varphi^\mu\partial_b\eta^\mu+\tfrac{3}{4}\overline{\gamma}^{ab}\overline{\gamma}_{ab})\\ &+M^2h^{1/2}f''(M^2h^{1/2})[\tfrac{1}{8}(\gamma_a^a)^2+2(h^{ab}\partial_a\varphi^\mu\partial_b\eta^\mu)^2\\ &-2\gamma_a^a(h^{bc}\partial_b\varphi^\mu\partial_c\eta^\mu)]\}\Bigg] .\end{aligned} \tag{50}$$

When in (47) the integrations over da and $[d\gamma]$ are carried out, one obtains the result

$$\int_0^\infty da\, e^{-a/2} \int_{\eta|\partial\mathscr{D}\sim dc/dt}[d\gamma]e^{-I_\epsilon^{(f)}[\eta,\gamma]} \sim e^{-M^2A\frac{2}{3}} \exp\left\{-\epsilon^2 b(f)M^2 f'\left[\frac{M^2A}{\bar{a}}\right]\int_{\mathscr{D}} d^2z\, h^{1/2}[h_{ab}\partial_a\eta^\mu\partial_b\eta^\mu - \tfrac{1}{3}(\delta_1\bar{h}_{ab})^2] -2\epsilon^2 b(f)M^4\frac{A}{\bar{a}}f''\left[\frac{M^2A}{\bar{a}}\right]\int_{\mathscr{D}} d^2z\, h^{1/2}(h^{ab}\partial_a\varphi^\mu\partial_b\eta^\mu)^2\right\}, \quad (51)$$

where, as in the preceding case, $\gamma_a^a=0$ has been chosen as the quantum Weyl gauge. The term $\delta_1\bar{h}_{ab}$ is given by (41). Making use of (25), (39), and (40), Eq. (51) can be written as

$$\int_0^\infty da\, e^{-a/2} \int_{\eta|\partial\mathscr{D}\sim dc/dt}[d\gamma]e^{-I_\epsilon^{(f)}[\eta,\gamma]} \sim e^{-M^2A\frac{2}{3}} \exp\left\{-\epsilon^2 M^2 \int_{\mathscr{D}} d^2z\, h^{1/2}\partial_a\eta^\mu \left[M_{\mu\nu}^{\perp ab} + 2\bar{\alpha}\frac{f''(\bar{\alpha})}{f'(\bar{\alpha})}M_{\mu\nu}^{\|ab}\right]\partial_b\eta^\nu\right\}, \quad (52)$$

where

$$\bar{\alpha} \equiv \frac{M^2A}{a}. \quad (53)$$

Let us now compare Eqs. (52) and (38). We see that, apart from the factor in front of $M^{\|}$, which in the general case (21) is different from that of the particular *Ansatz* (12), the rest of these expressions are exactly the same. In particular, the transverse part is identical in both cases. As a consequence, and except for a global factor, the result one obtains for the string propagator (27) in the semiclassical approximation (47) is the same (43), (46) that one gets for the semiclassical approximation (36) of the more particular *Ansatz* (26). Therefore, the semiclassical approximation of (27) is also the same as the semiclassical approximations of the string propagators (8)—(10). And this is true for *any* analytical function f in (27).

V. ANALYSIS FOR SOME PARTICULAR FUNCTIONS f

The scope of different, in principle admissible (i.e., classically and semiclassically equivalent to the already known ones) string actions, has been widely implemented. Of course, in general, the quantization (26), (27) of these models will have to face the same difficulties one encounters in the Nambu and Eguchi cases. Nevertheless, due to the extreme arbitrariness in choosing the function f, a door is left open to some particular *Ansatz* which could be free from such troubles.

For the sake of concretion, let us now compute $\bar{\alpha}$, $\bar{a}$, and $b(f)$, as well as the constant in front of $M^{\|}$, for some particular functions f.

(i) For $f(u)=u^\nu$ we must recover the particular case (26). In fact, $f'(u)=\nu u^{\nu-1}$, $f''(u)=\nu(\nu-1)u^{\nu-2}$, and we get

$$\bar{a}=\frac{2M^2A\nu}{\nu+2M^2A/\bar{a}},\quad \bar{a}=2\frac{\nu-1}{\nu}M^2A\,,$$
$$b(f)=\frac{1}{\nu}\left[\frac{M^2A}{\bar{a}}\right]^{1-\nu}=\left[\frac{2(\nu-1)}{\nu}\right]^{\nu-1}\frac{1}{\nu}\,, \quad (54)$$

which coincide with (20), and

$$\bar{\alpha}=\frac{M^2A}{\bar{a}}=\frac{2\nu}{\nu-1},\quad 2\bar{\alpha}\frac{f''(\bar{\alpha})}{f'(\bar{\alpha})}=2(\nu-1)\,, \quad (55)$$

from where we recover (38).

In the particular case $\nu=1$, this is the Brink-Di Vecchia-Howe[8] and Polyakov[9] model, and we get only the transverse part in (38) [see Eqs. (4.20) and (4.29) in Ref. 3].

(ii) Take $f(u)=e^u-1$. In this case $f'(u)=f''(u)=e^u$, and Eqs. (24), (25) turn out to be

$$1=\frac{2\bar{\alpha}e^{\bar{\alpha}}}{3e^{\bar{\alpha}}-2},\quad (2\bar{\alpha}-3)e^{\bar{\alpha}}+2=0,\quad b(f)=e^{-\bar{\alpha}}\,. \quad (56)$$

Solving the transcendent equation in $\bar{\alpha}$, we get the approximate solution

$$\bar{\alpha}=1.198,\quad b(f)=0.302\,. \quad (57)$$

Moreover,

$$2\bar{\alpha}\frac{f''(\bar{\alpha})}{f'(\bar{\alpha})}=2\bar{\alpha}=2.396 \quad (58)$$

is the coefficient of the longitudinal part in (52).

(iii) As the third and last example, let us consider $f(u)=(u^2+c^2)^{-1}-c^{-2}$, with $c=\text{const}$. Now $f'(u)=-2u/(u^2+c^2)^2$, $f''(u)=2(3u^2-c^2)/(u^2+c^2)^3$, and we obtain from (24)

$$(2\bar{\alpha}-1)f'(\bar{\alpha})=2f(\bar{\alpha}),\quad \bar{\alpha}^3-c^2\bar{\alpha}+c^2=0\,. \quad (59)$$

We could give $\bar{\alpha}$ in terms of c^2. Instead, for simplicity, we shall set $c^2=8$. Then

$$\bar{\alpha}=2,\quad b(f)=-36,\quad 2\bar{\alpha}\frac{f''(\bar{\alpha})}{f'(\bar{\alpha})}=\frac{2}{3}\,. \quad (60)$$

For this particular value of c^2, both the transverse and longitudinal parts contribute in Eq. (52).

On the contrary, setting $c^2=\frac{27}{4}$, we obtain

$$\bar{\alpha}=\tfrac{3}{2},\quad b(f)=-27,\quad 2\bar{\alpha}\frac{f''(\bar{\alpha})}{f'(\bar{\alpha})}=0\,. \quad (61)$$

In this case only the transverse part in Eq. (52) contributes.

VI. CONCLUSIONS

Guided by the aspect of the different Bose string actions proposed by several authors [Eqs. (1)–(3)], we have constructed a very large family of new actions (21), which contains Polyakov's[8,9] action (3) as a particular case, and contains also a subfamily of actions (12), which corresponds to the Eguchi family[7] "modified" according to Polyakov's procedure [just in the same way that Polyakov's action (3) is a "modification" of Nambu's action (1)].

All the new actions constructed have been proven to be both classically and semiclassically equivalent to the already existing ones (which, at these levels, are also equivalent among themselves[3]). Actually, this statement is true, in principle, only if one forgets about the boundary conditions implied by the different actions,[11] a question that has not been dealt with in this paper. A careful analysis of this point could provide very interesting results.

The quantization of the models presented in this work (at least of some of them, for particular types of function f) is also a challenging task to be done. It may happen that the same difficulties of the Nambu[6] and Eguchi[7] models would show up here, in general. But one can still hope to be able to simplify the calculations—for particular analytical functions f—in the sense of Polyakov.[9] Moreover, an important role will be played in the quantization by the topology of the embedding of $\mathscr{D}$ in R^d (number of holes and handles, see Ref. 5).

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